Systemic Risk and the Optimal Seniority Structure of Banking Liabilities*

Spiros Bougheas† Alan Kirman‡
University of Nottingham EHESS

August 2017

Abstract

The paper argues that systemic risk must be taken into account when designing optimal bankruptcy procedures in general, and priority rules in particular. Allowing for endogenous formation of links in the interbank market we show that the optimal policy depends on the distribution of shocks and the severity of fire sales.

Keywords: Banks; Priority rules; Systemic Risk

JEL: G21, G28

*Acknowledgement:** We would like to thank participants at the Financial Risk and Network Theory Conference, University of Cambridge, 2015 for helpful comments and suggestions. We would like to acknowledge financial support from COST Action IS1104 "The EU in the new economic complex geography: models, tools and policy analysis".

†(Corresponding Author) School of Economics, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom; e-mail: spiros.bougheas@nottingham.ac.uk

‡École des Hautes Études en Sciences Sociales, CAMS - EHESS 190-198, avenue de France, 75244 Paris Cedex 13, France,Tel:+33 612241766; e-mail: alan.kirman@ehess.fr

1 Introduction

An important issue that the design of any bankruptcy procedure must resolve is the allocation of priority rights among the various claimants of the corresponding entity's assets. By definition the value of the assets of a bankrupt entity is below the value of its liabilities and, therefore, a rule is needed for allocating those assets to the various claimants. Often the allocation of priority gives rise to a complex hierarchy among the creditors having at the top holders of secured debt and right down at the bottom the providers of equity. There is an extensive literature in financial economics[1] that studies the optimal design of bankruptcy procedures. The main aim of the design is to make investment attractive to creditors by shifting sufficient risk on those agents who have control over decision-making. In a setting with multiple stages of financing by multiple creditors the design of priority rights takes the form of a hierarchical structure.

A policy area that has attracted a lot of attention, especially in the aftermath of the 2008 crisis, is the design of priority rules for banks. What is striking is the variety of rules applied around the globe (Lenihan, 2012; Wood, 2011). Many countries in response to the crisis have either only recently introduced or are in the process of introducing depositor preference rules. These include Greece, Portugal, Hungary, Latvia and Romania that have to implement such rules as part of the conditions that they need to meet in order to participate in EU/IMF programmes. In the UK the Vickers report recommends the introduction of a depositor preference rule (ICB Report 2011).The arguments supporting the rules in place are mainly about the incentives that these rules provide to depositors and other creditors to monitor the activities of banks. The lower a creditor is on the priority list, the less likely is that she will receive (at least full) compensation in the case of bankruptcy and therefore the stronger are her incentives to ensure that the borrower does not take excessive risks. While nobody disagrees with the validity of the last statement there is substantial variation in opinions about which is the most suitable party to perform the monitoring service.

In this paper, we argue that systemic risk is another aspect of banking that must be taken into account when designing bankruptcy procedures. In general, the design of optimal priority rules is focused on the welfare of a bank's creditors, namely, its depositors, other debt holders and equity owners. However, when systemic risk is a concern, the design must also take into consideration the welfare of third parties and, in particular, all those who provided funds to the rest of the banking system. The degree to which a bank's credit providers will be affected when the bank becomes insolvent it depends on their position on the priority ladder. When among those creditors that are not getting paid in full are other banks, as long as these banks stay solvent the losses will be absorbed by equity holders. However, when these banks become insolvent the losses will be absorbed by their creditors and this process will continue till either the system clears or all banks become bankrupt. The total systemic losses will depend on the value of assets that can be recovered when banks become insolvent and

[1] See Berglöf *et al.* (2010) for a review of the relevant literature.

thus go into liquidation. The sale of assets in depressed markets, 'fire sales' as they are known in the literature, further deteriorates balance sheets and thus enhances the fragility of the financial system.[2] It is straightforward to show that if the value of assets is fully recovered under liquidation, that is the market value is equal to the book value, then the seniority structure does not matter. This is because the total losses are limited to the value of initial losses.[3] With this in mind, fire sales are going to be important in our work.

Recent work on systemic risk, models contagion in banking following a network approach.[4] One important finding of this literature is that the structure of the network matters. Thus far, this structure is exogenously given. This can be fine as long as the seniority structure of liabilities is fixed.[5] However, our aim is to compare the welfare implications of two alternative priority structures, namely, depositor seniority and bank seniority, and to do so we need to take into consideration the impact of seniority structure on the formation of the interbank network. Again, it is straightforward to show that if the network structure of the intertbank market could not be affected by the choice of the seniority structure then the optimal option would be to allocate seniority to banks. The reason is that as more of the losses are absorbed by depositors the lower will be the losses absorbed by the interbank market and thus the lower the risk of further insolvencies. [6] However, this is no longer true when the network structure is endogenous. The problem is that allowing for endogenous network formation complicates significantly the analysis and therefore we will ignore the incentives for monitoring that each seniority structure offers. However, this is an important issue and any related policy debate needs to include it. We are going to offer our thoughts on this issue in the concluding section of the paper.

We will analyze a model of the banking system where banks finance their investments by two types of borrowing, namely, retail deposits and loans from other banks. Given that our main goal is to understand how alternative priority policies might affect the network structure we focus on the optimal lending policy of a bank that has some excess liquidity. The main disadvantage of adopting a partial equilibrium approach is that by definition the network structure that we consider does not satisfy conventional equilibrium solutions. However, existing analytical models of systemic risk have predominantly focused on networks with symmetric structures.[7] Although, it is simple to demonstrate that symmetric

[2] In traditional models of bank panics the fear of liquidation costs are what drives depositor runs (see Diamond and Dybvid, 1983; Allen and Gale, 2000). See Shleifer and Vishny (2010) for a review of the related literature.

[3] The only thing affected by the seniority structure is the distribution of losses between depositors and bank owners. However, the seniority structure should minimize total losses leaving their distribution to other policy instruments.

[4] For reviews of the literature see Allen and Babus (2009) and Bougheas and Kirman (2015).

[5] In the literature so far the analysis is carried out under the supposition that in the case of bankruptcy depositors have priority. See, for example, Acemoglu *et al.* (2015).

[6] We are intentionally ignoring concerns about the impact of the priority structure on the likelihood of liquidity runs that can lead to insolvencies and hence systemic risk. The reason is that there are other instruments, such as deposit insurance, that are more appropriate to deal with such concerns (see, Diamond and Dybvig, 1983).

[7] See for example, Acemoglu *et al.* (2015) and Allen and Gale (2000).

structures can be stable, symmetry is not a characteristic feature of actual financial networks.[8] The main advantage of our approach is that it allows us to compare all the options that a bank could have when considering forming a new link. We show that as long as the network structure is fixed our results are consistent with those in the existing literature. However, we will also be able to go a step further by examining what happens when the policy regime can affect the formation of links.

The bank in our model has three lending options: the first one is to offer the loan to a bank that has zero net obligations to the rest of the banking system, the second option is to offer the loan to a bank that is a net borrower, and the third option is to offer the loan to a net lender. We derive the lender bank's profit maximizing choice under both priority rules and then we derive the corresponding social optimum choice. Given that priority rules only matter when a bank fails, our results will be sensitive to (a) the size of the shock and (b) the probability distribution of shocks across the banking system. We carry out the analysis under two alternative scenaria regarding the distribution of shocks. In one case, we assume that each bank is hit by a shock with the same probability but bigger banks are hit by proportionally bigger shocks, while in the other case we assume that the probability that a bank is hit by a shock is proportional to its size but the size of the shocks are the same for each bank. The former scenario corresponds to the case where a bank's various asset returns are strongly correlated (for example, when the portfolio is dominated by regional mortgages) in which case a shock would affect the balance sheet uniformly. The latter scenario corresponds to the case where bank portfolios are well diversified in which case a shock would only affect a fraction of the asset side of a bank's balance sheet.[9] Gabaix (2011) has extensively studied and compared such shocks and analyze their implications for the distribution of firm size. In this paper, we examine how such shocks can affect a small network of banks.

There are two main results. In general, under bank seniority the profit maximizing network structure is also the one that maximizes social welfare. The intuition of this result is straightfoward. Having the depositors absorb as much of the losses as possible allows banks to fulfill their obligations against each other thus minimizing systemic risk. However, there is a very important exception. When the joint likelihood of (a) extreme high initial losses, (b) catastrophic fire sales, (c) low profitability, and (d) lack of asset diversification, is very high

[8] See the review article by Bougheas and Kirman (2015) for examples.

