Constitutive modelling of hot deformation behaviour of metallic materials

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Abstract

The complexity of hot deformation behaviours of metallic materials is acknowledged in decades of study. The present work uses a new constitutive model by Zhu-Ou-Popov (ZOP model) and its modification to predict the hot deformation behaviours of metals. The ZOP model and its modifications are introduced firstly. The basic idea of these models is centred on a set of piecewise and transition functions: the piecewise functions are used to predict flow stress at different strain ranges, whilst the transition functions enable a smooth shifting from small strain to large strain ranges. The methods for identification of four such variant models are developed and given in detail. Hot compressive flow stress curves of 42CrMo at different strain rates and temperatures are used to show the validity of these models. Results show that, all of the developed models are able to predict the hot compressive behaviour of 42CrMo. Using the Arrhenius type equation and modified Zener-Holloman parameter in the ZOP model to predict the flow stress from the yield point, the Modified model III gives the most favourable prediction accuracy with $R^2$ of 0.9569 whilst other models are also effective with $R^2$ higher than 0.91. The modifications of the ZOP model can be used to predict the yield stress and to reflect the peak stress. In addition, the applicability and advancement of the presented constitutive models are discussed, and the models are considered to be effective in reflecting the occurrence and completion of dynamic recrystallization (DRX) in hot deformation of metallic materials. It is shown that the studied models are capable of predicting the peak strain and DRX completion strain, with comparable results obtained to experimental data.

Keywords: Constitutive modelling; Metallic materials; Hot deformation behaviour; Work hardening; Dynamic softening
1 Introduction

As metallic materials are widely used for extensive engineering applications, different metal forming processes have been developed to manufacture raw metallic materials to various shapes and dimensions. The study of the deformation behaviour of metallic materials is of a vital importance to metal forming processes. The flow stress curves at high temperature show specific complexity as compared with ones at room temperature. It has been established that four stages of hot deformation of metallic materials can be summarised in the flow behaviour: work hardening (WH), transition, dynamic softening and steady state [1, 2]. The complex deformation behaviour makes it difficult to develop a constitutive model to predict the flow behaviour at high temperature. However, a proper constitutive model is of significance to the numerical simulation and sometimes analytical study of hot forming processes of metallic materials. Numerous researchers have worked on the development of effective constitutive models for the hot deformation behaviour of metals for decades and many excellent works have been carried out and reported. Generally, there are three main types of constitutive models [1]: phenomenological constitutive model [3-10], physically based constitutive model [11-15] and neural network based constitutive model [16-18].

Among all the phenomenological models, the Johnson-Cook model [3] has been extensively used to predict the flow stress curves at different strains, strain rates and temperatures of metals. Its simple form and easy identification of model parameters make the Johnson-Cook model widely used for many metallic materials. However, at a given strain rate and temperature, although it is able to predict the trend of work hardening (WH), the Johnson-Cook model is shown less robust in representing the softening in hot deformation of metals. This severely limits its applicability to hot deformation behaviours of metallic materials and many researchers have tried to make modifications considering the effect of strain softening [19, 20].

Both the phenomenological Johnson-Cook model and some physically based constitutive models like the Zerilli-Armstrong model [11] have the limitation to give a good representation of the complex deformation behaviour at high temperature including the hardening and softening behaviours in different strain ranges. To solve this problem, a number of researchers used high-order polynomials to predict the effect of stain on flow stress [21-23]. In many cases, high-order polynomials are able to give a good prediction of the strain hardening and softening behaviours, but the disadvantages are the requirements for many trials to determine too many parameters and there is no clear meaning for the prediction functions. Therefore, there is still a need to develop
new constitutive models to predict the hardening-transition-softening-steady behaviour in the hot deformation of metallic materials.

In an earlier study by the authors, a new phenomenological constitutive model, the Zhu-Ou-Popov model (ZOP model) was proposed to predict the tensile and compressive behaviour of thermoplastics at different temperatures and strain rates [24]. The similarity of the deformation behaviour between thermoplastics and metals at high temperatures is that both have initial hardening followed by softening stages. The only difference is that for thermoplastics, another hardening stage comes after the softening stage, whilst for hot deformation of metals, steady flow stress comes afterwards. However, the steady state could be considered as a zero hardening rate and the difference could be neglected. Therefore, the ZOP model has the potential to be used in the prediction of hot deformation behaviour in metallic materials.

The current work aims to test the applicability of the ZOP model in modelling the hot deformation behaviour in metallic materials and to modify the ZOP model for better prediction accuracy. In the following sections, the ZOP model is firstly introduced and used to predict the hot deformation behaviour of 42CrMo carbon alloy steel. Then three modified models are developed to achieve better prediction results. The development strategy and the comparison between different models are demonstrated. Discussions are given to show the applicability and advancement of the developed constitutive models and how the different parts of the constitutive models represent the microstructure evolution in the hot deformation of metals. Finally, the studied models are also used to predict the peak strain and DRX completion strain with the results and comparisons also given in the discussion section.

2 Constitutive modelling of hot deformation behaviour

2.1 ZOP model

The ZOP model was developed by Zhu et al. [24] using phenomenological equations to mathematically predict the stress of thermoplastics at different strains, strain rates and temperatures, an approach similar to some widely-used model such as Johnson-Cook model [3], Voce-Kocks model [8, 25] the models developed by Khan et al. [4, 5, 26, 27]. The prediction of deformation behaviour of this model is based on the concept of piecewise functions and transition functions. It can be considered to have four parts: 1) function to predict the effect of strain rate and temperature; 2) function to predict the initial hardening and transition and following
dynamic softening stages; 3) function to predict the steady state or additional hardening stage; 4) functions to enable a smooth transition from small strain stage to large strain stage. Details are given in the following.

2.1.1 Effect of strain rate and temperature

In the hot deformation behaviour of metals, with the rise of strain rate and the decrease of temperature, the flow stress trends to be enhanced. As a result, in the ZOP model, the recognised G’Sell-Jonas model [28] is modified to predict the effect of strain rate and temperature as following:

\[ h(\dot{e}, T) = (\dot{e}/\dot{e}_{\text{ref}})^m \cdot \exp[a \cdot (1/T - 1/T_{\text{ref}})] \]  

(1)

where \( \dot{e} \) is the strain rate, \( \dot{e}_{\text{ref}} \) is the reference strain rate, \( T \) is the temperature, \( T_{\text{ref}} \) is the reference temperature, \( m \) is the model parameter related to the effect of strain rate and \( a \) is the parameter related to the effect of temperature. The use of reference strain rate and temperature gives the equation an explicit meaning of the change of flow stress when different strain rates and temperatures from reference ones are applied.

2.1.2 Initial hardening, transition and softening stages

The stress in the small strain range under initial hardening, transition and softening behaviours are predicted by the function \( f(\varepsilon, \dot{e}, T) \) as following:

\[ f(\varepsilon, \dot{e}, T) = K_1 \cdot \varepsilon^n \cdot \exp\{-\varepsilon/\left[\mu \cdot h(\dot{e}, T)\right]\} \]  

(2)

where \( \varepsilon \) is the true strain, \( \dot{e} \) is the strain rate, \( T \) is the temperature, \( K_1, n \) and \( \mu \) are model parameters. In this function, \( h(\dot{e}, T) \) is used to reflect the effect of strain rate and temperature on the hardening, transition and softening stages. The mathematic illustration of \( f(\varepsilon, \dot{e}, T) \) at small strains can be found as the red curve in Fig. 1(a).
2.1.3 Steady state

The steady state can be treated as a hardening stage where the hardening rate is zero. Considering that a slight hardening behaviour may occur in this state, function \( g(\varepsilon, \dot{\varepsilon}, T) \) referring to the one in Duan-Saigal-Greif-Zimmerman (DSGZ) model [29] is still applicable to predict the stress in large strain range. The function is given in the following expression:

\[
g(\varepsilon, \dot{\varepsilon}, T) = K_2 \cdot [\exp(-C_1 \cdot \varepsilon) + \exp(C_2)] \cdot [1 - \exp(-\alpha \cdot \varepsilon)] \cdot h(\dot{\varepsilon}, T) \quad (3)
\]

where \( \varepsilon \) is the true strain, \( \dot{\varepsilon} \) is the strain rate, \( \dot{\varepsilon}_{\text{ref}} \) is the reference strain rate, \( T \) is the temperature, \( T_{\text{ref}} \) is the reference temperature, \( K_2, C_1, C_2 \) and \( \alpha \) are model parameters. Its mathematical meaning is illustrated as the blue curve in Fig. 1(a). This function is shown to be flexible to describe rising or steady curves at large strains.

