Computational analysis of axially loaded thin-walled rectangular concrete-filled stainless steel tubular short columns incorporating local buckling effects

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Abstract

This paper investigates the structural performance of concrete-filled stainless steel tubular (CFSST) columns composed of rectangular and square sections. A fiber-based mathematical model is developed to simulate the nonlinear performance of such columns loaded concentrically accounting for the local buckling of steel tube. An accurate lateral pressure model, as well as a strength degradation parameter are proposed based on the existing test results and incorporated in the mathematical modeling developed. A large test dataset is used to validate the accuracy of the numerical prediction. The mathematical model is employed to study the sensitivities of important column parameters on their axial behavior. The accuracy of the existing confinement model of rectangular CFST columns is evaluated in predicting the performance of CFSST short columns. Furthermore, the accuracy of the existing codified design models as well as and the simplified design model proposed in this study to quantify the ultimate compressive strength of such columns is investigated. An accurate artificial neural network (ANN) model along with the GUI feature is developed for the design engineers as a tool to predict their ultimate axial strengths.

Keywords: CFST columns; Stainless steel; Short columns; Local buckling, Artificial neural network.

1. Introduction

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Concrete-filled steel tubular (CFST) sections composed of the rectangular have been widely used in the construction of composite structures owing to their improved strength, fire resistance, ductility, and good constructability compared to traditional reinforced concrete columns. Some of the notable applications of such columns included the construction of building Di Wang, Shenzhen, China (height-384m), Central Plaza, Hongkong (height-292m). Besides, the use of stainless steel in composite construction gained wide attraction due to its excellent corrosion resistance and aesthetic appearance. A rectangular CFSST column as illustrated in Fig. 1 offers improved structural performance compared to the conventional CFST columns made of carbon steel. Thus, such a column has the enormous potentiality to be widely utilized in designing future composite columns. However, the investigations performed towards understanding their structural performance have been very limited.

Young and Ellobody [1] performed tests on short CFSST columns composed of either rectangular or square section. The sensitivities of plate thickness, cross-sectional shape, and the strength of concrete are examined. The failure of the test specimens was due to the buckling of the steel plate associated with concrete crushing. The sensitivities of the geometry and the concrete strength on the axial performance of CFSST columns were examined by Lam and Gardner [2]. Test results showed that the ductility of the CFSST columns is more pronounced for columns with lower strength concrete. Uy et al. [3] carried out an extensive test program to study the performance of CFSST columns made of either circular and square sections. Test parameters including slenderness ratio, eccentricity ratio, and the strength of concrete were investigated. The improvement in the ductility and the residual strengths of the columns was observed for the CFSST columns compared to their carbon steel CFST counterparts. The influences of the plate thickness and the strength of concrete in CFSST square columns were investigated by Dai et al. [4]. Increasing either the thickness or the concrete strength increased the ultimate loads of the columns. The test results reported by Chen and Huang [5] found that CFSST columns have good plastic deformation capacity. However, the compressive strengths of the columns decreased as the height to width ratio or the width-to-thickness ratio increased. Recently, Azad et al. [6] investigated the responses of compact and slender CFSST columns under axial compression, bending, and combined axial load with bending and proposed criteria for
categorizing compact and slenderness of the columns. Furthermore, Li [7] performed some tests on rectangular CFSST short columns made of ferritic stainless steel. The test specimens were made of concrete strength ranged between 42.6 to 114.6 MPa.

The numerical models developed for CFSST columns to investigate their nonlinear performance have been very limited. Patel et al. [8] developed a mathematical model to study the performance of CFSST rectangular columns loaded biaxially accounting for the influences of local buckling. Only limited test data was used to validate the numerical model developed. The material constitutive laws of concrete originally proposed for the carbon steel CFST columns were adopted to simulate the behavior of confined concrete. A finite element model using the software package ABAQUS was developed by Tao et al. [9] to study the nonlinear performance of such a column. A large discrepancy between the test and numerical predictions can be observed during the validation. Yan et al. [10] developed FE models for rectangular CFSST columns composed of ferritic grades under axial compression and validated against the test results reported by Li [7]. The only FE model that was developed by Ellobody and Young [11] considered the concrete confinement for the core concrete in CFSST columns composed of the rectangular section based on the lateral pressure formula originally developed for the carbon steel CFST columns by Hu et al. [12]. The FE models developed by Tao et al. [9] and Ellobody and Young [11] did not consider the interaction of local buckling of steel plates which may overestimate their ultimate strengths. Recently, Ahmed et al. [13] developed a numerical model for elliptical CFST columns composed of stainless steel. The fiber-based numerical model is found to be effective in response analysis of composite columns.

This study investigates the performance of rectangular CFSST columns that are loaded concentrically. A mathematical model is developed using the technique of fiber analysis that considers the important features of such columns including local buckling, and concrete confinement. An accurate confinement model including a lateral pressure model and a strength degradation parameter is proposed from the existing test results. The accuracy of the existing confinement models and the codified design models is examined against a large test database collected. A simplified design equation is suggested as
well as an accurate artificial neural network (ANN) model is developed to predict the ultimate compressive strengths of such columns loaded axially.