[9] Correlation here refers to the asset returns of a bank's portfolio. Our intention is to study how the priority structure of banking libilities affects the transmission of shocks from one bank to another and thus we are ignoring correlations of asset returns across the banking system where multiple banks fail simultaneously. As long as the correlation of returns across the banking system is not perfect our main argument is still valid although the results become weaker. Of course, as Acharya (2009) has demonstrated, any design of new policies needs to consider the incentives that these policies offer to banks to have their portfolios correlated and thus enhancing systemic risk. The argument is that if banks expect that it is more likely that they will be bailed out during systemic events then they have an incentive to increase correlation.

the profit maximizing network structure under bank seniority is not optimal. Although the likelihood that all four conditions are met is low when they are all met systemic losses can be catastrophic. These four conditions provide a fair characterization of the state of the US banking system before the 2008 financial crisis. Taking the above results together we find that the optimal policy would, *ceteris paribus*, depend on the likelihood of such events. In the next section of the paper we review some other work in the literature that has identified other factors that must be considered when designing optimal bankruptcy policies.

We organize our work as follows. After a brief review of the related theoretical literature in Section 2 we describe our model and in Section 3 we present our results. In the final section we conclude. All derivations are provided in the Appendix.

Related Literature Responding to the 1980s Savings and Loans crisis, the US Congress enacted the 1991 Federal Deposit Insurance Corporation Improvement Act followed by the 1993 Depositor Preference Act. The introduction of these Acts motivated a long debate among financial economists, legal scholars and policymakers that is reviewed in Bougheas and Kirman (2016). Here we restrict our attention to some theoretical developments that are more closely related to this present work.

Some experts argue that non-depositor priority rights provide strong incentives to depositors to discipline the banks. Actually, Calomiris and Kahn (1991) have suggested that by its very nature demandable debt, which allows depositors to withdraw their funds at will, offers the required market discipline device. However, this argument is weakened when we allow for the possibility that uninformed depositors of nevertheless safe banks might also panic thus generating a wide-spread panic throughout the banking system.

Others believe that banks and other creditors are more suitable monitors. For example, Rochet and Tirole (1996) argue that interbank exposures generated through transactions in the interbank market provide strong incentives for banks to monitor other banks and therefore interbank loans should be junior to deposits. Kaufman (2014) has challenged this claim by arguing that how effective the banks are as monitors depends on their beliefs about the likelihood that the government would intervene in times of crises in which case banks would consider transactions in the interbank market as bearing low risk. Birchler (2000) also supports depositor preference on the grounds that banks have an informational advantage relative to a large number of small depositors. Moreover, he argues that raising funds by offering to depositors a standardized product with priority rights is a more efficient than having each depositor sign a bilateral contract with a bank. Thus, the introduction of a priority list reduces the amount of resources devoted to socially inefficient information gathering and such an arrangement seems to be ideal for banks that raise funds from a large number of uninformed investors.

In contrast to the above mentioned studies, Freixas *et al.* (2004) offer a mixed view. In their model banks provide two services, namely, screening and monitoring. By screening potential applicants they improve the pool of loans that they offer while by monitoring firms that have been offered loans banks ensure that these firms perform well. The optimal seniority structure depends on which of the two moral hazard problems associated with the two services is more pressing.

Most of the work on the seniority status of bank loans has focused on the interbank market where such loans are not secured. However, on the liability side of their balance sheet banks have other claims by financial institutions that are secured and therefore have top priority. Bolton and Oehmke (2015) analyze the seniority status of some types of derivatives and conclude that while these claims provide risk sharing opportunities, their position on the top of the priority ladder can lead to inefficiencies as it transfers risk to other bank creditors such as depositors.[10]

2 The Banking Network

In order to assess the welfare implications of each of the two alternative priority rights allocation policies we need first to understand how each of these options would affect the structure of the interbank network which in turn would depend on the profit maximizing decisions of each bank in the network. The dynamic formation of the interbank market network is a difficult problem that has recently attracted some attention (Babus, 2015; Cohen-Cole *et al.*, 2010). This work takes the priority ladder of banking liabilities as exogenously given. Our work is further complicated by the dependence of the network structure on the priority ladder that requires making very complex welfare comparisons. With that in mind we will examine a very simple banking network that will allow us to make such comparisons and then we will consider its relevance for more realistic environments.

There are two types of risk-neutral agents: bank owners and depositors. There are four banks which, for mnemonic reasons, we denote as N, L, B, O. We keep bank balance sheets very simple. On the asset side bank i has customer loans, L_i, and may have loans offered to other banks. Let $l_i^j (j \neq i)$ denote loans from bank i to bank j. On the liability side bank i has customer deposits, D_i, may have deposits from other banks and equity E_i. Let $d_i^j (j \neq i)$ denote deposits in bank i from bank j. Balance sheets must satisfy the constraints

$$L_i + \sum_{j \neq i} l_i^j = D_i + \sum_{j \neq i} d_i^j + E_i, \quad \forall i$$

and the interbank market must satisfy the constraints

$$l_i^j = d_j^i, \quad \forall i \text{ and } \forall j.$$

[10] There are some proposals in favor of subordinated debt but Blum (2000) casts some doudt on their efficacy.

The net interest rate on consumer loans is equal to $z < 1$, the interest rate on deposits is equal to 0 and interbank interest rate is also equal to 0.[11]

The initial balance sheets of the four banks are as follows:

$L_N = 1$	$D_N = 1$
$Assets = 1$	$Liabilities = 1$
$L_L = 1$	$D_L = 2$
$l_L^B = 1$	
$Assets = 2$	$Liabilities = 2$
$L_B = 2$	$D_B = 1$
	$d_B^L = 1$
$Assets = 2$	$Liabilities = 2$
$L_O = 1$	$D_O = 1$
$Assets = 1$	$Liabilities = 1$

Each bank has funded one unit of loans with its own deposits. In addition, bank L had an extra unit of deposits that it loaned to bank B that used it to finance an extra unit of loans. Thus, the balance sheets of banks N, L and B are constructed so that they capture the three possible cases of net interbank exposures. Bank N's (neutral) net exposure is zero, bank L is a net lender and bank B is a net borrower.[12]

Suppose that bank O (decision-maker) obtains an extra unit of deposits that is willing to loan to another bank. All other three banks can fund an extra unit of consumer loans. In what follows we provide answers to the following four questions:

- Assuming that depositors have priority, to which other bank will bank O offer the loan to maximize its profits?
- Assuming that depositors have priority, which bank should receive the loan so that social welfare is maximized?
- Assuming that banks have priority, to which other bank will bank O offer the loan to maximize its profits?
- Assuming that banks have priority, which bank should receive the loan so that social welfare is maximized?

The answers to the four questions only matter when there is a banking crisis and in particular when a bank other than O goes into liquidation. Thus, we assume that one of the other three banks has to write off some of its assets. The answers will also depend on many other modeling choices such as the likelihood

[11] Given the interest rate on consumer loans, the interbank rate in equilibrium could take any value in the interval $[0, z]$. It will become clear that the exact value is not important for our conclusions and, therefore, to keep things simple we set it equal to 0.

[12] It will will become clear below that the addition of bank N is not important (it is also the least realistic case).

and size of shocks and the expectations of bank O about future changes in each bank's balance sheets including changes in the interbank network. We will begin by analyzing a benchmark case that provides simple and intuitive answers. Later we will discuss how our results might be affected if we move to more complex variations of our model. Our benchmark model satisfies the following restrictions:

Assumption 1 *(Myopic Expectations) After bank O offers the loan to one of the other three banks it does not expect any further changes in any of the balance sheets.*[13]

Assumption 2 *(Catastrophic Fire Sales) When any bank, other than the bank initially hit by a shock, goes bankrupt (systemic losses) the value of customer loans on the its bank's balance sheet is completely wiped out.*

The first assumption is made for analytical convenience. It is hard to see how we can make any progress within our framework by considering any alternative. Given that decisions matter only in case of a systemic event any forward looking solutions would depend on the likelihood of such events after each possible choice that banks could make in the future. Having said that, a bank would always face one of the three choices that we are considering in this work. Our assumption would not be too restrictive as long as banks consider the indebtedness status (net borrower or net lender) of a potential borrower as more important than the size of net indebtedness.

The second assumption is only imposed to minimize the number of cases that we need to consider but it will become clear that our main conclusions do not depend on it.

Clearly, the answers to the four questions will also depend on the beliefs that bank O has about the size distribution of the initial shock. Suppose that bank i is the one inflicted by the initial shock. Then, let ψ_i denote bank i's liquidation value of customer loans. Thus, the size of the shock is given by $L_i - \psi_i$. We will compare the answers to the four questions under two alternative scenaria related to the distribution of shocks across the banking system.[14]

Proportional Shocks The probability that any of the banks N or L or B becomes insolvent is equal to $\frac{1}{3}$. Shocks are proportional to the value of the inflicted bank's customer loans ($\frac{\psi_i}{L_i} = \psi \; \forall i$).

Identical Shocks $L_i - \psi_i = L_j - \psi_j \leqslant 1$, for every bank i or j; The probability that a bank becomes insolvent is proportional to the value of its customer loans.

