Analysis of the Ill-posedness in Double Differential Ambiguity Resolution of BDS

S.G. Pan[1]*, W. Gao[2], S.L. Wang[1,3] ,X.Meng[4] and, Q.Wang[1]

Abstract The ill-posedness in variance-covariance matrix will directly determine the convergence speed and accuracy of integer ambiguities. Unlike GPS or GLONASS, BDS (BeiDou Navigation Satellite System) consists of not only MEO satellites but also GEO and IGSO satellites, both of which are high-orbit satellites. The angular velocities of the GEO and IGSO satellites are much smaller compared with MEO satellites. The changes of the geometric structure between satellites and stations of the high-orbit satellites GEO/IGSO in BDS are not obvious during short observational spans due to their relatively small angular velocity. This results in stronger correlation of equations between adjacent epochs while calculating ambiguities, leading to serious ill-posedness. In this paper the ill-posedness of double differential (DD) ambiguity resolution (AR) of the current BDS was analysed. And on this basis, some different combinations of GEO, IGSO and MEO satellites of BDS were used in the AR experiments to reveal the characteristic of ill-posedness. Moreover, AR experiments of GPS, GLONASS and BDS/GPS/GLONASS fusion were also carried out for comparison with BDS. These experiments indicate that AR of the current BDS is a more serious ill-posed problem, therefore takes much more time for AR fixing than GPS or GLONASS. The fusion with GPS or GLONASS, however, will solve the ill-posed problem effectively and improve the AR much more, achieving fixes even instantaneously.

Key words BDS; ill-Posedness; double differential ambiguity resolution; condition number; fusion with GPS, GLONASS

1 Introduction

BeiDou Navigation Satellite System (BDS) is among one of the four current Global Navigation Satellite Systems (GNSS) in the world that offers various services including surveying, timing, navigation of transportation, etc [Yang et al., 2010；Yang et al., 2011].BDS has now completed its second step of the three stages in total [Yang et al.,2011; Ran，2012], and can provide regional navigation and positioning service for Asian-Pacific region. Unlike GPS, GLONASS and other existing GNSS systems, BDS includes not only the frequently-used MEO satellites, but also 5 GEO (geostationary orbit) satellites and 5 IGSO (inclined geosynchronous satellite orbit) satellites in its constellation [Zhao et al., 2013; Zhou et al., 2012]. Both of them are high-orbit satellites and their orbital altitude is about 36000km, while the orbital altitude of BDS MEO satellites is just about 21150km [Gao et al.,2013].

The ill posedness is widespread in the model of GNSS double differential (DD) ambiguity resolution (AR) due to the existence of ambiguity parameters and other parameters (e.g.

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1. School of Instrument Science and Engineering, Southeast University, Nanjing, China
2. School of Transportation, Southeast University, Nanjing, China
3. School of Surveying and Mapping Engineering, Anhui University of Science & Technology, Huainan, China
4. Nottingham Geospatial Institute, the University of Nottingham, UK
* Correspondence Author e-mail: panshuguo@gmail.com

baseline vectors) [Shen et al., 2007; Gao et al., 2013]. A major solution to reduce the ill-posedness of the model is to change the geometrical relationships between satellites and stations from multi-epoch observations. The GEO/IGSO satellites of BDS are high-orbit satellites and the angular velocity relative to earth of the satellites is much slower than that of MEO satellites. Among these high-orbit satellites, the angular velocity relative to earth of IGSO is about 30%~50% of that of MEO satellites. And the angular velocity relative to earth of GEO is only about 0~2%. Therefore, the geometrical relationships between the GEO/IGSO satellites and observation stations change much slower, which leads to a stronger correlation of the DD equations between adjacent epochs. This further aggravates the ill posedness of the AR process. So it is more difficult to get the precise floating ambiguities and the reasonable covariance matrix [Teunissen et al., 1994; Hao et al.,2010; Xu et al., 2006]. So a clear understanding of the ill-posedness in DD AR process of the current BDS will be significant to the AR strategy.

In this paper, the ill-posedness in the AR of the current BDS was analysed. And then some AR experiments with different combinations of GEO, IGSO and MEO satellites of BDS were carried out to verify the characteristic of ill-posedness. AR experiments of GPS and GLONASS were also carried out at the same time for comparison. At last the fusion AR experiments of BDS and GPS or GLONASS were done to test the AR effect of multiple-constellation fusion.

2 Model of DD Ambiguity Resolution

The model of DD-carrier phase observables can be written as:

$$\lambda \Delta\nabla\varphi_{A,B}^{p,q}(t_i) = \Delta\nabla R_{A,B}^{p,q}(t_i) + \lambda\Delta\nabla N_{A,B}^{p,q} + \delta_{A,B}^{p,q}(t_i) \tag{1}$$

$$\delta_{A,B}^{p,q}(t_i) = -\Delta\nabla I_{A,B}^{p,q}(t_i) + \Delta\nabla T_{A,B}^{p,q}(t_i) + \Delta\nabla\delta m_{A,B}^{p,q}(t_i) + \Delta\nabla\varepsilon_{A,B}^{p,q}(t_i) \tag{2}$$

Where

$\Delta\nabla(\bullet)_{A,B}^{p,q}$ is the double-difference operator from observation station A, B and satellite p, q; λ is the nominal carrier phase wavelength; φ is the carrier phase observables; R is the geometric range from a receiver and a satellite; N is the integer ambiguity; I and T are the dispersive atmospheric (ionospheric) delay and non-dispersive atmospheric delay; m is the multipath on carrier; ε is the noise on carrier and other errors.