2. The mathematical model

The mathematical model is developed based on the concept of the fiber analysis method which is simple yet computationally efficient [14-19]. The cross-section of a CFSST column composed of the rectangular section is divided into steel and concrete elements as shown in Fig. 2. The stress for each fiber is calculated from the corresponding strain using the uniaxial material laws described in the following section. The axial load \( P \) is determined by integrating stresses over the whole cross-section. The mathematical model assumes that the plane section remained plane under deformation and the influences of concrete creep and shrinkage are ignored. The nonlinear analysis is performed by incrementally increasing the axial strain and compute the stresses using the material laws. The analysis is continued until either the axial load is less than 50% of the maximum load or the axial strain exceeds the maximum concrete strain \( \varepsilon_{cu} \) prescribed. The computational analysis procedures are as follows:

1. Input the cross-sectional details and material properties of CFSST columns.
2. Divide the cross-section into fine fibers.
3. Initialize axial strains \( \varepsilon = \Delta \varepsilon \).
4. Determine the stresses of fibers using uniaxial material laws.
5. Check for local buckling and update the stresses of fibers.
6. Determine the resultant axial force \( P \).
7. Increase strains as \( \varepsilon = \varepsilon + \Delta \varepsilon \).
8. Repeat steps 4-7 until \( P \leq 0.5 P_{\text{max}} \) or \( \varepsilon \geq \varepsilon_{cu} \).
9. Plot the load-axial strain curves.

3. Stress-strain relationships of stainless steel

The stress-strain curves of stainless steel proposed by Quach et al. [20] were employed in this study. The three-stage stress-strain curve is found to be more accurate compared to the other models with two-
stage stress-strain curves such as the one proposed by Rasmussen [21] to capture the stress-strain relationships of stainless steel with all three grades namely Austenitic, duplex/lean duplex and ferritic as discussed by Quach et al. [20], Patel et al. [22], and Tao et al. [9]. Abdella et al. [23] proposed the inversion of the stress-strain curves proposed by Quach et al. [20] and suggested formulations to calculate the stress as a function of strain which is illustrated in Fig. 3. The equations to calculate the full stress-strain curves of stainless steel are given by Ahmed et al. [13].

4. Constitutive laws of concrete

4.1. General

The idealized stress-strain curves of concrete are illustrated in Fig. 4. Formulas suggested by Mander et al. [24] are used to determine the longitudinal stress ($\sigma_c$) of the concrete in the ascending branch which can be written as

$$\sigma_c = \frac{f_{cc} (\varepsilon_c / \varepsilon_{cc}) \lambda}{(\varepsilon_c / \varepsilon_{cc})^4 + \lambda - 1} \quad \text{for} \quad 0 \leq \varepsilon_c \leq \varepsilon_{cc}$$

(1)

$$\lambda = \frac{E_c \varepsilon_{cc}}{E_c \varepsilon_{cc} - f_{cc}}$$

(2)

in which $\varepsilon_c$ refers to the longitudinal strain; $f_{cc}$, and $\varepsilon_{cc}$ are the compressive strength of confined concrete and the compressive strain. The elastic modulus of concrete ($E_c$) is determined as Lim and Ozbakkaloglu [25]

$$E_c = 4400 \sqrt{\gamma_c f_{cc}} \quad \text{(MPa)}$$

(3)

where $\gamma_c$ is the size reduction factor for concrete suggested by Liang [26] as $\gamma_c = 1.85D_c^{-0.135}$, where $D_c$ is taken as the larger of $(D - 2t)$ or $(B - 2t)$. The expressions given by Lim and Ozbakkaloglu [25] given in Eq. (4) are used to determine the descending branch of the stress-strain curve of concrete shown in Fig. 4.
\[
\sigma_c = f'_{ce} - \frac{f'_{ce} - f_{er}}{1 + \left(\frac{e_{cr} - e_{ec}}{e_{ci} - e_{ec}}\right)^2} \quad \text{for} \quad \varepsilon_c > \dot{\varepsilon}_{ec}
\] 

(4)

in which \(f_{er}\) is the residual concrete strength and \(\varepsilon_{ci}\) is the strain at the inflection point taken as 0.007 [27].

4.2. Compressive strength and strain of confined concrete

To determine the confined concrete strength \((f'_{ce})\) and the corresponding strain \((\varepsilon'_{ce})\), formulas proposed by Mander et al. [24] are utilized which are written as

\[
f'_{ce} = \left(1 + \frac{4.1f_{np}}{\gamma_{f_c}}\right)\gamma_{f_c}f'_{c}
\] 

(5)

\[
\varepsilon'_{ce} = \varepsilon'_{c} + \frac{20.5f_{np}\varepsilon'_{c}}{\gamma_{f_c}f'_{c}}
\] 

(6)

\[
\varepsilon'_{c} = 0.00076 + \sqrt{(0.626\gamma_{f_c} - 4.33) \times 10^{-7}}
\] 

(7)

where, \(f_{np}\) is the lateral pressure inserted by the steel tube to concrete. By interpreting the test results reported by Young and Ellobody [1], Uy et al. [3], Lam and Gardner [2], Dai et al. [4], Chen and Huang [5], Azad et al. [6], and Liao et al. [28], the expression to calculate \(f_{np}\) is proposed in this study. Firstly, the section capacity of the stainless steel tube is subtracted from the ultimate load of the column which was taken as the first peak load or the axial load at 1% strain. Eq. (5) was then utilized to calculate the test lateral pressure \((f_{np,teu})\). Simple linear relationships between the test lateral pressures of the tested columns and the confinement factors \((\xi)\) are established and an expression for \(f_{np}\) is obtained using the simple linear regression tool as shown in Fig. 5 which is expressed as

\[
f_{np} = 2.841 \xi - 1.773 \quad (f_{np} \geq 0)
\] 