The above two scenaria regarding the distribution of shocks across the banking system capture two polar cases. With proportional shocks the underlying

[13] The alternative assumption would be to have farsighted banks. The literature on farsighted networks is very young. There is some progress made in proving existence results but not much on characterizing the solutions (see, for example, Dutta *et al.*, 2005).

[14] For the moment we do not impose any restrictions on the joint distribution of z and ψ_i.

assumption is that scale does not lead to diversification. Put differently, as a bank grows (in terms of customer loans) it replicates its existing portfolio (extreme specialization). Thus, as a bank's customer loans grow in size so does the bank's exposure to the risks associated with the particular sector financed by the bank. Under the assumption that each sector is equally likely to be inflicted by a negative shock and thus the corresponding firms become unable to meet their obligations with their bank, shocks are proportional. In contrast, the implication of the assumption of identical shocks is extreme diversification. Thus, as a bank doubles in size (again, in terms of customer loans) it also doubles the number of sectors it finances. Under the same supposition as above, that is each sector is equally likely to be inflicted by a negative shock, now all shocks have the same size (assuming each sector has similar financial needs) but as a bank doubles the number of sectors that it finances it also doubles the probability that it will be impacted by a shock.

3 Results

In this section, we describe and offer some intuitive explanations of our main results. The detailed derivations can be found in the Appendix. The solution of the model proceeds in two steps. Firstly, we derive for each priority case and for each option that bank O has for offering the loan, bank O's profits and social welfare. The latter is defined as total bank profits plus total available deposits after any bank resolution.[15] This exercise is completed for all admissible values of net interest on loans, z, and liquidation values, ψ_i. These derivations are very straightforward but tedious and appear in the Appendix under the subsection heading 'Preliminary Derivations'.

Secondly, we use the calculations from the first step to derive bank O's profit maximizing choice and also the loan offer that maximizes welfare. The analysis is completed by choosing the priority rights policy that would maximize welfare conditional on bank O's profit maximizing choice. This step is repeated for each of the two restrictions on the distribution of shocks across the banking system.

The following two Propositions summarize the results.

Proposition 1 *(Proportional Shocks):*

(a) Under depositor seniority, for any values of z and ψ, it is never optimal for bank O to offer the loan to bank L.

(b) Under bank seniority, for any values of z and ψ, offering the loan to bank L weakly dominates the alternative two options.

[15] To simplify our analysis we have assumed linear utility. The absence of any curvature matters only for one important case where we carefully discuss the various trade-offs. We have also assumed equal weighting between equityholders and depositors. Our model does not distinguish between bank managers and equityholders and as we have argued above there is no *a priori* for the social planner to favor depositors over equityholders. If one of the aims is to protect depositors in order to guarantee adequate liquidity then this can be achieved by alternative policy instruments such as deposit insurance.

(c) Offering the loan to bank L maximizes welfare for any values of z and ψ except the worst case scenario of very high initial losses and very low profitability.

It is clear that, keeping the structure of the network fixed, bank O's expected profits are higher under bank priority for any of the three loan offer options as more of the losses are absorbed by depositors.

Parts (a) and (b) are important as they demonstrate that the expected profit maximizing choice under bank priority is not the same as under depositor priority. The result can be best understood by considering bank O's expected profits when it offers the loan to bank L which has already offered a loan to bank B. Under depositor priority bank O is potentially exposed to failures of either bank L or bank B. In contrast, under bank priority there is a buffer of deposits at bank L protecting bank O.

To understand part (c) of the Proposition we observe that as long as the network structure is fixed bank seniority maximizes welfare. (Social welfare is at least as high under bank seniority for all values of the shocks.) This is because having depositors absorb the losses prevents the spread of the crisis to other banks. Ignoring for the moment the worst case scenario (low z and low ψ), we also find that offering the loan to bank L is also the social welfare maximizing case. Form the point of view of social welfare we care about both depositors and equityholders. Given that bank L has a higher value of deposits, under bank seniority, offering the loan to this bank reduces the likelihood that the crisis spreads. Certainly, this would mean that depositors suffer most of the losses. But as aggregate losses are low this is only a distributional issue.

The above argument is not true for the worst case scenario (low z and low ψ). In that case offering the loan to bank L does not maximize welfare. When the shocks are large it is better for the network not to be too connected (Acemoglu *et al.*, 2015a; Cabrales *et al.* 2014). Low connectivity reduces contagion. However, from the point of view of bank O offering the loan to bank L is never dominated by the other two choices. The reason is that bank O does not take into account the effect of its choice on depositors. This creates a conflict between the equilibrium profit-maximizing choice and the one that maximizes social welfare. Of course, from an *ex ante* point of view everything depends on the relative likelihood of these extreme events (fat tails).

Thus, the profit maximizing choice in not necessarily the same as the social welfare maximizing choice when the structure of the network is affected by the allocation of priority rights. As we have already argued that is not the case when the network structure is fixed. Next, we consider the alternative structure for the distribution of shocks.

Proposition 2 *(Identical Shocks):*

(a) Under depositor seniority the optimal choice of bank O would be either to offer the loan to bank N or bank B depending on the distribution of shocks.

(b) Under bank seniority bank O will be indifferent across the three choices.

(c) Welfare is maximized by offering the loan either to bank N or bank L.

This structure for the distribution of shocks would arise when banks have well diversified portfolios. Banks with bigger balance sheets would be inflicted by shocks more often, however, the size of the shocks would not depend on the size of the balance sheets. With depositor seniority, the optimal choice of bank O, would depend on the exact specification of the distribution of shocks, and it would be either to offer the loan to bank N or to bank B, but never to bank L. This is because by offering the loan to bank L, bank O is exposed to any shocks inflicting either bank L or bank B (indirectly). In contrast, under bank seniority given that the size of the shocks are relatively small, and thus the losses are absorbed by depositors, expected profits are identical under all three choices.

Social welfare is maximized by avoiding offering the loan to bank B. This is because in the latter case there is a high concentration of loans in bank B and thus when this bank fails the potential for losses is higher. Thus, when shocks are relatively small bank seniority induces networks that maximize welfare.

Considering the two Propositions together we draw the following conclusions.

Corollary 1 *(a) The structure of the network is not invariant to the policy regime.*

(b) The profit maximizing choice in not necessarily the same as the social welfare maximizing choice when the structure of the network is affected by the policy regime.

We regard the two alternative restrictions that we have imposed on the distribution of shocks across the banking system as two polar cases of a much broader space of such distributions. Considering together the results of Propositions 2 and 3 we find that under bank seniority bank O would offer the loan to bank L. In general, this would also be the social welfare maximizing choice. However, this would not be the case if the joint likelihood of (a) extreme high initial losses, (b) catastrophic fire sales, (c) low profitability, and (d) lack of asset diversification, is very high. These four conditions provide a fair characterization of the status of the US banking system before the 2008 financial crisis. Our results suggest that while under most circumstances bank seniority would offer more protection against systemic risk, the higher connectivity that it encourages, might exacerbate the systemic consequences of extreme events.

It is important to keep in mind that our analysis has neglected other important factors that must be considered when designing bankruptcy rules. The issues are very complex to be considered in one unified model. However, by focusing on just a single aspect we hope that we have identified some of the trade-offs that need to be considered when considering such a complicated design problem.

4 Conclusion

Responding to the 2008 financial crisis policy-makers around the world have introduced, or are in the process of introducing, new bank resolution procedures, knows as 'bail-ins' that would require a failing bank's creditors to bear the costs of restoring it back to health. In USA the relevant legislation has been introduced through the Dodd-Frank Act while in the European Union such measures where enacted through the European Stability Mechanism and the Bank Recovery and Resolution Directive. The purpose of these new policy measures is to shift the burden of bank failures away from taxpayers (bail-outs) and onto unsecured creditors. When resolution takes place under a bail-in procedure, creditors instead of directly receiving the proceeds from the liquidation of the failing bank's assets as it would happen under normal bankruptcy procedures would now have their claims converted into equity. But still the exact value of their equity holdings will depend on their place in the priority ladder. Avgouleas and Goodhart (2015) offer a critical review of these policies and discuss how each of three groups of creditors, namely, depositors, other financial institutions and bondholders, is going to bear the burden. By focusing on issues related to systemic risk our work contributes in this discussion given that these policies were designed to eliminate the systemic consequences of failures of large financial institutions.

We have not delivered an unconditional optimal policy recommendation. This is not too surprising given that we already know that while some types of network structures are better at protecting the system during mild episodes the same structures can prove catastrophic during extreme events (Acemoglu *et al.*, 2015a, Cabrales *et al.* 2014).

In order to keep our analysis simple, we have completely ignored other reasons for supporting one or another priority rights policy. However, there is a substantial body of work that has examined the incentives that such policies offer to various types of creditors to monitor banks. What has been absent from this discussion so far are their potential systemic risk implications. Keeping the analysis tractable also meant that we had to impose a couple of strong restrictions on our model.