The error equation of baseline solution can be written as:

$$\begin{pmatrix} v^{1,q} \\ v^{2,q} \\ \vdots \\ v^{p,q} \end{pmatrix} = \begin{pmatrix} l^{1,q} & m^{1,q} & n^{1,q} \\ l^{2,q} & m^{2,q} & n^{2,q} \\ \vdots & \vdots & \vdots \\ l^{p,q} & m^{p,q} & n^{p,q} \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} + \begin{pmatrix} \lambda & & & \\ & \lambda & & \\ & & \ddots & \\ & & & \lambda \end{pmatrix} \begin{pmatrix} \Delta\nabla N^{1,q} \\ \Delta\nabla N^{2,q} \\ \vdots \\ \Delta\nabla N^{p,q} \end{pmatrix} - \begin{pmatrix} \Delta\nabla L^{1,q} \\ \Delta\nabla L^{2,q} \\ \vdots \\ \Delta\nabla L^{p,q} \end{pmatrix} \tag{3}$$

Where

p is the number of a satellite; q is the number of the reference satellite; $\Delta\nabla L$ is the constant term including DD carrier observables and other modelled errors; dx, dy, dz are the coordinate increments; $l^{p,q}, m^{p,q}, n^{p,q}$ are the coefficients of the coordinate vector, which are usually expressed as:

$$l^{p,q} = -\frac{1}{2}(\frac{\Delta X_A^p}{\rho_A^p} + \frac{\Delta X_B^p}{\rho_B^p} - \frac{\Delta X_A^q}{\rho_A^q} - \frac{\Delta X_B^q}{\rho_B^q})$$

$$m^{p,q} = -\frac{1}{2}(\frac{\Delta Y_A^p}{\rho_A^p} + \frac{\Delta Y_B^p}{\rho_B^p} - \frac{\Delta Y_A^q}{\rho_A^q} - \frac{\Delta Y_B^q}{\rho_B^q}) \qquad (4)$$

$$n^{p,q} = -\frac{1}{2}(\frac{\Delta Z_A^p}{\rho_A^p} + \frac{\Delta Z_B^p}{\rho_B^p} - \frac{\Delta Z_A^q}{\rho_A^q} - \frac{\Delta Z_B^q}{\rho_B^q})$$

ΔX_i^n, ΔY_i^n, ΔZ_i^n are the difference of corresponding coordinates from satellite n and observation station i. Eq. (3) can be simplistically expressed as:

$$V = Aa + Bb - L , \ P \qquad (5)$$

Where a and b represent the unknown parameter vectors (baseline vector and ambiguity vector respectively); A and B are the corresponding coefficient matrix which are assumed known; P is the weight matrix.

The procedure for solving the model like Eq. (3) can be divided into three steps. In the first step, float solution of ambiguity parameters and the covariance matrix can be obtained through the standard least-squares adjustment as follows [Teunissen, 2007]:

$$\begin{bmatrix} a \\ \hat{b} \end{bmatrix} = N^{-1}W \quad , \qquad \begin{bmatrix} Q_a & Q_{a\hat{b}} \\ Q_{\hat{b}a} & Q_{\hat{b}} \end{bmatrix} = N^{-1} \qquad (6)$$

$$\begin{cases} N = (A \ B)^T P(A \ B) \\ W = (A \ B)^T PL \end{cases} \qquad (7)$$

Where N represents the normal matrix; W represents the constant term matrix.

The second and the third step are to search the various combinations of integer ambiguities and fix the integer ambiguity respectively according to the float solution and the covariance matrix. A widely used method in the two later steps is the Least-Square Ambiguity Decorrelation Adjustment (LAMBDA) method [Teunissen, 1995; Teunissen,1997], and this method will be adopted in ambiguity searching and fixing in the next part of the paper.

If the matrix N and W contain minor errors δN and δW, an error δX of the corresponding parameters vector is produced. Then we can get the following equation:

$$(N + \delta N)(X + \delta X) = W + \delta W \qquad (8)$$

Then by some transformation of matrixes and norms, the following formula can be achieved [Liu et al., 2009; Wang et al., 2013]:

$$\frac{\| \delta X \|}{\| X \|} \le \frac{\| N^{-1} \| \bullet \| N \|}{1 - \| N^{-1} \| \bullet \| N \| \frac{\| \delta N \|}{\| N \|}} (\frac{\| \delta W \|}{\| W \|} + \frac{\| \delta N \|}{\| N \|}) \qquad (9)$$

The condition number of a $n \times n$ matrix N is expressed as $cond(N)$:

$$cond(N) = \| N^{-1} \| \bullet \| N \| \qquad (10)$$

We can infer form Eq. (9) that if the condition number of N is too large, it may result in a large bias or major instability of the parameter solution even though the disturbance of N or W is

small. So the condition number is often used as the characterization index of an ill-posed system [Li et al., 2010], and it will be also adopted in this paper.

