State-of-art in modelling methods of membrane-based liquid desiccant heat and mass exchanger: A comprehensive review

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Abstract

Air dehumidification is of vital importance in building air conditioning and production safety. Semi-permeable membrane module is a novel heat and mass exchanger, which separates the air and liquid desiccant to overcome desiccant droplet carry-over problem in traditional direct-contact systems. Recently, some research works have been carried out in mathematical modelling and experimental testing of membrane-based liquid desiccant dehumidification technology. Compared with the experimental testing, the mathematical modelling has advantages of significant time and cost reductions, practically unlimited level of detail, more profound understanding of physical mechanism and better investigation of critical situation without any risks. This paper presents a comprehensive review of various modelling methods for two types of membrane-based liquid desiccant modules: flat plate and hollow fiber.

Keywords: membrane; dehumidification; liquid desiccant; heat and mass transfer

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34 Nomenclature

A	membrane surface area (m^2)
c_p	specific heat capacity (J/kgK)
C	heat capacity rate
C_r^*	capacitance ratio
D_v	diffusivity (m^2/s)
h	convective heat transfer coefficient ($\text{W/m}^2\text{K}$)
h_{fg}	enthalpy of phase change for saturated water (J/kg)
h_v	specific heat of evaporation of vapour (kJ/kg)
H^*	operating factor
k	thermal conductivity (W/m K)
m^*	solution to air mass flow rate ratio
$\dot{m}$	mass flow rate (kg/s)
m_{lat}^*	latent heat ratio
$\dot{m}_{rr}$	moisture removal rate
$\dot{m}_v$	moisture emission rate
NTU	number of heat transfer units
NTU_m	number of mass transfer units
T	temperature ($^\circ\text{C}$)
U	overall heat transfer coefficient ($\text{W/m}^2\text{K}$)
U_m	overall mass transfer coefficient ($\text{kg/m}^2\text{s}$)
$\dot{V}$	volumetric flow rate (l/min)
W	humidity ratio (kg/kg dry air)

Greeks

ϵ	effectiveness
δ	thickness of membrane (m)
ρ	density (kg/m^3)

Superscripts

$*$	dimensionless
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Subscripts

<i>air</i>	air flow
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<i>in</i>	inlet
<i>lat</i>	latent
<i>m</i>	mass transfer
<i>max</i>	maximum
<i>mem</i>	membrane
<i>min</i>	minimum
<i>num</i>	numerical
<i>out</i>	outlet
<i>sen</i>	sensible
<i>sol</i>	solution flow
<i>tol</i>	total

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1. Introduction

Buildings are responsible for a significant part of global energy consumption. In particular, heating, ventilation and air-conditioning (HVAC) systems account for around 50% of energy consumed in buildings [1]. As a matter of fact, 20-40% of the total energy are consumed in the dehumidification process, and it can be even higher when 100% fresh air ventilation is required for better indoor environment [2]. In coastal areas where the humidity level is significantly high, air dehumidification is a necessity otherwise living would be seriously affected by the humid climate. The ASHRAE Standard 62-2001 recommends the relative humidity of 30-60% for indoor environment [3]. However in some coastal regions, the outside air humidity can reach 80-90% continuously for a dozen of days, and latent cooling load accounts for 20-40% of total energy consumption of HVAC system [4]. High humidity level would lead to discomfort and affect the body surface temperature. Furthermore, production safety and quality would be seriously influenced by too high humidity level [2]. It has been reported that building energy consumption could be decreased by 20-64% using efficient dehumidification technologies [5].

There are many air dehumidification methods, including cooling coils, solid desiccant dehumidification and liquid desiccant dehumidification [2]. The traditional cooling coil system is inefficient in dealing with latent heat load. Furthermore, the air leaving the cooling coil is normally overcooled and needs to be re-heated to an appropriate supply temperature. Therefore, this combined process consumes a considerable amount of energy to cool (typically using a vapour compression system) and heat (using hot water or electricity) the supply air [6]. Liquid desiccant system has gained much progress recently due to the coherent virtues compared with others, for example high efficient without liquid water condensation. Liquid desiccant can be regenerated using low-grade heat such as solar energy, and the regenerated solution can be used as energy storage medium as well [7]. Membrane-based dehumidification stands out for its continuously working mode, reliability and non-direct contacting of air with working substance that avoids the problem of carry-over [8, 9]. As a result, the membrane-based air dehumidification system has been studied extensively both experimentally and numerically. Several literature reviews have been conducted in this area [2, 7, 10]. Most of the reviews focus on the structures of liquid-to-air contractor, applications of membrane-based air dehumidification system, membrane materials and simple theory

models. Numerical modelling methods of different types of membrane-based heat and mass exchanger have not been reviewed. This paper gives a comprehensive analysis of various numerical modelling methods to assess energy performance of the membrane-based heat and mass exchanger.

2. Performance evaluation

2.1. Designed operating parameters

Numerical and experimental investigations have revealed that the moisture removal rate and various effectiveness of the membrane-based heat and mass exchanger depend on its operating conditions significantly, so the main designed operating parameters are explored at first.

2.1.1. Capacitance ratio (C_r^*)

Heat capacity rate C is defined as the product of specific heat capacity and mass flow rate (W/K). Thus the heat capacities of desiccant solution and air can be calculated by Eqs. (1) - (2) [11].

$$C_{sol} = \dot{m}_{sol} c_{p,sol} \quad (1)$$

$$C_{air} = \dot{m}_{air} c_{p,air} \quad (2)$$

Where $\dot{m}_{sol}$ is the solution mass flow rate (kg/s), $\dot{m}_{air}$ is the air mass flow rate (kg/s), $c_{p,sol}$ is the solution specific heat capacity (J/kgK) and $c_{p,air}$ is the air specific heat capacity (J/kgK).

Then the capacitance ratio (or ratio of heat capacity) can be calculated by Eq. (3) [7].

$$C_r^* = \frac{C_{sol}}{C_{air}} = \frac{\dot{m}_{sol} c_{p,sol}}{\dot{m}_{air} c_{p,air}} \quad (3)$$

Specifically, another parameter Cr is defined as:

$$Cr = \frac{1}{C_r^*} \quad (4)$$

2.1.2. Solution to air mass flow rate ratio (m^*)

Solution to air mass flow rate ratio is used to indicate relative flow rate of two heat exchanging fluids, which is defined as:

$$m^* = \frac{\dot{m}_{min}}{\dot{m}_{max}} = \frac{\dot{m}_{air}}{\dot{m}_{sol}} \quad (5)$$

2.1.3. Operating factor (H^*)

The operating factor is a dimensionless parameter defined as the ratio between the latent and sensible heat differences between the air and desiccant solution at the exchanger inlet [12].

$$H^* = \frac{\Delta H_{lat}}{\Delta H_{sen}} \approx 2500 \frac{W_{air,in} - W_{sol,in}}{T_{air,in} - T_{sol,in}} \quad (6)$$

Where $T_{air,in}$ and $T_{sol,in}$ are air and solution temperatures respectively ($^{\circ}\text{C}$), $W_{air,in}$ is air humidity ratio (kg/kg dry air) and $W_{sol,in}$ is solution equilibrium humidity ratio (kg/kg dry air).

2.1.4. Number of heat transfer units (NTU)

Effectiveness-NTU method is one of the most commonly used approaches for heat exchanger analysis. Compared with *log-mean-temperature-difference* method, it provides a superior way to analyse heat exchanger performance in terms of non-dimensional variables [12]. *NTU* has a considerable impact on the sensible effectiveness of energy exchanger. The sensible effectiveness increases with *NTU* [8, 13].

$$NTU = \frac{UA}{C_{min}} \quad (7)$$

$$U = \left[\frac{1}{h_{air}} + \frac{\delta}{k_{mem}} + \frac{1}{h_{sol}} \right]^{-1} \quad (8)$$

Where U is the overall heat transfer coefficient ($\text{W/m}^2\text{K}$), A is the membrane surface area (m^2), C_{min} is the minimum value of heat capacity rate of air and desiccant solution (W/K), h_{air} is the convective heat transfer coefficient of air ($\text{W/m}^2\text{K}$), h_{sol} is the convective heat transfer coefficient of desiccant solution ($\text{W/m}^2\text{K}$), δ is the thickness of membrane (m), k_{mem} is the thermal conductivity of membrane (W/mK).

2.1.5. Number of mass transfer units (NTU_m)

The number of mass transfer units has an important effect on the latent effectiveness of energy exchanger. The latent effectiveness increases with NTU_m [7, 13].

$$NTU_m = \frac{U_m A}{\dot{m}_{min}} \quad (9)$$

$$U_m = \left[\frac{1}{h_{m,air}} + \frac{\delta}{k_m} + \frac{1}{h_{m,sol}} \right]^{-1} \quad (10)$$

Where U_m is the overall mass transfer coefficient ($\text{kg/m}^2\text{s}$), $\dot{m}_{min}$ is the minimum mass flow rate of air and desiccant solution (kg/s), $h_{m,air}$ is the air convective mass transfer coefficient ($\text{kg/m}^2\text{s}$), $h_{m,sol}$ is the convective mass transfer coefficient of the desiccant solution ($\text{kg/m}^2\text{s}$), k_m is the membrane water permeability (kg/m s). It is found that the convective mass transfer coefficient of the desiccant solution is much higher than that of the air, thus $\frac{1}{h_{m,sol}}$ can be neglected for the simplicity [12].

2.1.6. Moisture emission rate (m_v)

The moisture emission rate m_v is defined by Zhang [14] as the moisture flux through the membrane.

$$m_v = \rho_{air} D_{vm} \frac{W_{mem,1} - W_{mem,2}}{\delta} \quad (11)$$

Where ρ_{air} is the air density (kg/m^3), D_{vm} is the moisture diffusivity in the membrane (m^2/s), $W_{mem,1}$ and $W_{mem,2}$ are the humidity ratios of the membrane on the air and solution sides (g/kg dry air).

2.1.7. Latent heat ratio (m_{lat}^*)

Latent heat ratio is firstly introduced by Zhang [15] to show how the water temperature is affected by the moisture evaporation heat.

$$m_{lat}^* = \frac{\rho_{air} V_{air} h_v (W_{w,in} - W_{air,in})}{\rho_w V_w c_{p,w} (T_{w,in} - T_{air,in})} \quad (12)$$

Where h_v is the specific heat of evaporation of vapour (kJ/kg).

2.2. Moisture removal rate ($\dot{m}_{rr}$)

Moisture removal rate of the membrane-based heat and mass exchanger is defined as the mass transfer rate of moisture between the air and desiccant solution (kg/s).

$$\dot{m}_{rr} = \dot{m}_{air} |W_{air,out} - W_{air,in}| \quad (13)$$

Where $\dot{m}_{air}$ is the mass flow rate of dry air (kg/s).

