A Model for Fresh Produce Shelf Space Allocation and Inventory Management with Freshness Condition Dependent Demand

Ruibin Bai, Graham Kendall

Automated Scheduling, Optimisation and Planning (ASAP) Research Group
School of Computer Science & IT, University of Nottingham, Nottingham NG8 1BB, UK
rzb/gxk@cs.nott.ac.uk

Abstract: A significant amount of work has investigated inventory control problems associated with fresh produce. Much of this work has considered deteriorating inventory control with many models having been proposed for the various situations that exist. However, no researchers have specifically studied fresh produce which has its own special characteristics. Most research categorise fresh produce into more general deteriorating categories with random lifetimes and non-decaying utilities. However, this classification is not reasonable or practical because the freshness condition usually plays a very important role in influencing the demand for the produce, which drops gradually over time. In this paper, a single-period inventory and shelf space allocation model is proposed for fresh produce. These items usually have a very short lifetime. The demand rate is assumed to be deterministic and dependent on both the displayed inventory (the number of facings of items on the shelves) and the items' freshness conditions. The freshness condition drops continuously over time according to a known function. Several problem instances of different sizes are given and solved by a modified generalised reduced gradient (GRG) algorithm.

1. Scope and Purpose

The profit on general foods, such as cans, frozen vegetables, fruit juice, etc., is gradually decreasing due to highly competitive retail conditions. The demand for these products is also slowing. On the other hand, the demand for some other merchandise, such as fresh produce, organic food and children clothes, has increased dramatically owing to improving living standards. This requires retailers to concentrate more in these areas (Johnson 2002). In this paper, we formulate a mathematical model in order to assist in ordering and shelf allocation decisions for the retail of fresh produce, such as vegetables, fruits, fresh meats, etc. The main characteristics of these items are their very short shelf-life and decaying utilities over time. Preliminary experiments are conducted (using a modified generalised reduced gradient algorithm) in order to demonstrate that good quality solutions can be found. Most of the literature have treated fresh produce as deteriorating items with a random lifetime and non-decaying utilities (Nahmias 1982, Goyal and Giri 2001). In this paper, we assume that the produce has a continuous utility and physically deteriorates over time. Freshness is one of the main criteria to evaluate a product's quality and could dramatically affect its demand if its condition is inferior. To obtain a good financial performance from fresh goods requires the adoption of strict temperature control and intelligent inventory and shelf management systems. Furthermore, although a large number of deteriorating inventory models have been proposed in previous research, most of them are based on the analysis of a single item excluding the constraints of shelf space which come when considering a range of goods. No researchers have integrated a deteriorating inventory model with a shelf space allocation model (which plays a very important role in retail decision making due to the scarce shelf space resources). In this paper, we formulate a fresh produce management model which can simultaneously decide the ordering policy as well as allocate shelf space among different items, together with consideration of utility (i.e. freshness) deterioration.

2. Background

Perishable inventory has been intensively studied and a large number of models have been proposed in the literature. See (Nahmias 1982, Raafat 1991, Goyal and Giri 2001) for comprehensive reviews. However, most models assume that a fixed fraction of the inventory deteriorates over time but the utilities of the items do not decay before their expiration dates. Few models

specifically consider the fresh produce with the characteristics we mentioned in section 1. In summary, these models have the following drawbacks: 1). Most models (Liu 1990, Jain and Silver 1994) assume that fresh produce, such as vegetables, fruits and fresh meats, have a random lifetime (normally assuming an exponentially distributed lifetime) but the item utilities do not decay over time. Hence different ages of items capture the same demand however fresh they are as long as they are not completely spoilt. This is contradictory to the common sense view that freshness is one of the most important quality criteria for fresh produce. 2). Some models (Mandal and Phaujdar 1989, Giri et al. 1996) formulate the demand as a deterministic function of instantaneous inventory with the assumption that all stock could be displayed on the shelves. However, this situation seldom occurs in most supermarkets because the shelf space for fresh food is normally limited. It is also expensive due to the low temperature requirements. Therefore, only a part of the inventory can be displayed on the shelf. Shelf space allocation among different items is especially important in this situation. The significance of shelf space allocation for non-perishable merchandise has already been addressed in previous research (Kotzan and Evanson 1969, Curhan 1972, Borin et al. 1994, Urban 1998, Yang and Chen 1999, Bai and Kendall 2005). 3). The approaches that were used to optimise the models (Ben-Daya and Raouf 1993, Kar et al. 2001) disregarded the integer nature of the solution and assumed that the objective function is a quasi-concave function and is differentiable. The last assumption is usually too strict for problems involving many constraints.