(8)

where \(\xi\) is the confinement factor defined as
\[
\xi = \frac{A_c \sigma_{0.2}}{A_s \gamma_c f_c}
\]  \hspace{1cm} (9)

where \( A_c \) and \( A_s \) are the cross-sectional areas of concrete and steel, respectively. An \( R^2 \) value of 0.89 is obtained from the statistical analysis as seen in Fig. 5.

4.3. Residual strength of confined concrete

Parameter study shows that the residual strength of confine concrete (\( f_{cr} \)) is influenced by the confinement factor (\( \xi \)) and calculated as

\[
f_{cr} = \beta_c f_{cc}
\]  \hspace{1cm} (10)

in which \( \beta_c \) refers to the strength degradation parameter and ranged between 0 to 1. The expression to calculate \( \beta_c \) is derived from the test results of Uy et al. [3], Lam and Gardner [2], Dai et al. [4], and Liao et al. [28]. The trial and error method was used to determine the value of \( \beta_c \) to best fit the load-axial strain curves of test columns. Similar to the development of the lateral pressure model, the test values of \( \beta_c \) for tested columns are plotted with the corresponding values of confinement factors (\( \xi \)) and a simple linear relationship is established to obtain a formula to calculate \( \beta_c \) suggested herein as

\[
\beta_c = 0.459 \xi + 0.046 \quad (0 < \beta_c \leq 1)
\]  \hspace{1cm} (11)

Figure 6 shows the validation of the proposed formula where an \( R^2 \) value of 0.85 is obtained.

5. Local buckling of rectangular steel tube

As can be seen from the literature review, the failure mode of the CFSST columns is associated with the local buckling of the flat steel plate and thus should be considered in the mathematical modeling to obtain accurate estimations. In this study, the progressive localized buckling of flat steel walls is designed using the guidelines given by Liang et al. [29]. The effective and ineffective widths of the flat plates are used to simulate the post-local buckling mode of the steel plates as shown in Fig. 7 which are estimated using the effective width formulas proposed by Liang et al. [29].

6. Model validation
6.1. Test results

Table 1 summarises the test data used to verify the accuracy of the mathematical model developed. A total of 109 test columns are collected from the literature which covers a wide range of column parameters including: $B/t = 11-100$, $f' = 19-114.6$ MPa, $\sigma_{0.2} = 258-634$ MPa, and $\xi = 0.4-5.5$. The test results cover all three grades of stainless steel as shown in Table 1. In comparison, the ultimate axial load is taken as the first peak load if the measured load-axial strain ($P-\varepsilon$) curve has a strain-softening branch whereas the ultimate load is defined as the axial load at the onset of the strain at 0.01 if the $P-\varepsilon$ curve has no obvious strain-softening branch [30, 31]. From the comparisons of the ratio $P_{u,\text{num}}/P_{u,\text{exp}}$ presented in Fig. 8, it is seen that the mathematical model incorporating the proposed material laws can reasonably predict the ultimate strengths of CFSST columns. The mean $P_{u,\text{num}}/P_{u,\text{exp}}$ is calculated as 1.00 with a standard deviation of 0.07.

The comparisons between the measured and predicted $P-\varepsilon$ curves of CFSST columns are given in Fig. 9. The test specimens S-3-L-DS-1 and S-3-H-SS-1 were tested by Liao et al. [28], specimen D250 was tested by Azad et al. [6], specimen SHS 100x100x2-C60 was reported by Lam and Gardner [2], and specimens 304-t12c70 and 304-t12c80 were tested by Dai et al. [4]. In general, the mathematical model is capable of predicting the initial stiffness and the residual strength of such columns with reasonable accuracy.

6.2. Comparisons with existing confinement models

The accuracy of existing confinement models for carbon steel CFST columns listed in Table 2 proposed by Liang [26], Thai et al. [32], Hu et al. [12], and Lai and Varma [15] is evaluated in simulating the axial performance of CFSST columns. Figure 10 presents the comparisons between the test and the predicted ultimate strengths of such columns obtained using various confinement models. It is seen that the existing confinement models fail to accurately predict the ultimate strengths of some columns with a large discrepancy. Although the confinement model proposed by Hu et al. [12] provides an average value of 1, the ultimate strengths of some columns are overestimated as much as 30% as can be seen from Fig. 10.
From the comparisons of the measured and predicted $P - \varepsilon$ curves of CFSST columns using various confinement models presented in Fig. 11, it is observed that the existing model can not accurately yield the $P - \varepsilon$ curves of such a column loaded concentrically. On the contrary, the confinement model proposed can reasonably predict the $P - \varepsilon$ curves of such a column.

7. Parameter study

The influences of important column parameters on the axial performance of CFSST columns are studied in this section. The details of the reference columns are given in Table 3. The Young’s modulus of the steel tube is taken as 200GPa.