The analysis in the paper was carried out under the assumption that fire sales are catastrophic. We have also carried out the analysis for the case of fire sales that are not catastrophic and it turns out that the general message does not change. As we have pointed out in the Introduction, in the absence of fire sales, priority rights are irrelevant for the social welfare implications of systemic risk. For values of fire sales in the intermediate range we get many more cases to consider making the presentation of the results cumbersome. However, given that we are interested in comparing choices *ex ante* much of the intermediate variation vanishes out.

What is more worrisome is our assumption of myopic expectations. What it implies is that bankers do not expect any further changes in the network structure. Although, recently there has been some progress in the direction of endogenizing the formation of banking networks (Babus, 2015; Cohen-Cole *et al.* 2010), the complexity of the issues that we have attempted to address in this paper it does not allow us to use their methods. We chose our network structure as it captures the three alternative types of borrowers that a lending bank might meet (positive exposures, negative exposures, zero exposures). Any other lending bank would be facing similar options. Clearly, as the network structure gets more complicated a bank's decisions will not depend only on the net exposures of their potential lenders but also on the exposures further down the line.

In summary, our results have been derived from a very simple network structure under naive behavioral assumptions. Having said that we believe that the results are very intuitive and at the very least are a good starting point for addressing important policy issues.

References

[1] D. Acemoglu, A. Ozdaglar, A. Tahbaz-Salehi, 2015. Systemic risk and stability in financial networks. *American Economic Review* 105, 564-608

[2] V. Acharya, 2009. A theory of systemic risk and the design of prudential bank regulation. *Journal of Financial Stability* 5, 224-255.

[3] F. Allen, A. Babus, 2009. Networks in finance. In P. Kleindorfer, Y. Wind, R. Gunther (eds) The Network Challenge:Strategy, Profit, and Risk in an Interconnected World. Pearson Education, New Jersey, p. 367-382

[4] F. Allen, D. Gale, 2000. Financial contagion. *Journal of Political Economy* 108, 1-33

[5] E. Avgouleas, C. Goodhart, 2015. Critical reflections on bank bail-ins. *Journal of Financial Regulation* 1, 3-29.

[6] A. Babus, 2016. The formation of financial networks. *Rand Journal of Economics* 47, 239-272

[7] E. Berglöf, G. Roland, E.-L. von Thadden, 2010. Optimal debt design and the role of bankruptcy. *Review of Financial Studies* 23, 2648–2679

[8] U. Birchler, 2000. Bankruptcy priority for bank deposits: A contract theoretic explanation. *Review of Financial Studies* 13, 813-840

[9] J. Blum, 2002. Subordinated debt, market discipline, and banks' risk taking. *Journal of Banking and Finance* 26, 1427–1441

[10] P. Bolton, M. Oehmke, 2015. Should derivatives be privileged in bankruptcy? *Journal of Finance* (in press)

[11] S. Bougheas, A. Kirman, 2015. Complex financial networks and systemic risk: A review. In P. Commendatore, S. Kayam, I. Kubin (eds.), Complexity and Geographical Economics: Topics and Tools. Springer, Heidelberg

[12] S. Bougheas, A. Kirman, 2016. Bank insolvencies, priority claims and systemic risk. In P. Commendatore, M. Matilla-Garcia, L. Varela and J. Canovas (eds.), Complex Networks and Dynamics: Social Economic Interactions. Springer, Heidelberg

[13] A. Cabrales, P. Gottardi, F. Vega-Redondo, 2014. Risk-sharing and contagion in networks. CESIFO Working Paper 4715

[14] C. Calomiris, C. Kahn, 1991. The role of demandable debt in structuring optimal banking arrangements. *American Economic Review* 81, 497-513

[15] E. Cohen-Cole, E. Patacchini, Y, Zenou, 2010. Systemic risk and network formation in the interbank market. Carefin WP 25/2010, University of Bocconi

[16] D. Diamond, P. Dybvig, 1983. Bank runs, depoit insurance and liquidity. *Journal of Political Economy* 91, 401-419

[17] B. Dutta, S. Ghosal, D. Ray, 2005. Farsighted network formation, *Journal of Economic Theory* 122, 143-164

[18] X. Freixas, J.-C. Rochet, B. Parigi, 2004. The lender of last resort: A twenty-first century approach. *Journal of the European Economic Association* 2, 1085-1115

[19] X. Gabaix, 2011. The granular origins of aggregate fluctuations. *Econometrica* 79, 733-772

[20] G. Kaufman, 2014. Too big to fail in banking: What does it mean? *Journal of Financial Stability* 13, 214-223

[21] N. Lenihan, 2012, Claims of depositors, subordinated creditors, senior creditors and central banks in bank resolutions. Speech delivered at the Association Européenne pour le Droit Bancaire et Financière Conference, Athens, 5–6 October

[22] J.-C. Rochet, J. Tirole,1996. Interbank lending and systemic risk. *Journal of Money, Credit and Banking* 28, 733–62

[23] A. Shleifer, R. Vishny, 2010. Fire sales in finance and macroeconomics. NBER Working Paper No. 16642

[24] P. Wood, 2011. The bankruptcy ladder of priorities and the inequalities of life, *Hofstra Law Review* 40, (1), Article 9

5 Appendix

Throughout our analysis we assume that when a bank becomes insolvent its assets are distributed to its creditors according to priority rules and all parties at the same priority level share their allocated assets in proportion to their corresponding claims. Let Π_i denote bank i's profits and C_i final withdrawals (consumption) by the depositors of bank i. As a result of potential bank liquidations we have $C_i \leqslant D_i$.

We begin by deriving bank O's profits, Π_O, and social welfare (post-bankruptcy customer deposits plus total bank profits), W, for each of the two priority cases and for each of the three options of bank O, for all admissible values of z and ψ_i. Then we compare bank O's profit maximizing choice with the social welfare maximizing choice for each seniority structure and for each of the two restrictions on the distribution of shocks across the banking system.

5.1 Preliminary Derivations

5.1.1 Depositor Seniority

(a) Loan from Bank O to Bank N

The new balance sheets of banks N and O are given by:[16]

$L_N = 2$	$D_N = 1$
	$d_N^O = 1$
$Assets = 2$	$Liabilities = 2$

$L_O = 1$	$D_O = 2$
$l_O^N = 1$	
$Assets = 2$	$Liabilities = 2$

We consider separately the three cases of initial insolvencies:

Case 1 *Bank N goes bankrupt*

In this case the shock only affects bank N (direct hit) and its creditor bank O. Depending on the value of $\psi_N \in [0, 2]$ we need to consider two cases:

(a) $\psi_N \leqslant 1$. Depositor seniority implies that $C_N = \psi_N$. Thus, bank O will lose its deposits at bank N, go bankruptcy itself and, by Assumption 2, $C_O = 0$. Thus, we have

$$\Pi_O = 0 \text{ and } W = 3(1 + z) + \psi_N$$

given that $C_L = 2$, $\Pi_L = z$, $C_B = 1$ and $\Pi_B = 2z$.

(b) $\psi_N > 1$. In this case the depositors of bank N get all their deposits back, $C_N = 1$, and bank O recovers $\psi_N - 1$ of its loan to bank N. There are two cases to consider depending on whether or not bank O remains solvent:

(i) $z + \psi_N < 2$. In this case the bank gets liquidated as the value of its assets that is the sum of the value of its loans $1 + z$ plus the funds that managed to

[16] The balance sheets of the other two banks are not directly affected by this transaction.

recover from bank N are less than the value of its liabilities which is equal to 2. The depositors of the bank will get $C_O = \psi_N - 1$ and thus we have as in the previous case

$$\Pi_O = 0 \text{ and } W = 3(1+z) + \psi_N$$

but now the higher value of ψ_N is shared between the depositors of the two affected banks.

(ii) $z + \psi_N \geqslant 2$. Now bank O is solvent and thus

$$\Pi_O = z + \psi_N - 2 \text{ and } W = 4(1+z) + \psi_N$$

where for the derivation of welfare we notice that the depositors of all banks got their funds back and the sum of the profits of the two unaffected banks is equal to $3z$.

Case 2 *Bank L goes bankrupt*

The bankruptcy only affects bank L whose assets include a loan of 1 unit to bank B. Therefore, its depositors will receive $\psi_L \in [0,1]$ plus a deposit of 1 unit at bank B. Given that all banks other than B have not been affected, we have:

$$\Pi_O = z \text{ and } W = 5(1+z) + \psi_L$$

Case 3 *Bank B goes bankrupt*

The bankruptcy will also affect bank L that has offered a loan to bank B. Symmetry implies that the welfare results in this case are exactly the same as those derived from the case when bank N goes bankrupt. When bank N receives the new loan from bank O there is a symmetric network structure, namely, two banks offering a loan and two banks receiving a loan. Moreover, banks N and B are the two banks receiving the loans. The only difference is that in this case bank O is solvent. Thus, we have the following cases:

(a) $\psi_B \leqslant 1$.

$$\Pi_O = z \text{ and } W = 3(1+z) + \psi_B$$

(b) $\psi_B > 1$.