Different deteriorating inventory models have been classified into two types in the literature: fixed lifetime models and random lifetime models. Examples of fixed lifetime models include photographic films, medicine, computer chips, canned food, etc. A major characteristic of this type of model is that inventory allowed for different ages of items with either a First-In-First-Out (FIFO) or Last-In-First-Out (LIFO) issuing policy (Nandakumar and Morton 1990, Liu and Lian 1999). However, fresh produce was usually treated as a typical example of a random lifetime product due to the uncertain spoilage (Liu 1990, Jain and Silver 1994). These models usually assumed a constant fraction of inventory decay or obsolesce over time (called *exponential decay* in some publications).

Since fresh produce only has a very limited shelf life, most of the literature employed a single-period inventory model although different forms of demand function are used. Both stochastic and deterministic demand inventory models were proposed for the perishable products. Ben-

Daya and Raouf (1993) proposed a multi-item, single-period perishable inventory model with a uniform distribution demand. The objective was to maximise the total profit of all the items during one period. The "optimal" solution was calculated by a Lagrangian optimisation with the assumption that the objective is differentiable. The integer nature of the variables was also disregarded. Furthermore, the method is not efficient when there are a large number of constraints. Rajan et al. (1992) proposed a dynamic pricing and ordering decision making model for decaying produce, in which the demand was assumed to be deterministic and dependent on the selling price. The products are assumed to have an exponential deterioration. Abad (1996) formulated the demand function as a function of instantaneous price. A closed-form mathematical procedure was carried out to solve the problem and parameter sensitivities were analysed. However, the approach is heavily dependent on the mathematical description of the model so that even adding a single constraint could result in this approach becoming invalid. Some other models formulated the demand as a deterministic function of instantaneous inventory. Mandal and Phaujdar (1989) formulated a single-period inventory model for deteriorating items. The demand rate was linearly dependent on the instantaneous inventory level and the inventory deteriorated according to a given function. Backordering was allowed and holding and shortage costs were also considered in the model. The objective was to minimise the average cost. The model was optimised by applying the derivative to the objective function. The variables included the time slots for different inventory stages and maximal stock level and maximal stock deficit. Giri et al. (1996) formulated the demand as a polynomial function of the instantaneous inventory in their perishable inventory model which also assumed an exponential decay. The objective is to maximise the profit, with order quantity and reorder point (or cycle time) as decision variables. Some time-dependent demand functions were also proposed in deteriorating inventory models to describe changing demand over time. Xu and Wang (1990) assumed a linear time-dependent demand function within a limited time horizon. Exponentially time-dependent demand were also proposed to simulate a rapidly increasing/declining market (Hollier and Mak 1983, Zhou et al. 2003). Yet Urban and Baker (1997) used a multiplicative demand function of price, time and inventory level in their single-period inventory model with the aim of finding optimal ordering and pricing policies for non-perishable products.

The first research to consider the effect of utility deterioration on demand was described by (Fujiwara and Perera 1993) in the formulation of an Economic Order Quantity (EOQ) perishable

inventory model. In this publication, an exponential penalty function $\alpha(e^{\beta t} - 1)$ $(\alpha > 0, \beta > 0)$ was used to measure the cost of keeping an aging item in inventory. A closed form of economic order quantities was obtained by a quadratic approximation of exponential terms. The results show that this model is consistent with other EOQ models with exponential decay. Sarker et al. (1997) also attempted to incorporate the negative effect of aging inventory on demand. In their production-inventory model, the demand function in the inventory build-up phase and depletion phase considered a constant term and a negative term which is proportional to the instantaneous inventory (i.e. $f(t) = D - \beta I(t)$, where $f(t)$ is the demand function, $\beta > 0$, D is constant demand and $I(t)$ is the instantaneous inventory level). However, illogically, the demand during the inventory depletion phase is actually an increasing function due to the continuous decrease of the inventory $I(t)$ over time. This contradicts the authors' initial intention to represent a declining demand with the aging of the inventory.

Almost all of the models described above only consider a single item without any constraints being included, with the optimal solution normally obtained by some mathematical derivations. Recently, researchers have begun to incorporate the shelf space allocation technologies into their inventory systems. Kar et al. (2001) proposed a single-period inventory model for multi-deteriorating items with the constraints of shelf space and investment. The problem considers selling the deteriorating items from two stores. At the beginning of the period, the ordered items are separated into fresh items and items that have begun to deteriorate. The fresh items are shipped to the main store, selling with a high price and the deteriorating items are delivered to the second store and sold at a lower price. During the period, all decayed items in the main store are retained and delivered to the second store. The demand rate in the first store was formulated as a function of the item selling price and instantaneous inventory. However, the demand in the second store was only dependent on the selling price. A Generalised Reduced Gradient (GRG) method was used to optimise the model. However, as stated in (Lasdon et al. 1978), GRG may not be efficient or robust for larger problem sizes and can only guarantee a local optimum. Besides, the non-integer variables and continuous objective assumption are the major drawbacks of this approach in solving many NP-hard problems with integer variables. Hence, some meta-heuristic approaches (Glover and Kochenberger 2003) have been introduced into this area to optimise these models. Borin et al. (1994) used a simulated annealing approach to solve a product assortment and shelf space allocation problem. Genetic algorithms were employed in Urban's

publication (Urban 1998) to solve an integrated product assortment, inventory and shelf space allocation model.