7.1. Effects of the steel ratio ($\rho_s$)

The sensitivities of the steel ratio ($\rho_s$) defined as $\rho_s = (A_s / A_c) \times 100\%$ were analyzed using columns C1 to C4 in Group G1 where $\rho_s$ ratio varied from 5% to 20%. The increasing $\rho_s$ ratio increases the strength and the ductility of the columns as can be seen from Fig. 12. The increase in the ultimate strength is calculated as 166.3% when the $\rho_s$ ratio increases from 5% to 20%. Furthermore, improvement in the residual strength of confined concrete can be observed for the increase of $\rho_s$ ratio. The CFSST column with a larger steel ratio is less prone to local buckling, hence improved strength and ductility can be obtained. The load distributions of the steel and concrete in CFSST columns with varying $\rho_s$ ratio were also captured using the mathematical model and illustrated in Fig. 13. It is found that increasing $\rho_s$ ratio increases the load contribution of the steel tube at the ultimate load but decreases the contribution of the confined concrete. The load contribution of the steel tube and concrete is 26.6% and 73.4% of the ultimate load for the column with $\rho_s$ ratio of 5% while it is 66.5% and 33.5% for $\rho_s$ ratio of 20%.

7.2. Effects of concrete strength

The strength of concrete for the columns in Group G2 varied from 40 MPa to 70 MPa to study the sensitivities of the strength of concrete on the axial behavior of such columns. Figure 14 presents the $P - \varepsilon$ curves of columns with varying strength of concrete. It can be seen that the increase in the concrete strength
leads to an improvement in the ultimate strength of the columns. However, the decrease in the ductility of the columns is observed due to the brittle nature of high-strength concrete. The increase of the ultimate load is estimated at 44.2% by increasing the strength of concrete from 40 MPa to 70 MPa. The influences of concrete strength on the ultimate loads of short columns with varying $B/t$ ratio are presented in Fig. 15. The ultimate axial load of the column composed of 40 MPa concrete was considered as the benchmark value. Interestingly, it is observed that the influences of concrete strengths on the strength enhancement of columns increase with the increase of the $B/t$ ratio.

7.3. Effects of the width-to-thickness ratio of the steel tube

Columns in Group G3 had the width-to-thickness ($B/t$) ratio of the steel tube varied from 40 to 100 by varying the tube thickness. From the $P - \varepsilon$ curves of these columns presented in Fig. 16, it is observed that increasing $B/t$ ratio remarkably decreases the ultimate loads of the columns. The ultimate load of the column with a $B/t$ ratio of 100 is 24.3% less than the column with a $B/t$ ratio of 40. The CFSST column with a larger $B/t$ ratio is susceptible to local buckling and can not provide effective confinement to the core concrete.

7.4. Effects of steel proof stress ($\sigma_{0.2}$)

Columns in Group G4 had the steel proof stress ($\sigma_{0.2}$) varied from 205MPa to 430 MPa. This covers all three grades of stainless steel prescribed in AS/NZS 4673:2001 [33] namely duplex, austenitic, and ferritic stainless steel. From the $P - \varepsilon$ curves with varying proof stress presented in Fig. 17, it is seen that the initial stiffness and the ultimate compressive strength of CFSST columns increase for the increase of the proof stress. When the proof stress increases from 250 MPa to 430 MPa, the percentage of the increase in the ultimate strength is calculated as 35.3%. Figure 18 illustrates the effects of steel proof stress on the ultimate capacities of short columns for varying $B/t$ ratio where the ultimate axial load of the column with steel proof stress of 205MPa was taken as the benchmark value. It is seen that the rate of increase in the ultimate strength for varying steel yield stress reduces as the $B/t$ ratio of the columns increases.

7.5. Effects of concrete confinement
To study the influences of concrete confinement, Column C4 is analyzed with and without considering the effects of concrete confinement. It is obvious from Fig. 19 that neglecting the effects of confinement results in the underestimation of the ultimate strength of the column remarkably. When the confinement effects were neglected, the ultimate strength was underestimated as much as 12.19%. From Fig 20(a), it is observed that for columns with \( B/t \) ratio of 40 onwards or the concrete strength of 80MPa or above, the effects of concrete strength on the concrete confinement can be neglected. On the contrary, for varying steel proof stress, the effects of concrete confinement are pronounced for columns with a smaller \( B/t \) ratio as can be seen in Fig. 20(b). However, the effects of steel yield stress on the concrete confinement can be neglected from the onwards of \( B/t \) ratio of 80. The effects of local buckling on the concrete confinement of CFSST columns are also investigated in Fig. 21. It is seen that local buckling of outer steel tube has a significant influence on the peak compressive strength and ductility of the columns particularly for columns with large \( B/t \) ratio. When the effects of local buckling on the concrete confinement were neglected, the ultimate strength of CFSST columns was overestimated by 0%, 7.3%, 7.9% and 8.1% for column with a \( B/t \) ratio of 16.6, 40, 60 and 80, respectively.