2.1.4 Transition functions

As given above, the flow stress at small and large strains is predicted using two different functions. To achieve a smooth transition between the two functions at around a critical strain \( \varepsilon_{\text{critical}} \), two transition functions are used in the ZOP model as follow:

\[
u(\varepsilon, \dot{\varepsilon}, T) = 1/\{1 + k \cdot \exp[w \cdot \varepsilon - \lambda \cdot h(\dot{\varepsilon}, T)]\} \quad (4)
\]

\[
u(\varepsilon, \dot{\varepsilon}, T) = 1/\{1 + \exp[\lambda \cdot h(\dot{\varepsilon}, T) - w \cdot \varepsilon]\} \quad (5)
\]

where \( \varepsilon \) is the true strain, \( \dot{\varepsilon} \) is the strain rate and \( T \) is the temperature, \( k, w \) and \( \lambda \) are model parameters. The values of these functions stabilised at ‘1’ and ‘0’ with different strains, as shown in Fig. 1(b). The combination of the transition functions \( u(\varepsilon, \dot{\varepsilon}, T) \) and \( v(\varepsilon, \dot{\varepsilon}, T) \) contributes to the transition from the small strain stage to the large strain stage of the hot deformation behaviour. The expression of the ZOP model is given as the following control equation:

\[
\sigma(\varepsilon, \dot{\varepsilon}, T) = f(\varepsilon, \dot{\varepsilon}, T) \cdot u(\varepsilon, \dot{\varepsilon}, T) + g(\varepsilon, \dot{\varepsilon}, T) \cdot v(\varepsilon, \dot{\varepsilon}, T) \quad (6)
\]

A piecewise model can be given using the above expression, where the flow stress can be predicted using \( f(\varepsilon, \dot{\varepsilon}, T) \) at small strain stage and using \( g(\varepsilon, \dot{\varepsilon}, T) \) at large strain stage. In this model, there are twelve
parameters including $k$, $w$, $\lambda$, $n$, $\mu$, $C_1$, $C_2$, $\alpha$, $K_1$, $K_2$, $m$ and $a$, and reference strain and temperature should also be selected.

2.2 Modified model I

The ZOP model was initially proposed for the depiction of stress-strain curves of thermoplastics, which has a much smaller Young’s modulus than metals. For thermoplastics, the transition from elasticity to the yield and plasticity is gradual and smooth because of the existence of non-linear elasticity, whilst in hot deformation of metals, a quicker transition from elastic to plastic deformation is acknowledged and it is less smooth than thermoplastics. Therefore, the modification of the ZOP model is to give a reasonable representation of the flow stress from the yield point instead of prediction from the zero-stress point.

The modification of the ZOP model is made in the expression of function $f(\varepsilon, \dot{\varepsilon}, T)$ and the modified function $f^*(\varepsilon, \dot{\varepsilon}, T)$ is expressed as below:

$$f^*(\varepsilon, \dot{\varepsilon}, T) = K_1 \cdot \dot{\varepsilon}^n \cdot \exp\left\{-\varepsilon/[\mu \cdot h(\dot{\varepsilon}, T)]\right\} + B \cdot h(\dot{\varepsilon}, T)$$  \hspace{1cm} (7)

where parameter $B$ is used to enable the prediction starting from the yield point. With the modification from $f(\varepsilon, \dot{\varepsilon}, T)$ to $f^*(\varepsilon, \dot{\varepsilon}, T)$, Modified model I may be given in the following equation:

$$\sigma(\varepsilon, \dot{\varepsilon}, T) = f^*(\varepsilon, \dot{\varepsilon}, T) \cdot u(\varepsilon, \dot{\varepsilon}, T) + g(\varepsilon, \dot{\varepsilon}, T) \cdot v(\varepsilon, \dot{\varepsilon}, T)$$  \hspace{1cm} (8)

Thirteen parameters should be derived for Modified model I. Besides, selected reference strain rate and temperature should be given as well.

2.3 Modified model II

The modification made from the ZOP model to Modified model I is applied to improve the prediction accuracy of the original model at the initial small strain stage. Even though the ZOP model and Modified model I are both phenomenological constitutive models. They do not have any physical meaning to support the flow stress prediction and reflect the physical mechanism. To improve the prediction accuracy, physical method may also be used to modify ZOP model. For hot deformation of metals, the well-established Arrhenius equation can be used to predict the effect of strain rate and temperature related to the Zener-Hollomon ($Z$) parameter on the flow stress [30, 31], which is given as follows:
\[ Z = \dot{\varepsilon} \cdot \exp[Q/(R \cdot T)] \]  
\[ \dot{\varepsilon} = F(\sigma_p) \cdot \exp[-Q/(R \cdot T)] \]

where the function \( F(\sigma_p) \) can be given in the form of a power function, exponential function or hyperbolic sine function as shown below [32]:

\[
F(\sigma_p) = \begin{cases} 
A' \cdot \sigma_p^N & \alpha \cdot \sigma_p < 0.8 \\
A'' \cdot \exp(\beta \cdot \sigma_p) & \alpha \cdot \sigma_p > 1.2 \\
A \cdot \left[ \sinh(\gamma \cdot \sigma_p) \right]^N & \text{for all } \sigma_p
\end{cases}
\]

where \( \dot{\varepsilon} \) is the strain rate, \( R \) is the universal gas constant (8.31 J·mol\(^{-1}\)·K\(^{-1}\)), \( T \) is the absolute temperature (K), \( Q \) is the activation energy of hot deformation (kJ·mol\(^{-1}\)), \( \sigma_p \) is the peak stress (MPa), \( A, A', A'', \gamma, \beta \) and \( N \) are the material constants, where \( \gamma = \beta/N \).