3. Model Formulation

Instead of assuming that fresh food has a random lifetime with an exponential decay, here we assume that fresh food has predictable expiry dates but their freshness condition decreases continuously according to a known function over time. The demand for the fresh produce is deterministic and is both dependent on the displayed inventory level and their freshness condition. The main difference between these two assumptions is that the former assumed that all items that have not yet deteriorated capture the same demand however fresh they are. This may sound reasonable for long lifetime perishable items (like photographic films and medicine) but is unrealistic for fresh produce as freshness is one of the most important aspects in evaluating their quality. In this paper, all fresh items are assumed to have a fixed, but very short, lifetime and will not entirely lose utilities before their expiration date. However, freshness keeps deceasing over time, which has an effect on demand. It should be noted that the assumption of a fixed lifetime of fresh produce, with decreasing utilities is realistic considering the advances in food planting, packing and conservation technologies, especially the introduction of temperature control systems in most supermarkets.

The following notations are used in our model:

- $D_i(t)$ is the demand function of item i over time.
- $f_i(t)$ is a decreasing function (within range [0,1]) representing the freshness condition of item i over time.
- $I_i(t)$ is the inventory level of item i at time t.
- q_i is the procurement quantity of item i.
- s_i is the number of the facings assigned to item i.
- r_i is the surplus of item i at the end of the cycle.
- W is the total shelf space available.
- a_i is the space required for one facing of item i.
- p_i is the unit selling price of item i.

- p_{di} is the unit discount price of item i. This price should be low enough such that all of the remaining items at the end of period can be sold out in a very short time at this price.

- c_{ai} is the unit acquisition cost of item i (or unit procurement price).

- c_{hi} is the unit holding cost of item i (including the costs caused by inventory loses, damage, maintenance, interest, insurance, etc.).

- c_s is the shelf cost per unit space.

- C_{oi} is the constant order cost of item i (independent of the order quantity).

- T_{ei} is the lifetime of item i after which the item is rotten (i.e. cannot be sold).

- L_i is the lower bound of the number of facings of item i.

- U_i is the lower bound of the number of facings of item i.

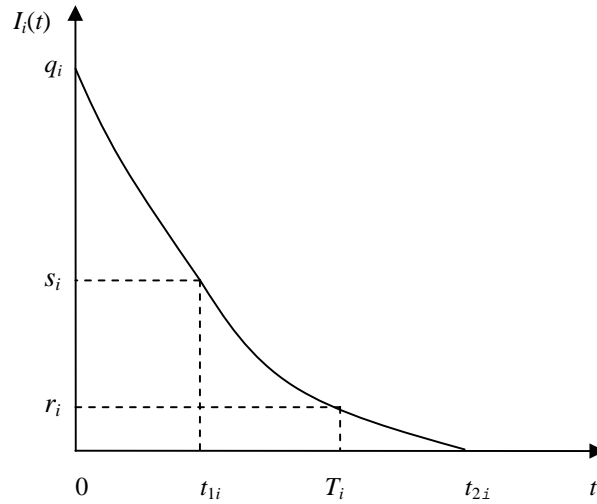- T_i is the length of the cycle period of item i.

Figure 1: Graphical Representation of Inventory Level Changes Over Time

Many researchers (e.g. Kar et al. 2001, Urban 2002) use the function depicted in figure 1 to describe the change of inventory level over time t. From time 0 to t_{1i}, s_i facings of item i are displayed on the shelf with some of the stock stored in the backroom. As sales are made, the items in the backroom are moved to the shelf until the stock in backroom reaches zero (corresponding to the point when time reaches t_{1i}). Therefore, during this period, the shelf is fully stocked and the demand is only a function of product freshness. From time t_{1i} to t_{2i}, the shelf is only partly stocked and the demand is both dependent on the freshness and the instantaneous inventory

level. Once the time reaches point T_i, a new order of quantity q_i is placed for item i (assuming no lead time) and the r_i surplus of item i are sold at a discount price p_{di}. In this research, we will adopt this representation together with a polynomial demand function that is widely used in many shelf space allocation models (Corstjens and Doyle 1981, Giri et al. 1996, Urban and Baker 1997, Urban 1998):

$$D_i^*(t) = \begin{cases} \alpha_i s_i^{\beta_i} & 0 \le t \le t_{1i} \\ \alpha_i [I_i(t)]^{\beta_i} & t_{1i} < t \le t_{2i} \end{cases} \tag{1}$$