7.6. Influences of local buckling

The sensitivities of local buckling on the axial performance of CFSST columns are studied by analyzing Column C17 with and without considering the effects of local buckling. From the \( P-\varepsilon \) curves presented in Fig. 22, it is observed that ignoring the effects of local buckling overestimates the ultimate load and the ductility of CFSST columns considerably. The ultimate load was overestimated by 6.7% when the interaction of local buckling was neglected during mathematical modeling. The sensitivities of local buckling effects on the nonlinear analysis of CFSST columns were investigated by varying \( B/t \) ratio and steel proof stress of the columns. From Fig. 23 it is observed that the influences of local buckling on the strength curves of CFSST columns become significant as the \( B/t \) ratio of the columns increases particularly from the onwards of \( B/t \) ratio of 40.

8. Design models for rectangular CFSST columns

8.1. Codified design models
The accuracy of the design guidelines provided by the various design codes including AISC 360-16 [34], Eurocode 4 [35], ACI 318-19 [36], and DBJ 13-51-2010 [37] for conventional carbon steel CFST columns is evaluated in this section. However, the yield stress of the steel tube is replaced with the proof stress of stainless steel during the strength predictions. The design specifications of the design codes are summarized in Table 4. A database of 516 rectangular CFSST short columns including 109 test results presented in Table 1 and 407 FE simulations is developed for the validation of the design models. The FE simulations cover a wide range of column parameters. Table 5 presents the statistical background of the FE simulations developed. The comparisons between the design ultimate strength and the ultimate strengths of the columns obtained from the test or FE are presented in Fig. 24 where it is seen that there are large discrepancies between the design predictions and the actual measurement. The prediction-to-test/numerical ultimate strength is estimated as 1.06, 0.97, 0.96, and 1.05 for Eurocode 4, ACI 318-19, AISC 360-16 and DBJ 13-51-2010, respectively. Although the ratio is close to 1 for the design codes of ACI 318-19 and AISC 360-16, the standard deviation values calculated were very high. These design codes underestimate the ultimate strength by as much as 30% as seen in Fig. 24.

8.2. Proposed design model

A simplified design equation similar to the ones suggested by the researchers earlier such as [10, 38-40] is suggested to predict the ultimate compressive load of CFSST columns defined as

\[ P_{u,des} = A_{se} \sigma_{0.2} + f'_{cc} A_c \]  \hspace{1cm} (12)

where, \( A_{se} \) is the effective cross-sectional area for the steel tube. From the comparisons of the predicted ultimate loads presented in Fig. 25, it is seen that the design proposed model can provide a reasonable estimation where the average prediction-to-test ultimate strength is calculated as 0.94 with a corresponding SD value of 0.06.

9. Development of the ANN model

Recently, artificial neural network (ANN), one of the robust methods in the area of artificial intelligence, has been widely used to solve many complex engineering problems [41]. Generally, the main
elements in an ANN are neurons (or units), an input layer, hidden layer(s), an output layer, activation functions, weights, and biases [42, 43]. Figure 26 shows a schematic of a back-propagation neural network with 2 input neurons ($I_1$ and $I_2$), a hidden layer with 3 hidden neurons ($H_1$, $H_2$, and $H_3$), and 2 output neurons ($O_1$ and $O_2$).

In the ANN, the input neurons receive the information from the input variables without conducting any computation. The hidden or output neuron receives information as inputs on one side and provides outputs from the other side using the weights, a bias, and an activation function, as shown in Fig. 27.

Mathematically, the output of a hidden neuron is obtained using the following equation:

$$ f_j = \varphi_h \left( \sum_{i=1}^{n} w_{ij} I_i + b_j \right) $$

(13)

where, $n$ denotes the number of input neurons; $I_i$ indicates the $i^{th}$ input value; $w_{ij}$ is the weight between the $i^{th}$ neuron in the input layer and the $j^{th}$ neuron in the hidden layer; $b_j$ known as the bias of $j^{th}$ neuron; and $\varphi_h$ is the activation function in the hidden layer.

The output of an output neuron is obtained as follows:

$$ f_o = \varphi_o \left( \sum_{j=1}^{k} w_{jk} H_j + b_k \right) $$

(14)

where, $h$ denotes the number of hidden neurons; $H_j$ indicates the $j^{th}$ hidden value; $w_{jk}$ is the weight between the $j^{th}$ neuron in the hidden layer and the $k^{th}$ neuron in the output layer; $b_k$ known as the bias of $k^{th}$ neuron; and $\varphi_o$ is the activation function in the output layer.

After determining the ANN structure, the training algorithm is performed to find optimum values for weights and biases. This procedure comprises two phases: forward propagation and backward
propagation. In the forward propagation phase, the error between the predicted value and the target value is determined. In the backward propagation phase, the weights and biases are readjusted to reduce the error. This process is iterated until minimal error is obtained.

In this study, the ANN model has an input layer with 6 input variables are $L, B, D, t, \sigma_{\text{eq}},$ and $f'$. The output variable is the ultimate loads of the CFSST columns ($P_u$). The activation functions employed in the ANN model are the hyperbolic tangent sigmoid function (TANSIG) in the hidden layer and a linear function (PURELIN) in the output layer. The Levenberg–Marquardt (LM) algorithm is chosen during the training of the data [44, 45]. The database of 516 CFSST columns (109 test data and 407 FE data) is randomly divided into three groups to train, test, and validate the ANN model developed. The early stopping technique is employed to prevent over-fitting during the training stage. The data is scaled in a range of $(-1, 1)$ before training the network based on the recommendations of Tran and Kim [45]. In the development of an accurate ANN model, it is vital to select the appropriate number of hidden layers, and the number of neurons in each hidden layer [44-46]. Based on the trial and error method that was analyzed to study the best splitting ratios along with the number of neurons in the hidden layers, the best ANN model comprises of two hidden layers with ten neurons for each, 70% training data, 15% test data, and 15% validation data is chosen. The structure of the developed ANN model is shown in Fig. 28.