Eq. (13) is the most commonly used equation for all stress levels. From Eq. (13), the following equation can be obtained:

\[
\sigma_p = 1/\gamma \cdot \sinh^{-1}\left[ \left( \dot{\varepsilon} \cdot \exp(Q/(R \cdot T))/A \right)^{1/N} \right]
\]

Meanwhile, in the ZOP model, partial differentiation of \( f(\varepsilon, \dot{\varepsilon}, T) \) of \( \varepsilon \) gives the following relationship:

\[
\partial f(\varepsilon, \dot{\varepsilon}, T)/\partial \varepsilon = K_1 \cdot \varepsilon^{n-1} \cdot \exp[-\varepsilon/[\mu \cdot h(\dot{\varepsilon}, T)]] \cdot \{n - \varepsilon/[\mu \cdot h(\dot{\varepsilon}, T)]\}
\]

From Eq. (15), when the strain increases to a peak strain \( \varepsilon_p = n \cdot \mu \cdot h(\dot{\varepsilon}, T) \) (defined as the strain where the peak stress appears), the peak stress is reached by using Eq. (2):

\[
\sigma_p = K_1 \cdot [n \cdot \mu \cdot h(\dot{\varepsilon}, T)/\varepsilon]^n
\]

Combining Eqs (14) and (16), the following equation can be obtained:

\[
1/\gamma \cdot \sinh^{-1}\left[ \left( \dot{\varepsilon} \cdot \exp(Q/(R \cdot T))/A \right)^{1/N} \right] = K_1 \cdot [n \cdot \mu \cdot h(\dot{\varepsilon}, T)/\varepsilon]^n
\]

which leads to the following results for the expression of \( h(\dot{\varepsilon}, T) \):

\[
h(\dot{\varepsilon}, T) = e/(n \cdot \mu) \cdot \left[ 1/(\gamma \cdot K_1) \right]^{1/n} \cdot \left[ \sinh^{-1}\left( Z/A \right)^{1/N} \right]^{1/n}
\]
In Eq. (18), the value of $e/(n \cdot \mu) \cdot [1/(\gamma \cdot K_1)]^{1/n}$ is constant. For simplification, the term $h(\dot{e}, T)$ used in the constitutive model to predict the effect of strain rate and temperature can be replaced by the following expression without consideration of the term $e/(n \cdot \mu) \cdot [1/(\gamma \cdot K_1)]^{1/n}$:

$$h^*(\dot{e}, T) = \{\sinh^{-1}\left[(Z/A)^{1/N}\right]\}^{1/n} = \{\sinh^{-1}\left[(\dot{e} \cdot \exp(Q/(R \cdot T))/A)^{1/N}\right]\}^{1/n}$$

(19)

Or it may be given in the following format:

$$h^*(\dot{e}, T) = \left[\ln \left[(Z/A)^{1/N} + ((Z/A)^{2/N} + 1)^{1/2}\right]\right]^{1/n}$$

(20)

With the modification from $h(\dot{e}, T)$ to $h^*(\dot{e}, T)$, Modified model II has the following formula:

$$\sigma(\epsilon, \dot{\epsilon}, T) = f^*(\epsilon, \dot{\epsilon}, T) \cdot u^*(\dot{\epsilon}, \dot{\epsilon}, T) + g^*(\epsilon, \dot{\epsilon}, T) \cdot v^*(\epsilon, \dot{\epsilon}, T)$$

(21)

where,

$$f^*(\epsilon, \dot{\epsilon}, T) = K_1 \cdot \epsilon^n \cdot \exp\left(-\epsilon/[\mu \cdot h^*(\dot{\epsilon}, T)]\right) + B \cdot h^*(\dot{\epsilon}, T)/h^*(\dot{\epsilon}_{ref}, T_{ref})$$

(22)

$$g^*(\epsilon, \dot{\epsilon}, T) = K_2 \cdot [\exp(-\alpha \cdot \epsilon) + \exp C_2] \cdot [1 - \exp(-\alpha \cdot \epsilon)] \cdot h^*(\dot{\epsilon}, T)$$

(23)

$$u^*(\dot{\epsilon}, \dot{\epsilon}, T) = 1/[1 + k \cdot \exp[w \cdot \epsilon - \lambda \cdot h^*(\dot{\epsilon}, T)]$$

(24)

$$v^*(\epsilon, \dot{\epsilon}, T) = 1/[1 + \exp[\lambda \cdot h^*(\dot{\epsilon}, T) - w \cdot \epsilon]]$$

(25)

In the Modified model II, a total of fourteen parameters should be derived. The identification procedures of parameters $k, w, \lambda, n, \mu, C_1, C_2, \alpha, K_1, K_2$ and $B$ are the same as the ZOP and the Modified model I. The parameters $Q, A$ and $N$ related to the effect of strain rate and temperature need to be determined first. Besides, the reference strain rate and temperature should also be determined prior to the identification of the model parameters.

2.4 Modified model III

From the prediction results of Modified model II as given in Section 4, the Zener-Holloman parameter used in the constitutive model is not able to reflect the effect of strain rate and temperature with high prediction accuracy. Therefore, compensation for either the strain rate or temperature should be used to enhance their effect on the flow stress. In this study, compensation for the strain rate in $Z$ parameter is carried out via
modifying the exponent of the strain rate in $Z$ parameter. The modified $Z$ parameter can be written in the following format [22]:

$$Z^* = \dot{\varepsilon}^M \cdot \exp[Q / (R \cdot T)]$$  \hspace{1cm} (26)

And the modified term $h^{**}(\dot{\varepsilon}, T)$ can be expressed as follows:

$$h^{**}(\dot{\varepsilon}, T) = \left\{ \sinh^{-1}\left[ (Z^* / A)^{1/N} \right] \right\}^{1/n} = \left\{ \ln \left[ (Z^* / A)^{1/N} + ((Z^* / A)^{2/N} + 1)^{1/2} \right] \right\}^{1/n}$$  \hspace{1cm} (27)

Then the following expression is given as Modified model III:

$$\sigma(\varepsilon, \dot{\varepsilon}, T) = f^{***}(\varepsilon, \dot{\varepsilon}, T) \cdot u^{**}(\varepsilon, \dot{\varepsilon}, T) + g^{**}(\varepsilon, \dot{\varepsilon}, T) \cdot v^{**}(\varepsilon, \dot{\varepsilon}, T)$$  \hspace{1cm} (28)

where,

$$f^{***}(\varepsilon, \dot{\varepsilon}, T) = K_1 \cdot \varepsilon^n \cdot \exp\left\{ -\varepsilon / [\mu \cdot h^{**}(\dot{\varepsilon}, T)] \right\} + B \cdot h^{**}(\dot{\varepsilon}, T) / h^{**}(\dot{\varepsilon}_{ref}, T_{ref})$$  \hspace{1cm} (29)

$$g^{**}(\varepsilon, \dot{\varepsilon}, T) = K_2 \cdot [\exp(-C_1 \cdot \varepsilon) + \exp C_2] \cdot [1 - \exp(-\alpha \cdot \varepsilon)] \cdot h^{**}(\dot{\varepsilon}, T)$$  \hspace{1cm} (30)

$$u^{**}(\varepsilon, \dot{\varepsilon}, T) = 1 / [1 + k \cdot \exp[\lambda \cdot h^{**}(\dot{\varepsilon}, T)]]$$  \hspace{1cm} (31)

$$v^{**}(\varepsilon, \dot{\varepsilon}, T) = 1 / [1 + \exp[\lambda \cdot h^{**}(\dot{\varepsilon}, T) - w \cdot \varepsilon]]$$  \hspace{1cm} (32)

In Modified model III, fifteen parameters are used, but only parameter $M$ is required to be identified as other parameters are already given in Modified model II. Similarly, the reference strain rate and temperature are needed but this can be referred to as the ones used in Modified model II.

### 3 Identification of model parameters

To validate the developed constitutive models for deformation behaviour in hot deformation of metals, compression testing results of 42CrMo by Lin et al. [33] are used. The testing results are presented in true stress – strain curves with processing temperature from 850 °C to 1050 °C and strain rate from 0.01 s$^{-1}$ to 50 s$^{-1}$. The identification of model parameters is performed based on the above testing results. It is noteworthy that the testing results at strain rate of 0.1 s$^{-1}$ and different temperatures are not used in the parameter identification in order to test the real predictive capability of the developed models in Section 4.

#### 3.1 ZOP model
The identification procedures for the model parameters in the ZOP model can be found in detail in the previous work of Zhu et al. [24]. Six steps should be followed to determine all the twelve parameters, as summarised below. To derive the parameters for the hot compressive behaviour of 42CrMo, the reference strain and temperature are selected as 1 s⁻¹ and 1050 °C, respectively.