where α_i and β_i are scale parameters and the space elasticity of item i respectively and $\alpha_i > 0$, $0 < \beta_i < 1$. In this paper, we assume that the demand function conforms to a multiplicative form of the instantaneous inventory and the item's freshness condition, i.e. $D_i(t) = D_i^*(t) \cdot f_i(t)$ where $f_i(t)$ is a continuously decreasing function over time and $0 \le f_i(t) \le 1$. $f_i(t)$ could be a linear, quadratic or exponential function of time. During the beginning of the period, the items are fresh and the value of freshness function is almost 1. The demand rate is only affected by the displayed inventory level. However, as time elapses, $f_i(t)$ gradually decreases and the demand is scaled down according to how long an item has been kept in inventory. To be consistent with the exponential decay assumption in the literature, here, we assume that an items' freshness condition decreases exponentially over time, i.e. $f_i(t) = e^{-\sigma_i t}$, where σ_i is a constant decay rate and $\sigma_i > 0$. Hence we have:

$$D_i(t) = D_i^*(t) \cdot f_i(t) = \begin{cases} \alpha_i s_i^{\beta_i} e^{-\sigma_i t} & 0 \le t \le t_{1i} \\ \alpha_i [I_i(t)]^{\beta_i} e^{-\sigma_i t} & t_{1i} < t \le t_{2i} \end{cases} \tag{2}$$

Based on the assumptions above, the inventory level of item i can be described by the following differential equation:

$$dI_i(t)/dt = -D_i(t) \tag{3}$$

During time $[0, t_{1i}]$, we have

$$dI_i(t)/dt = -\alpha_i s_i^{\beta_i} e^{-\sigma_i t} \tag{4}$$

with the boundary conditions $I_i(0) = q_i$ and $I_i(t_{1i}) = s_i$. The solution of eq. (4) is:

$$I_i(t) = q_i + \frac{\alpha_i s_i^{\beta_i}}{\sigma_i}(e^{-\sigma_i t} - 1) \tag{5}$$

and

8

$$t_{1i} = -\frac{1}{\sigma_i} \ln(1 - \frac{(q_i - s_i)\sigma_i}{\alpha_i s_i^{\beta_i}}) \tag{6}$$

During time $[t_{1i}, t_{2i}]$, we have the following differential equation:

$$dI_i(t)/dt = -\alpha_i [I_i(t)]^{\beta_i} \cdot e^{-\sigma_i t} \tag{7}$$

with the boundary conditions $I_i(t_{1i}) = s_i$ and $I_i(t_{2i}) = 0$. The solution of eq. (7) is:

$$I_i(t) = [\frac{\alpha_i(1-\beta_i)}{\sigma_i} e^{-\sigma_i t} + K_i]^{\frac{1}{(1-\beta_i)}} \tag{8}$$

and

$$t_{2i} = -\frac{1}{\sigma_i} \ln(1 - \frac{(q_i - \beta_i(q_i - s_i))\sigma_i}{\alpha_i(1-\beta_i)s_i^{\beta_i}}) \tag{9}$$

where $K_i = [q_i - \beta_i(q_i - s_i)]s_i^{-\beta_i} - \mu_i$ and $\mu_i = \frac{\alpha_i(1-\beta_i)}{\sigma_i}$.

In general, we have the following inventory function:

$$I_i(t) = \begin{cases} q_i + \frac{\alpha_i s_i^{\beta_i}}{\sigma_i}(e^{-\sigma_i t} - 1) & 0 \leq t \leq t_{1i} \\ \\ [\mu_i e^{-\sigma_i t} + K_i]^{\frac{1}{(1-\beta_i)}} & t_{1i} < t \leq t_{2i} \end{cases} \tag{10}$$

The length of cycle period T_i ($I_i(T_i) = r_i$) is:

$$T_i = -\frac{1}{\sigma_i} \ln\left[\frac{1}{\mu_i}(r_i^{(1-\beta_i)} - K_i)\right] \tag{11}$$

The holding cost during $[0, t_{1i}]$ is:

$$HC_{1i} = c_{hi} \int_0^{t_{1i}} (q_i + \frac{\alpha_i s_i^{\beta_i}}{\sigma_i}(e^{-\sigma_i t} - 1))dt = c_{hi}[(q_i - \frac{\alpha_i s_i^{\beta_i}}{\sigma_i})t_{1i} + (1 - e^{-\sigma_i t_{1i}})\frac{\alpha_i s_i^{\beta_i}}{\sigma_i^2}] \tag{12}$$

The holding cost during $[t_{1i}, T_i]$ is:

$$HC_{2i} = c_{hi} \int_{t_{1i}}^{T_i} ([\mu_i e^{-\sigma_i t} + K_i]^{\frac{1}{(1-\beta_i)}})dt \tag{13}$$

The approximate expression of HC_{2i} is given in the Appendix. However, calculation results show that this part is very small and a simpler approximation is used in this paper (using $(s_i + r_i)/2$ as an approximation of average inventory during $[t_{1i}, T_i]$):

$$HC_{2i} = c_{hi}[s_i + r_i](T_i - t_{1i})/2 \tag{14}$$

Therefore, the average profit of item i per unit time is the total income less any costs involved divided by the time of the period, we have:

$$M_i = \frac{1}{T_i}[p_i(q_i - r_i) + p_{di}r_i - c_{ai}q_i - C_{oi} - HC_{1i} - HC_{2i}] - c_s s_i a_i \tag{15}$$