2.3. Effectiveness

Effectiveness is the most important parameter used to evaluate the performance of a heat and mass exchanger [16]. Three types of effectiveness have been defined: sensible effectiveness (ϵ_{sen}), latent effectiveness (ϵ_{lat}) and total effectiveness (ϵ_{tot}). The sensible effectiveness is defined as the ratio between the actual and maximum possible rates of sensible heat transfer inside the heat exchanger, the latent effectiveness is defined as the ratio between the actual and the maximum possible moisture transfer rates inside the mass exchanger, and the total effectiveness is the ratio between the actual and maximum possible energy (enthalpy) transfer rates inside the heat and mass exchanger. In the air side, the capacity rate of desiccant solution is higher than that of the air, which means $Cr^* \geq 1$, the sensible, latent and total effectiveness for the air flow can be calculated by Eqs. (14) - (16) [17].

$$\varepsilon_{air, sen} = \frac{T_{air, in} - T_{air, out}}{T_{air, in} - T_{sol, in}} \quad (14)$$

$$\varepsilon_{air, lat} = \frac{W_{air, in} - W_{air, out}}{W_{air, in} - W_{sol, in}} \quad (15)$$

$$\varepsilon_{air, tol} = \frac{\varepsilon_{sen} + H^* \varepsilon_{lat}}{1 + H^*} \quad (16)$$

For the regenerator where the main focus is on the solution, solution-side effectiveness are more important than air side effectiveness. The solution side sensible, latent and total effectiveness can be calculated by [18]:

$$\varepsilon_{sol, sen} = \frac{(\dot{m}c_p)_{sol}(T_{sol, out} - T_{sol, in}) - \dot{m}_{salt}h_{fg}(X_{sol, out} - X_{sol, in})}{(\dot{m}c_p)_{air}(T_{air, in} - T_{sol, in})} \quad (17)$$

$$\varepsilon_{sol, lat} = \frac{\dot{m}_{salt}(X_{sol, out} - X_{sol, in})}{\dot{m}_{air}(W_{air, in} - W_{sol, in})} \quad (18)$$

$$\varepsilon_{sol, tot} = \frac{(\dot{m}c_p)_{sol}(T_{sol, out} - T_{sol, in})}{(\dot{m}c_p)_{air}(T_{air, in} - T_{sol, in}) + \dot{m}_{air}h_{fg}(W_{air, in} - W_{sol, in})} \quad (19)$$

Where h_{fg} is the enthalpy of phase change for saturated water (J/kg).

3. Modelling methods

There are three main types of modelling methods for the membrane-based liquid desiccant heat and mass exchanger: effectiveness NTU ($e - NTU$), finite difference and conjugate heat and mass transfer methods. In this section these will be discussed in details.

3.1. Effectiveness NTU ($e - NTU$) method

3.1.1. Flat-plate membrane module

An analytical model is presented by Shah and London [19] based on pure analogy for a flat-plate heat exchanger, but only the sensible effectiveness is predicted. The model is valid for the flat-plate heat exchanger at steady-state laminar flow operating condition. Based on their study, the sensible effectiveness of pure cross and counter flow heat exchangers are given by:

$$\varepsilon_{sen, cross} = 1 - \exp\left\{\frac{NTU^{0.22}}{Cr} [\exp(-Cr \cdot NTU^{0.78}) - 1]\right\} \quad (20)$$

$$\varepsilon_{sen, counter} = \frac{1 - \exp[-NTU(1 - Cr)]}{1 - Cr \cdot \exp[-NTU(1 - Cr)]} \quad (21)$$

For an innovative counter-cross flow exchanger, the sensible effectiveness can be calculated by combining Eqs. (20) and (21) [20]:

$$\varepsilon_{sen,counter-cross} = \left(\frac{A_{cross}}{A_{counter-cross}} \right) \varepsilon_{sen,cross} + \left(\frac{A_{counter}}{A_{counter-cross}} \right) \varepsilon_{sen,counter} \quad (22)$$

Where A is the active area of the exchanger (m^2).

However, the above analytical solution does not take mass transfer into consideration, which means this method is unable to predict the dehumidification performance. Thus the latent effectiveness for cross, counter and counter-cross flow exchangers are produced in literature [21] based on the heat and mass transfer analogy from Eqs. (20) - (22):

$$\varepsilon_{lat,cross} = 1 - \exp \left\{ \frac{NTU_m^{0.22}}{m^*} [\exp(-m^* \cdot NTU_m^{0.78}) - 1] \right\} \quad (23)$$

$$\varepsilon_{lat,counter} = \frac{1 - \exp[-NTU_m(1-m^*)]}{1 - m^* \cdot \exp[-NTU_m(1-m^*)]} \quad (24)$$

$$\varepsilon_{lat,counter-cross} = \left(\frac{A_{cross}}{A_{counter-cross}} \right) \varepsilon_{lat,cross} + \left(\frac{A_{counter}}{A_{counter-cross}} \right) \varepsilon_{lat,counter} \quad (25)$$

The above analytical solution is used for the traditional flat plate heat exchanger. As for membrane-based liquid desiccant heat and mass exchanger, a novel technique is firstly presented in 1996 by Isetti et al. [22] to stabilize the relative humidity of air in a museum by using a synthetic hydrophobic membrane coupled with a hygroscopic solution in a plane plate membrane contractor, as shown in Fig. 1.

Fig. 1. A synthetic hydrophobic membrane separating the air and liquid phase [22]

Flat plate membrane module is the most common type of membrane modules for simple structure and easy fabrication [10], and used as air-to-air heat and mass exchanger previously [23, 24]. Effectiveness correlations are introduced by Zhang and Niu [25] for a flat plate exchanger with membrane cores as indicated in Fig. 2.

Fig.2. Schematic of a cross-flow enthalpy exchanger with membrane cores [25]

As shown in Fig. 2, two fluids flow in thin, parallel, alternating membrane layers to transfer heat and mass from one fluid to another. In their research, some assumptions are made to build governing equations, these include: no lateral mixing of the two fluids, no heat conduction and vapour diffusion in the fluids, constant heat conductivity and water diffusivity in the membrane, and one-dimensional heat and mass transfer in the membrane. The sensible and latent effectiveness correlations are developed as:

$$\varepsilon_{sen} = 1 - \exp \left[\frac{\exp(-NTU^{0.78} R_1) - 1}{NTU^{-0.22} R_1} \right] \quad (26)$$

$$\varepsilon_{lat} = 1 - \exp \left[\frac{\exp(-NTU_{lat}^{0.78} R_1) - 1}{NTU_{lat}^{-0.22} R_1} \right] \quad (27)$$

Where

$$R_1 = \frac{(\dot{m}c_{pa})_{min}}{(\dot{m}c_{pa})_{max}} \quad (28)$$

$$R_2 = \frac{\dot{m}_{min}}{\dot{m}_{max}} \quad (29)$$

NTU_{lat} is the total number of transfer units for moisture which can be calculated by:

$$NTU_{lat} = \beta \cdot NTU \quad (30)$$

Where

$$\beta = \frac{1}{1+\alpha} \quad (31)$$

$$\alpha = \frac{\gamma_m}{\gamma_c} \quad (32)$$

$$\gamma_c = \frac{2}{k_s} \quad (33)$$

Where γ_c is the convective moisture transfer resistance, α is the ratio of diffusive resistance to convective resistance of the membrane.

3.1.2. Hollow fiber membrane module

The parallel-plate membrane module is simple in structure and easy to be fabricated, however, the packing density is around $500\text{ m}^2/\text{m}^3$, which is not large enough. Compared with the parallel-plate membrane module, the hollow fiber membrane module is more attractive for its higher packing density and larger heat and mass transfer capacity [2]. A hollow fiber module prototype is firstly built at the University of Genoa by Bergero and Chiari [26]. As shown in Fig. 3, 800 polypropylene hollow fibers are assembled together in a rectangular geometry, LiCl and water are used as the working fluids for air dehumidification and humidification.

Fig. 3. Structure and dimensions of the hollow fiber membrane module prototype built at the University of Genoa [26]

Many studies have been conducted based on the fluid flow and heat and mass transfer through membranes, in tube side or in the shell side. In literature [27], it is identified that for microporous membranes, the mechanism of the common gas-liquid membrane contractors is based on the combined Knudsen and ordinary diffusions. For the nonporous membranes, they are highly hydrophobic, and Knudsen diffusion is the predominant mechanism for water vapour transporting through the membrane. The resistance in the nonporous membrane is considered as a lumped parameter, for the fluid flow inside the hollow fibers, a set of differential equations are obtained from

mass balance inside the fibers. The Graetz-Levegue solution [28, 29] is used to predict the fiber-side mass transfer coefficient, and a general classification for Graetz-Levegue solution is given by Kreulen et al. [30] by curve fitting of Graetz and Levegue solutions, which can be applied for transition region not covered by Graetz-Levegue solution. The heat and mass transfer outside the hollow fiber tubes is relatively more complex than that in inner fiber tubes. The coupled heat and mass transfer in the hollow fiber membrane contractor is studied by Zhang [31], and an analytical solution is obtained which is convenient to use in the practice, the hollow fiber membrane contractor used in this research is given in Fig. 4.

Fig. 4. Schematic of the hollow fiber membrane module for air dehumidification [31]

As shown in Fig. 4, this hollow fiber membrane module is similar to the traditional shell-and-tube heat exchanger, air flows outside the fibers while the solution flows in the fibers, and they are in counter flow configuration. The differential equations of heat and moisture transfer in the air and solution flows are established, the sensible and latent effectiveness can be obtained:

$$\varepsilon_{sen} = 1 - C_1 e^{\lambda_1} - C_2 e^{\lambda_2} \quad (34)$$

$$\varepsilon_{lat} = 1 + K_1 C_1 e^{\lambda_1} - K_2 C_2 e^{\lambda_2} \quad (35)$$

Where C_1 and C_2 are the coefficients that are functions of constant coefficients: a_{11} – a_{22} . λ_1 and λ_2 are the roots of characteristic equations. K_1 and K_2 are the intermedia coefficients of the analytical solution.

This analytical solution based on solely algebraic correlations is accurate and convenient to use to estimate the sensible and latent effectiveness. However the solution outlet temperature and concentration need to be assumed in order to start the calculation, which increases the calculation load. Zhang's analytical solution is extended by

assuming that the desiccant solution mass flow rate and concentration are constant [32], since the moisture contents absorbed/desorbed by the solution are negligible compared with desiccant mass flow rate. This assumption is verified by experimental results for air dehumidification and desiccant regeneration. In literature [32], a flat-plate counter-cross-flow liquid-to-air membrane energy exchanger (LAMEE) is adopted, as shown in Fig. 5.

Fig. 5. Schematic of a single counter-cross-flow LAMEE [32]

The solution properties and its equilibrium humidity ratio are only related to the solution temperature based on the assumption mentioned earlier, and the slope (E_T) is only associated with iterated solution temperature. The slope calculation method is different from Zhang's analytical solution [31], in which E_T is related to both iterated solution temperature and concentration. Furthermore, Zhang's analytical solution is for hollow fiber membrane contractor, thus in order to use this solution U and U_m need to be calculated based on a flat-plate membrane contractor and substituted into Zhang's analytical correlations.

To sum up, there are few literatures using $e - NTU$ method. Compared to finite difference method and conjugate heat and mass transfer method, $e - NTU$ method is more effective, time-saving but less accurate accordingly.

3.2. Finite difference method

The finite difference method is widely used for its high accuracy [33]. The coupled heat and mass transfer in the air, solution flows and membrane of a counter flow LAMEE is numerically modelled by Moghaddam et al. [34], the steady state effectiveness of the LAMEE is analysed. The schematic of the LAMEE is shown in Fig. 6.