From the performance of the ANN model illustrated in Fig. 29, it is seen that the best training performance of the ANN model was achieved at the 15th epoch. The statistical analysis of the performance of the ANN model is illustrated in Fig. 30, where the $R^2$ values are 1.0, 0.997, 0.999, and 0.999 for training data, test data, validation data, and all data, respectively. Figure 31 shows the comparisons of the ANN and design models in predicting the ultimate strengths of CFSST columns. While it is seen that both ANN and design models provide very close results for the test columns, however, ANN model provides a better estimation for the FE results.
For a practical application of the developed ANN model, a Graphical User Interface (GUI) as shown in Fig. 32 is also developed using the language program MATLAB. The GUI tool can be readily available to structural design engineers for designing such a column.

10. Conclusions

This study investigates the axial performance of rectangular CFSST columns loaded concentrically using an efficient computational model developed. A new lateral pressure model and a strength degradation parameter are suggested for the confined concrete of CFSST columns composed of the rectangular section by interpreting the available test data. Unlike the existing confinement models, the material laws proposed can reasonably yield the performance of CFSST columns observed experimentally. Parameter study shows that $B/t$ ratio of rectangular CFSST columns is the key parameter that influences their performance under axial loading. The increase in the ultimate strengths of rectangular CFSST columns for the increase of the concrete strength, proof stress of steel tube is found to be significant for columns with a smaller $B/t$ ratio. Furthermore, it is found that the influences of concrete confinement and local buckling effects on their axial performance are also controlled by the $B/t$ ratio of the columns. For columns with $B/t$ ratio of 40 onwards or the concrete strength of 80MPa or above, the effects of concrete strength on the concrete confinement can be neglected. Similarly, the effects of steel yield stress on the concrete confinement can be neglected from the onwards of $B/t$ ratio of 80. From the comparisons of the design model, it is found there are large discrepancies between the codified design predictions and the test/numerical results. On the contrary, the simple design formula suggested can yield a more reasonable estimation than the codified methods. Lastly, the developed ANN model is recommended to be used by the practical design engineers for designing rectangular CFSST columns loaded concentrically owing to its excellent accuracy.

The range of column parameters investigated and used to develop the database to train and validate the ANN model in this study is limited to the following: $B = 40–1200$ mm, $D = 51–1200$ mm, $t = 1–40$ mm, $B/t = 10–100$, $D/t = 16–100$, $f' = 19–115$ MPa, $\sigma_{y2} = 205–634$ MPa, and
However, the theoretical model developed was validated against the test results of CFSST columns made of all different grades of stainless steel including austenitic, duplex and ferritic grades. Furthermore, this study investigates the performance of CFSST columns composed of rectangular sections, however, considering the more effective confinement effects compared to their rectangular counterparts, further studies should investigate the axial performance of circular CFSST columns.

References


### FIGURES AND TABLES

**Table 1** Summary of the test data utilized for validation purposes.

<table>
<thead>
<tr>
<th>Number of specimens</th>
<th>B (mm)</th>
<th>D (mm)</th>
<th>t (mm)</th>
<th>B/t</th>
<th>Stainless steel grade</th>
<th>σ_{0.2} (MPa)</th>
<th>f_c (MPa)</th>
<th>Ref.</th>
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<td>160</td>
<td>160</td>
<td>2.9-4.8</td>
<td>33-56</td>
<td>Austenitic</td>
<td>415-446</td>
<td>37.4-43.7</td>
<td>[25]</td>
</tr>
<tr>
<td>14</td>
<td>60-100</td>
<td>60-100</td>
<td>1.0</td>
<td>60-100</td>
<td>Austenitic</td>
<td>258</td>
<td>19.3-25</td>
<td>[5]</td>
</tr>
<tr>
<td>18</td>
<td>295.5-303.7</td>
<td>296-304</td>
<td>7.8-12.5</td>
<td>24-38</td>
<td>Austenitic and Duplex</td>
<td>293-634</td>
<td>41-69.8</td>
<td>[4]</td>
</tr>
<tr>
<td>6</td>
<td>99.3-101.6</td>
<td>100-101</td>
<td>2.0-4.9</td>
<td>20-50</td>
<td>Austenitic</td>
<td>385-458</td>
<td>30-74</td>
<td>[2]</td>
</tr>
<tr>
<td>22</td>
<td>51-150</td>
<td>51-150</td>
<td>1.8-5.1</td>
<td>18-53</td>
<td>Austenitic</td>
<td>268-440</td>
<td>21.5-34.9</td>
<td>[3]</td>
</tr>
<tr>
<td>6</td>
<td>80.1-150.5</td>
<td>140.2-150.5</td>
<td>3.1-5.8</td>
<td>26</td>
<td>Duplex</td>
<td>486-497</td>
<td>46.6-83.5</td>
<td>[1]</td>
</tr>
<tr>
<td>4</td>
<td>98.8-249.5</td>
<td>98.8-249.5</td>
<td>3.07-3.08</td>
<td>32-81</td>
<td>Austenitic and Lean Duplex</td>
<td>266-511</td>
<td>37</td>
<td>[6]</td>
</tr>
<tr>
<td>15</td>
<td>40.1-79.9</td>
<td>59.9-120</td>
<td>1.9-3.8</td>
<td>11-29</td>
<td>Ferritic</td>
<td>381-479</td>
<td>42.6-114.6</td>
<td>[7]</td>
</tr>
</tbody>
</table>
Table 2. Various concrete confinement models of rectangular CFST columns utilized for response simulation of CFSST columns.