3 Analysis of ill posedness in BDS Ambiguity Resolution

3.1 Analysis of satellite angular velocity

The running angular velocity of GEO, IGSO satellite from BDS and MEO satellite from GPS are shown in Fig.1. It should be noted that the MEO satellites from BDS have similar orbital altitude and operating characteristic, so the angular velocities of two kinds MEO satellites are nearly the same in values.

(a) GEO satellite of BDS

(b) IGSO satellite of BDS

(c) MEO satellite of GPS

(d) MEO satellite of GLONASS
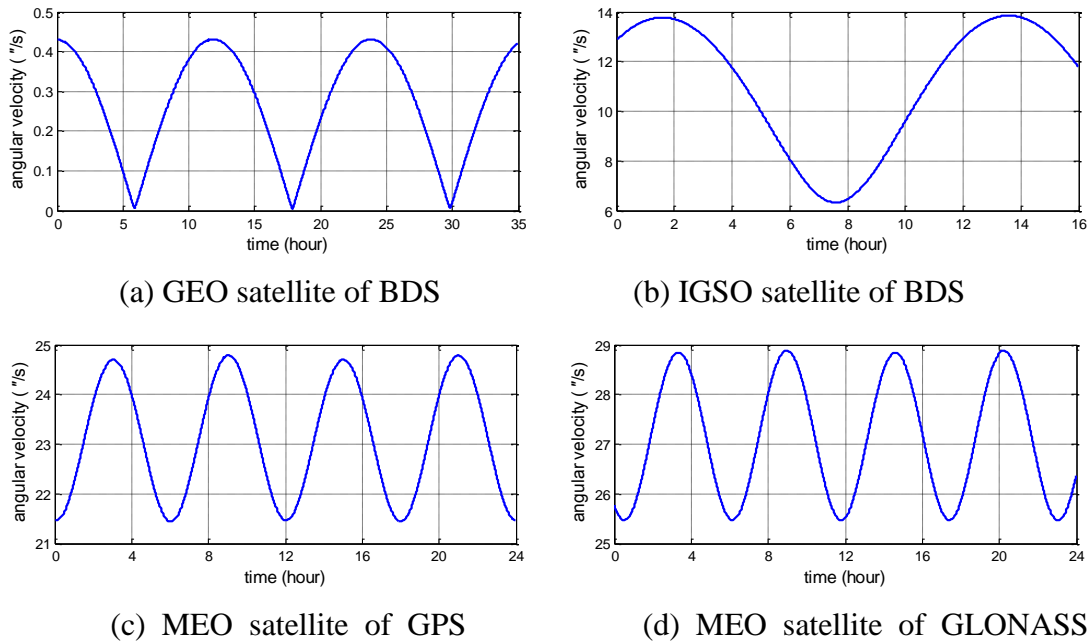
Fig.1 Satellite angular velocity

As can be seen from Fig.1, the angular velocity of GEO satellites are nearly zero due to its earth-synchronous character. And the angular velocity of IGSO satellites is just about 30-50 percentages of that of MEO satellites. This is consistent with their orbital altitude and operating characteristics. If the satellite has a larger angular velocity, the geometric relationship between the satellite and the station would change much more in the process of ambiguity resolution within the same observation time [Gao,2013].So the correlation between observations from adjacent epochs is mainly decided by the satellite running angular velocity. Due to the existence of ambiguity parameters and other parameters (e.g. baseline vectors), the AR system is usually ill-posed, so it generally requires observations from epochs over a long period. There are five GEO satellites and five IGSO satellites in the current BDS, while only four MEO satellites around the earth. In China, the visible satellites which can be observed are mainly GEO and IGSO satellites during most of the time. So it can be inferred that there is more serious ill-posedness in AR of BDS compared to GPS or GLONASS.

3.2 Ill-posednes Experiments for BDS AR

A group of short-distance (about 3m) baseline data including BDS, GPS and GLONASS observations was collected for experiments. The date was collected on Aug 29th 2013 in

Southeast University, Nanjing and the sample interval is 1s. In order to collect complete and detailed data, observation times lasted for 20-minutes (1200 continuous epochs), and for the short baseline data we used in the paper, 20 minutes may be enough for ambiguity resolution. During the observation period, we can use 4 GEO, 4 IGSO and 3 MEO satellites from BDS, 7 MEO satellites from GPS, 7 MEO satellites from GLONASS. The sky plot of visible satellites can be seen in Fig.2.