Fig. 6. Schematic of a counter-flow small-scale single-panel LAMEE and system coordinates [34]

As can be seen in Fig. 6, the air and solution channels are separated by a semi-permeable membrane, and they are in counter flow configuration. To simplify the numerical modelling, some assumptions are made, including: well-insulated exchanger, steady state, fully developed flows, constant thermal properties and convective heat and mass transfer coefficients, uniform velocity and temperature at inlets, no axial conduction (i.e. $Pe > 20$ in the air and solution channels [35, 36]) and phase change heat gain or loss only occur in the solution side. The normalized governing equations of mass and energy balances for the solution side are given by:

$$\frac{dX_{sol}}{dx^*} - W_0 NTU_m \cdot m^* (1 + X_{sol}) (\varphi_{air} - \varphi_{sol,mem}) = 0 \quad (36)$$

$$\frac{d\theta_{sol}}{dx^*} - NTU_m \cdot H^* C_r (\varphi_{air} - \varphi_{sol,mem}) - NTU \cdot C_r (\theta_{air} - \theta_{sol}) = 0 \quad (37)$$

The normalized governing equations for the air side are given by:

$$\frac{d\varphi_{air}}{dx^*} + 2NTU_m (\varphi_{air} - \varphi_{sol,mem}) = 0 \quad (38)$$

$$\frac{d\theta_{air}}{dx^*} + 2NTU (\theta_{air} - \theta_{sol}) = 0 \quad (39)$$

The normalized governing equations for the membrane are:

$$NTU_m W_0 (\varphi_{air} - \varphi_{sol,mem}) = NTU_{m,sol} (C_{salt} - C_{salt,mem}) \quad (40)$$

$$NTU (\theta_{air} - \theta_{sol,mem}) + NTU_m H^* (\varphi_{air} - \varphi_{sol,mem}) = NTU_{sol} (\theta_{sol,mem} - \theta_{sol}) \quad (41)$$

In Eqs. (36) - (41), the number of heat transfer unit NTU , number of mass transfer unit NTU_m , Cr , mass flow rate ratio m^* , and operating factor H^* have been introduced in section 2. φ, θ and x^* are dimensionless humidity content, temperature and length respectively, which are given by:

$$\varphi = \frac{W - W_{air,in}}{W_{sol,in} - W_{air,in}} \quad (42)$$

$$\theta = \frac{T - T_{air,in}}{T_{sol,in} - T_{air,in}} \quad (43)$$

$$x^* = \frac{x}{x_0} \quad (44)$$

In Eq. (36), X_{sol} is the solution mass fraction, which can be obtained by:

$$X_{sol} = \frac{\text{mass of water}}{\text{mass of salt}} \quad (45)$$

W_0 is defined as:

$$W_0 = W_{sol,in} - W_{air,in} \quad (46)$$

The required boundary conditions are the inlet air temperature and humidity ratios, and water temperature and mass fraction in the solution side. The governing equations are then solved by finite difference method, the numerical model is validated by experimental test data for summer test conditions, a good agreement between numerical results and experimental data is achieved.

Parallel flow structure is not easy for sealing the liquid fluids [37, 38], while across flow arrangement is more favourable for practical application. A cross flow heat and moisture exchanger is modelled by Fan et al. [39], the geometry of one pair of flow channels of this module and the coordinate system are shown in Fig. 7.

Fig. 7. Schematic of a cross-flow LAMEE (a) and the coordinate system (b) [39]

The same modelling method is used in this study as that in literature [34]. Due to the cross flow configuration, the dimensionless length in y direction is defined as:

$$y^* = \frac{y}{y_0} \quad (47)$$

As mentioned previously, the counter flow structure is difficult to seal the solution. Furthermore, the counter flow exchanger with simple headers located adjacent to each other is hard to construct in the limited space available in HVAC systems [35]. However, the efficiency of the counter flow exchanger is approximately 10% higher than that of the cross flow configuration [40]. Thus a module that incorporates counter flow heat exchanger with cross flow inlet and outlet headers is developed by Vali et al [35, 41], as shown in Fig. 8.

Fig. 8. Schematic of a flat-plate counter-cross flow LAMEE [35, 41]

Heat transfer in the counter-cross flow LAMEE is firstly studied [35], the air temperature T_{air} and liquid solution temperature T_{sol} are obtained from heat balance equations. The bulk mean liquid velocity is determined by applying the second-order Laplace equation for the stream function (ψ):

$$\nabla^2 \psi = 0 \quad (48)$$

Implicit finite difference method is used to solve the governing equations. Then this study is extended by considering both heat and mass transfer problems [41], and the governing equations in the air and solution sides are similar to Eqs. (36) - (41). The determination of heat transfer coefficient h and mass transfer coefficient h_m are based on an approximation that heat and mass flux on the membrane are uniform. h can be derived from the correlation for Nusselt number:

$$Nu = \frac{h d_h}{k} \quad (49)$$

Where d_h is the hydraulic diameter (m), k is thermal conductivity ($W/m^{-1}K^{-1}$), Nu is constant and equal to 8.24 [42, 43]. Similarly, h_m can be derived from the correlation for Sherwood number [40]:

$$Sh = NuLe^{-1/3} \quad (50)$$

$$Sh = \frac{h_m d_h}{D} \quad (51)$$

Where Le is Lewis number given by the ratio of thermal diffusivity α to the mass diffusivity D . Implicit finite difference method is also used to solve the governing equations.

A similar numerical modelling for cross flow membrane contractor is produced by Das and Jain [44], but the calculation process of Nu and mass transfer resistance in the membrane is improved. Most of the previous studies only focus on the laminar regime, the approximate correlations is used to consider transition regime in literature [44]. Nusselt number is obtained from correlations for laminar, transition and turbulent conditions in literature [45]. The mass transfer resistance in the membrane can be calculated by:

$$\phi_{mem} = \frac{\tau \delta}{D \epsilon \rho_{air}} \quad (52)$$

Where ϵ is the membrane porosity, τ is the tortuosity which can be obtained by the empirical relation in literatures [46, 47]:

$$\tau = \frac{(2-\epsilon)^2}{\epsilon} \quad (53)$$

Heat transfer resistance in the membrane, convective heat and mass transfer coefficients and Sherwood number Sh are calculated in the same ways as those introduced previously.

A two-dimensional transient model is developed by Seyed-Ahmadi et al. [36, 48] for the coupled heat and mass transfer in a run-around heat and moisture exchanger system (RAMEE), as shown in Fig.9.

Fig. 9. Schematic of RAMEE system (a) and schematic of cross flow LAMEE (b) [36, 48]

In Fig. 9(a), a RAMEE consists two LAMEEs, two storage tanks, two pumps and connecting tubing. Compared to the steady-state modelling, the transient effect is of vital importance during the system operation since the thermal and mass capacities of the liquid are larger than that of air. In this study, governing equations are similar to those developed in literatures [34-35, 39, 41]. Then a characteristic dimensionless time (τ) is defined relative to the transport time for the bulk solution to flow through both LAMEEs without considering the storage tanks or connecting tubes:

$$\tau = \frac{1}{t_{sol,s}^{*-1} + t_{sol,e}^{*-1}} \quad (54)$$

τ indicates the number of complete volume circulations of the solution in both LAMEEs, which is used to interpret the transient response of the system at different operating conditions. In order to investigate the transient behaviour, the performance of RAMEE is studied for a sufficient time until the system attains quasi-steady state. Two different criteria are adopted to define quasi-steady state for different initial solution concentrations.

The first criterion is based on energy and mass balances for the air side and applied when $\Delta C_{salt} = 0$:

$$\left| \frac{(W_{air,in,s} - W_{air,out,s}) - (W_{air,out,e} - W_{air,in,e})}{(W_{air,in,s} - W_{air,in,e})} \right| \leq 1 \times 10^{-2} \quad (55)$$

$$\left| \frac{(H_{air,in,s} - H_{air,out,s}) - (H_{air,out,e} - H_{air,in,e})}{(H_{air,in,s} - H_{air,in,e})} \right| \leq 1 \times 10^{-2} \quad (56)$$

The second criterion is based on rate of change in the effectiveness and applied when $\Delta C_{salt} \neq 0$:

$$416 \quad \left| \frac{\partial \varepsilon}{\partial \tau} \right| \leq 5 \times 10^{-6} \quad (57)$$

417 The governing equations are discretised by applying implicit finite difference method
 418 for the time derivative and upwind scheme for the first order spatial derivative, and
 419 solved by using Gauss-Seidal iteration method.

420 Following the similar method, the transient model is extended to a RAMEE system
 421 consisting two counter-cross flow LAMEEs by Namvar et al. [49], the time constant is
 422 treated as an important parameter in transient response of the LAMEE under the
 423 summer test conditions. Furthermore, it is found that the buoyancy forces affect the
 424 LAMEE performance under the winter testing conditions. The modelling results show
 425 that Grashof number Gr is higher than Reynolds number under the winter testing
 426 conditions, so the buoyancy forces should not be neglected.

427 To sum up, the finite difference method provides better accuracy in modelling
 428 compared with $\varepsilon - NTU$ method, its governing equations are discretised by applying
 429 finite difference method and solved by using iteration method. The heat and mass
 430 transfer coefficients are derived from correlations of fundamental data such as Nusselt
 431 and Sherwood numbers, which are borrowed from well-known books [40, 50].
 432 However, these data are obtained under the uniform temperature (heat flux) or
 433 concentration (mass flux) boundary conditions, which are unable to accurately reflect
 434 real heat and mass transfer properties in the membrane module. That is because the
 435 boundary conditions on the membrane surface are neither uniform temperature (heat
 436 flux) nor uniform concentration (mass flux), instead, they are naturally formed by the
 437 coupling of the air and solution flows [2]. In order to solve this problem, conjugate heat
 438 and mass transfer modelling method is required, which is introduced in the next section.

439

440 *3.3. Conjugate heat and mass transfer method*

441 *3.3.1. Flat plate membrane module*

442 As mentioned previously, the drawback of the finite difference modelling can be
 443 avoided by using the conjugate heat and mass transfer modelling method. A
 444 mathematical model is established by Huang and Zhang [51] for a cross-flow flat plate
 445 membrane module, the schematic of the membrane module and its coordinate system
 446 are given in Fig. 10 for the unit cell with one membrane and two neighbouring flow
 447 channels. Some assumptions are made in this model including: laminar flow mode for

both the air and liquid solution, Newtonian fluids with constant thermophysical properties, hydrodynamically fully developed while developing both thermally and in concentration for both flows, no heat and mass diffusion along the main flow direction.

Fig. 10. Schematic of a cross-flow LAMEE (a) and the coordinate system (b) [51]

The normalized governing equations in the model for the air side are:

$$\frac{\partial^2 u_a^*}{\partial x^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 u_a^*}{\partial y^{*2}} = -\frac{4b^2}{D_h^2} \quad (58)$$

$$\frac{\partial^2 \theta_a}{\partial x^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 \theta_a}{\partial y^{*2}} = U_a \frac{\partial \theta_a}{\partial z_h^*} \quad (59)$$

$$\frac{\partial^2 \xi_a}{\partial x^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 \xi_a}{\partial y^{*2}} = U_a \frac{\partial \xi_a}{\partial z_m^*} \quad (60)$$

The normalized governing equations for the solution side are:

$$\frac{\partial^2 u_s^*}{\partial x'^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 u_s^*}{\partial y'^{*2}} = -\frac{4b^2}{D_h^2} \quad (61)$$

$$\frac{\partial^2 \theta_s}{\partial x'^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 \theta_s}{\partial y'^{*2}} = U_s \frac{\partial \theta_s}{\partial z_h'^*} \quad (62)$$

$$\frac{\partial^2 \Theta_s}{\partial x'^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 \Theta_s}{\partial y'^{*2}} = U_s \frac{\partial \Theta_s}{\partial z_m'^*} \quad (63)$$

Where u^* , θ , ξ , Θ and U are dimensionless velocity, dimensionless temperature, dimensionless humidity, dimensionless mass fraction and dimensionless velocity coefficient respectively, as explained below:

$$u^* = -\frac{\mu u}{D_h^2 \frac{dP}{dz}} \quad (64)$$

Where μ is the dynamic viscosity ($pa \cdot s$), D_h is the hydraulic diameter (m) and defined as:

$$D_h = \frac{4A_c}{2(2a+2b)} \quad (65)$$

Where A_c is the cross-section area of the channel (m^2).

$$\theta = \frac{T - T_{ai}}{T_{si} - T_{ai}} \quad (66)$$

Where T_{ai} is the inlet air temperature (K), and T_{si} is the inlet solution temperature (K).

$$\xi = \frac{\omega - \omega_{ai}}{\omega_{si} - \omega_{ai}} \quad (67)$$

Where ω_{ai} is the inlet air humidity (kg/kg), and ω_{si} is the inlet equilibrium solution humidity (kg/kg).