<table>
<thead>
<tr>
<th>Confinement model</th>
<th>Compressive concrete strength ( (\hat{f}_{cc}) )</th>
<th>Strength reduction factor for compressive concrete ( (\beta_c) )</th>
</tr>
</thead>
</table>
| Liang [26]        | \( f_{cc} = \gamma f_c \)                         | \[
\beta_c = \begin{cases} 
1 & \text{for } B/t \leq 24 \\
1.5 - \frac{1}{48} B/t & \text{for } 24 < B/t \leq 48 \\
0.5 & \text{for } B/t \geq 48 
\end{cases}
\] |
| Hu et al. [12]    | \[
\frac{f_{cc}}{f_c} = 1 + 4.1 \left( \frac{f_{sp1}}{f_c} \right)^{0.8} \\
\] \[
\frac{f_{sp1}}{f_c} = \begin{cases} 
0.055048 - 0.001885(B/t) & \text{for } 17 \leq B/t \leq 29.2 \\
0 & \text{for } 29.2 < B/t \leq 150 
\end{cases}
\] | \[
\beta_c = \begin{cases} 
0.00178(B/t)^2 - 0.02492(B/t) + 1.2722 & \text{for } 17 \leq B/t \leq 70 \\
0.4 & \text{for } 70 < B/t \leq 150 
\end{cases}
\] |
| Thai et al. [32]  | \[
\frac{f_{cc}}{f_c} = 1 + 3.24 \left( \frac{f_{sp2}}{f_c} \right)^{0.8} \\
\] \[
\frac{f_{sp2}}{f_c} = \begin{cases} 
(195.118 + 40.611 f_c) e^{-0.0(B/t)} & \text{for } B/t \leq 15 \\
988 - 0.01962 f_c & \text{for } B/t > 15 
\end{cases}
\] | \( \beta_c = 0.1 \) |
| Lai and Varma [15]| \[
\frac{f_{cc}}{f_c} = 0.8 + 0.18 \left( \frac{B/t}{100} + \frac{f_{cc}}{f_c} \right) \leq 1.10 
\] | The post-peak behavior of concrete is similar to the unconfined concrete model |
Table 3 Details of the rectangular and square CFSST short columns utilized for the parameter study.

<table>
<thead>
<tr>
<th>Group</th>
<th>Column</th>
<th>B (mm)</th>
<th>D (mm)</th>
<th>t (mm)</th>
<th>$\sigma_{0.2}$ (MPa)</th>
<th>$\sigma_w$ (MPa)</th>
<th>n</th>
<th>$f'_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>C1</td>
<td>300</td>
<td>400</td>
<td>4.3</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>300</td>
<td>400</td>
<td>8.8</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>300</td>
<td>400</td>
<td>13.4</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td>300</td>
<td>400</td>
<td>18.1</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td>G2</td>
<td>C5</td>
<td>500</td>
<td>500</td>
<td>10</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C6</td>
<td>500</td>
<td>500</td>
<td>10</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C7</td>
<td>500</td>
<td>500</td>
<td>10</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>C8</td>
<td>500</td>
<td>500</td>
<td>10</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>70</td>
</tr>
<tr>
<td>G3</td>
<td>C9</td>
<td>400</td>
<td>400</td>
<td>10</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>C10</td>
<td>400</td>
<td>400</td>
<td>6.67</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>C11</td>
<td>400</td>
<td>400</td>
<td>5</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>C12</td>
<td>400</td>
<td>400</td>
<td>4</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>50</td>
</tr>
<tr>
<td>G4</td>
<td>C13</td>
<td>500</td>
<td>500</td>
<td>10</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C14</td>
<td>600</td>
<td>600</td>
<td>10</td>
<td>275</td>
<td>450</td>
<td>9.00</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C15</td>
<td>600</td>
<td>600</td>
<td>10</td>
<td>350</td>
<td>520</td>
<td>8.5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C16</td>
<td>600</td>
<td>600</td>
<td>10</td>
<td>430</td>
<td>590</td>
<td>5.5</td>
<td>40</td>
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<tr>
<td>G5</td>
<td>C17</td>
<td>500</td>
<td>500</td>
<td>5</td>
<td>275</td>
<td>450</td>
<td>8.5</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 4 Codified design formulas for rectangular CFSST short columns.