Step 1:

By selecting points at the same large strain on curves at the same temperature but different strain rates, four values \(m_1, m_2, m_3\) and \(m_4\) can be calculated by fitting the relationship between the stress and strain rate with Eq. (33) as shown in Fig. 2(a). \(m\) is identified to be 0.1081 by taking the average value of the above.

\[
\sigma(\varepsilon, \dot{\varepsilon}, T) = \sigma_{\text{ref}}(\varepsilon, \dot{\varepsilon}_{\text{ref}}, T) \cdot \left(\dot{\varepsilon}/\dot{\varepsilon}_{\text{ref}}\right)^m
\]

where \(\sigma(\varepsilon, \dot{\varepsilon}, T)\) is the stress of selected points and \(\sigma_{\text{ref}}(\varepsilon, \dot{\varepsilon}_{\text{ref}}, T)\) is the selected stress at the reference strain rate.
Step 2:

Points are selected at the same large strain on curves at the same strain rate but different temperatures. Different values of \(a\) can be determined by fitting the relationship between the stress and temperature using Eq. (34) as shown in Fig. 2(b). \(a\) is calculated as the average value and is identified as 2600.75.

\[
\sigma(\varepsilon, \dot{\varepsilon}, T) = \sigma_{\text{ref}}(\varepsilon, \dot{\varepsilon}, T_{\text{ref}}) \cdot \exp[a \cdot (1/T - 1/T_{\text{ref}})]
\]  

(34)

where \(\sigma_{\text{ref}}(\varepsilon, \dot{\varepsilon}, T_{\text{ref}})\) is the selected stress at the reference temperature.

Step 3:
As shown in Fig. 2(c), by fitting the relationship between the stress and strain in the large strain range at reference strain rate and temperature with Eq. (3), \( C_1, C_2, \alpha \) and \( K_2 \) can be obtained as: 0.275, 0.2966, 46.14 and 58.52, respectively.

Step 4:

From all the flow stress curves, peak strains and stresses can be extracted, and the values of \( h(\dot{\varepsilon}, T) \) at different conditions can be calculated. By fitting the relationship between \( \sigma_p, \varepsilon_p \) and \( h(\dot{\varepsilon}, T) \) with Eq. (35), \( \mu \) can be identified to be 0.2076 as shown by Fig. 2(d).

\[
\sigma_p = \sigma_p^{\text{ref}} \cdot \left(\frac{\varepsilon_p}{\varepsilon_p^{\text{ref}}}\right)^{\varepsilon_p/\mu h(\dot{\varepsilon}, T)} \tag{35}
\]

where \( \varepsilon_p^{\text{ref}} \) and \( \sigma_p^{\text{ref}} \) are peak strain and stress at reference strain rate and temperature.

Step 5:

After the identification of \( \mu \), by fitting the relationship between stress and strain in small strain range at reference strain rate and temperature with Eq. (36), \( n \) is determined to be 1.327 as illustrated in Fig. 2(e).

\[
\sigma(\varepsilon, \dot{\varepsilon}, T) = \sigma_p \cdot \left(\frac{\varepsilon}{\varepsilon_p}\right)^n \cdot e^n \cdot \exp\left\{-\varepsilon/\left[\mu \cdot h(\dot{\varepsilon}, T)\right]\right\} \tag{36}
\]

Step 6:

Finally, as shown in Fig. 2(f), the fitting of the relationship between stress and strain in the whole range at reference strain rate and temperature with Eq. (6) gives the identification results of \( k, w, \lambda \) and \( K_1 \) as 0.6564, 4.524, 0 and 2681, respectively.

Following the above procedures, the model parameters of the ZOP model were derived and are given in Table 1.

Table 1. Parameters of the ZOP model for the hot compressive behaviour of 42CrMo.

<table>
<thead>
<tr>
<th>ID</th>
<th>Model parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( k )</td>
<td>0.6564</td>
</tr>
<tr>
<td>2</td>
<td>( w )</td>
<td>4.524</td>
</tr>
<tr>
<td>3</td>
<td>( \lambda )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>( n )</td>
<td>1.327</td>
</tr>
<tr>
<td>5</td>
<td>( \mu )</td>
<td>0.2076</td>
</tr>
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</table>
The identification process of the parameters of Modified model I is almost identical to the ZOP model. The only difference is that parameter $B$ is required to be identified during the fitting of the flow stress at small strains along with the identification process of parameter $n$. In the studied hot compressive behaviour of 42CrMo, the reference strain rate is selected as $1 \text{ s}^{-1}$ and reference temperature as $1050 \degree C$. Given the above conditions, the parameters of Modified model I were identified according to the procedures and are listed in Table 2.

Table 2. Material parameters of Modified model I for the hot compressive behaviour of 42CrMo.

<table>
<thead>
<tr>
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<th>Value</th>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>$w$</td>
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<tr>
<td>3</td>
<td>$\lambda$</td>
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</tr>
<tr>
<td>4</td>
<td>$n$</td>
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</tr>
<tr>
<td>5</td>
<td>$\mu$</td>
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</tr>
<tr>
<td>6</td>
<td>$C_1$</td>
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</tr>
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<td>7</td>
<td>$C_2$</td>
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<tr>
<td>8</td>
<td>$\alpha$</td>
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<td>9</td>
<td>$K_1$ (MPa)</td>
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</tr>
<tr>
<td>10</td>
<td>$K_2$ (MPa)</td>
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</tr>
<tr>
<td>11</td>
<td>$m$</td>
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</tr>
<tr>
<td>12</td>
<td>$a$</td>
<td>2600.75</td>
</tr>
<tr>
<td>13</td>
<td>$B$ (MPa)</td>
<td>74.28</td>
</tr>
</tbody>
</table>
### 3.3 Modified model II

The identification process of Modified model II is different as the effect of strain rate and temperature is predicted by function $h^*(\dot{\varepsilon}, T)$, which is given in Eqs (19) and (20). The following procedure is used to determine the parameters of Modified model II.

#### N:

From Eqs (10) and (11), by taking the logarithm, the following equation can be obtained [33]:

$$\ln \dot{\varepsilon} = N \cdot \ln \sigma_p + \ln A' - Q/(R \cdot T)$$  \hspace{1cm} (37)

From the above equation, it can be clearly seen that the value of $1/N$ can be determined as the slope of $\ln \sigma_p$ versus $\ln \dot{\varepsilon}$ curves in linear fitting, as shown in Fig. 3(a). $N$ is calculated as 8.1755 MPa$^{-1}$ by taking the average value of the ones obtained at different temperatures.

![Graphs showing parameter calculation of $h^*(\dot{\varepsilon}, T)$ in Modified model II.](image)

**Fig. 3.** Parameter calculation of $h^*(\dot{\varepsilon}, T)$ in Modified model II. Relationship between (a) $\ln \sigma_p$ and $\ln \dot{\varepsilon}$, (b) $\sigma_p$ and $\ln \dot{\varepsilon}$, (c) $\ln[\sinh(\sigma_p/K_1 \cdot (e/(n \cdot \mu))^n)]$ and $1/T$, and (d) $\ln[\sinh(\sigma_p/K_1 \cdot (e/(n \cdot \mu))^n)]$ and $\ln Z$.