$$\theta = \frac{X - X_{ei}}{X_{si} - X_{ei}} \quad (68)$$

Where X_{si} is the inlet solution mass fraction (kg water/kg solution), and X_{ei} is the equilibrium air mass fraction (kg water/kg solution).

$$U = \frac{u^*}{u_m^*} \frac{4b^2}{D_h^2} \quad (69)$$

Where u_m^* is the average dimensionless velocity on a cross-section, which can be obtained by:

$$u_m^* = \frac{\iint y^* dA}{A_c} \quad (70)$$

In Eqs. (58) - (63), x^* , y^* , z_h^* and z_m^* are the dimensionless coordinates:

$$x^* = \frac{x}{2b} \quad (71)$$

$$y^* = \frac{y}{2a} \quad (72)$$

$$z_h^* = \frac{z}{RePrD_h} \quad (73)$$

$$z_m^* = \frac{z}{ReScD_h} \quad (74)$$

The governing equations of the membrane surface are different from previous model's, the real heat and mass boundary conditions on the membrane surface are numerically obtained by simultaneous solution of momentum, energy and concentration equations for the air and solution. Firstly, some assumptions are made: (1) temperature differences in the membrane thickness are neglected since it is rather thin; (2) absorption heat is released on the membrane surface on the solution side. Then the normalized heat balance equation on the membrane surface between the air and solution is:

$$\lambda^* \frac{\partial \theta_a}{\partial y^*} \Big|_{y^*=0} + h_{abs}^* \frac{\partial \xi_a}{\partial y^*} \Big|_{y^*=0} = \frac{\partial \theta_s}{\partial y'^*} \Big|_{y'^*=0} \quad (75)$$

Where the dimensionless absorption heat and heat conductivity are defined as:

$$h_{abs}^* = \frac{\rho_a D_{va} h_{abs}}{\lambda_s} \left(\frac{\omega_{si} - \omega_{ai}}{T_{si} - T_{ai}} \right) \quad (76)$$

$$\lambda^* = \frac{\lambda_a}{\lambda_s} \quad (77)$$

Where λ_a and λ_s are the heat conductivity in the air and solution sides respectively ($W/m^{-1}K^{-1}$).

The mass boundary conditions on the air side membrane surface are:

$$y^* = 0 \text{ and } y^* = 1, q(x^*, z_G^*) = \dot{m}_v \quad (78)$$

Then the heat flux on the membrane surfaces on the air and solution sides is equal to:

$$q_h = -\lambda \frac{\partial T}{\partial y} \Big|_{y=0,2a} \quad (79)$$

The mass boundary conditions on the solution side membrane surface are:

$$y'^* = 0 \text{ and } y'^* = 1, q(x'^*, z_G'^*) = \dot{m}_v \quad (80)$$

Where z_G^* is the dimensionless geometric position and defined as:

$$z_G^* = \frac{z}{2b} \quad (81)$$

$\dot{m}_v$ is the moisture emission rate through the membrane at point (x^*, z_G^*) , which is determined by diffusion equation in the membrane as:

$$\dot{m}_v = \rho_a D_{vm} \frac{\omega_{m,a} - \omega_{m,s}}{\delta} \quad (82)$$

Where D_{vm} is the moisture diffusivity in the membrane (m^2/s), δ is the thickness of membrane (m).

Then moisture emissions from the membrane surface on the air and solution sides can be obtained by:

$$q_m = -\rho_a D_{va} \frac{\partial \omega_a}{\partial y} \Big|_{y=0,2a} \quad (83)$$

$$q_m = -\rho_s D_{ws} \frac{\partial X_s}{\partial y'} \Big|_{y'=0,2a} \quad (84)$$

All governing equations and their relevant boundary conditions are solved by using finite difference method, iterative techniques are also adopted in the process. Numerical results are validated by comparing with experimental results, the discrepancies between the numerical and experimental data are less than 6.0%. Thus generally speaking, this model predicts the heat and mass transfer well. Fundamental data of Nusselt and Sherwood numbers under different aspect ratios (b/a) are then calculated and listed in Table 1.

Table 1

Fully developed (fRe), Nuseelt and Sherwood numbers for different aspect ratios [51]

Aspect ratios (b/a)	Nu_H	Nu_T	Nu_C	Sh_C	$Nu_{C,a}$	$Sh_{C,a}$	$Nu_{C,s}$
References	[40,42,43]	[40,42,43]	[23]	[23]	[51]	[51]	[51]

1.0	3.61	2.98	1.88	1.89	3.12	3.30	3.41
1.43	3.73	3.08	2.49	2.52	3.23	3.41	3.64
2.0	4.12	3.39	6.06	3.10	3.48	3.65	4.05
3.0	4.79	3.96	4.06	4.00	4.15	4.28	4.74
4.0	5.33	4.44	4.64	4.52	4.61	5.11	5.35
8.0	6.49	5.60	6.06	6.03	5.79	5.83	6.41
50.0	-	-	7.78	7.81	7.54	7.74	7.91
100.0	-	-	8.07	8.05	7.70	7.98	8.08
∞	8.23	7.54	-	-	-	-	-

527

528

529 It is found from Table 1 that Nusselt number under the conjugate heat and mass transfer
530 condition in the air side ($Nu_{C,a}$) is between that under uniform temperature condition
531 (Nu_T) and heat flux condition (Nu_H). Furthermore, the Nusselt number under the
532 conjugate heat and mass transfer condition in the solution side ($Nu_{C,s}$) is approximately
533 15% higher than that in the air side.

534 It is assumed in previous studies that the flow is hydrodynamically fully developed to
535 avoid difficulties in solving Navier-Stokes equations. However, this assumption would
536 underestimate the performance seriously since the developing entrance length accounts
537 for a huge proportion of the duct length. A step forward to investigate the effects of the
538 developing entrances on the fluid flow and conjugate heat and mass transfer in a cross-
539 flow flat plate membrane module is made by Huang et al. [52], the membrane module
540 structure in this study is the same as that in literature [51]. More accurate governing
541 equations are established and solved directly by considering the influences of the flow,
542 heat and mass transfer developing entrances. For the air flow, the mass, momentum and
543 heat governing equations are given by:

$$544 \quad \frac{\partial u_a^*}{\partial x^*} + \frac{\partial v_a^*}{\partial y^*} + \frac{\partial w_a^*}{\partial z^*} = 0 \quad (85)$$

$$545 \quad u_a^* \frac{\partial u_a^*}{\partial x^*} + v_a^* \frac{\partial u_a^*}{\partial y^*} + w_a^* \frac{\partial u_a^*}{\partial z^*} = -\frac{\partial p_a^*}{\partial x^*} + \left(\frac{\partial^2 u_a^*}{\partial x^{*2}} + \frac{\partial^2 u_a^*}{\partial y^{*2}} + \frac{\partial^2 u_a^*}{\partial z^{*2}} \right) \quad (86)$$

$$546 \quad u_a^* \frac{\partial v_a^*}{\partial x^*} + v_a^* \frac{\partial v_a^*}{\partial y^*} + w_a^* \frac{\partial v_a^*}{\partial z^*} = -\frac{\partial p_a^*}{\partial y^*} + \left(\frac{\partial^2 v_a^*}{\partial x^{*2}} + \frac{\partial^2 v_a^*}{\partial y^{*2}} + \frac{\partial^2 v_a^*}{\partial z^{*2}} \right) \quad (87)$$

$$547 \quad u_a^* \frac{\partial w_a^*}{\partial x^*} + v_a^* \frac{\partial w_a^*}{\partial y^*} + w_a^* \frac{\partial w_a^*}{\partial z^*} = -\frac{\partial p_a^*}{\partial z^*} + \left(\frac{\partial^2 w_a^*}{\partial x^{*2}} + \frac{\partial^2 w_a^*}{\partial y^{*2}} + \frac{\partial^2 w_a^*}{\partial z^{*2}} \right) \quad (88)$$

$$548 \quad u_a^* \frac{\partial \theta_a}{\partial x^*} + v_a^* \frac{\partial \theta_a}{\partial y^*} + w_a^* \frac{\partial \theta_a}{\partial z^*} = \frac{1}{Pr_a} \left(\frac{\partial^2 \theta_a}{\partial x^{*2}} + \frac{\partial^2 \theta_a}{\partial y^{*2}} + \frac{\partial^2 \theta_a}{\partial z^{*2}} \right) \quad (89)$$

The governing equations of momentum and heat transfer for the solution flow are in the same forms as those for the air flow. The governing equations of mass transfer for the solution flow is:

$$u_s^* \frac{\partial \theta_s}{\partial x'^*} + v_s^* \frac{\partial \theta_s}{\partial y'^*} + w_s^* \frac{\partial \theta_s}{\partial z'^*} = \frac{1}{Sc_s} \left(\frac{\partial^2 \theta_s}{\partial x'^*{}^2} + \frac{\partial^2 \theta_s}{\partial y'^*{}^2} + \frac{\partial^2 \theta_s}{\partial z'^*{}^2} \right) \quad (90)$$

The governing equations and conjugate heat and mass transfer boundary conditions are solved by applying a finite volume method, iterative techniques are also used to solve interacted problems. The model is validated by comparing to the experimental data, the maximum difference between the calculated values and experimental data is 4.0%, which means this model is able to predict heat and mass transfer for the cross-flow flat plate membrane module. Then fundamental data of Nusselt and Sherwood numbers under different aspect ratios (b/a) are calculated, a comparison of these numbers and those obtained without considering developing entrance is made in Table 2.

Table 2

Comparison of fully developed (fRe), Nuseelt and Sherwood numbers for different aspect ratios with and without considering developing entrance

Aspect ratios (b/a)	$Nu_{C,a}$	$Nu_{C,s}$	$Nu_{C,a}$	$Nu_{C,s}$	$Nu_{m,a}$	$Nu_{m,s}$
References	[51]	[51]	[52]	[52]	[52]	[52]
1.0	3.12	3.41	3.37	3.64	3.87	3.95
1.43	3.23	3.64	3.48	3.78	3.99	4.18
2.0	3.48	4.05	3.73	4.19	4.25	4.59
3.0	4.15	4.74	4.39	4.88	4.95	5.28
4.0	4.61	5.35	4.86	5.49	5.34	5.89
8.0	5.79	6.41	6.04	6.55	6.58	6.95
50.0	7.54	7.91	7.79	8.02	8.32	8.45
100.0	7.70	8.08	7.95	8.22	8.49	9.62
∞	-	-	-	-	-	-

As displayed in Table 2, $Nu_{C,a}$ and $Nu_{C,s}$ obtained with considering developing entrance are about 10% higher than those without this consideration. Furthermore, the mean Nusselt numbers along the whole duct ($Nu_{m,a}$ and $Nu_{m,s}$) are about 10% higher than the local values ($Nu_{C,a}$ and $Nu_{C,s}$), and approximately 20% higher than those calculated in literature [51]. This means thermally developing entrance has substantial effect on heat transfer in the duct.

The membrane-based dehumidifiers are adiabatic in the previous researches, and the solution is heated in the dehumidifier due to the absorption heat released on the solution side. An internally-cooled membrane-based liquid desiccant dehumidifier (IMLDD) is studied by Huang et al. [53], which consists feed air, solution, cooling water and sweep air. The structure of IMLDD is given in Fig. 11.

Fig. 11. Structure of a cross-flow internally-cooled membrane-based liquid desiccant dehumidifier (IMLDD): (a) Space diagram; (b) planform [53]

As shown in Fig. 11, the feed air and solution are in cross-flow arrangement and separated by membranes. Water is sprayed vertically along plastic plates in cooling channels which are adjacent to the solution channels. The sweep air flows over the water falling film in a co-current flow arrangement. The heat released on the solution side is swiftly taken away by falling water and sweeping air. In this modelling, a unit cell consists of a feed air channel, a piece of membrane, a solution channel, one plastic plate and half of cooling channel are selected as the calculation domain, and the coordinate system is given in Fig. 12.