As we are only interested in the ill-posedness caused by the satellite angular velocity, the DD carrier observations were processed to avoid the influence caused by other factors (such as gross errors of observations, atmospheric delay and so on). The real and reliable ambiguities and station coordinates are achieved by processing the long-time observations. Each DD observation was processed to have the same precision with a random noise ranging from -1cm to 1cm according to Eq. (1) and Eq. (5).
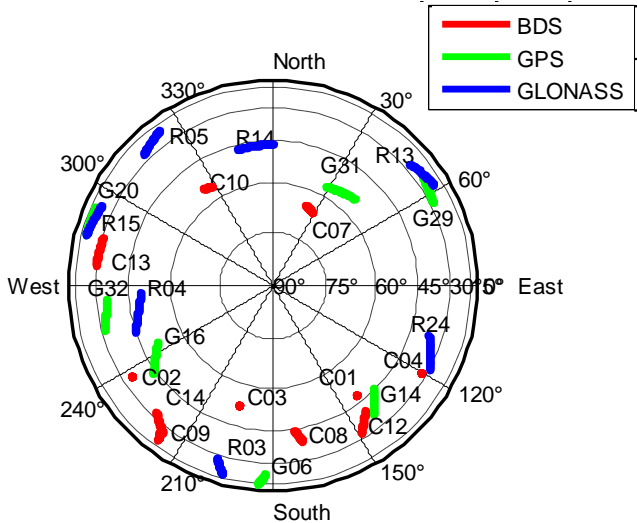

Fig.2 The sky plot of BDS/GPS/GLONASS visible satellites

Table 1 Experiments details with different satellites combination

Experiment No.		Satellites Number	Satellite Combination
BDS	1	7	4GEO+3IGSO
	2	7	3GEO+4IGSO
	3	7	3GEO+2IGSO+2MEO
	4	7	2GEO+3IGSO+2MEO
	5	7	1GEO+4IGSO+2MEO
	6	7	4IGSO+3MEO
GPS	7	7	7GPS
GLONASS	8	7	7GLONASS
GPS+BDS	9	7+7	7GPS+4GEO+3IGSO
GLONASS+BDS	10	7+7	7GLONASS+4GEO+3IGSO
GPS+GLONASS+BDS	11	7+7+7	7GPS+7GLONASS+4GEO+3IGSO

So as to reflect the different influence from different satellites, the following experiments listed in table 1 are carried out. In all of the experiments, the ionosphere-free observations combined of the basis dual frequency observations were used, and the different DD observations are treated as the same weight.

In each experiment we calculate the float ambiguity bias (FloatA for short), the condition number at last with sequential adjustment using 1200 epochs (CondSA for short), the needed time to get correct fixed ambiguities with LAMBDA method (FixTime for short). The results are listed in Table 2. Because the satellite number of BDS reaches 11 in the observation, some combinations in Table 1 are not exclusive. For example, we may have four different choices for Experiment 1 due to the selections of 3 IGSO satellites from the total 4 IGSO satellites. So the results in Table 2 are the mean value when the combinations are not exclusive in each experiment.

Table 2 Calculation results of different experiments with 7 satellites

Experiment No.		Time1/s (FloatA<1)	Time2/s (FloatA<0.5)	FixTime/s	CondSA
BDS	1	None	None	120	1.948e+008
	2	527	1161	87	2.336e+007
	3	413	642	80	9.496e+006
	4	386	560	74	6.788e+006
	5	366	521	74	5.592e+006
	6	336	476	65	1.530e+006
GPS	7	277	329	52	4.437e+005
GLONASS	8	170	315	45	3.802e+005

Note: 'None' means that the FloatA cannot be within ± 1 or ± 0.5 in the 1200 epochs

In the table FloatA<1 and FloatA<0.5 means that the absolute value of FloatA is smaller than 1 or 0.5, and the later results also meet the corresponding condition. The time of FloatA<1 or FloatA<0.5 indicates the convergence speed of float ambiguity solution. As mentioned previously, the observations has been processed to the same precision (-1cm~1cm). So the AR difference among the several experiments is mainly caused by the different satellite composition.

In Experiment1-6, MEO or IGSO satellites with larger angular velocity from BDS were added gradually to replace IGSO or GEO satellites. Seven satellites were used in all six experiments. As we can see from Table2, Time1 and Time2 become gradually shorter. This indicates that the convergence of ambiguities become faster. Moreover, the FixTime with LAMBDA method is also shorter in general, which indicates that the correlation among unknown parameters is weakened. Besides the LAMBDA method, other ambiguity search method may get the similar result as the ill-posedness of the variance-covariance matrix is an inherent property of itself. This is also reflected in the condition number in the last column of Table 2, where the condition number becomes smaller and smaller. The reason for the fact is that the larger running angular velocity will make the geometric relationship between the satellite and the station change more rapidly, so the ill-posedness can be reduced faster.