(i) $z + \psi_B < 2$.

$$\Pi_O = z \text{ and } W = 3(1+z) + \psi_B$$

(ii) $z + \psi_B \geqslant 2$.

$$\Pi_O = z \text{ and } W = 4(1+z) + \psi_B$$

(b) Loan from Bank O to Bank L

The new balance sheets of banks L and O are given by:

$L_L = 2$	$D_N = 2$
$l_L^B = 1$	$d_L^O = 1$
$Assets = 3$	$Liabilities = 3$
$L_O = 1$	$D_O = 2$
$l_O^L = 1$	
$Assets = 2$	$Liabilities = 2$

Case 1 *Bank N goes bankrupt*

Given that other banks are not affected, we have:

$$\Pi_O = z \text{ and } W = 5(1+z) + \psi_N$$

where $\psi_N \in [0,1]$.

Case 2 *Bank L goes bankrupt*

We need to consider two cases depending of the value of $\psi_L \in [0,2]$:

(a) $\psi_L < 1$. The depositors of bank L receive ψ_L plus a deposit of 1 unit at bank B. Bank O's loan to bank L will not be repaid and therefore bank O will go bankrupt and, by Assumption 2, $C_O = 0$. Thus,

$$\Pi_O = 0 \text{ and } W = 3(1+z) + \psi_L$$

(b) $\psi_L > 1$. In this case the depositors of bank L are fully compensated by receiving ψ_L plus $2 - \psi_L$ of deposits at bank B. Bank O recovers $\psi_L - 1$ of its loan to bank L and there are two cases to consider depending on whether or not bank O remains solvent. The analysis is exactly the same as in the case when bank O offers the loan to bank N and the latter becomes insolvent. The only difference is that we need to replace ψ_N with ψ_L. Thus,

(i) $z + \psi_L < 2$.

$$\Pi_O = 0 \text{ and } W = 3(1+z) + \psi_L$$

(ii) $z + \psi_N \geqslant 2$.

$$\Pi_O = z + \psi_N - 2 \text{ and } W = 4(1+z) + \psi_L$$

Case 3 *Bank B goes bankrupt*

The bankruptcy will also affect bank L that has offered bank B a loan and potentially bank O that has offered bank L a loan. Note that $\psi_B \in [0,2]$.

(a) $\psi_B < 1$. The payoff of the depositors of bank B is given by $C_B = \psi_B$ and bank L's loan is not repaid. There are two cases to consider depending on whether or not bank L remains solvent.

(i) $z < \frac{1}{2}$. Given that the assets of bank L are equal to $2(1+z)$ and the liabilities are equal to 3, the inequality implies that bank L also goes bankrupt. The bankruptcy implies that that $C_L = 0$ which, in turn, implies that bank O goes bankrupt and thus $C_O = 0$. Only the isolated bank N survives.

$$\Pi_O = 0 \text{ and } W = 1 + z + \psi_B$$

(ii) $z \geqslant \frac{1}{2}$. Bank L is solvent and $C_L = D_L = 2$ and $\Pi_L = 2z - 1$. Bank O is not affected.

$$\Pi_O = z \text{ and } W = 4(1 + z) + \psi_B$$

(b) $\psi_B \geqslant 1$. Now the depositors of bank B are fully paid, $C_B = 1$ and bank L recovers $\psi_B - 1$. Once more, there are two cases to consider depending or not bank L remains solvent. The only difference with the above case is that now the assets of bank L are larger by $\psi_B - 1$. Thus, now we have

(i) $2z + \psi_B < 2$.

$$\Pi_O = 0 \text{ and } W = 1 + z + \psi_B$$

(ii) $2z + \psi_B \geqslant 2$.

$$\Pi_O = z \text{ and } W = 4(1 + z) + \psi_B$$

(c) Loan from Bank O to Bank B

The new balance sheets of banks B and O are given by:

$L_B = 3$	$D_B = 1$
	$d_B^L = 1$
	$d_B^O = 1$
$Assets = 3$	$Liabilities = 3$
$L_O = 1$	$D_O = 2$
$l_O^B = 1$	
$Assets = 2$	$Liabilities = 2$

Case 1 *Bank N goes bankrupt*

This is similar to the case when bank O offers the loan to bank L. Given that other banks are not affected, we have:

$$\Pi_O = z \text{ and } W = 5(1 + z) + \psi_N$$

where $\psi_N \in [0, 1]$.

Case 2 *Bank L goes bankrupt*

The payoff to depositors of bank L is equal to ψ_L, where $\psi_L \in [0, 1]$, plus a deposit of 1 unit at bank B. All other banks are not affected.

$$\Pi_O = z \text{ and } W = 5(1 + z) + \psi_L$$

Case 3 *Bank B goes bankrupt*

We need to consider the following two cases depending on the value of $\psi_B \in [0,3]$.

(a) $\psi_B < 1$. The payoff of depositors of bank B is given by $C_B = \psi_B$ and the loans to banks L and O are not repaid and thus both banks go bankrupt with only bank N surviving.

$$\Pi_O = 0 \text{ and } W = 1 + z + \psi_B$$

(b) $\psi_B \geqslant 1$. Now the depositors of bank B are fully paid, $C_B = D_B = 1$, and banks L and O each recover $\frac{\psi_B - 1}{2}$ from their one 1 unit of loan to bank B. What happens to the two banks depends on the value of ψ_B. Each of these two banks have assets equal to $1 + z + \frac{\psi_B - 1}{2} = z + \frac{1}{2} + \frac{\psi_B}{2}$ and liabilities equal to 2. Thus, we have two cases to consider:

(i) $z + \frac{\psi_B}{2} < \frac{3}{2}$. Bank L and O are insolvent.

$$\Pi_O = 0 \text{ and } W = 1 + z + \psi_B$$

(ii) $z + \frac{\psi_B}{2} \geqslant \frac{3}{2}$. Bank L and O are solvent.

$$\Pi_O = z + \frac{\psi_B}{2} - \frac{3}{2} \text{ and } W = 3(1+z) + \psi_B$$

5.1.2 Bank Seniority

(a) Loan from Bank O to Bank N

See Table 1 for the balance sheets of banks L and B and Table 2 for the balance sheets of banks N and O.

Case 1 *Bank N goes bankrupt*

Depending on the value of $\psi_N \in [0,2]$ we need to consider two cases:

(a) $\psi_N < 1$. Given that banks have seniority $C_N = 0$ and bank O receives a payoff equal to ψ_N and depending on its value we need to consider two cases:

(i) $z < 1 - \psi_N$. Bank O goes bankrupt and $C_O = \psi_N$.

$$\Pi_O = 0 \text{ and } W = 3(1+z) + \psi_N$$

(ii) $z \geqslant 1 - \psi_N$. Bank O is solvent $C_O = 2$ and $\Pi_O = z + \psi_N - 1$.

$$\Pi_O = z + \psi_N - 1 \text{ and } W = 4(1+z) + \psi_N$$

(b) $\psi_N \geqslant 1$. $C_N = \psi_N - 1$ and bank O's loan to bank N is fully repaid.

$$\Pi_O = z \text{ and } W = 4(1+z) + \psi_N$$

Case 2 *Bank L goes bankrupt*

Given that the only liabilities of bank L are customer deposits, the outcome is exactly the same as the case with depositor seniority.

$$\Pi_O = z \text{ and } W = 5(1+z) + \psi_L$$

Case 3 *Bank B goes bankrupt*

The bankruptcy will also affect bank L that has offered a loan to bank B. Symmetry implies that the welfare results in this case are exactly the same as those derived from the case when bank N goes bankrupt. When bank N receives the new loan from bank O there is a symmetric network structure, namely, two banks offering a loan and two banks receiving a loan. Moreover, banks N and B are the two banks receiving the loans. The only difference is that in this case bank O is solvent. Thus, we have the following cases:

(a) $\psi_B \leqslant 1$.

(i) $z < 1 - \psi_N$.

$$\Pi_O = z \text{ and } W = 3(1+z) + \psi_B$$

(ii) $z \geqslant 1 - \psi_N$.

$$\Pi_O = z \text{ and } W = 4(1+z) + \psi_N$$

(b) $\psi_B > 1$.

$$\Pi_O = z \text{ and } W = 4(1+z) + \psi_B$$

(b) Loan from Bank O to Bank L

See Table 1 for the balance sheets of banks N and B and Table 2 for the balance sheets of banks L and O.