Fig. 12. Coordinate system of the unit cell for the heat and mass transfer model in IMLDD. (a) Space diagram; (b) planform [53].

In this modelling, the interface between the water film and sweep air is at thermodynamic equilibrium state, and the velocity of the sweeping air at interface is equal to that of water flow. Furthermore, gravitational force of the feed air is neglected, while water falling film is formed only due to its gravitational force. Using the same method introduced in literature [51], the governing equations of the feed air, solution, water falling film and sweep air are developed respectively and solved by a finite volume numerical scheme. Experiment tests are conducted to validate the numerical model, but only the outlet parameters of the feed air, solution, water falling film and sweep air are measured due to the difficulty in measuring parameters inside the contractor. The maximum deviation between numerical and experimental results is less than 8.0%, which means numerical model is capable of predicting the performance of IMLDD well.

3.3.2. Hollow fiber membrane module

Even though many researches focusing on heat and mass transfer in the fiber tubes are carried out [26-30], the conjugate heat and mass transfer needs to be investigated seriously. The heat and mass transfer in a parallel flow tube-and-shell heat exchanger is investigated previously [40, 50]. However the heat and mass transfer coefficients in these studies are simply borrowed from well-established Nusselt and Sherwood correlations for traditional tube-and-shell heat exchangers, which are not suitable for membrane-based liquid desiccant dehumidifier since the membrane surface boundary

conditions are different [2]. More accurate conjugate heat and mass transfer is discussed subsequently.

3.3.2.1. Free surface model

To model the whole bundle of a fiber-to-fiber basis directly is difficult since the number of fibers are numerous, normally 200-600 in a small shell 4 cm in diameter. This problem can be solved by using Happel's free surface model [54]. In Happel's model, each fiber in the shell is surrounded by a fluid envelop, and there is no momentum, heat or mass transfer on the outer free surface. The fibers are assumed to be distributed homogeneously and the flow is axial. The bundle consists of a series of free surface cells, which have only one fiber in the centre and surrounded by homogeneous fluid. The free surface models in literature [54] are shown in Fig. 13.

Fig. 13. Free surface models for axial flow [54]

Many researches are conducted based on Happel's free surface model to investigate the correlations of the basic Nusselt number [55-63]. However, in these researches, the Nusselt numbers are obtained under uniform temperature (concentration) or heat flux (mass flux) boundary condition, which are not accurate since the membrane surface boundary condition is naturally formed and neither uniform temperature (concentration) nor uniform heat flux (mass flux). To have a better understanding of the conjugate heat and mass transfer for hollow fiber membrane module, the hollow fiber membrane modules with parallel and cross flow configurations based on free surface method are numerically analysed by Zhang et al. [14, 64]. The structures and representative cell with a free surface are shown in Fig. 14.

Fig. 14. Schematic of the hollow fiber membrane module and cell with a free surface model. (a) Parallel flow type; (b) Cross flow type; (c) The cell with a free surface [14, 64]

Both parallel and cross flow types have their advantages and disadvantages. The advantages of the parallel module are simple for manufacturing, well known fluid dynamic in shell and tube side, and easy for mass transfer estimation [65]. However this configuration has a disadvantage of rather large pressure drop. Furthermore, Nusselt and Sherwood numbers of a cross-flow configuration are much higher than that of a counter-flow configuration when Reynolds number is large. As a result, the cross flow hollow fiber membrane module is more attractive than the parallel flow type. The parallel flow module has 200-600 fibers in a small shell with 4cm in diameter, while the cross flow module has 2000-12000 fibers in a contractor 20×20 cm in cross section [14, 64]. To solve the problem of numerous fibers, Happel's free surface method is used. The free surface radius of a single fiber can be obtained [54, 66]:

$$r_f = r_o \cdot \left(\frac{1}{\varphi}\right)^{1/2} \quad (91)$$

Where r_o is the fiber outer radius (m), φ is the packing fraction of the module. For the counter and cross flow types, φ can be calculated by:

$$\varphi_{counter} = \frac{\eta_{fiber} r_o^2}{R_0^2} \quad (92)$$

Where η_{fiber} is the number of fibers, R_0 is the counter flow module shell radius (m).

$$\varphi_{cross} = \frac{\eta_{fiber} \pi r_o^2}{ab} \quad (93)$$

Where a is the cross flow module shell width (m), b is the cross flow module shell height (m).

In this modelling, one-fourth of a cell is selected as the calculation domain for the sake of symmetry and simplicity for the counter flow module. As for the cross flow module, only half of a cell is selected as the calculation domain. The coordinate systems of the unit cell for both counter and cross flow modules are shown in Fig. 15.

Fig. 15. The coordinate system of the calculated domain: The three dimensional coordinate system for (a) counter flow; (d) cross flow. The physical plane for the channel cross section for (b) counter flow; (e) cross flow. The computational plane for (c) counter flow; (f) cross flow [14, 64]

Some assumptions are made for both modelling: (1) the fibers are evenly distributed, thus results obtained from a single fiber can be generalized to the whole module [67]; (2) both air and solution flows are considered to be laminar since the Reynolds numbers are much less than 2300; (3) both air and solution flows are considered to be Newtonian with constant thermal-physical properties; (4) both air and solution flow are assumed hydrodynamically fully developed, but developing both thermally and in concentration, (5) both air and solution flows are laminar, axial heat and mass diffusions for two flows are negligible.

The developing of governing equations follows the same method as used in literature [51]. Compared to Eqs. (58) - (63), the geometric properties $2a$ and $2b$ are replaced by the free surface radius r_f , then a new hydraulic dimeter is defined as:

$$D_h = \frac{4A_c}{P_d} \quad (94)$$

Where P_d is the setted perimeter of the channels (m).

Body-fitted coordinate systems are generated to convert the physical plane into numerical one. For the counter flow, the cross section as given in Fig. 15(b) is transformed to the numerical plane, as shown in Fig. 15(c). The air and solution flows are coupled on BC and EH. Similarly, the physical plane of the cross flow module is transformed to the numerical plane, and two flows are coupled on ABC and FKN. After the set-up of boundary-fitted coordinate system, the governing equations are transformed into the corresponding governing equations, which are then discretized by using finite volume method. For the conjugate heat and mass transfer problem, alternating direction implicit (ADI) techniques are used to solve the governing equations. The local and mean Nusselt and Sherwood numbers are obtained and then experimentally validated. The experimental results show a good agreement with calculated data. Discrepancies between the numerical and tested data are below 5% for the counter flow module, and 6% for the cross type, which means this numerical model can be successfully used to simulate heat and mass transfer in the counter and cross flow hollow fiber membrane modules.

The above model is conducted based on the assumption of laminar flow for both air and solution flows. In practical applications, the solution Reynolds number is normally below 10, thus the assumption of laminar flow for the solution is reasonable. However, although the Reynolds number for the air flow is still below 2300, the air flow would be turbulent because of the continuous disturbances from the fine tubes, which means a turbulent model for the solution side is needed [68]. The small local turbulent Reynolds numbers (less than 150) make it inaccurate to use a standard $k - \varepsilon$ model. A low-Re $k - \varepsilon$ turbulent model is employed to describe the turbulent flow and heat and mass transfer across the fiber bundle in a cross flow hollow fiber membrane module [69]. Happel's free surface model is used again to overcome the difficulty of direct modelling of numerous number of fibers. Thus the free surface radius and packing fraction are still the same as the laminar model's. Since the solution flow is in laminar mode along z direction, normalized governing equations of fluid flow, heat and mass

conservation are the same as those in previous two models. For the air side, the turbulent kinetic energy and dissipation rate are calculated through the transport equations, and solved simultaneously with the air flow conservation equations. The normalized governing momentum, heat and mass equations of the air flow are given below:

$$\frac{\partial u_x^*}{\partial x^*} + \frac{\partial u_y^*}{\partial y^*} = 0 \quad (95)$$

$$\frac{\partial(u_x^* u_x^*)}{\partial x^*} + \frac{\partial(u_y^* u_x^*)}{\partial y^*} = \frac{\partial}{\partial x^*} \left(\Gamma_x \frac{\partial u_x^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_x \frac{\partial u_x^*}{\partial y^*} \right) - \frac{1}{2} \frac{\partial p_a^*}{\partial x^*} + \frac{\partial}{\partial x^*} \left(\Gamma_x \frac{\partial u_x^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_x \frac{\partial u_y^*}{\partial x^*} \right) \quad (96)$$

$$\frac{\partial(u_x^* u_y^*)}{\partial x^*} + \frac{\partial(u_y^* u_y^*)}{\partial y^*} = \frac{\partial}{\partial x^*} \left(\Gamma_y \frac{\partial u_y^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_y \frac{\partial u_y^*}{\partial y^*} \right) - \frac{1}{2} \frac{\partial p_a^*}{\partial y^*} + \frac{\partial}{\partial x^*} \left(\Gamma_y \frac{\partial u_x^*}{\partial y^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_y \frac{\partial u_y^*}{\partial y^*} \right) \quad (97)$$

$$\frac{\partial(u_x^* k_a^*)}{\partial x^*} + \frac{\partial(u_y^* k_a^*)}{\partial y^*} = \frac{\partial}{\partial x^*} \left(\Gamma_k \frac{\partial k_a^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_k \frac{\partial k_a^*}{\partial y^*} \right) + \frac{\mu_t V_a}{\rho_a r_f k_a} G_k^* - \frac{r_f \varepsilon_a}{V_a k_a} \varepsilon_a^* - E^* \quad (98)$$

$$\frac{\partial(u_x^* \varepsilon_a^*)}{\partial x^*} + \frac{\partial(u_y^* \varepsilon_a^*)}{\partial y^*} = \frac{\partial}{\partial x^*} \left(\Gamma_\varepsilon \frac{\partial \varepsilon_a^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_\varepsilon \frac{\partial \varepsilon_a^*}{\partial y^*} \right) + \frac{c_1 \mu_t V_a}{\rho_a r_f k_a} \frac{\varepsilon_a^* G_k^*}{k_a^*} - \frac{c_2 r_f \varepsilon_a}{V_a k_a} \frac{\varepsilon_a^{*2}}{k_a^*} + F^* \quad (99)$$

$$\frac{\partial(u_x^* \theta_a)}{\partial x^*} + \frac{\partial(u_y^* \theta_a)}{\partial y^*} = \frac{\partial}{\partial x^*} \left(\Gamma_T \frac{\partial \theta_a}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_T \frac{\partial \theta_a}{\partial y^*} \right) \quad (100)$$

$$\frac{\partial(u_x^* \xi_a)}{\partial x^*} + \frac{\partial(u_y^* \xi_a)}{\partial y^*} = \frac{\partial}{\partial x^*} \left(\Gamma_\omega \frac{\partial \xi_a}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left(\Gamma_\omega \frac{\partial \xi_a}{\partial y^*} \right) \quad (101)$$

In Eqs. (98) - (99), G_k^* , E^* , and F^* are defined as follows [70, 71]:

$$G_k^* = 2 \left[\left(\frac{\partial u_x^*}{\partial x^*} \right)^2 + \left(\frac{\partial u_y^*}{\partial y^*} \right)^2 \right] + \left(\frac{\partial u_x^*}{\partial y^*} + \frac{\partial u_y^*}{\partial x^*} \right)^2 \quad (102)$$

$$E^* = \frac{2\mu_{air}}{\rho_{air} r_f V_{air}} \left(\frac{\partial(k_a^*)^{1/2}}{\partial y^*} \right)^2 \quad (103)$$

$$F^* = \frac{2\mu_t \mu_a V_{air}}{\rho_{air}^2 r_f^3 \varepsilon_{air}} \left(\frac{\partial^2 u_y^*}{\partial y^{*2}} \right)^2 \quad (104)$$

E^* is added due to the anisotropy of kinetic energy in the viscous layer, F^* is added for corresponding to the experimental values.