In order to reflect the difference of the AR effect in detail, the selected three representative experimental results of BDS (Experiment 1, 3, 6, 4GEO+3IGSO, 3GEO+2IGSO+2MEO, 4IGSO+3MEO respectively) are shown in Fig.3. In the three experiments, the numbers of GEO satellites change from 4 to 0, while the numbers of MEO satellites change from 0 to 3. The condition number of the covariance matrix during the calculating process of the six experiments of BDS is shown in Fig.4.
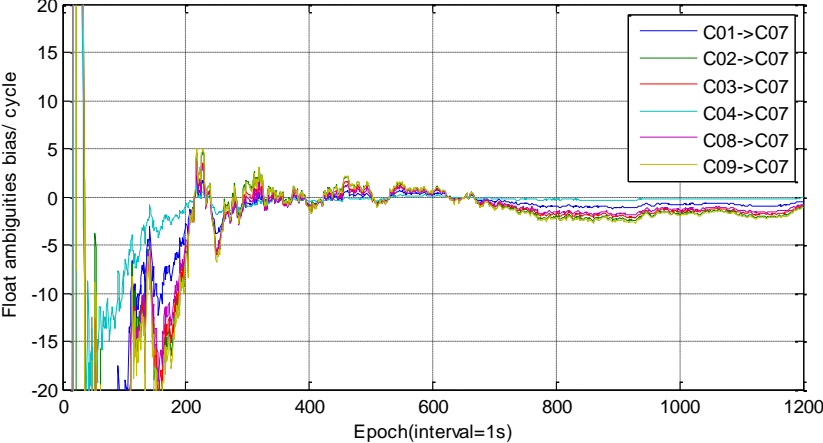
(a) Experiment 1
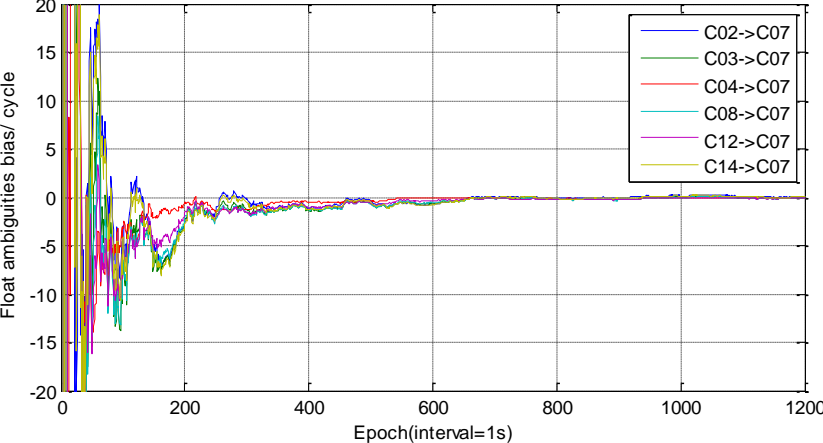
(b) Experiment 3
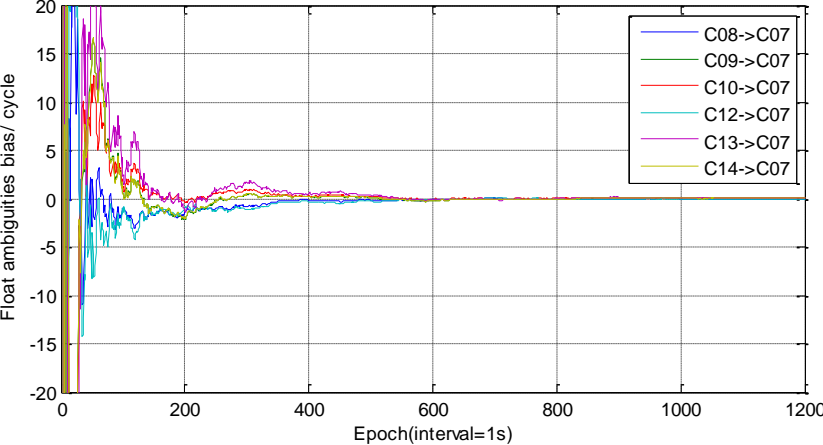
(c) Experiment 6

Fig.3 Float ambiguities bias of the three representative experiments of BDS. Experiment 1 contains 4 GEO satellites and 3 IGSO satellites (all high-orbit satellites); Experiment 4 contains

2 GEO satellites, 3 IGSO satellites and 2 MEO satellites; Experiment 6 contains 4 IGSO satellites and 3 MEO satellites (without GEO satellite).
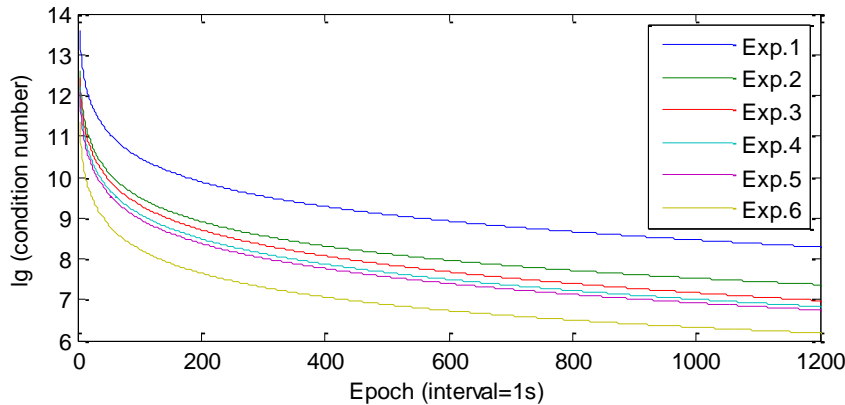