#### $\beta$:

From Eqs. (10) and (12), by taking the logarithm, it gives [33]:

---
\[ \ln \dot{\varepsilon} = \beta \cdot \sigma_p + \ln A'' - Q/(R \cdot T) \] (38)

Therefore, by performing linear fitting, \(1/\beta\) can be determined as the slope of \(\sigma_p\) versus \(\ln \dot{\varepsilon}\) curves as shown in Fig. 3(b). \(\beta\) is calculated as 0.0687 MPa\(^{-1}\), which is given by the average value of the ones at different deformation temperatures.

\[ K_1 \cdot (n \cdot \mu/e)^n: \]

Substituting Eq. (19) into Eq. (16), the peak stress in Modified model II can be expressed as follows:

\[ \sigma_p = K_1 \cdot (n \cdot \mu/e)^n \cdot \sinh^{-1} \left[ (Z/A)^{1/N} \right] \] (39)

Combining Eqs (14) and (39), the value of \(K_1 \cdot (n \cdot \mu/e)^n\) can be calculated as 119.0029 MPa using the following equation:

\[ K_1 \cdot (n \cdot \mu/e)^n = 1/\gamma = N/\beta \] (40)

\[ Q: \]

From Eq. (39), the following equation can be derived:

\[ Q/T = R \cdot N \cdot \ln \{\sinh[\sigma_p/K_1 \cdot (e/(n \cdot \mu))^n]\} + R \cdot \ln A - R \cdot \ln \dot{\varepsilon} \] (41)

Partially differentiating both sides of \(1/T\) in Eq. (41), it gives the following equation:

\[ Q = R \cdot N \cdot \frac{\partial \ln \{\sinh[\sigma_p/K_1 \cdot (e/(n \cdot \mu))^n]\}}{\partial (1/T)} \] (42)

And therefore:

\[ \frac{\partial \ln \{\sinh[\sigma_p/K_1 \cdot (e/(n \cdot \mu))^n]\}}{\partial (1/T)} = Q/(R \cdot N) \] (43)

In the above equation, the value of \(1/K_1 \cdot [e/(n \cdot \mu)]^n\), \(R\) and \(N\) are already known, therefore \(Q\) can be determined as the slope of the linearly fitted relationship between \(\ln \{\sinh[\sigma_p/K_1 \cdot (e/(n \cdot \mu))^n]\}\) and \(1/T\) as shown in Fig. 3(c). By taking the average value of the ones for different strain rate conditions, the value of \(Q\) was calculated as 482.1344 kJ·mol\(^{-1}\).

\[ A: \]

From Eqs (9) and (40), the following equations can be derived:
\[ Z = \dot{\varepsilon} \cdot \exp[Q/(R \cdot T)] = A \cdot \left\{ \sinh\left[\frac{\sigma_p}{K_1} \cdot \left( \frac{e}{n \cdot \mu} \right)^n \right] \right\}^N \]  

(44)

After taking the logarithm of both sides, the following relationship is obtained:

\[
\ln\left[ \sinh\left[\frac{\sigma_p}{K_1} \cdot \left( \frac{e}{n \cdot \mu} \right)^n \right] \right] = \ln Z / N - \ln A / N
\]

(45)

As given in Eq. (45), \( \ln A / N \) can be determined by the intercept of the linearly fitted \( \ln\left[ \sinh\left[\frac{\sigma_p}{K_1} \cdot \left( \frac{e}{n \cdot \mu} \right)^n \right] \right] \) versus \( \ln Z \) curve. Since the value of \( N \) has been identified in the above steps, \( A \) can be calculated as \( 6.37431 \times 10^{21} \, \text{s}^{-1} \) from the linear fitting result as shown in Fig. 3(d).

From the above identification procedure, the parameters related to the effect of strain rate and temperature can be determined. Other parameters in Modified model II can be referred to the identification methods of the ZOP model and Modified model I. Before this, the reference strain rate and temperature need to be determined and they are selected as 1 \( \text{s}^{-1} \) and 1050 \( ^\circ \text{C} \) in the studied hot compressive behaviour of 42CrMo. Identified parameters of Modified model II are listed in Table 3.

Table 3. Material parameters of Modified model II for the hot compressive behaviour of 42CrMo

<table>
<thead>
<tr>
<th>ID</th>
<th>Model parameters</th>
<th>Value</th>
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<tr>
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<tr>
<td>2</td>
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<td>2.969</td>
</tr>
<tr>
<td>4</td>
<td>( n )</td>
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</tr>
<tr>
<td>5</td>
<td>( \mu )</td>
<td>0.3545</td>
</tr>
<tr>
<td>6</td>
<td>( C_1 )</td>
<td>0.282</td>
</tr>
<tr>
<td>7</td>
<td>( C_2 )</td>
<td>0.2965</td>
</tr>
<tr>
<td>8</td>
<td>( \alpha )</td>
<td>45.67</td>
</tr>
<tr>
<td>9</td>
<td>( K_1 ) (MPa)</td>
<td>973.4</td>
</tr>
<tr>
<td>10</td>
<td>( K_2 ) (MPa)</td>
<td>106.3</td>
</tr>
<tr>
<td>11</td>
<td>( N ) (MPa(^{-1}))</td>
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</tr>
<tr>
<td>12</td>
<td>( Q ) (kJ·mol(^{-1}))</td>
<td>482.1344</td>
</tr>
<tr>
<td>13</td>
<td>( A ) (s(^{-1}))</td>
<td>6.37431\times 10^{21}</td>
</tr>
<tr>
<td>14</td>
<td>( B ) (MPa)</td>
<td>70.06</td>
</tr>
</tbody>
</table>

3.4 Modified model III
The identification process of Modified model III is based on the results of Modified model II. To determine the exponent of strain rate ($M$) for compensation of its effect on the flow stress, the following steps are followed.

With the modification from $Z$ to $Z^*$, the expression of the peak stress is updated in the following:

$$\sigma_p = K_1 \cdot (n \cdot \mu / e)^n \cdot \sinh^{-1}\left[\left(\frac{Z^*}{A}\right)^{1/N}\right]$$  \hspace{1cm} (46)

After derivation, Eq. (46) can be expressed as Eq. (47).

$$\dot{\varepsilon}^M = A \cdot \left[\sinh\left[\frac{\sigma_p}{K_1 \cdot (e/(n \cdot \mu))^n}\right]\right]^{N \cdot \exp[-Q/(R \cdot T)]}$$  \hspace{1cm} (47)

By taking logarithm of both sides, the following equation can be derived in the following:

$$\ln\left\{\sinh\left[\frac{\sigma_p}{K_1 \cdot (e/(n \cdot \mu))^n}\right]\right\} = M/N \cdot \ln \dot{\varepsilon} - \ln A/N + Q/(N \cdot R \cdot T)$$  \hspace{1cm} (48)

As shown in Eq. (48), $M/N$ can be identified as the slope of the linearly fitted relationship between the $\ln\left\{\sinh\left[\frac{\sigma_p}{K_1 \cdot (e/(n \cdot \mu))^n}\right]\right\}$ and $\ln \dot{\varepsilon}$. The relationships and linear fitting results at different deformation temperatures are given in Fig. 4. As $N$ is already determined in the identification process of Modified model II, by taking the average value, $M$ can be calculated as 1.4011.

![Fig. 4](image_url)

Fig. 4. Calculation of parameter $M$ in Modified model III: the relationship between $\ln\left\{\sinh\left[\frac{\sigma_p}{K_1 \cdot (e/(n \cdot \mu))^n}\right]\right\}$ and $\ln \dot{\varepsilon}$.

The above proposed identification method of $M$ is different from the trial-and-error method used in the studies of Lin et al. [22] and Peng et al. [34]. The advantage is obvious as an optimal solution of $M$ can be obtained to achieve a higher prediction accuracy at the lower computing cost. The parameter of Modified model III can be found in Table 4.