In Eqs. (96) - (101), Γ_x , Γ_k , Γ_T , Γ_ε and Γ_ω are all associated diffusion coefficients.

In Eq. (99), c_2 is defined by [70, 71]:

$$c_2 = 1.92(1 - 0.3 \exp(-Re_t^2)) \quad (105)$$

Other coefficients are

$$c_\mu = 0.09, c_1 = 1.44, \sigma_k = 1.0, \sigma_\varepsilon = 1.3, \sigma_T = 0.95, \sigma_\omega = 1.0$$

As mentioned before, for the air side, the turbulent kinetic energy and dissipation rate are calculated through the transport equations, thus the dimensionless turbulent kinetic energy of the air flow is given by:

$$k_a^* = \frac{k_a}{k_{ai}} \quad (106)$$

Where k_{ai} is the turbulent kinetic energy of the approaching air flow (m^2/s^2), which can be calculated by [72]:

$$k_{ai} = 0.01(V_{ai}^2/2) \quad (107)$$

The dimensionless turbulent dissipation rate of the air flow is defined by:

$$\varepsilon_a^* = \frac{\varepsilon_a}{\varepsilon_{ai}} \quad (108)$$

Where ε_{ai} is the turbulence dissipation rate of the approaching air flow (m^2/s^2), which can be obtained by [72]:

$$\varepsilon_{ai} = \frac{500c_\mu k_{ai}^2}{V_{ai}D_{h,a}} \quad (109)$$

The solution procedure is the same as that used for laminar flow. Then the overall mean Nusselt, Sherwood numbers of both flows, and total drag coefficient of the air flow are calculated and compared with the experimental results. To evaluate the accuracy of low-Re $k - \varepsilon$ turbulent model, the laminar model is also used for comparison. The results show that when $Re_a < 300$, both the laminar and turbulent models agree with the test data well, and the maximum difference is less than 6.0% for three indices. However when $Re_a > 300$, the turbulent model shows better fitting than the laminar one. As a result, the low-Re $k - \varepsilon$ turbulent model is successful in modelling fluid flow, heat and mass transfer in the cross flow hollow fiber membrane module, especially with relatively large Re_a .

3.3.2.2. Periodic unit cell model

The free surface model illustrated previously is a coarse approximation since each single fiber is enclosed into a hypothetical concentric envelop, the effect of fiber-to-fiber interaction is not considered. To study the interaction among neighbouring fibers, the momentum and energy conservation equations are solved analytically or numerically for periodic arrays with different geometrical arrangements, such as in-line and staggered arrangements. More researches are conducted to investigate the conjugate heat and mass transfer for counter and cross hollow fiber membrane modules [73, 74]. Schematics of the membrane tube bank with in-line and staggered arrangements are given in Fig. 16. The areas surrounded by dashed lines are selected as the calculation domains.

Fig. 16. Schematic of a counter flow hollow fiber membrane tube bank: (a) In-line; (b) staggered, and a cross flow hollow fiber membrane tube bank: (c) In-line; (d) staggered [73, 74]

The new packing fraction is defined as:

$$\varphi = \frac{\pi r_0^2}{S_L S_T} \quad (110)$$

Where r_0 is the fiber outer radius (m), S_L and S_T are the longitudinal and transverse pitches (m).

The developing of governing equations follows the same procedure used in literature [51]. Compared with the free surface model, the free surface radius r_f is replaced by the longitudinal pitch S_L . Then the hydraulic diameters of the air and solution flows are obtained by:

$$D_{h,a} = \frac{2(S_L S_T - \pi r_0^2)}{\pi r_0}, D_{h,s} = 2r_i \quad (111)$$

Where r_i is the fiber inner radius (m).

Due to the complexity of the geometric constructions of the unit cells, the body fitted coordinate transformation method is employed to convert the physical domains into the rectangular calculating domains as depicted in Fig. 17.

Fig. 17. The coordinate systems of the cross-sections of the unit cells for counter flow contractor: (a) (c) physical planes for in-line and staggered configurations; (b) (d) computational planes for in-line and staggered configurations; and for cross flow contractor: (e) (g) physical planes for in-line and staggered configurations; (f) (h) computational planes for in-line and staggered configurations [73, 74]

For the counter flow contractor, the physical planes as shown in Fig. 17 (a)(c) are transformed into the computational planes, as displayed in Fig. 17(b)(d). Similarly, for the cross flow contractor, physical planes in Fig. 17(e)(g) are transformed into the computational planes in Fig. 17(f)(h). For the in-line configuration of the cross flow contractor, the air and solution flows are conjugated on DE, FC, HK and GL, while they are conjugated on FC, DE, HK, GL, OPQ and RWT for the staggered configuration in the cross flow contractor. After the set-up of body-fitted coordinate systems, the governing equations are solved by using finite volume method via a self-built code developed by FORTRAN. Then the fundamental data, such as friction factor, Nusselt and Sherwood numbers are obtained and compared with those calculated from the free surface model. It is found that the friction factor and Nusselt number from the free surface model deviate 8-50% from the periodic unit cell model for the counter flow contractor, and 20% for the cross flow contractor. The free surface model is only applicable when the packing fraction is less than 0.25.

The periodic unit cell model is developed by Zhang et al. [75] based on the same structure of the cross flow hollow fiber module, the low- Re $k - \varepsilon$ model is adopted rather than the laminar model to investigate the hollow fiber module performance. The model governing equations are the same as Eqs. (95)-(101), but the free surface radius r_f is replaced by the longitudinal pitch S_L as well. The solution procedure is the same

as that used for the laminar flow. The numerical results of the laminar model are also compared with the turbulent model's. It is found that when $Re_a < 300$, the results from both the laminar and turbulent models agree with the experimental data well, the maximum difference is less than 7.0%. However, when $Re_a > 300$, the turbulent model shows a better fitting than the laminar model.

In fact, the flow is usually impinged with a skewed angle to the fibers as a result of the duct and exchanger structure limitations, both the parallel and cross flows are ideal. The impinging angle has significant influence on the hollow fiber module performance. One research is conducted by Ali and Vafai [76] to analyse heat and mass transfer between the air and desiccant film in an inclined parallel counter flow model. The inclined parallel counter flow channels are shown in Fig. 18.

Fig. 18. Schematic of inclined parallel (a) and counter (b) flow configurations [76]

To solve the air flow mass, momentum and energy conservation governing equations, the coordinate transformation method is used:

$$\xi = x_1 \text{ and } \eta = \frac{y_1}{\delta_a(x_1)} \quad (112)$$

Where

$$\delta_a(x_1) = l_1 - x_1 \tan \theta \text{ for the inclined parallel flow channel} \quad (113)$$

$$\delta_a(x_1) = l_1 + x_1 \tan \theta \text{ for the inclined counter flow channel} \quad (114)$$

Then the finite difference method is adopted to solve the governing equations. The results show that the inclination angle is capable of enhancing the dehumidification performance significantly compared to the pure parallel flow. The periodically fully-developed laminar heat transfer and fluid flow characteristics of an array of uniform plate length is investigated by using body-fitted coordinate system [77]. The fluid flow and heat transfer phenomena around a rectangular cylinder are studied at an inclination angle from 0° to 20° [78], the heat transfer coefficients are summarized by empirical

equations. However, the above researches only focus on the skewed flow around a single fiber, In order to investigate the interactions among neighbouring fibers, the heat transfer across a skewed hollow fiber membrane module is studied based on the uniform temperature boundary conditions on the membrane surface [79]. The heat and mass transfer in a skewed flow fiber bank is also investigated in another research [80], the boundary conditions on a membrane surfaces are naturally formed by coupling between the air and solution flows. The schematic of a skewed flow in a hollow fiber membrane bundle with inline and staggered arrangements is given in Fig. 19.

Fig. 19. Schematic of a skewed flow in a hollow fiber membrane bundle: (a) the bundle with inline arrangement; (b) the bundle with staggered arrangement [80]

In this study, two periodic cells containing two or three fibers for different arrangements along the air stream are selected for the sake of simplicity and symmetry, as shown in Fig. 20. The interactions among neighbouring fibers and the coupling between the air and solution flows are considered.

Fig. 20. Schematic of the periodic cells for modelling: (a) inline array and (b) staggered array [80]

The governing equations are similar to those in literatures [73, 74]. The skewed angle α is defined as:

$$\alpha = \arctan(u_x/u_z) \quad (115)$$

It is of vital importance to obtain the air-side transport data in the periodic cell. For the skewed flow configuration specifically, the air-side mean Nusselt and Sherwood numbers are obtained by energy and mass conservation analyses [74]:

$$Nu_{c,a} = \frac{S_T L \sin \alpha}{2A_m} Re_a Pr_a \frac{(\theta_{b,a})_{EFF'E'} - (\theta_{b,a})_{ABB'A'}}{\Delta \theta} \quad (116)$$

$$Sh_{c,a} = \frac{S_T L \sin \alpha}{2A_m} Re_a Sc_a \frac{(\xi_{b,a})_{EFF'E'} - (\xi_{b,a})_{ABB'A'}}{\Delta \xi} \quad (117)$$

The above equations are applicable to $0 < \alpha \leq 90^\circ$. $\Delta \theta$ and $\Delta \xi$ are the logarithmic mean temperature and humidity differences between the membrane surface and air flow respectively, which are given by:

$$\Delta \theta = \frac{(\theta_{w,a} - \theta_{b,a})_{EFF'E'} - (\theta_{w,a} - \theta_{b,a})_{ABB'A'}}{\ln[(\theta_{w,a} - \theta_{b,a})_{EFF'E'} / (\theta_{w,a} - \theta_{b,a})_{ABB'A'}]} \quad (118)$$

$$\Delta \xi = \frac{(\xi_{w,a} - \xi_{b,a})_{EFF'E'} - (\xi_{w,a} - \xi_{b,a})_{ABB'A'}}{\ln[(\xi_{w,a} - \xi_{b,a})_{EFF'E'} / (\xi_{w,a} - \xi_{b,a})_{ABB'A'}]} \quad (119)$$

Where the subscripts “w” and “b” represent “wall mean” and “bulk” respectively.

To solve the governing equations, the body fitted coordinate transformation method is employed to convert the physical domains to the rectangular calculating domains as well, the governing equations are discretised by finite volume method. It is found that the skewed angle has significant impact on heat and mass transfer in the bundle. The periodically mean friction factors, Nusselt and Sherwood numbers increase with the skewed angle. Furthermore, the skewed heat and mass transfer rates and friction factors are higher than those of the counter flow, but lower than the cross flow's.

To sum up, the conjugate heat and mass transfer method provides better accuracy in modelling compared with $\varepsilon - NTU$ and finite difference methods. The boundary conditions on the membrane surface are naturally formed in this method rather than uniform temperature or concentration boundary conditions. Fundamental heat and mass transfer data can be obtained directly from the conjugate heat and mass transfer governing equations. As a matter of convenience, a detail comparison of three different modelling methods is summarized in Table 3.