The objective is to maximise the overall profit of all items during the unit time:

$$\max \qquad \sum_{i=1}^{n} M_i(s_i, q_i, r_i) \tag{16}$$

$$\text{subject to} \quad \sum_{i=1}^{n} s_i a_i \leq W \tag{17}$$

$$L_i \leq s_i \leq U_i \qquad i = 1, 2, ..., n \tag{18}$$

$$r_i \leq s_i \leq q_i \qquad i = 1, 2, ..., n \tag{19}$$

$$r_i < q_i \qquad i = 1, 2, ..., n \tag{20}$$

$$0 < T_i \leq T_{ei} \qquad i = 1, 2, ..., n \tag{21}$$

$$s_i, q_i \in \{1, 2, 3, ...\} \quad i = 1, 2, ..., n \tag{22}$$

$$r_i \in \{0, 1, 2, ...\} \qquad i = 1, 2, ..., n \tag{23}$$

The decision variables are shelf space, order quantity and the amount of surplus at the end of the cycle. Constraint (17) ensures that the total shelf space allocated to each item is no more than the total available shelf space. Constraint (18) makes sure that the space allocated to each item must be within an upper and a lower bound. Constraint (19) ensures sure that the order quantity of each item must be greater than the shelf displayed quantity which itself should be greater than the number of surplus. Constraint (21) ensures that the span of one cycle period must be less than the product validity period. Constraint (22) and (23) ensures than the number of facings, order quantity and the number of surplus are integers. The model is a non-linear combinatorial optimisation problem and is difficult to optimise by utilising conventional mathematical approaches.

Suppose we have n products, the total number of variables is $3 \times n$. From the model, we have the upper and lower bounds of variables r_i ($0 < r_i \leq s_i$) and s_i ($L_i < s_i \leq U_i$ and lower bound of q_i ($q_i \geq s_i$). The upper bound of q_i can be obtained from constraint (21). Since

$$T_i = -\frac{1}{\sigma_i} \ln\left[\frac{1}{\mu_i}(r_i^{(1-\beta_i)} - K_i)\right] \leq T_{ei} \tag{24}$$

we have

$$q_i \leq \frac{1}{(1-\beta_i)} r_i^{(1-\beta_i)} s_i^{\beta_i} + \frac{\alpha_i}{\sigma_i} s_i^{\beta_i} - \frac{\beta_i}{(1-\beta_i)} s_i - \frac{\alpha_i}{\sigma_i} e^{-\sigma_i T_{ei}} s_i^{\beta_i} \tag{25}$$

Let $\lfloor x \rfloor$ represents the largest integer no greater than value x, the upper bound of order quantity q_i^{ub} is

$$q_i^{ub} = \left\lfloor \frac{1}{(1-\beta_i)} r_i^{(1-\beta_i)} s_i^{\beta_i} + \frac{\alpha_i}{\sigma_i} s_i^{\beta_i} - \frac{\beta_i}{(1-\beta_i)} s_i - \frac{\alpha_i}{\sigma_i} e^{-\sigma_i T_{ei}} s_i^{\beta_i} \right\rfloor \tag{26}$$

An interesting derivation of the model is that inventory depletes exponentially over time (see eq.(10)), which is consistent with the exponential decay models in the literature. In addition, when $\sigma_i \to 0$, $e^{-\sigma_i t} \to 1 - \sigma_i t$, the inventory function becomes the same polynomial function derived in (Urban 2002).

4. Optimisation of the Model

We use a generalised reduced gradient (GRG) algorithm to search for a good quality solution to the problem subject to the model (16). The underlying ideas of the algorithm were described in (Gabriele and Ragsdell 1977, Lasdon et al. 1978). The GRG algorithm has been shown to be efficient in solving non-linear programming problems with smooth objective functions and its applications in optimising the inventory and shelf space allocation model include (Urban 1998, Kar et al. 2001), with good results being reported. The GRG algorithm is imbedded in many spreadsheet software packages. The one we used is called Solver that is included in Microsoft Excel 2002. However, GRG algorithm has two major drawbacks: 1. it can only solve continuous-variable models. Although the package included in Microsoft Excel 2002 can deal with integer variables, it takes too long for the search to converge (1800 seconds computation time is needed even for a problem with 6 items, running on a PC with Pentium IV 1.8GHZ and 256MB RAM. For a problem with 18 products, the algorithm does not converge even after one hour). 2. GRG usually only gives a local optimum which is closest to the initial solution. Some preliminary experiments showed that, if the initial solution is not carefully chosen, GRG performed very badly. To solve these shortcomings, in this application, we used a multi-thread GRG algorithm together with a solution repair heuristic to optimise the model. Each thread of the algorithm can be divided into three sub-procedures: initialisation, GRG calling and solution repair, described in figure 2.