Case 1 *Bank N goes bankrupt*

All other banks are not affected by the shock. $C_N = \psi_N$.

$$\Pi_O = z \text{ and } W = 5(1+z) + \psi_N$$

Case 2 *Bank L goes bankrupt*

The creditor bank O has its loan repaid by obtaining 1 unit of deposits at bank B. The payoff of depositors of bank L is equal to $C_L = \psi_L$.

$$\Pi_O = z \text{ and } W = 4(1+z) + \psi_L$$

Case 3 *Bank B goes bankrupt*

The bankruptcy will also affect bank L that has offered bank B a loan and potentially bank O that has offered bank L a loan. Note that $\psi_B \in [0, 2]$.

(a) $\psi_B < 1$. The payoff of the depositors of bank B is given by $C_B = 0$ and bank L receives ψ_B. There are two cases to consider depending on whether or not bank L remains solvent.

(i) $z < \frac{1-\psi_B}{2}$. Given that the assets of bank L are equal to $2(1+z) + \psi_B$ and the liabilities are equal to 3, the inequality implies that bank L also goes bankrupt. The bankruptcy implies that that $C_L = 0$ and bank O receives ψ_B.

However, the inequality implies that bank O with assets equal to $1+z+\psi_B$ and liabilities equal to 2 also goes bankrupt, hence $C_O = \psi_B$.

$$\Pi_O = 0 \text{ and } W = 1+z+\psi_B$$

(ii) $z \geqslant \frac{1-\psi_B}{2}$. Bank L is solvent and hence bank O is not affected.

$$\Pi_O = z \text{ and } W = 4(1+z)+\psi_B$$

(b) $\psi_B \geqslant 1$. Bank L's loan is fully repaid and $C_B = \psi_B - 1$.

$$\Pi_O = z \text{ and } W = 4(1+z)+\psi_B$$

(c) Loan from Bank O to Bank B

See Table 1 for the balance sheets of banks N and L and Table 2 for the balance sheets of banks B and O.

Case 1 *Bank N goes bankrupt*

As in the case when bank O offers the loan to bank L the only bank affected is bank N.

$$\Pi_O = z \text{ and } W = 5(1+z)+\psi_N$$

Case 2 *Bank L goes bankrupt*

The payoff to depositors of bank L is equal to ψ_L plus 1 unit of deposits at bank B.

$$\Pi_O = z \text{ and } W = 5(1+z)+\psi_L$$

Case 3 *Bank B goes bankrupt*

Depending on the value of $\psi_B \in [0,3]$ we need to consider two cases:

(a) $\psi_B < 2$. In this case banks L and O each receive $\frac{\psi_B}{2}$ and $C_B = 0$. What happens to banks L and O depends on the value of ψ_B. Each bank's assets are equal to $1+z+\frac{\psi_B}{2}$ while the corresponding liabilities are equal to 2. Then, once more, we need to consider tow cases:

(i) $z < 1-\frac{\psi_B}{2}$. Both banks go bankrupt and $C_L = C_O = \frac{\psi_B}{2}$.

$$\Pi_O = 0 \text{ and } W = 1+z+\psi_B$$

(ii) $z \geqslant 1-\frac{\psi_B}{2}$. Banks L and O are solvent, $C_L = C_O = D_L = D_O = 2$, and $\Pi_L = \Pi_O = z+\frac{\psi_B}{2}-1$.

$$\Pi_O = z+\frac{\psi_B}{2}-1 \text{ and } W = 3(1+z)+\psi_B$$

(b) $\psi_B \geqslant 2$. The loans of banks L and O are fully repaid and $C_B = \psi_B - 2$.

$$\Pi_O = z \text{ and } W = 3(1+z)+\psi_B$$

5.2 Comparing Bank Seniority to Depositor Seniority (Proportional Shocks)

5.2.1 The Optimal Choice of Bank O

Depositor Seniority For the case of depositor seniority and under the restriction that shocks are proportional, Table 1 shows the expected profits of bank O, $E[\Pi_O]$, for each of the 3 loan offer options and for all possible values of per unit of loan profits, z, and liquidation value per unit of loan, ψ. The last column of the table indicates the optimal choice, that is the one that maximizes bank O's expected profits, under the supposition that shocks per unit of loans and profits per unit of loan can only take the value that corresponds to that particular row of the table.

[Please insert Table 1 about here]

The restriction that shocks are proportional implies that, for example, the probability that a bank with 2 units of customer loans will be inflicted by a shock of size x is equal to the probability that a bank with 3 units of customer loans will be inflicted by a shock of size $\frac{3}{2}x$. Then, in deriving the above table if, for example, bank N has 2 units of customer loans and thus $\psi_N \in [0, 2]$ we have set $\psi_N = 2\psi$, where $\psi \in [0, 1]$, in the derivations of the last section.

Consider Table 1A. For the first three rows we have $\psi < \frac{1}{3}$ and $z < \frac{1}{2}$. For the derivation of the first row we focus on sub-section 5.1.1.(a) that is when bank O offers the loan to bank N. From case 1(a) we find that if bank N goes bankrupt $\Pi_O = 0$. Similarly, from case 2(a) we find that if bank L goes bankrupt $\Pi_O = z$ and if bank B goes bankrupt $\Pi_O = z$. Under the supposition of proportional shocks each bankruptcy event is equiprobable and thus we conclude that $E[\Pi_O] = \frac{2}{3}z$. Similarly, for the derivation of the second row we focus on sub-section 5.1.1.(b) that is when bank O offers the loan to bank L and in particular cases 1, 2(a) and 3(a)(i) and for the derivation of the third row we focus on sub-section 5.1.1.(c) that is when bank O offers the loan to bank L and in particular cases 1, 2 and 3(a). Comparing the three rows we find that under the supposition that liquidation values and profits satisfy $\psi < \frac{1}{3}$ and $z < \frac{1}{2}$ bank O would be indifferent between offering the loan to banks N and B. For the next three rows of the table we now have $z \geqslant \frac{1}{2}$. The only difference between this case and the one considered above is that when bank O offers the loan to bank L we need to consider case 3(a)(ii) instead of case 3(a)(i). Now all three choices result in the same expected profits.

Next, consider Table 1B. When $\frac{1}{3} \leqslant \psi < \frac{1}{2}$ and bank O offers the loan to bank B, if the latter goes bankrupt the liquidation value of its assets is higher than its obligations to depositors. Therefore, its remaining assets will be equally distributed to its two creditor banks, namely, L and O, and what happens to these banks it depends on the values of both z and ψ. Given that $\psi < \frac{1}{2}$, as long as $z < \frac{3}{4}$, we have $\psi < 1 - \frac{2}{3}z$ and thus the relevant case is 3(b)(i). The derivation of expected profits follows exactly the same logic as the one used for the derivations of Table 1A. In contrast, when $z \geqslant \frac{3}{4}$ what happens depends on the relative values of z and ψ and the results are captured by the last six rows of the table. Notice that in the last row the expected profits of bank O are boosted by the fact that it recovers a sufficient amount of funds when bank B goes bankrupt to stay solvent. In fact the inequalities imply that $\frac{2}{3}z \leqslant z + \frac{1}{2}(\psi - 1) < z$.

Lastly, consider Table 1C. We compare the first three rows of the table and the next three rows for the case when bank O offers the loan to bank L. All depends on whether bank L stays solvent in which case bank O also stays solvent (compare cases 3(b)(i) and 3(b)(ii)). Next, focusing on the middle six rows of the table we observe that there is a difference between the first three rows of this group and the last three rows when bank O offers the loan to bank B. This is exactly the same case discussed in the previous paragraph and the results depend on whether or not bank O can remain solvent after the bankruptcy of bank B. Finally, comparing the last six rows of the table we find that there is a difference between the first three rows of this group and the last three rows when bank O offers the loan to either bank N or bank L. It all depends on whether or not bank O remains solvent after the bank that was offered the loan went into bankruptcy.

Bank Seniority Table 2 shows the expected profits of bank O for the case of bank seniority.

[Please insert Table 2 about here]

Consider Table 2A. Focusing on the first three rows we observe that when the level of profits z is very low, bank O always becomes insolvent when the bank to which it offered the loan becomes insolvent. Thus, with probability $\frac{2}{3}$ the bank remains solvent. Comparing the first three rows with the next three rows we find that the expected profits of bank O increase when it offers the loan to bank L. Comparing cases 3(a)(i) and 3(a)(ii) we find that for sufficiently high values of z bank L remains solvent after the bankruptcy of bank B. Next, we compare the middle six rows and find that there is a difference depending on whether or not bank O becomes insolvent when it offers the loan to bank N and the latter becomes insolvent (cases 1(a)(i) and 1(a)(ii)). Moving to the last six rows we find that there is a difference in expected profits when bank O offers the loan to bank B. As z moves above the threshold that separates the last three rows from the three rows above them bank O remains solvent despite the bankruptcy of bank B.

Differences in the other two tables also capture how further increases in ψ and z affect the expected profits of bank O when it offers the loan to bank B and the latter goes bankrupt. In Table 2B we have exactly the same results as for the case above. In contrast, in Table 2C, ψ is sufficiently high for bank O to fully recover its loan to bank B.