Table 3

Comparison of different modelling methods for membrane-based heat and mass exchanger

Modelling method	exchanger type	Flow pattern	System stability	Cooling type	Fiber-fiber interaction (for hollow fiber exchangers only)	Air flow assumption (for hollow fiber exchangers only)	Solutions to solve governing equations	Experimental validation	References			
$\varepsilon - NTU$ method	Flat plate	cross	Steady-state	adiabatic	-			$\leq 7.3\%$ (sensible) $\leq 8.6\%$ (latent)	[25]			
		Counter-cross						$\leq 10\%$	[32]			
	Hollow fiber	cross						$\leq 5\%$ (sensible) $\leq 12\%$ (latent)	[31]			
Finite difference method	Flat plate	Counter	Steady state	adiabatic	-		Finite difference	Good agreement	[34]			
		cross	transient						[39]			
		Counter-cross	Steady state						[36, 48-49]			
									[35, 41]			
Conjugate heat and mass transfer method	Flat plate	cross	Steady-state	adiabatic	-		Finite volume	$\leq 6.0\%$ (hydrodynamically fully developed) $\leq 4.0\%$ (Consider effects of developing entrances)	[51, 52]			
				Internally-cooled				$\leq 8\%$	[53]			
	Hollow fiber	parallel		adiabatic	Free-surface method	laminar		$\leq 5\%$	[14]			
		cross				Turbulent (low-Re $k - \varepsilon$ model)		$\leq 6\%$	[64]			
		cross						$\leq 6\%$	[69]			
		Parallel						$\leq 5\%$	[73]			
		Cross						$\leq 7.5\%$	[74]			
		Skewed			Periodic unit cell method	laminar		For heat transfer $\leq 6.7\%$ For mass transfer $\leq 7.5\%$ For friction factor $\leq 5.8\%$	[80]			
		cross				Turbulent (low-Re $k - \varepsilon$ model)		$\leq 7\%$	[75]			

4. Modelling of randomly placed fiber distribution

Different modelling methods for the hollow fiber membrane modules are discussed in previous section. In those studies the fibers are assumed to be orderly packed for the ease of calculations. However, in practice the fibers are distributed densely and randomly due to the numerous number of fibers in a shell and fibers are normally fine [81]. Thus the nature of non-uniformity in fiber distribution needs to be considered to study the influence of randomly placed fibers on heat and mass transfer. Methods used to simulate the fiber distribution are discussed in this section.

4.1. Voronoi Tessellation method

Recently, Varonoi Tessellation is employed to model the randomly packed hollow fiber modules [82-86]. In this method, each fiber is surrounded by a polygonal cell whose boundaries are defined by perpendicular bisectors of lines joining each fiber with its nearest neighbour, and a typical Voronoi polygon is shown in Fig. 21.

Fig. 21. Voronoi tessellation of flow area [82]

An exponential probability density function f is applied to approximate the probability that there is no other fiber within a polygonal area ϕ surrounded by s nearest fibers [82]:

$$f(\phi) = \frac{s^s}{\langle \phi \rangle^s} \frac{\phi^{s-1}}{(s-1)!} e^{-(s\phi)/\langle \phi \rangle} \quad (120)$$

Where

$$\phi = a - a_f \text{ and } \langle \phi \rangle = a_0 - a_f \quad (121)$$

a is the cross-sectional area of a polygonal cell (m^2), a_0 is the average cross-sectional polygonal cell area (m^2), a_f is the cross-sectional area of a fiber (m^2). The typical value of s is 4-6 [85]. At lower packing density, $s = 4$ appears to fit the simulation results better; at higher packing density, the obtained distribution agrees with $s = 6$, but the influence of s on results is not evident [86].

The probability density function can be written in terms of λ and the normalized probability P , for a fixed value of s of cell having an area between a_1 and a_2 [83]:

$$P = \frac{1}{(1-(a_f/a_0))} \int_{\lambda_1}^{\lambda_2} s^2 \frac{\lambda^{s-1} e^{-s\lambda}}{(s-1)!} d\lambda \quad (122)$$

Where

$$\lambda_1 = \frac{a_1 - a_f}{a_0 - a_f} \text{ and } \lambda_2 = \frac{a_2 - a_f}{a_0 - a_f} \quad (123)$$

The average mass transfer coefficient $\langle K \rangle$ is estimated from the local mass transfer coefficient K_i in each polygonal cell, which is obtained by the Sherwood number:

$$Sh_{f_i} = 1.62 \left(\frac{LD}{u_i d_{t_i}^2} \right)^{-0.33} = \frac{K_i d_{h_i}}{D} \quad (124)$$

Where Sh_{f_i} is the length-averaged feedside Sherwood number in the i th flow category, D is the diffusion coefficient, d_{t_i} is the internal diameter of hollow fiber (m). In Eq. (124), the local axial flow velocity u_i is obtained by:

$$\Delta P = \left(\frac{2\mu L}{\rho} \right) \left(\frac{(fRe)_e}{\phi d_h^2} \right) = \frac{4\rho u_i^2 L}{d_{h_i}} \quad (125)$$

Where L is the flow length (m), ρ and μ are the fluid density (kg/m^3) and viscosity (Ns/m^2) respectively, d_h and d_{h_i} are the hydraulic diameters in the whole bundle and the i th flow category (m) respectively.

For the situation where Voronoi cells or polygons are irregular, it is difficult to give the velocity profiles in the cell, which means it is impossible to obtain the mass transfer coefficient in each cell as well as the whole randomly packed module [87]. The Voronoi Tessellation method is extended by applying Happel's free surface model [54], a shear free boundary condition is developed in the imaginary outer boundary of the cell. The schematic of the module cross-section sub-division and free surface model is shown in Fig. 22.

Fig. 22. Schematic of the module cross-section sub-division and free surface model [87]

The probability density function is also applied in this study. The relationship between the cell area and local packing density is:

$$a = \frac{1}{4} \pi d_f^2 = \frac{a_f}{\phi} = \frac{1}{4} \frac{\pi d_0^2}{\phi} \quad (126)$$

Where ϕ is the local packing fraction, d_f and d_0 are the free surface diameter and fiber outer diameter (m) respectively. The probability density distribution function of the local cell packing fraction is calculated as:

$$f(\phi) = \frac{s^s}{(s-1)!} \frac{\phi_0^s}{(1-\phi_0)^s} \frac{(1-\phi)^{s-1}}{\phi^{s+1}} \times \exp \left[-s \frac{\phi_0(1-\phi)}{\phi(1-\phi_0)} \right] \quad (127)$$

Which fulfils the criteria:

$$\int_0^1 \phi f(\phi) d\phi = \phi_0 \text{ and } \int_0^1 f(\phi) d\phi = 1 \quad (128)$$

Then a function is defined according to the free surface model $g(\phi)$:

$$g(\phi) = \frac{8(\phi - \phi^2)}{4\phi - \phi^2 - 3 - 2\ln\phi} \quad (129)$$

Accordingly, the probability cumulative distribution function of cell void area $H(\phi)$ and probability cumulative distribution function of fluid flow $J(\phi)$ are defined as:

$$H(\phi) = \frac{f(\phi)(1/\phi - 1)}{(1/\phi_0 - 1)} \quad (130)$$

$$J(\phi) = \frac{f(\phi)(1/\phi - 1)/g(\phi)}{\int_0^1 f(t)(1/t - 1)/g(t) dt} \quad (131)$$

4.2. Fractal method

The Voronoi Tessellation method provides a possibility to calculate heat and mass transfer in a randomly packed module quantitatively. However, the degree of irregularity is not truly reflected in this method because this method assumes a uniform probability function for all muddles, while in reality the degree of irregularity is changed for different modules even under the same packing fraction [81]. As a result, it is impossible to know how exactly the irregularity influences on the flow and transport phenomena in a module.

Another study is conducted by using the fractal model approach to analyse the non-uniformity problem [81], where a “imaginary free-surface cells” technique is employed to set up the model. The empty volume on the shell side is divided by the number of fibers, and each volume is called a cell. Each cell has two concentric cylinders: one is the outer diameter of the fiber, the other is a hypothetic free surface, and all flows are concentrated inside the imaginary cells. The concept is shown in Fig. 23.

Fig. 23. The concept of imaginary free-surface cells model: (a) a bundle of equivalent hypothetical cells in the shell, and (b) a hypothetical cell in the shell [81]

The cell geometric properties are defined as:

The local packing density: $\phi = \frac{d_0^2}{d_e^2}$ (132)

The local porosity: $\varepsilon = 1 - \phi$ (133)

The equivalent void diameter: $\lambda = \varepsilon^{1/2} d_e \left(\frac{\varepsilon}{1-\varepsilon} \right)^{1/2} d_0$ (134)

The hydrodynamic dimeter: $d_h = \frac{4A_c}{Pe} = \frac{\varepsilon}{1-\varepsilon} d_0 \frac{\lambda^2}{d_0}$ (135)

The cumulative size distribution of voids follows the power law relation [88, 89]:

$$N(\lambda) = \left(\frac{\lambda_{max}}{\lambda} \right)^{D_f} \quad (136)$$

Where λ is the void diameter (m), $N(\lambda)$ is the total number of voids with diameter greater than λ , λ_{max} is the maximum void diameter (m), D_f is the fractal dimension of self-similar fractal structures.

Accordingly, the following relationship is obtained:

$$-\frac{dN}{N_f} = D_f \lambda_{min}^{D_f} \lambda^{-(D_f+1)} d\lambda = f(\lambda) d\lambda \quad (137)$$

Where $f(\lambda)$ is the probability density function:

$$\int_{-\infty}^{+\infty} f(\lambda) d\lambda = \int_{\lambda_{min}}^{\lambda_{max}} f(\lambda) d\lambda = 1 - \left(\frac{\lambda_{min}}{\lambda_{max}} \right)^{D_f} \quad (138)$$

Based on the numerical data with fractal model, the correlation of Sherwood number is given by:

$$Sh = (14.06\phi^4 - 29.21\phi^3 + 22.59\phi^2 - 7.71\phi + 1.03)Re^{0.33}Sc^{0.33}\psi_k \quad (139)$$

$$\psi_k = 0.882D_f - 0.535 \quad (140)$$

Where ψ_k is the correction factor for mass transfer, which takes into account of the irregularity. The area dimension D_f is determined based on the box-counting method introduced in literature [83]. The established correlations for Sherwood number are summarized in Table 4.

Table 4
Correlations for predicting shell side mass transfer coefficient in a randomly packed hollow fiber membrane module [90]

Correlations	Packing density (ϕ , %)	Reynolds number range	Reference
$Sh = 1.25 \left(\frac{ReD_h}{L} \right)^{0.93} Sc^{0.33}$	2.5, 26	0-500	Yang and Cussler [65]
$Sh = 5.85(1 - \phi) \left(\frac{d_h}{L} \right) Re^{0.6} Sc^{0.33}$	4, 8.7, 19.7, 40	0-500	Prasad and Sirkar [91]
$Sh = (0.53 - 0.58\phi) Re^{0.53} Sc^{0.33}$	31.9-75.8	25-300	Castello et al. [92]
$Sh = 0.41 \left(\frac{1}{N_f} \right) \left(\frac{D_s}{d_o} \right)^2 \left(\frac{d_h}{L} \right)^{0.25} Re^{0.33} Sc^{0.33}$	5.06-15.7	10-300	Nii and Takeuchi [93]
$Sh = (Sh_1^3 + Sh_2^3 + Sh_3^3)^{1/3}$ $Sh_1 = 3.66 + 1.2(\sqrt{\phi})^{-0.8}$ $Sh_2 = 1.615(1 + 0.14(\sqrt{\phi})^{-0.5})^3 \left(\frac{ReScd_h}{L} \right)^{1/2}$ $Sh_3 = \left(\frac{2}{1+22Sc} \right)^{1/6} \left(\frac{ReScd_h}{L} \right)^{1/2}$	10-75	Laminar	Lipnizki et al. [94]
$Sh = 1.62 \left(ReSc \frac{d_h}{L} \right)^{1/3}$	Tube flow	Laminar, $Gz \geq 25$	Leveque equation [95, 96]
$Sh = (0.31\phi^2 - 0.34\phi + 0.1) Re^{0.9} Sc^{0.33}$	29.78, 49.78, 69.78, 91.6	32-1287	Wu and Chen [83]
$Sh = (0.163 + 0.27\phi) \left(\frac{u}{D_h L} \right)^{0.6}$	20, 30, 40, 50	178-1194	Zheng et al. [87]
$Sh = \left(14.06\phi^4 - 29.21\phi^3 + 22.59\phi^2 - 7.71\phi + 1.03 \right) Re^{0.33} Sc^{0.33} \psi_k$ $\psi_k = 0.882D_f - 0.535$	10-80	Laminar	Zhang [81]

It is found that Sh_m increases with D_f , the correlations in this study reflect the influence of the irregularity degree on the module performance, while the correlations by other studies cannot.