To prevent the GRG getting stuck at a local optimum, *MaxIter* runs of GRG were executed using different initial states (solutions) and the best solution was output as the final solution. In this application, we set *MaxIter* = 5 after some preliminary experiments. The initialisation

---

Set *MaxIter*;
Set *iter* = 0;
Loop
    //Initialisation sub-procedure
    For each item i ($1 \leq i \leq n$) set $s_i = L_i$, $q_i = s_i$, $r_i = 0$;
    Loop
        Select a random item *j*;
        $s_j = s_j + 1$;
    Until no more facings can be added without violating the space constraint (17);
    For each item *i*
        Increase q_i until no improvement can be obtained in the objective value;
        Increase r_i until no improvement can be obtained in the objective value;
    Output solution $S_0(q_i, s_i, r_i)$

    //GRG calling sub-procedure
    S'=Solver(*S₀*);

    //Solution repair sub-procedure
    Round every s_i, q_i, r_i ($1 \leq i \leq n$) in *S'* to their nearest integers
    While space constraint (17) is violated
        Rank the items by their unit space profit value $M_i/(a_i s_i)$;
        Delete one facing of the item with the smallest unit space profit value (if this operation causes a constraint violation, the next item in the ranking list is considered);
    If free shelf space > the size of the smallest item
        Loop
            Rank the items by their unit space profit value $M_i/(a_i s_i)$;
            Add one facing of the item with the largest unit space profit value (the next item in the ranking list is considered if the operation generates a constraint violation);
        Until no more facings can be added without violating the space constraint (17);
    For each item i ($1 \leq i \leq n$)
        Increase/decrease q_i until no improvement can be obtained in the objective value;
        Increase/decrease r_i until no improvement can be obtained in the objective value;
    Remember the best solution (*S_best*) found so far;
    iter++;
Until *iter* = *MaxIter*;
Output *S_best*;

Figure 2: Pseudo Code of the Multi-start GRG Algorithm

sub-procedure was used to generate a set of diverse solutions that can be used by GRG. Note that because GRG is only efficient in handling continuous variables, a relaxed model (ignoring integer constraints (22) and (23)) was input into the Excel Solver. Therefore, the solution output by GRG lost feasibility. The solution repair sub-procedure was used to recover the feasibility of the solution and further improve it by using a simple local search method described in figure 2 (several other rounding heuristics were tried and the one presented in this paper generally performs best across the five problem instances we tested). All results were averaged over ten runs on a PC with a Pentium IV 1.8GHZ CPU and 256MB RAM, running Microsoft Windows 2000 professional Version 5.

5. A Numerical Example

To allow a better understanding of the model and the solution procedure described above, a numerical example with 6 items was generated (denoted by BORIN94/6). The problem scale parameters (α_i) and space elasticities (β_i) are taken from (Borin et al. 1994) and the other parameters are listed in table 1. The GRG algorithm described in section 4 was run 10 times with different initial random solutions. The algorithm consistently returned the same solution which is shown in table 2. For the purpose of comparison, an exhaustive search was also carried out to get an optimal solution which is listed in table 2. It can be seen that for this numerical example, the solution obtained by GRG is very close to the optimal solution. The relative deviation from optimality is only 0.04% ($\frac{347.58 - 347.45}{347.58} \cdot 100\%$).

Table 1: Parameters of the Numerical Example

Item	a_i	p_i	c_{ai}	c_{hi}	p_{di}	C_o	α_i	β_i	σ_i
1	0.028	5.03	2.46	0.19	1.23	34.3	28.53	0.1532	0.06
2	0.061	9.37	5.67	0.20	2.84	48.9	23.62	0.2273	0.07
3	0.025	5.10	2.70	0.26	1.35	35.6	25.59	0.2089	0.06
4	0.060	11.48	6.11	0.16	3.06	47.9	22.40	0.2143	0.04
5	0.036	6.74	3.53	0.30	1.77	33.9	15.62	0.2955	0.03
6	0.033	5.97	3.41	0.27	1.71	39.1	10.50	0.3104	0.03

W=0.608(m^2), c_s=5.0(pounds/m^2/unit time), L_i =1, U_i =12, T_{ei} =7(days)

Table 2: Solution of the Numerical Example

Item	Solution by GRG				Optimal Solution			
	q_i	s_i	r_i	T_i	q_i	s_i	r_i	T_i
1	83	3	0	2.68	81	2	0	2.78
2	78	2	0	3.17	78	2	0	3.17
3	77	3	0	2.61	77	3	0	2.61
4	88	3	0	3.35	88	3	0	3.35
5	64	3	0	3.17	64	3	0	3.17
6	50	1	0	5.19	56	2	0	4.68
Objective	347.45				347.58			