5.2.2 Social Welfare

Depositor Seniority For the case of depositor seniority and under the restriction that shocks are proportional, Table 3 shows the expected social welfare, $E[W]$, for each of the 3 loan offer options and for all possible values of per unit of loan profits, z, and liquidation value per unit of loan, ψ. The last column of the table indicates the optimal choice, that is the one that maximizes expected social welfare, under the supposition that shocks per unit of loans and profits per unit of loan can only take the value that corresponds to that particular row of the table.

[Please insert Table 3 about here]

Notice that the second term of all expressions is the same. The reason is that at any time there are 5 units of consumer loans in the books of banks N, L and B and under proportional shocks $\frac{5}{3}\psi$ equals the expected liquidation value of these loans. Thus, from now on we focus on the first term. For each pair of values for z and ψ and for each possible loan offer by bank O we derive the first term by adding the welfare results for the corresponding three bankruptcy cases divide by 3 given that each of the three banks becomes insolvent with the same probability. The various cutoffs are the same as the ones derived for the derivation of bank O's expected profits under depositor seniority. As an example, consider the first entry of Table 3A. The loan is offered to bank N and, therefore, we focus on Section 5.1.1.(a) and take the average of the welfare values of cases 1(a), 2 and 3(a). Notice that in this case $\psi_N = 2\psi$, $\psi_L = \psi$ and $\psi_B = 2\psi$ and thus, as we pointed above, the average liquidation value is equal to $\frac{5}{3}\psi$. As another example, consider the last entry of Table 3C. The loan is offered to bank B and, therefore, we focus on Section 5.1.1.(c) and take the average of the welfare values of cases 1, 2 and 3(b)(ii). Notice that in this case $\psi_N = \psi$, $\psi_L = \psi$ and $\psi_B = 3\psi$ and thus, once more, the average liquidation value is equal to $\frac{5}{3}\psi$.

Bank Seniority Table 4 below shows the expected social welfare for the case of bank seniority.

[Please insert Table 4 about here]

The cut-off values correspond to those of Table 2. For the derivations we have followed the same steps as for the case of depositor seniority but this time we used the results of Section 5.1.2.

5.2.3 Depositor vs Bank Seniority

Proof of Proposition 1 Parts (a) and (b): The proof follows from a direct comparison of Tables 1 and 2. When comparing Tables 1 and 2 we need to keep in mind that what matters is the value of expected profits under each policy regime. However, it is clear that the exact probability weights do not matter.

Table 1 shows that under depositor seniority it is never optimal for bank O to offer the loan to bank L. For any values of z and ψ by offering the loan to bank N, bank O makes at least as high profits as when offering the loan to bank L. Then, bank O optimal choice will be either to offer the loan to bank N or to bank B with the optimal choice depending on the distributions of z and ψ. In contrast, under bank seniority, Table 2 shows that offering the loan to bank L is the dominant choice for any values of z and ψ.

Part (c): The proof follows from the proofs of parts (a) and (b) and the following observations: According to Table 4, bank seniority that provides incentives to bank O to offer the loan to bank L, maximizes welfare in all cases but the top one which corresponds to the worst case scenario of very high initial losses and very low profitability. For this particular case, welfare would be maximized by the depositor seniority option (see Table 3) that provides incentives to bank O to offer the loan to either bank N or bank B.

5.3 Comparing Bank Seniority to Depositor Seniority (Identical Shocks)

With identical shocks we have $L_i - \psi_i = L_j - \psi_j \leqslant 1$, for every bank i or j. Then, we define $\psi \equiv \psi_i - L_i + 1$. Thus, if $L_i = 1$, $\psi = \psi_i$, if $L_i = 2$, $\psi = \psi_i - 1$ and if $L_i = 3$, $\psi = \psi_i - 2$. Put differently, given that the losses are restricted to be at most equal to 1 unit of consumer loans, ψ equals 1 minus these losses and is identical across banks.

5.3.1 The Optimal Choice of Bank O

Depositor Seniority For the case of depositor seniority and under the restriction that shocks are identical, Table 5 below shows the expected profits of bank O, $E[\Pi_O]$, for each of the 3 loan offer options and for all possible values of per unit of loan profits, z, and ψ values defined above. The last column of the table indicates the optimal choice, that is the one that maximizes bank O's expected profits, under the supposition that shocks per unit of loans and profits per unit of loan can only take the value that corresponds to that particular row of the table.

Table 5: Bank O's Optimal Choice: Depositor Seniority; Identical Shocks

z	$E[\Pi_O]$	OC
$z < \frac{1-\psi}{2}$	$N : \frac{3}{5}z$	*
	$L : \frac{1}{5}z$	
	$B : \frac{3}{5}z$	*
$\frac{1-\psi}{2} \leqslant z < 1-\psi$	$N : \frac{3}{5}z$	
	$L : \frac{3}{5}z$	
	$B : z + \frac{3}{5}\left(\frac{\psi-1}{2}\right)$	*
$1-\psi \leqslant z$	$N : z + \frac{2}{5}(\psi-1)$	*
	$L : z + \frac{2}{5}(\psi-1)$	*
	$B : z + \frac{3}{5}\left(\frac{\psi-1}{2}\right)$	

Consider the first row of the table that is derived using the results of Section 5.1.1.(a) Given that the losses cannot exceed one unit and banks N and B have two units of loans each, the relevant cases are 1(b)(i), 2 and 3(b)(i) . Thus, for the derivations we let $\psi_N = \psi_B = 1 + \psi$ and $\psi_L = \psi$ and the entry in the first column follows from $z + \psi_N < 2 \Leftrightarrow z < 1 - \psi$ and from $z < \frac{1-\psi}{2} \Rightarrow z < 1 - \psi$. The total number of customer loan units on the books of banks N, L and B is equal to 5 and, thus, the probability that bank N will go bankrupt is equal to $\frac{2}{5}$, the corresponding probability for bank L is equal to $\frac{1}{5}$ and the corresponding probability for bank B is equal to $\frac{2}{5}$. Then, $E[\Pi_O] = \frac{2}{5} \times 0 + \frac{1}{5} \times z + \frac{2}{5} \times z = \frac{3}{5}z$.

For the second row we use the results of Section 5.1.1.(b) and in particular cases 1, 2 and 3(b)(i). In this case we have $\psi_L = \psi_B = 1 + \psi$ and $\psi_N = \psi$. For the case 2(b)(i) the cut-off value for z is the same as above but for case 3(b)(i) we have a new cut-off value given by $\frac{1-\psi}{2}$. Lastly, the probabilities that banks N, L and B become insolvent are equal to $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{2}{5}$, respectively. For the third row we use the results of Section 5.1.1.(c) and in particular cases 1, 2(b)(i) and 3(b)(i). In this case we have $\psi_N = \psi_L = \psi$ and $\psi_B = 2 + \psi$. For the case 3(b)(i) the cut-off value for z is given by $2z + \psi_B < 2 \Leftrightarrow z < 1 - \psi$. Lastly, the probabilities that banks N, L and B become insolvent are equal to $\frac{1}{5}$, $\frac{1}{5}$ and $\frac{3}{5}$, respectively.

Following the same steps we have completed the table.

Bank Seniority Table 6 shows the expected profits of bank O for the case of bank seniority.

Table 6: Bank O's Optimal Choice: Bank Seniority; Identical Shocks

$E[\Pi_O]$	OC
$N : z$	*
$L : z$	*
$B : z$	*

Bank seniority completely protects bank O form the insolvencies of any other bank when the size of the shocks are restricted to be less than 1.

5.3.2 Social Welfare

Depositor Seniority Table 7 shows the social welfare results for the depositor seniority case. For the derivation of the entries we follow exactly the same steps as those that we followed for the derivation of Table 5. The only difference is that now we use the results for W instead of $E[\Pi_O]$.

Table 7: Expected Social Welfare: Depositor Seniority; Identical Shocks

z	$E[W]$	OC
$z < \frac{1-\psi}{2}$	$N : \frac{21}{5} + \frac{17}{5}z + \psi$	*
	$L : \frac{17}{5} + \frac{13}{5}z + \psi$	
	$B : \frac{19}{5} + \frac{13}{5}z + \psi$	
$\frac{1-\psi}{2} \leqslant z < 1 - \psi$	$N : \frac{21}{5} + \frac{17}{5}z + \psi$	
	$L : \frac{23}{5} + \frac{19}{5}z + \psi$	
	$B : 5 + \frac{19}{5}z + \psi$	*
$1 - \psi \leqslant z$	$N : 5 + \frac{21}{5}z + \psi$	*
	$L : 5 + \frac{21}{5}z + \psi$	*
	$B : 5 + \frac{19}{5}z + \psi$	

Then the result of the first row is obtained from cases 1(b)(i), 2 and 3(b)(i) of Section 5.1.1.(a) In particular, we have $\frac{2}{5}(3(1+z)+\psi_N)+\frac{1}{5}(5(1+z)+\psi_L)+\frac{2}{5}(3(1+z)+\psi_B) = \frac{2}{5}(3(1+z)+1+\psi)+\frac{1}{5}(5(1+z)+\psi)+\frac{2}{5}(3(1+z)+1+\psi) = \frac{21}{5}+\frac{17}{5}z+\psi$. As another example, consider the last row of the table where the relevant cases are 1, 2 and 3(b)(ii) of Section 5.1.1.(c). In this case we have $\frac{1}{5}(5(1+z)+\psi_N)+\frac{1}{5}(5(1+z)+\psi_L)+\frac{3}{5}(3(1+z)+\psi_B) = \frac{1}{5}(5(1+z)+\psi)+\frac{1}{5}(5(1+z)+\psi)+\frac{3}{5}(3(1+z)+2+\psi) = 5+\frac{19}{5}z+\psi$.