Table 4. Material parameters of the modified model III for the hot compressive behaviour of 42CrMo.
### Table 1: Model parameters

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</tr>
<tr>
<td>15</td>
<td>$M$</td>
<td>1.4011</td>
</tr>
</tbody>
</table>

### 4 Validation of the developed constitutive models

The testing results of 42CrMo in the study of Lin et al. [33] are given as the solid curves in the figures (Figs 5-8). From the testing results, the four stages of the stress-strain curves are presented clearly: work hardening, transition, dynamic softening and steady state. It also shows that with the increase of temperature and strain rate, the true stress of 42CrMo will be reduced and increased, respectively. The prediction results of the ZOP model, the modified models I, II and III are presented and validated in this section. The testing cases at 0.1 s$^{-1}$ and different temperatures are also used in the validation although they are not involved in the parameter identification. This is used to evaluate the predictive capability of the developed models.

#### 4.1 Prediction and validation of ZOP model

Using the determined model parameters in Table 1, the predicted flow stress curves at different strain rates and temperatures are depicted in Fig. 5. It is shown that the ZOP model represents well the flow stress at different strain ranges including hardening, transition, softening and steady state. More specifically, most predicted curves follow the experimental data to a high extent from the peak stress point, i.e., the transition point between the hardening and the softening stages. However, deficiency occurs at small strains. The predicted curves are
not able to describe the sudden change of the slope where the elastic deformation completes and yielding happens. This is unavoidable for the ZOP model because it was originally proposed for thermoplastics, the small strain deformation of which has much smoother curves than metals. For the magnitude of flow stress curves, the ZOP model is capable to predict the effect of strain rate and temperature except from the curves of 1050 °C and 1150 °C at the strain rate of 0.01 s⁻¹, and the curve of 950 °C at the strain rate of 0.1 s⁻¹. This indicates the effectiveness of the strain rate and temperature related term \( h(\dot{\varepsilon}, T) \) in the ZOP model but there is still scope for improvement to achieve better prediction for the shifting of the flow stress curves.

4.2 Prediction and validation of Modified model I

The comparison between the experimental data and prediction results of Modified model I is depicted in Fig. 5. True stress – strain curves of 42CrMo and prediction results of ZOP model at strain rate of (a) 0.01 s⁻¹, (b) 0.1 s⁻¹, (c) 1 s⁻¹, (d) 10 s⁻¹, and (e) 50 s⁻¹.
6. Modified model I shows a noted improvement on the prediction accuracy especially at the initial hardening and the following softening stages as compared with the ZOP model. It indicates that the proposed strategy works well in the prediction of the flow stress from the yield stress instead of from zero stress. It shows that the use of the piecewise functions and the transition functions is an effective way to predict the effect of strain on the flow stress in hot deformation behaviour of metals. However, on the other hand, at the conditions of high temperature and low strain rate and the conditions of lower temperature and higher strain rate, the predicted curves are unable to follow the experimental curves well. The predicted results at temperature of 1150 °C and strain rate of 0.01 s⁻¹ and at temperature of 950 °C and strain rate of 0.1 s⁻¹ show considerable deviation from the testing result, which are shown by the red dotted curve in Fig. 6(a) and the green dotted curve in Fig. 6(b), respectively. The same observations are also presented at temperature of 850 °C and strain rate of 0.01 s⁻¹ and some other conditions. This leads to conclusion that the term \( h(\dot{\varepsilon}, T) \) is unable to give an accurate prediction of the effect of strain rate and temperature.
4.3 Prediction and validation of Modified model II

The same as Modified model I, Modified model II also starts the prediction from the yield point, as shown in Fig. 7, which shows a good agreement of the overall shape of the flow stress curves. Especially at different temperature conditions at strain rates of 1 s\(^{-1}\) and 10 s\(^{-1}\), the prediction curves follow the experimental curves very well. However, at strain rates of 0.01 s\(^{-1}\), 0.1 s\(^{-1}\) and 50 s\(^{-1}\), the predicted and experimental curves do not match up sufficiently well. In another word, the modified term \(h^*(\dot{\varepsilon}, T)\) in Modified model II is unable to predict the effect of strain rate and temperature with satisfactory accuracy, especially at small strain rate such as 0.01 s\(^{-1}\), as shown in Fig. 7(a). At the strain rate of 0.01 s\(^{-1}\), the deviation between the predicted and the experimental results becomes larger with the decrease of deformation temperature. Even if the trend and shape
of the predicted flow stress curves have a good agreement with the experimental results, the inaccuracy could be improved. The above observation indicates that the use of the function $h^*(\dot{\varepsilon}, T)$ is still not sufficient for Modified model II to accurately predict the effect of strain rate and temperature in a wide range, even though its original format has been used widely with metallic materials.

Fig. 7. True stress – strain curves of 42CrMo and prediction results of Modified model II at strain rate of (a) 0.01 s$^{-1}$, (b) 0.1 s$^{-1}$, (c) 1 s$^{-1}$, (d) 10 s$^{-1}$ and (e) 50 s$^{-1}$.

4.4 Prediction and validation of Modified model III

To further improve Modified model II, the modified term $h^{**}(\dot{\varepsilon}, T)$ is used to predict the effect of strain rate and temperature with the addition of the exponent of strain rate in the $Z$ parameter to compensate for the effect of strain rate. The prediction results of Modified model III are shown in Fig. 8 and a clearly improved prediction can be observed. Different deformation stages in hot deformation are predicted and presented properly due to
the use of piecewise and transition functions and the change of prediction starting point at yield point instead of the zero-stress point. In the meantime, the shifting of the flow stress curves due to the change of strain rate and temperature is given satisfactorily because of the compensation for the effect of strain rate and the optimal identification of the exponent $M$. Although at the conditions of 0.1 s$^{-1}$ at 950 °C and 50 s$^{-1}$ at 850 °C, the predicted curves deviate from the experimental ones to a degree, the capability of Modified model III to predict the effect of a wide range of strain rate and temperature is enhanced significantly as compared to Modified model II. Overall, Modified model III presents the best prediction effect over the ZOP model and the other two modified models.

![Graphs showing true stress-strain curves](image)

Fig. 8. True stress – strain curves of 42CrMo and prediction results of Modified model III at strain rate of (a) 0.01 s$^{-1}$, (b) 0.1 s$^{-1}$, (c) 1 s$^{-1}$, (d) 10 s$^{-1}$ and (e) 50 s$^{-1}$.

4.5 Prediction of peak and yield stresses
The comparison of the peak stress at different strain rates from experiment data and prediction by the ZOP model, Modified models I, II and III is given in Fig. 9. In general, all models are able to predict the varying trend of the peak stress with the change of temperature and strain rate. The ZOP model gives accurate predictions of the peak stresses as compared with experimental results at different strain rates and temperatures. Modified model I is able to give favourable predictions in most conditions, while deviation becomes larger at the strain rate of 10 s⁻¹ and 50 s⁻¹ and temperature of 850 °C. Similarly, Modified model II works quite well in most cases except in the case of 0.01 s⁻¹ strain rate. Finally, Modified model III shows a noted improvement as compared with Modified model II, and at all conditions, in predicting the peak stress with a high degree of accuracy. From the above observations, the ZOP model and Modified model III has a better capability in the prediction of the peak stress.