Fig.4 Condition number of the six different experiments of BDS

From Fig.3(a) we can see that in Experiment 1 with all high-orbit satellites, the float ambiguity biases are much larger and unstable during the whole process. The float ambiguity biases cannot be reduced to within 1 cycle or 0.5 cycles even by the end of the 20 minutes. This means that some straightforward method without search process (e.g. rounding-off method) cannot be applied to obtain the correct ambiguities results of the nearly zero-distance baseline in 20 minutes. Fig.3(b) and Fig.3(c) show the AR effect becomes better. This is mainly reflected on that the ambiguity biases become smaller while the stability is improved. Fig.4 shows the condition number of the six experiments of BDS in the whole AR process. We can see the ill-posedness levels also match the AR effects well.

3.3 Comparison between BDS and GPS, GLONASS

In order to analyze the ill-posedness of BDS with a different approach, the AR experiments of GPS and GLONASS for comparison with BDS were also carried out. The satellite number of GPS and GLONASS are both seven. The calculating results are listed in the last two rows of Table 2. The float ambiguities biases are shown in Fig.5 and Fig.6, and the comparison of condition number with BDS (Experiment 6) is in Fig.7.
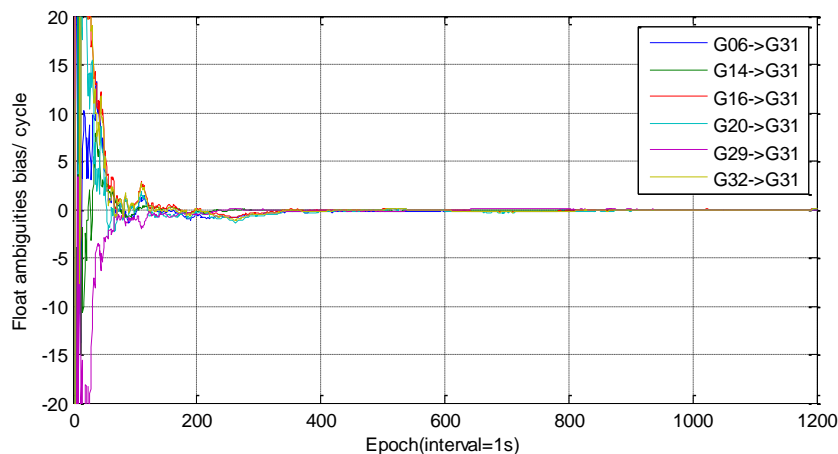

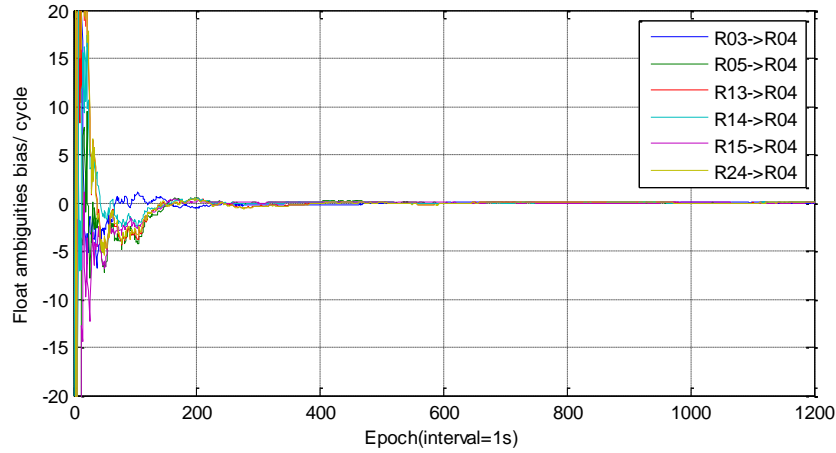Fig.5 Float ambiguities bias of Experiment 7 (GPS)

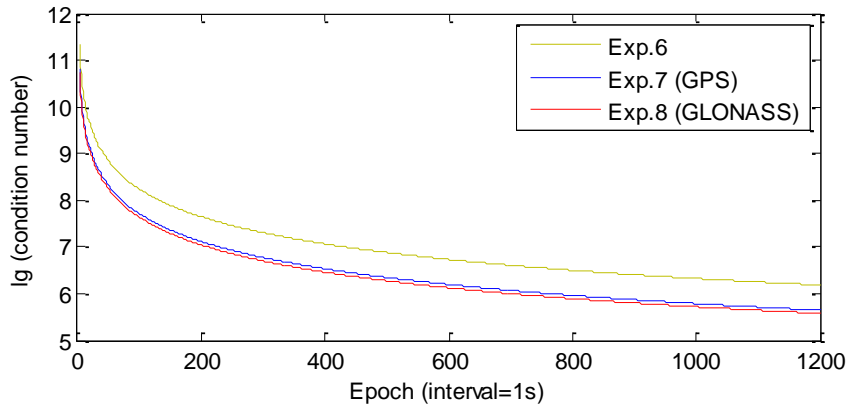Fig.6 Float ambiguities bias of Experiment 8 (GLONASS)