6. Larger Problem Instances

Although numerical examples are helpful in understanding the model and testing the performance of the solution procedure, it is necessary to test the algorithm over larger problem instances. For this purpose, we created four benchmark problem instances using the parameters in table 3. The problem size ranges from 18 to 64 products. Those datasets can be downloaded from website: http://www.cs.nott.ac.uk/~gxk/research. Here we provide the computational results of the modified GRG algorithm, shown in table 4. It can be seen that the modified GRG algorithm used

Table 3: Parameters of Problem Instances

Parameters	Values	Parameters	Values
n	18/32/49/64	L_i	1
α_i	U(10, 30)	U_i	12
β_i	U(0.15, 0.3)	p_{di}	$0.5c_{ai}$
σ_i	U(0.03, 0.1)	c_s	5.0 pounds/m^2/day
a_i	U(0.01,0.09) m^2	C_o	U(30, 50) pounds
c_{ai}	N(100a_i, 0.4) pounds	T_{ei}	7 days
p_i	N(1.8c_{ai}, 0.4) pounds	W	2.5* *minSpace*
c_{hi}	U(0.1,0.3) pounds		

U(a, b): Uniform Distribution   N(c, d): Normal Distribution
minSpace: the minimal space requirement to satisfy products' the number of facings lower bounds

in this paper is quite robust on the five tested problem instances. With the problem instance BORIN94/6 and FRESH2, all of ten runs consistently returned the same solution although each run started from different, random initial solutions. For the other three instances, the difference

between the best solution and worst solution among the ten runs are very small and the standard deviations are less than 1, a very small value compared with the objective values. Note that all the solutions obtained by the algorithm satisfy the integer constraints and are therefore feasible solutions.

Table 4: The Computational Results of the GRG Algorithm on Five Problem Instances

	BORIN94/6	FRESH2	FRESH3	FRESH4	FRESH5
n	6	18	32	49	64
av. obj.	347.45	1129.60	2056.46	3163.98	4387.16
best obj.	347.45	1129.60	2057.15	3164.59	4387.73
worst obj.	347.45	1129.60	2055.17	3163.33	4386.66
std. dev	0.00	0.00	0.97	0.51	0.43
av. cpu	3.2	73.6	74.3	179.2	209.7

av. obj.: the average objective value over 10 runs
best obj.: the best objective value over 10 runs
worst obj: the worst objective value over 10 runs
std. dev.: absolute standard deviation of 10 results obtained by GRG
av. cpu: average cpu time consumed by GRG (in seconds)

7. Conclusions

A single-period inventory and shelf space allocation model has been proposed for fresh produce. The demand is assumed to be deterministic and conforms to a multiplicative form of the displayed stock-level and items' freshness conditions. The items' freshness condition is assumed to drop exponentially over time but could still capture some demand. The model is consistent with deteriorating inventory models reported in literature, in which an exponential decay in the inventory is assumed. Unlike other researches, the proposed model considers the integer nature of the solution. Five benchmark problem instances were generated for the fresh produce inventory control and shelf space allocation problem. A modified GRG algorithm was used to search for good quality solutions and their computational results were reported. The algorithm used in this paper ensures the integrality of the decision variables.

Appendix

Denote $y(t) = [\mu_i e^{-\sigma_i t} + K_i]^{\frac{1}{(1-\beta_i)}}$. Divide range $[t_{1i}, T_i]$ into k identical ranges by point $x_0 = t_{1i}$, x_1, $x_2, \ldots, x_k = T_i$. We have:

$$HC_{2i} = c_{hi} \int_{t_{1i}}^{T_i} ([\mu_i e^{-\sigma_i t} + K_i]^{\frac{1}{(1-\beta_i)}}) dt$$

$$\approx \frac{c_{hi}(T_i - t_{1i})}{k} [\frac{1}{2}(y(x_0) + y(x_k)) + y(x_1) + \ldots + y(x_{k-1})]$$

References

Abad, P. L. 1996. Optimal Pricing and Lot-sizing Under Conditions of Perishability and Partial Backordering. *Management Sci.* 42(8) 1093-1104.

Bai, R., G. Kendall. 2005. An Investigation of Automated Planograms Using a Simulated Annealing Based Hyper-heuristics. T. Ibaraki, K. Nonobe, M. Yagiura eds. *Metaheuristics: Progress as Real Problem Solvers* - (Oper. Res./Comput. Sci. Interfaces, Vol. 32). Springer, Berlin, Heidelberg, New York. 87-108.

Ben-Daya, M., A. Raouf. 1993. On the Constrained Multi-item Single-period Inventory Problem. *Internat. J. of Oper. & Production Management* 13(11) 104-112.

Borin, N., P. W. Farris, J. R. Freeland. 1994. A Model for Determining Retail Product Category Assortment and Shelf Space Allocation. *Decision Sci.* 25(3) 359-384.