Fig. 9. Comparison of peak stress from experimental results and predicted results from the ZOP model, Modified models I, II and III at deformation strain rate of (a) 0.01 s⁻¹, (b) 0.1 s⁻¹, (c) 1 s⁻¹, (d) 10 s⁻¹ and (e) 50
The predicted yield stresses are compared with the experimental results as shown in Fig. 10. In the figure, the predicted yield stress is identified to be the flow stress when a plastic strain of “0” is applied in Modified model I, II and III. The ZOP model is not included in this comparison because it starts the prediction from the zero-stress point and the predicted yield stress is unable to determine accurately by observation. Although the studied models can predict the general trend of the yield stress variation and give a reasonable prediction at deformation strain rates of 0.01 s⁻¹, 0.1 s⁻¹ and 50 s⁻¹, deviation between the experimental data and predicted results is noticeable, especially in the conditions of 1 s⁻¹ strain rate at temperature of 850 °C and 950 °C, and of the case 10 s⁻¹ strain rate at temperature of 1050 °C and 1150 °C. This may be due to the inaccuracy in the extraction of the yield stress from the experimental data. Secondly, Modified models I, II and III may not be sufficiently suitable to predict the single yield stress in hot deformation of the 42CrMo steel, even though they are modified to start prediction from the yield point.
Fig. 10. Comparison of yield stress from experimental results and predicted results from Modified models I, II and III at deformation strain rate of (a) 0.01 s\(^{-1}\), (b) 0.1 s\(^{-1}\), (c) 1 s\(^{-1}\), (d) 10 s\(^{-1}\) and (e) 50 s\(^{-1}\).

4.6 Evaluation and comparison of prediction accuracy

The prediction accuracy of each model is evaluated by the relative error $\delta$, coefficient of determination ($R^2$) and root mean square error ($RMSE$). They are calculated using the following expressions:

$$\delta = \frac{|E_i - P_i|}{E_i} \cdot 100\% \quad (49)$$

$$R^2 = \frac{\sum_{i=1}^{N}(E_i - \bar{E})^2 - \sum_{i=1}^{N}(E_i - P_i)^2}{\sum_{i=1}^{N}(E_i - \bar{E})^2} \quad (50)$$

$$RMSE = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N}(E_i - P_i)^2} \quad (51)$$
where $E_i$ and $\bar{E}$ are the experimental and the mean values, respectively; $P_i$ is the predicted value; $N$ is the number of data used in the evaluation. Higher prediction accuracy is indicated by a larger value of $R^2$ and a smaller value of RMSE.

Fig. 11 gives the relative error distribution between the experimental stress and predicted results of the studied constitutive models. It is clearly shown that the ZOP model provides better prediction at the later stage of deformation, while at small strain range, the predicted stress of the ZOP model shows a large deviation from the experimental stress. This is because the ZOP model is unable to describe the elastic deformation of metals, Young’s modulus of which is normally large. In terms of Modified models I, II and III, better prediction is usually given in the initial stage of deformation, which benefits from the prediction at the start point of yield point instead of the zero-stress point. As shown in Fig. 11(b) and (c), a comparison between Modified models I and II indicates that the original $h(\dot{\varepsilon}, T)$ is more efficient than the modified function $h^*(\dot{\varepsilon}, T)$ on the prediction of the effect of strain rate and temperature, even if $h^*(\dot{\varepsilon}, T)$ is a function with the use of the Arrhenius type equation and $Z$ parameter, which are widely used in metallic materials. However, after further modification of the $Z$ parameter on the compensation of strain rate effect, the modified function $h^{**}(\dot{\varepsilon}, T)$ gives a much better prediction accuracy than $h(\dot{\varepsilon}, T)$ and $h^*(\dot{\varepsilon}, T)$, as shown in Fig. 11(d). The improvement in prediction results is due to not only the additional exponent of the strain rate but also the optimal solution of parameter $M$ introduced in this study.
Fig. 11. Comparison between the experimental stress and predicted stress by (a) the ZOP model, (b) Modified model I, (c) Modified model II and (d) Modified model III.

With the use of $R^2$ and $RMSE$ calculated by Eqs (50) and (51), the quantitative evaluation shows that among all four models, Modified model III is able to predict the flow stress with the highest degree of accuracy, and meanwhile, the predicted results of the ZOP model and Modified model II show least prediction accuracy in comparison. The $R^2$ value of Modified model III reaches 0.9569 and the $RMSE$ value is only 14.63 MPa, which means a great improvement from the original ZOP model after several steps of modification.

5 Discussion

5.1 Applicability of developed constitutive models

The present work demonstrates the applicability of the ZOP model and the modified models in predicting the hot deformation behaviour of 42CrMo. Results indicate that after several steps of modification, Modified model III is capable of predicting the flow stress under different strain, strain rate and temperature conditions with a high degree of accuracy. Although only the flow stress of 42CrMo is used to validate and compare the prediction accuracy, its testing results are representative and typical for the hot deformation behaviour of most metals. In the published literature, most metallic materials, such as steel [2, 33, 35-37], aluminium alloys [38-42], magnesium alloys [43-45], titanium alloys [46-49], superalloys [50-53], are similar in flow stress curves
and variation trends under tension [39, 54], compression [55, 56] and torsion [2, 57] at hot deformation when DRX occurs. The typical representation of the hot deformation behaviour of metals at a certain temperature and strain rate can be summarised in Fig. 12. In the illustration, the blue curve represents the true stress – true strain relationship under the effect of work hardening (WH) and dynamic recovery (DRV), and the red curve under the additional effect of dynamic recrystallization (DRX). As shown by the red curve, four different stages can be found including initial hardening or work hardening, transition, dynamic softening and steady state. The developed models are effective in dealing with this shape of flow stress curves by using the introduced piecewise functions and transition function as modelling tools. In terms of the hot deformation where only WH and DRV happen, the stress – strain behaviour has a much simpler shape as given by the blue curve in Fig. 12. The difference between the blue and red curves is that there is no softening stage before the achievement of steady state. Representation of this kind of curves can be achieved by the transition functions to allow a smooth stress change once the peak value is reached in small strain range. Therefore, the ZOP model and its modified models developed in this study, especially Modified model III, can be used to predict the hot deformation behaviour of many metallic materials.

Fig. 12. Typical hot deformation behaviour of metals.

Meanwhile, the improvement of the ZOP and the modified models is obvious in the prediction of flow stress in hot deformation of metals. Although the effect of strain rate and temperature on the flow stress can be predicted effectively, the effect of strain is mainly predicted by fitting high-order polynomials [21-23]. In comparison, the ZOP model and the modified models developed in this study show a clear degree of simplification and applicability. The piecewise functions used in the models provide a good level of accuracy in both small and large strain ranges, whilst the transition functions enable a smooth transition between small
and large strain deformation behaviours and the flexibility to reflect accumulated effects of WH, dynamic recovery (DRV) and dynamic recrystallization (DRX).