Fig.7 Comparison of Condition number between BDS and GPS, GLONASS

From Table 2 we can see the AR effects of GPS and GLONASS are both better than BDS, even the best-effect combination without GEO satellites. Although IGSO satellites run much faster than GEO satellites, their angular velocities are just about 30~50 percentage of MEO satellites. So there exists much more serious ill-posedness problem in the DD AR of BDS. Fig.5 and Fig.6 also indicate that the convergence of float ambiguities of GPS and GLONASS is much faster than BDS, and much more stable. The comparison of condition number in Fig.7 shows that BDS has the strongest ill-posedness whereas GLONASS has the slightest one in AR. And this also corresponds to their orbital altitude or angular velocities.

4 Fusion of BDS and GPS, GLONASS

From **Part 3**, we found that the current BDS has much more stronger ill-posedness in AR process especially when its MEO satellites cannot be observed. At present, BDS and GPS or GLONASS satellites can be used together for ambiguity resolution adjustment, as they have common unknown parameters, like the baseline vector or others. However, there are some differences among different systems, such as the time system, the coordinate reference system and so on. The time system can be unified according to [Deng et al., 2013]. There are small differences among the three coordinate reference systems so the absolute coordinates in the three coordinate reference systems are a little different. However, when it comes to baseline vectors (the difference value of coordinates), they can be treated as the same for the three systems. It is better to choose their own reference satellites respectively as this will avoid some

complex error terms like the error caused by different channels.

The worst-AR-effect one of BDS (i.e. Experiment 1) and GPS or GLONASS (Experiment 7 and Experiment 8) is combined together to calculate ambiguities. The calculating results are listed in Table 3, and the float ambiguity biases from Experiment 11 can be seen from Fig.8.

Table 3 Calculation results of fusion AR experiments

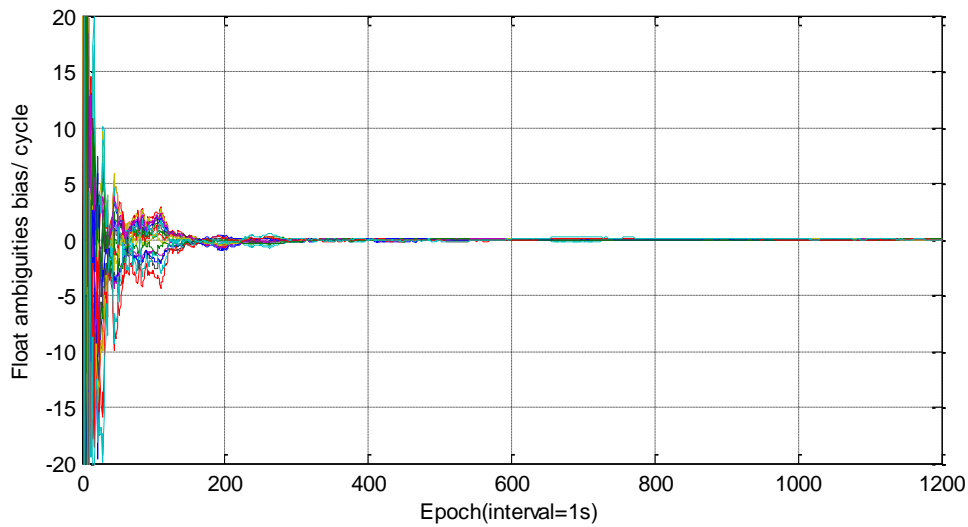| Experiment No.		Time1/s (FloatA<1)	Time2/s (FloatA<0.5)	FixTime/s	CondSA
GPS+BDS	9	195	285	7	1.187e+006
GLONASS+BDS	10	178	277	6	9.644e+005
GPS+GLONASS+BDS	11	139	270	2	7.859e+005

Fig.8 Float ambiguities bias of Experiment 11 (with all the three systems)

From Table 2 we can see that the AR effects from all the three fusion experiments become much better than all the six experiments of BDS, even better than that of GPS and GLONASS. In the FixTime with LAMBDA method, only a few epochs are required to achieve the correct AR solution. Furthermore, when all three systems are used together, just 2 epochs are needed to get the correct fixed ambiguities, which is almost instantaneous. Fig.8 also shows the faster and more stable ambiguity convergence. It should be noted that the condition number in Table 3 becomes larger than that of GPS or GLONASS alone. However, this does not mean that the ill-posedness has become more serious, because the number of unknown parameters or the dimensions of the covariance matrix have not been the same.

5 Conclusions

In the paper we analyse the ill-posedness in DD AR of the current BDS, two-thirds of whose main components are high-orbit satellites. The main conclusions are as follows.