4.3. Periodic unit cell method

A research is conducted into the fluid flow and heat transfer in an elliptical hollow fiber membrane module with random distribution [97]. It is found that the elliptical hollow fiber membrane module has better heat and mass transfer performance compared to the conventional one [98-100]. A normally distributed random error model is used to describe the random distribution of fibers. The Box-Muller transform [101, 102] is applied, which is a pseudo-random number sampling method for generating pairs of

independent, standard, normally distributed random numbers. Two independent random variables U_1 and U_2 are supposed, which are uniformly distributed in the interval $(0, 1]$.

$$Z_1 = \sqrt{-2\ln U_1} \cos(2\pi U_2) \text{ and } Z_2 = \sqrt{-2\ln U_1} \sin(2\pi U_2) \quad (141)$$

Where Z_1 and Z_2 are the independent random variables with standard normal distributions, E and S are the mean value and square deviation respectively, which can be manually adjusted. Then two groups of random numbers are calculated by:

$$Z_3 = E + Z_1 \cdot S \quad (142)$$

$$Z_4 = E + Z_2 \cdot S \quad (143)$$

The geometric centre coordinates of the elliptical fibers are given by Z_3 and Z_4 . Three unit cells are selected as the calculation domains, which are shown in Fig. 24.

Fig. 24. The calculated domains: (a) random 1; (b) random 2; (c) random 3 [97]

20 fibers are proper in each unit cell to represent the whole fiber bank. Then the longitudinal length L_{unit} and transverse length H_{unit} of the unit cell are calculated by:

$$L_{unit} = H_{unit} = \sqrt{\eta_{fiber} \pi ab / \varphi} \quad (144)$$

Where a and b are the elliptical semi-axis in y and x axes respectively, η_{fiber} is the number of fibers, and φ is the packing fraction.

The random distributions of three unit cells can be determined, the square deviations of three unit cells are 4.85, 8.94 and 16.49 respectively, while the mean values of them are all set to be 7.20. A renormalization group $k - \varepsilon$ turbulence model is used to develop governing equations, and the finite control volume method is adopted to solve governing equations. It is found that the air has lower flow resistances when the fiber

distribution becomes more concentrated, and Nusselt number becomes smaller, which means the fiber distribution has great influence on the fluid flow and heat transfer. Furthermore, the flow and heat transfer comprehensive capabilities of the regularly packed module are greater than those of the randomly packed module, meaning heat transfer performance is deteriorated for the fluid flow across randomly packed hollow fiber module. Therefore, the module performance can be enhanced by decreasing the elliptical semi-axis ratio (b/a).

5. Modelling of flow maldistribution

A uniform flow distribution is assumed in the previous studies, which means a representative single channel is targeted, and the real flow distribution in the whole exchanger is neglected. However, in reality the whole exchanger consists of inlet, outlets, exchanger shells, separating plates and core, as shown in Fig. 25 [90]:

Fig. 25. Schematic of a practical total heat exchanger [90].

Due to the complex ducting work in a practical exchanger shell, the flows would undergo turnarounds, expansions and contractions, which would lead to flow maldistribution across the core face [90]. Previous researches [103-105] on flow maldistribution normally assume one-dimensional flow distribution for both U-type and Z-type arrangements. Furthermore, the velocity distribution is determined without coupling with the core effect when solving fluid fields. More realistic and accurate analyses are required.

5.1. Plate-type modules

There are two main types of plate module regarding core structure: parallel plate and plate-fin types. The schematics of these two types are shown in Fig. 26:

Fig. 26. Schematics of plate-type module core: (a) parallel plate type [106]; (b) triangular plate-fin type [107, 108]

Several researches into these two types are carried out [106-108]. In these studies, the flow maldistribution is calculated by selecting the whole exchanger as the calculating domain. In the meanwhile, only the fresh air duct is selected as the calculating domain since the fresh air duct and exhaust air duct are in symmetry, which is given in Fig. 27.

Fig. 27. The calculating domain for flow distribution of the total heat exchanger fresh air duct [106-108]

In reality, a core has hundreds of channels, which makes the direct modelling of flow on the channel-to-channel basis difficult. To solve this problem, the core is treated as a porous media, which only permits one dimensional air flow along the channel length [106-108]. This method is reasonable due to the small channel pitch (1.5-5 mm) in the core. The core itself is similar to a porous media. The flow fields are predicted first

without considering heat transfer. The porous media are simulated by the addition of a momentum source term to the standard fluid flow equations:

$$S_i = \sum_{j=1}^3 D_{ij} \mu u_j + \sum_{j=1}^3 C_{ij} \frac{1}{2} \rho |u_j| u_j \quad (145)$$

Where S_i is the source term for the i th (x , y , or z) momentum equation. The first part is the viscous term (Darcy), the second part is the inertial loss term. For the case of simple homogeneous porous media:

$$S_i = \frac{\mu}{\alpha} u_i + C_2 \frac{1}{2} \rho |u_i| u_i \quad (146)$$

Where α is the permeability and C_2 is the inertial resistance factor.

For laminar flow, the pressure drop is typically proportional to velocity, and C_2 in Eq. (146) can be considered as zero:

$$\Delta P = \frac{\mu}{\alpha} u_i \Delta x \quad (147)$$

Where u_i is the face velocity in the core (in x direction) (m/s).

Then based on the definition of duct friction factor, the pressure drop and Re number are:

$$\Delta P = \frac{\Delta x}{D_h} f \frac{1}{2} \rho u_d^2 \quad (148)$$

$$Re = \frac{\rho u_d D_h}{\mu} \quad (149)$$

where u_d is the velocity in duct (m/s), which is twice the face velocity.

For fully developed laminar flow in duct:

$$f \cdot Re = C_3 \quad (150)$$

Where C_3 is the constant for a given cross-section, which can be found in literatures [40, 42].

The core equivalent permeability α is obtained from Eqs. (147), (148) and (150) when its structure is known, then the fluid flow is calculated, and two dimensional velocity distributions on the core surface are predicted. Afterwards, the heat transfer in the core is obtained. It is found that for plate type module, the channel pitch determines how serious the flow maldistribution is. The larger the channel pitch, the more serious the

flow maldistribution. Thermal deterioration factors caused by maldistribution could be as high as 10-20%.

5.2. Hollow fiber modules

In Section 4, the maldistribution caused by fibers' nonuniform distribution is discussed. In this section, the maldistribution caused by hollow fiber module structure will be addressed.

The flow maldistribution for cross and counter flow hollow fiber modules are investigated [109, 110], the structures of two types are shown in Fig. 28:

Fig. 28. Schematics of the hollow fiber module: (a) cross flow type [109]; (b) counter flow type [110].

The flows in the hollow fiber module are more complicated compared to the plate type module's, so three dimensional flow distributions are considered. The permeability α in Eq. (147) for the three directions is different due to the different resistances, for the transverse flow across the fiber (y and z directions):

$$\Delta P_i = N_i \chi_i \frac{\rho_a u_{max}^2}{2} f_i \quad (151)$$

$$u_{max,i} = \frac{\sigma_i}{\sigma_i - 1} u_{\infty,i} \quad (152)$$

Where N is the row number of fibers along the flow direction, χ is a correction factor [40], f is the friction factor across tube banks [46]. σ is the pitch to diameter ratio, and u_{∞} is face velocity (m/s).

For the flow along the fiber (x direction):

$$\Delta P_i = f_i \frac{x_F}{D_h} \frac{\rho_a u_i^2}{2} \quad (153)$$

1129 The friction factor f_i can be obtained from the correlations in literatures [27, 111]:

1130 $f_i \cdot Re_i = 41.3$ for cross flow configuration (154)

1131 $f_i = 0.079Re_i^{-0.25}$ for counter flow configuration (155)

1132 Where Re_i is obtained by:

1133 $Re_i = \frac{D_h u_i \rho_a}{\mu_a}$ (156)

1134 Where hydraulic diameter D_h is calculated by:

1135 $D_h = 4 \frac{A}{L_{wetted}}$ (157)

1136 Where A is the cross section area of hollow fiber membrane core (m^2):

1137 $A = y_F z_F - \frac{n_{fr} \pi d_0^2}{4}$ for cross flow configuration (158)

1138 $A = \frac{\pi d_{core}^2}{4} - \frac{n_{fr} \pi d_0^2}{4}$ for counter flow configuration (159)

1139 Where n_{fr} is the total number of fibers in the membrane core.

1140 L_{wetted} is the wetted perimeter (m), which can be calculated by:

1141 $L_{wetted} = 2(y_F + z_F) + n_{fr} \pi d_0$ for cross flow configuration (160)

1142 $L_{wetted} = \pi d_{core} + n_{fr} \pi d_0$ for counter flow configuration (161)

1143 Thus the permeability for each direction is obtained by Eqs. (147), (151) and (153)
1144 when the structure of the module is known. The effect on the flow distribution is
1145 determined, and the heat transfer rate in the core is calculated. It is found that flow
1146 maldistribution is mostly induced by inlet/outlet manifolds, the packing fraction
1147 influences flow distribution significantly. It is recommended to use module with higher
1148 packing density and straight inlet and outlet headers for less flow mal-distribution.

1149

1150 **6. Conclusion**

1151 This paper has presented a comprehensive review of different modelling methods for
1152 membrane-based liquid desiccant heat and mass transfer exchanger. The following
1153 conclusions can be drawn:

- 1) There are three predominant modelling methods for the membrane-based liquid desiccant heat and mass exchanger: effectiveness NTU ($e - NTU$), finite difference and conjugate heat and mass transfer methods.
- 2) The $e - NTU$ method is the most effective, time-saving but less accurate accordingly. The finite difference method provides better accuracy in modelling compared with $\varepsilon - NTU$ method, its heat and mass transfer coefficients are derived from correlations of fundamental data such as Nusselt and Sherwood numbers, which are borrowed from well-known books. The conjugate heat and mass transfer method has the best accuracy in modelling, its boundary conditions on the membrane surface are naturally formed, and the fundamental heat and mass transfer data are obtained directly from the conjugate heat and mass transfer governing equations.
- 3) In terms of modelling of hollow fiber membrane module, free surface model overcomes the difficulty of direct modelling of numerous fibers, while periodic unit cell model provides a more accurate modelling by considering the neighbouring fiber interactions.
- 4) Low-Re $k - \varepsilon$ turbulent model, which assumes the air flow to be turbulent, demonstrates a better accuracy in heat and mass transfer than the laminar flow model for hollow fiber membrane module.
- 5) Voronoi Tessellation method offers a possibility to analyse heat and mass transfer in a randomly packed module quantitatively. Fractal method reflects the effects of the irregularity degree on performance better compared to Voronoi Tessellation method.
- 6) Flow maldistribution can be calculated and analysed by selecting the whole exchanger as the calculating domain. For the plate type module, the larger the channel pitch, the more serious the flow maldistribution. For the hollow fiber type module, high packing density and straight inlet and outlet headers are recommended for less flow mal-distribution.

7. Future research on modelling method

Based on current reviews of the modelling methods for the membrane-based heat and mass transfer exchanger, the following suggestions are proposed for future research:

- 1) The modelling of application-scale membrane module for practical utilization.

- 1187 2) The modelling of hollow fiber module based on the transient state.
- 1188 3) The run-around system modelling for hollow fiber modules.
- 1189 4) The effects of fouling and crystallization of desiccant solution on the
- 1190 performance of membrane-based heat and mass transfer exchanger.

1191

1192 **References**

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