5.2 Physical meanings of the developed constitutive models

The validity of the developed constitutive models is confirmed in Section 4 and the applicability is discussed in Section 5.1. It is worthwhile to discuss the underlining mechanism and the relationship between the developed models and microstructure evolution under hot deformation conditions. It is well established that WH, DRV and DRX exist in metal deformation at high temperatures, as shown in Fig. 12. At the initial stage of deformation, WH dominates the flow stress increase, and this is followed by the softening behaviour due to DRX. More specifically, WH exists throughout the whole deformation process, while the work hardening rate is reduced because of the existence of DRV. With the increase of deformation strain, when a critical value of strain (defined as $\varepsilon_s$) is reached, DRX starts and dominates the material behaviour. In completion of DRX, a balance between WH, DRV and DRX is achieved, and this leads to the steady state of flow stress curves. In the ZOP model and the modified models, piecewise functions are used to predict the flow stress at small and large strains separately, and the transition functions are used to enable a smooth shifting between different prediction parts. Taking Modified model III as an example, $f^{***}(\varepsilon, \dot{\varepsilon}, T)$ is responsible for the prediction of small strain deformation behaviour before $\varepsilon_c$, while $g^{**}(\varepsilon, \dot{\varepsilon}, T)$ is applied to predict larger strain deformation behaviour after $\varepsilon_c$. Obviously, the predicted parts given by the function $f^{***}(\varepsilon, \dot{\varepsilon}, T)$ include the initial work hardening, transition and dynamic softening stages. If DRX continues even in large strains, the dynamic softening will continue as shown in the red curve in Fig. 1. However, the occurrence of DRX must have an endpoint. With the completion of DRX, the steady state appears, and the prediction may be given by function $g^{**}(\varepsilon, \dot{\varepsilon}, T)$. On this account, the completion strain of DRX (defined as $\varepsilon_c$) can be approximated as the critical strain $\varepsilon_{\text{critical}}$. In the process, transition functions $u^{**}(\varepsilon, \dot{\varepsilon}, T)$ and $v^{**}(\varepsilon, \dot{\varepsilon}, T)$ play an important role to approach smooth shifting from dynamic softening to the steady state, i.e., from DRX dominated deformation towards the steady state of flow stress. Also, the change of DRX completion strain at different strain rates and temperatures can be reflected by the transition functions because they provide flexibility in the change of the critical strain due to the involvement of strain rate and temperature related function $h^{**}(\dot{\varepsilon}, T)$ in $u^{**}(\varepsilon, \dot{\varepsilon}, T)$ and $v^{**}(\varepsilon, \dot{\varepsilon}, T)$. The above discussions show the meanings of different parts of the developed constitutive models and their relationship with microstructural evolution in hot deformation of metallic materials. This is
a useful rationality for the use of the ZOP model and the modified models in flow stress prediction in hot deformation of metals. This also gives physical meanings to the developed constitutive models and even makes it possible to predict the occurrence and completion of DRX.

It has been revealed that DRX initially starts before the peak stress point, and the strain at the start of DRX is related to a minimum amount of energy [58, 59], so the discussion of the strain at the start of DRX is not easy and out of the scope of the developed constitutive models. Even though, the peak strain \(\varepsilon_p\) is available to determine using the constitutive models and it is an important tool to study the behaviour and the strain at the start of DRX because these two important strains have a close relationship in which the starting strain of DRX is proportional to the peak strain [60] and could be approximated as \(0.8 \cdot \varepsilon_p\) as studied by Sellar [61] and Ouchi and Okita [62]. Meanwhile, the completion strain of DRX \(\varepsilon_c\) can be approximated as the critical strain \(\varepsilon_{critical}\) between the transition of the piecewise functions for small and large strain ranges, as at this point the flow stress curves enter into the steady state as discussed earlier. As a result, the peak strain \(\varepsilon_p\) and the completion strain of DRX \(\varepsilon_c\) are discussed below and predicted by the developed constitutive models.

Taking the ZOP model as an example, from Eqs (14) and (15), the result of the partial differentiation indicates that the peak strain \(\varepsilon_p\) can be predicted by the following function:

\[
\varepsilon_p = n \cdot \mu \cdot h(\dot{e}, T) \tag{52}
\]

Meanwhile, the combination of \(f(\varepsilon, \dot{e}, T)\) and \(g(\varepsilon, \dot{e}, T)\) expressed by Eqs (2) and (3) gives the following equation:

\[
K_1 \cdot \varepsilon^n \cdot \exp\{-\varepsilon/[\mu \cdot h(\dot{e}, T)]\} = K_2 \cdot [\exp(-C_1 \cdot \varepsilon) + \exp C_2] \cdot [1 - \exp(-\alpha \cdot \varepsilon)] \cdot h(\dot{e}, T) \tag{53}
\]

Solution of Eq. (53) leads to the critical strain \(\varepsilon_{critical}\) or the completion strain of DRX \(\varepsilon_c\). The prediction results of the peak strain \(\varepsilon_p\) and the completion strain of DRX \(\varepsilon_c\) and the comparison with experimental data are given in Figs 13 and 14, respectively.
Fig. 13. Comparison of peak strain from experimental results and predicted results from the ZOP model, Modified models I, II and III at deformation strain rate of (a) 0.01 s$^{-1}$, (b) 0.1 s$^{-1}$, (c) 1 s$^{-1}$, (d) 10 s$^{-1}$ and (e) 50 s$^{-1}$. 
As shown in Figs 13 and 14, from experimental testing at the same strain rate, with the elevation of temperature, both the peak strain and the completion strain of DRX show a decreasing trend; while with the decrease of strain rate, the studied strains show a decreasing trend. This was confirmed by microstructure observation in the study of Lin et al. [63]. At higher temperatures, the grain sizes were observed larger at a constant deformation degree, which indicates larger degree of DRX, because the start and completion of DRX process are accelerated while at low temperatures, more time is needed to complete DRX. At lower strain rates, although larger grain sizes were observed, it is a result of increased DRV rate. However, the required deformation time is longer at lower strain rates, so the DRX happens and completes at smaller strain than that at higher strain rates. In Fig. 13, the comparison between the experimental and predicted results shows that
the developed constitutive models are capable of predicting the peak strain with comparable results. Among all four studied models, Modified model III gives the most favourable prediction of the peak strain compared with experimental ones despite that the deviations at 50 s\(^{-1}\) are observed to become larger at lower deformation temperature. In Fig. 14, the completion strains of DRX are predicted by the developed models. All the developed models, especially Modified models II and III, give effective prediction results of the DRX completion strains extracted from experimental flow stress curves. The success in predicting the peak strain and the completion strain of DRX shows that the developed constitutive models can be used as a useful tool to study the DRX process and microstructural evolution in the hot deformation of metallic materials.

6 Conclusions

This study demonstrates that the ZOP model and its modifications can be used to represent in hot deformation behaviour of metallic materials. The prediction results of the ZOP model and its modified models are investigated and presented, followed by a discussion of the applicability and physical meanings of these models in hot deformation of metals. The main conclusions can be drawn as below.

(1) The ZOP model shows less prediction accuracy in the initial hardening stage but works well at later deformation stages. In comparison, Modified models I, II and III give a better prediction of the flow stress from the yield stress.

(2) The ZOP model can be modified to predict the hot deformation behaviour of metals at different temperatures, strain rates and strains. The most favourable prediction is given by Modified model III, in which Arrhenius type equation and modified Zener-Holloman parameter are used to predict the effect of strain rate and temperature.

(3) In Modified model III, the Zener-Holloman parameter is modified to strengthen the effect of strain rate. Unlike random attempts in previous studies, the determination method of the exponent of strain rate is given by linear curve fitting of the relationship between the \(\ln\left(\sinh\left[\sigma_p/K_1 \cdot \left(e/(n \cdot \mu)\right)^n\right]\right)\) and \(\ln \dot{\varepsilon}\). This is useful to obtain an optimum \(M\) value to achieve improved prediction accuracy.

(4) All of the developed models are able to predict the peak stress effectively. Among them, the ZOP model and Modified model III give better prediction accuracy than the others. However, the prediction of the yield
strain could only be given by Modified models I, II and III and there is still scope for improvement in terms of prediction accuracy.

(5) The piecewise functions enable the prediction of flow stress curves in different strain ranges, while the transition functions are used to shift the two piecewise functions smoothly from one to the other. In terms of physical meaning, the DRX induced softening behaviour is shifted to a steady state among WH, DRV and DRX via the developed transition functions.

(6) The peak strain and DRX completion strain can be predicted effectively by the developed models, especially Modified model III. This is of significant meaning to the study of the occurrence and completion of DRX in the hot deformation of metals.

Acknowledgements

The first author gratefully acknowledges the financial support from the University of Nottingham via Faculty of Engineering Research Excellence PhD Scholarship.

References


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