(1) The smaller angular velocities of high-orbit satellites lead into the stronger ill-posedness in DD AR model of BDS, especially when the only four MEO satellites cannot be observed. When just high-orbit satellites of BDS are used for AR, the float ambiguity bias even cannot be within ± 1 cycle in 20 minutes. With the increase of MEO or IGSO satellites, the ill-posedness

becomes weaker significantly.

(2) The ill-posedness in AR of GPS or GLONASS is much weaker than that of BDS. The numbers of epochs needed for AR fixing of the two systems are 52 and 45 respectively, which are both much fewer than anyone of the six experiments of BDS. This is mainly attributed to faster angular velocity of MEO satellite.

(3) The fused ambiguity resolution of BDS and GPS or GLONASS will solve the ill-posed problem effectively. When all the three systems are combined together, the fixing of integer ambiguity becomes almost instantaneous with just two epochs. So for the current BDS, especially when only high-orbit satellites can be observed, it is better to fuse GPS or GLONASS and BDS together to ambiguity resolution if possible.

Acknowledgments

Reference

[1] Deng, C. L., Tang, W. M., Liu, J. N., & Shi, C., 2013. Reliable single-epoch ambiguity resolution for short baselines using combined GPS/BeiDou system. *GPS Solutions*, http://link.springer.com/article/10.1007/s10291-013-0337-5.

[2] Gao, W., Gao, C. F., Pan, S. G., Wang, S. L., & Wang, D. H., 2013. The Analysis of Ill Posedness in GNSS High-Precision Differential Positioning. In: *China Satellite Navigation Conference (CSNC) 2013 Proceedings* (pp. 311-321). Springer Berlin Heidelberg.

[3] Hao, W. F., & Li, F., 2010. A new method for ill-conditioned diagnosis based on spatial analysis. *J Inf Comput Sci*, *7*(9), 1846-1853.

[4] Li, B. F., Shen, Y. Z., & Feng, Y. M., 2010. Fast GNSS ambiguity resolution as an ill-posed problem. *Journal of geodesy*, *84*(11), 683-698.

[5] Liu, C. S., & Chang, C. W., 2009. Novel methods for solving severely ill-posed linear equations system. *J. Marine Sci. Tech*, *17*, 216-227.

[6] Ran, C. Q., 2012. Development of the BeiDou Navigation Satellite System. In: *Global Navigation Satellite Systems. Report of the Joint Workshop of the National Academy of Engineering and the Chinese Academy of Engineering. Washington, DC*.

[7] Shen, Y. Z., & Li, B. F., 2007. Regularized solution to fast GPS ambiguity resolution. *Journal of Surveying Engineering*, *133*(4), 168-172.

[8] Teunissen, P. J. G., de Jonge, P. J., & Tiberius, C. C. J. M., 1994, September. On the spectrum of the GPS DD-ambiguities. In *Proceedings ION GPS-94, 7th International Technical Meeting of the Satellite Division of the Institute of Navigation, Salt Lake City, UT, Sept 20* (Vol. 23, pp. 115-124).

[9] Teunissen, P. J. G., 1995. The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. *Journal of Geodesy*, *70*(1-2), 65-82.

[10] Teunissen, P. J. G., De Jonge, P. J., & Tiberius, C. C. J. M., 1997. Performance of the LAMBDA method for fast GPS ambiguity resolution.*Navigation*, *44*(3), 373-383.

[11] Teunissen, P. J. G., 2007. Influence of ambiguity precision on the success rate of GNSS integer ambiguity bootstrapping. *Journal of Geodesy*, *81*(5), 351-358.

[12] Wang, S. L., Wang Q., Gao W., & Pan S. G., 2013. Analysis and valuation of ill-condition

in baseline solution of GNSS multi-system. *Journal of Southeast University (Natural Science Edition)*, *43*(4), 753-757.

[13] Xu, P. L., Shen, Y. Z., Fukuda, Y., & Liu, Y. M., 2006. Variance component estimation in linear inverse ill-posed models. *Journal of Geodesy*, *80*(2), 69-81.

[14] Yang, Y. X., Li, J. L., Xu, J. Y., Tang, J., Guo, H. R., & He, H. B., 2011. Contribution of the compass satellite navigation system to global PNT users. *Chinese Science Bulletin*, *56*(26), 2813-2819.

[15] Yang, Y. X., 2010. Progress, contribution and challenges of Compass/Beidou satellite navigation system. *Acta Geodaetica et Cartographica Sinica*, *39*(1), 1-6.

[16] Zhao, Q. L., Guo, J., Li, M., Qu, L. Z., Hu, Z. G., Shi, C., & Liu, J. N., 2013. Initial results of precise orbit and clock determination for COMPASS navigation satellite system. *Journal of Geodesy*, *87*(5), 475-486.

[17] Zhou, S., Cao, Y., Zhou, J., Hu, X., Tang, C., Liu, L., & Wu, B., 2012. Positioning accuracy assessment for the 4GEO/5IGSO/2MEO constellation of COMPASS. *Science China Physics, Mechanics and Astronomy*, *55*(12), 2290-2299.