Bank Seniority Table 8 below shows the expected social welfare for the case of bank seniority.

Table 8: Expected Social Welfare: Bank Seniority; Identical Shocks

$E[W]$	OC
$N : 5 + \frac{21}{5}z + \psi$	*
$L : 5 + \frac{21}{5}z + \psi$	*
$B : 5 + \frac{19}{5}z + \psi$	

In all 3 entries of the table the total welfare of depositors is equal to $5 + \psi$. There are 6 units of deposits in the banking network and each time a bank defaults its depositors lose $1 - \psi$. Profits are lower when bank O offers the loan to bank B because of the concentration of loans in one bank.

5.3.3 Depositor vs Bank Seniority

Proof of Proposition 2 Parts (a) and (b): This follows directly from Tables 5 and 6.

Part (c): From Table 8, it is clear that as long as bank O does not offer the loan to bank B expected welfare will be maximized. Table 5 shows that under depositor seniority the choice of bank O would depend on the distribution of shocks. In contrast, Table 6 shows that under bank seniority bank O will be indifferent across the three choices.

Table 1: Bank O's Optimal Choice: Depositor Seniority; Proportional Shocks

Table 1A: $\psi < \frac{1}{3}$

z	$E[\Pi_O]$	OC
$z < \frac{1}{2}$	$N : \frac{2}{3}z$	*
	$L : \frac{1}{3}z$	
	$B : \frac{2}{3}z$	*
$z \geqslant \frac{1}{2}$	$N : \frac{2}{3}z$	*
	$L : \frac{2}{3}z$	*
	$B : \frac{2}{3}z$	*

Table 5B: $\frac{1}{3} \leqslant \psi < \frac{1}{2}$

z	$E[\Pi_O]$	OC
$z < \frac{1}{2}$	$N : \frac{2}{3}z$	*
	$L : \frac{1}{3}z$	
	$B : \frac{2}{3}z$	*
$\frac{1}{2} \leqslant z < \frac{3}{4}$	$N : \frac{2}{3}z$	*
	$L : \frac{2}{3}z$	*
	$B : \frac{2}{3}z$	*
$\frac{3}{4} \leqslant z < 1; \psi < 1 - \frac{2}{3}z$	$N : \frac{2}{3}z$	*
	$L : \frac{2}{3}z$	*
	$B : \frac{2}{3}z$	*
$\frac{3}{4} \leqslant z < 1; \psi \geqslant 1 - \frac{2}{3}z$	$N : \frac{2}{3}z$	
	$L : \frac{2}{3}z$	
	$B : z + \frac{1}{2}(\psi - 1)$	*

Table 5C: $\frac{1}{2} \leqslant \psi$

z	$E[\Pi_O]$	OC
$z < 1 - \psi$	$N : \frac{2}{3}z$	*
	$L : \frac{1}{3}z$	
	$B : \frac{2}{3}z$	*
$1 - \psi \leqslant z < \frac{3}{2}(1 - \psi)$	$N : \frac{2}{3}z$	*
	$L : \frac{2}{3}z$	*
	$B : \frac{2}{3}z$	*
$\frac{3}{2}(1 - \psi) \leqslant z < 2(1 - \psi)$	$N : \frac{2}{3}z$	
	$L : \frac{2}{3}z$	
	$B : z + \frac{1}{2}(\psi - 1)$	*
$2(1 - \psi) \leqslant z$	$N : z + \frac{2}{3}(\psi - 1)$	*
	$L : z + \frac{2}{3}(\psi - 1)$	*
	$B : z + \frac{1}{2}(\psi - 1)$	

Table 2: Bank O's Optimal Choice: Bank Seniority; Proportional Shocks

Table 2A: $\psi < \frac{1}{2}$

z	$E[\Pi_O]$	OC
$z < \frac{1}{2} - \psi$	$N : \frac{2}{3}z$	*
	$L : \frac{2}{3}z$	*
	$B : \frac{2}{3}z$	*
$\frac{1}{2} - \psi \leqslant z < 1 - 2\psi$	$N : \frac{2}{3}z$	
	$L : z$	*
	$B : \frac{2}{3}z$	
$1 - 2\psi \leqslant z < 1 - \frac{3}{2}\psi$	$N : z + \frac{2\psi-1}{3}$	
	$L : z$	*
	$B : \frac{2}{3}z$	
$1 - \frac{3}{2}\psi \leqslant z$	$N : z + \frac{2\psi-1}{3}$	
	$L : z$	*
	$B : z + \frac{1}{2}\psi - \frac{1}{3}$	

Table 2B: $\frac{1}{2} \leqslant \psi < \frac{2}{3}$

z	$E[\Pi_O]$	OC
$z < 1 - \frac{3}{2}\psi$	$N : z$	*
	$L : z$	*
	$B : \frac{2}{3}z$	
$z \geqslant 1 - \frac{3}{2}\psi$	$N : z$	*
	$L : z$	*
	$B : z + \frac{1}{2}\psi - \frac{1}{3}$	

Table 2C: $\frac{2}{3} \leqslant \psi$

$E[\Pi_O]$	OC
$N : z$	*
$L : z$	*
$B : z$	*

Table 3: Expected Social Welfare: Depositor Seniority; Proportional Shocks

Table 3A: $\psi < \frac{1}{3}$

z	$E[W]$	OC
$z < \frac{1}{2}$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
	$L: \frac{9}{3}(1+z) + \frac{5}{3}\psi$	
	$B: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
$z \geqslant \frac{1}{2}$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$L: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	*
	$B: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	

Table 3B: $\frac{1}{3} \leqslant \psi < \frac{1}{2}$

z	$E[W]$	OC
$z < \frac{1}{2}$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
	$L: \frac{9}{3}(1+z) + \frac{5}{3}\psi$	
	$B: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
$\frac{1}{2} \leqslant z < \frac{3}{4}$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$L: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	*
	$B: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$\frac{3}{4} \leqslant z < 1; \psi < 1 - \frac{2}{3}z$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$L: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	*
	$B: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$\frac{3}{4} \leqslant z < 1; \psi \geqslant 1 - \frac{2}{3}z$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$L: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$B: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*

Table 3C: $\frac{1}{2} \leqslant \psi$

z	$E[W]$	OC
$z < 1 - \psi$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
	$L: \frac{9}{3}(1+z) + \frac{5}{3}\psi$	
	$B: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
$1 - \psi \leqslant z < \frac{3}{2}(1-\psi)$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$L: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	*
	$B: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$\frac{3}{2}(1-\psi) \leqslant z < 2(1-\psi)$	$N: \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$L: \frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$B: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
$2(1-\psi) \leqslant z$	$N: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$L: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$B: \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*

Table 4: Expected Social Welfare: Bank Seniority; Proportional Shocks

Table 4A: $\psi < \frac{1}{2}$

z	$E[W]$	OC
$z < \frac{1}{2} - \psi$	$N : \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
	$L : \frac{10}{3}(1+z) + \frac{5}{3}\psi$	
	$B : \frac{11}{3}(1+z) + \frac{5}{3}\psi$	*
$\frac{1}{2} - \psi \leqslant z < 1 - 2\psi$	$N : \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
	$L : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$B : \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$1 - 2\psi \leqslant z < 1 - \frac{3}{2}\psi$	$N : \frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$L : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$B : \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$1 - \frac{3}{2}\psi \leqslant z$	$N : \frac{12}{3}(1+z) + \frac{5}{3}\psi$	
	$L : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$B : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*

Table 4B: $\frac{1}{2} \leqslant \psi < \frac{2}{3}$

z	$E[W]$	OC
$z < 1 - \frac{3}{2}\psi$	$N : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$L : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$B : \frac{11}{3}(1+z) + \frac{5}{3}\psi$	
$z \geqslant 1 - \frac{3}{2}\psi$	$N : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$L : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
	$B : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*

Table 4C: $\frac{2}{3} \leqslant \psi$

$E[W]$	OC
$N : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
$L : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*
$B : \frac{13}{3}(1+z) + \frac{5}{3}\psi$	*