<table>
<thead>
<tr>
<th>Design codes</th>
<th>Design equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurocode 4 [35]</td>
<td>$P_{u,EC4} = \sigma_{0.2}A_s + A_cf'_c$</td>
</tr>
<tr>
<td>ACI 318-19 [36]</td>
<td>$P_{u,ACI} = \sigma_{0.2}A_i + 0.85A_i f'_c$</td>
</tr>
<tr>
<td>AISC 360-16 [34]</td>
<td>$P_{u,AISC} = \begin{cases} P_u \left[0.658^{\frac{P_u}{P_o}}\right] &amp; \text{for } P_u \geq 0.44P_o \ 0.877P_u &amp; \text{for } P_u &lt; 0.44P_o \end{cases}$</td>
</tr>
<tr>
<td>DBJ 13-51-2010 [37]</td>
<td>$P_{u,DBJ} = f'_{cs}(A_s + A_t)$</td>
</tr>
<tr>
<td></td>
<td>$f'<em>{cs} = f</em>{cs} \left(1.18 + 0.85 \xi\right)$</td>
</tr>
<tr>
<td></td>
<td>$f_{cs} = 0.67f_{cs}$</td>
</tr>
</tbody>
</table>

where $\xi = C_s = 0.85$
Table 5 Statistical data of CFSST short columns utilized for developing the ANN model.

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>L (mm)</th>
<th>B (mm)</th>
<th>D (mm)</th>
<th>t (mm)</th>
<th>( \sigma_{0.2} ) (MPa)</th>
<th>( f'_c ) (MPa)</th>
<th>( \xi )</th>
<th>( P_u ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>300</td>
<td>300</td>
<td>3</td>
<td>205</td>
<td>30</td>
<td>0.1</td>
<td>3741.3</td>
<td></td>
</tr>
<tr>
<td>3600</td>
<td>1200</td>
<td>1200</td>
<td>40</td>
<td>634</td>
<td>100</td>
<td>4.6</td>
<td>181059</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2199.5</td>
<td>628.0</td>
<td>733.2</td>
<td>15.9</td>
<td>369.1</td>
<td>55.6</td>
<td>1.1</td>
<td>39660</td>
</tr>
<tr>
<td>Standard Deviation (SD)</td>
<td>819.6</td>
<td>297.6</td>
<td>273.2</td>
<td>8.1</td>
<td>121.9</td>
<td>19.9</td>
<td>0.8</td>
<td>35182</td>
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<tr>
<td>Coefficient of Variance (CoV)</td>
<td>0.37</td>
<td>0.47</td>
<td>0.37</td>
<td>0.51</td>
<td>0.33</td>
<td>0.36</td>
<td>0.7</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Fig. 1. Cross-section of a rectangular CFSST column.

Fig. 2. Discretization of rectangular CFSST columns.
Fig. 3. Stress-strain relationships of stainless steel.

Fig. 4. Typical stress-strain curves for confined and unconfined concrete.
Fig. 5. Development of the formula for calculating lateral pressure \((f_p)\).

\[
y = 2.841x - 1.773 \\
R^2 = 0.89
\]

Fig. 6. Verification of the proposed equation for \(\beta_c\).
Fig. 7. The effective width of the rectangular steel tube.

Fig. 8. The comparisons of the test and numerical strengths of the rectangular and square CFSST short columns.
Fig. 9. Comparisons of predicted and measured axial load-strain curves of rectangular and square CFSST columns employing the proposed confinement models.
Fig. 10. Comparisons of the predicted and measured ultimate strengths of rectangular and square CFSST short columns for various concrete confinement models.
Fig. 11. Comparisons of the predicted and measured load-axial strain graphs of rectangular and square CFSST short columns for various concrete confinement models.
Fig. 12. Influences of $\rho_s$ ratio on the $P-\varepsilon$ curves of rectangular CFSST columns.

Fig. 13. Load-distributions in rectangular CFSST short columns with varying $\rho_s$ ratios.
Fig. 14. Influences of concrete strength on the $P - \varepsilon$ curves of rectangular CFSST columns.

Fig. 15. Influences of $B/t$ ratio on the concrete strength of CFSST columns.
Fig. 16. Influences of $B/t$ ratio on the $P-\varepsilon$ curves of rectangular CFSST columns.

Fig. 17. Influences of the proof stress of stainless steel on the $P-\varepsilon$ curves of rectangular CFSST columns.
Fig. 18. Influences of $B/t$ ratio on the proof stress of stainless steel of CFSST columns.

Fig. 19. Influences of the concrete confinement on the $P - \varepsilon$ curves of rectangular CFSST columns.
Fig. 20. Influences of the concrete confinement on the ultimate loads of rectangular CFSST columns for varying concrete strengths and steel proof stresses.
Fig. 21. Influences of the local buckling on the concrete confinement of rectangular CFSST columns.
Fig. 22. Influences of the local buckling on the $P - \varepsilon$ curves of rectangular CFSST columns.

Fig. 23. Influences of local buckling for the various $B/t$ ratio and steel proof stress of CFSST columns.
Fig. 24. Comparisons of the ultimate strengths of rectangular CFSST short columns against the code predictions.

Fig. 25. Comparisons of the ultimate strengths of rectangular CFSST short columns against the proposed design model.
Fig. 26. A sample of an ANN.

Fig. 27. An operation of an artificial neuron in the hidden and output layers.
Fig. 28. The structure of the ANN model.

Fig. 29. The performance of the ANN model.
Fig. 30. Regression relationship between target and output of the ANN model.
Fig. 31. Comparisons of the ultimate strengths of rectangular CFSST short columns against the design and ANN models.

Fig. 32. The GUI feature of the ANN model.