Corstjens, M., P. Doyle. 1981. A Model for Optimising Retail Space Allocations. *Management Sci.* 27(7) 822-833.

Curhan, R. 1972. The Relationship Between Space and Unit Sales in Supermarkets. *J. of Marketing Res.* 9 406-412.

Fujiwara, O., U. L. J. S. R. Perera. 1993. EOQ Models for Continuously Deteriorating Products Using Linear and Exponential Penalty Costs. *Eur. J. of Oper. Res.* 70(1) 104-114.

Gabriele, G. A., K.M. Ragsdell. 1977. The Generalized Reduced Gradient Method: A Reliable Tool for Optimal Design. *AMSE J. of Engrg. for Indust.* 99 384-400.

Giri, B. C., S. Pal, A. Goswami K. S. Chaudhuri. 1996. An Inventory Model for Deteriorating Items with Stock-dependent Demand Rate. *Eur. J. of Oper. Res.* 95 604-610.

Glover, F., G. A. Kochenberger. 2003. *Handbook of Meta-Heuristics*. Kluwer.

Goyal, S. K., B. C. Giri. 2001. Recent Trends in Modeling of Deteriorating Inventory. *Eur. J. of Oper. Res.* 134(1) 1-16.

Hollier, R. H., K. L. Mak. 1983. Inventory Replenishing Policies for Deteriorating Items in a Declining Market. *Internat. J. of Production Res.* 21(6) 813-826.

Jain, K., E. A. Silver. 1994. Lot Sizing for a Product Subject to Obsolescence or Perishability. *Eur. J. of Oper. Res.* 75(3) 287-295.

Johnson, M. 2002. Personal Correspondence. Retail Vision Ltd.

Kar, S., A. K. Bhunia, M. Maiti. 2001. Inventory of Multi-deteriorating Items Sold From Two Shops Under Single Management with Constraints on Space and Investment. *Comput. & Oper. Res.* 28 1203-1221.

Kotzan, J., R. Evanson. 1969. Responsiveness of Drug Store Sales to Shelf Space Allocations. *J. of Marketing Res.* 6 465-469.

Lasdon, L. S., A. D. Waren, A. Jain, M. Ratner. 1978. Design and Testing of a Generalized Reduced Gradient Code for Nonlinear Programming. *ACM Trans. on Math. Software* 4(1) 34-50.

Liu, L. 1990. (*s*,*S*) Continuous Review Models for Inventory with Random Lifetimes. *Oper. Res. Letters* 9(3) 161-167.

Liu, L., Z. Lian. 1999. Continuous Review Models for Inventory with Fixed Lifetimes. *Oper. Res.* 47(1) 150-158.

Mandal, B. N., S. Phaujdar. 1989. An Inventory Model for Deteriorating Items and Stock-dependent Consumption Rate. *J. of the Oper. Res. Soc.* 40(5) 483-488.

Nahmias, S. 1982. Perishable Inventory Theory: A Review. *Oper. Res.* 30(4) 680-708.

Nandakumar, P., T. E. Morton. 1990. Near Myopic Heuristic for the Fixed Life Perishability Problem. *Management Sci.* 39(12) 241-246.

Raafat, F. 1991. Survey of Literature on Continuously Deteriorating Inventory Model. *J. of the Oper. Res. Soc.* 42: 27-37.

Rajan, A., Rakesh, R. Steinberg. 1992. Dynamic Pricing and Ordering Decisions by A Monopolist. *Management Sci.* 38(2) 240-262.

Sarker, B. R., S. Mukherjee, C. V. Balan. 1997. An Order-level Lot Size Inventory Model with Inventory-level Dependent Demand and Deterioration. *Internat. J. of Production Econom.* 48 227-236.

Urban, T. L. 1998. An Inventory-Theoretic Approach to Product Assortment and Shelf-Space Allocation. *J. of Retailing* 74(1) 15-35.

Urban, T. L. 2002. The Interdependence of Inventory Management and Retail Shelf Management. *Internat. J. of Phys. Distribution & Logist. Management* 32(1) 41-58.

Urban, T. L., R. C. Baker. 1997. Optimal Ordering and Pricing Policies in a Single-period Environment with Multivariate Demand and Markdowns. *Eur. J. of Oper. Res*. 103 573-583.

Xu, H., H.-P. Wang. 1990. An Economic Ordering Policy Model for Deteriorating Items with Time Proportional Demand. *Eur. J. of Oper. Res*. 46(1) 21-27.

Yang, M.-H., W.-C. Chen. 1999. A Study on Shelf Space Allocation and Management. *Internat. J. of Production Econom.* 60(61) 309-317.

Zhou, Y.-W., H.-S. Lau, S.-L. Yang. 2003. A New Variable Production Scheduling Strategy for Deteriorating Items with Time-varying Demand and Partial Lost Sale. *Comput. & Oper. Res*. 30 1753-1776.