

The impact of thermal insulation on cooling energy consumption and optimal insulation thickness for underground tunnel

Yijiang Wang ^{a*}, Chaochao Wang ^a, Shang Gao ^a, Xiaofeng Zheng ^{b*}, Jo Darkwa ^b

^a State key laboratory for Geomechanics and Deep Underground Engineering, School of Mechanics and Civil Engineering, China University of Mining and Technology, Xuzhou 221116, China

^b Faculty of Engineering, University of Nottingham, University Park, Nottingham NG7 2RD, UK

Corresponding authors: Y. Wang (yjwang@cumt.edu.cn), X. Zheng(xiaofeng.zheng@nottingham.ac.uk)

Abstract: The provision of mechanical cooling in deep mines comes with a significant energy cost as a significant amount of heat transfers from surrounding rock to the airflow. Thermal insulation can be applied to reduce such heat transfer, thereby cutting down the cooling load. In this study, the impact of thermal insulation on reducing the heat flux through the rock was analytically investigated. The optimal insulation thickness, life cycle saving and the payback period were also evaluated by using the life cycle cost method as the economic benefit is heavily dependent on the insulation thickness. Results show that heat flux between tunnel and airflow can be significantly reduced by the use of thermal insulation, but the reduction varies with the tunnel and insulation conditions. The total cost associated with using the thermal insulation firstly decreases and then increases when the insulation thickness increases, implying an optimal insulation thickness. Nonetheless, both the optimal insulation thickness and maximum life cycle saving can be increased by a rising rock temperature, eventually leading to a reduced payback period.

Keywords: thermal insulation; cooling energy consumption; optimal insulation thickness; life cycle cost analysis; underground tunnel

23 **Nomenclature**

24	C_0	insulation cost (¥/m)
25	C_1	electricity cost (¥/m)
26	C_e	electricity price (¥/kWh)
27	C_t	total cost (¥/m)
28	c_p	specific heat capacity of air (kJ/(kgK))
29	COP	coefficient of performance of the cooling system
30	ΔC_1	difference between the electricity cost with and without insulation (¥/m)
31	D	ratio of down payment to initial investment
32	d	market discount rate
33	Fo	Fourier number ($\alpha_1 t / r_1^2$)
34	h	convection coefficient (W/(m ² K))
35	i	energy price rise rate
36	J_0, J_1	Bessel functions of the first kind of order 0 and 1
37	k_1	thermal conductivity of rock (W/(mK))
38	k_2	thermal conductivity of insulation layer (W/(mK))
39	LCS	life cycle saving (¥/m)
40	M_s	ratio of the annual maintenance and operation cost to initial investment
41	m	mass flow rate of air (kg/s)
42	m'	loan interest rate
43	N	payback period (year)
44	N_L	term of loan (year)

45	N_{\min}	years over which mortgage payments contribute to the analysis
46	Ne	analysis period (year)
47	P_1	ratio of cycle cost to the first-year fuel cost
48	P_2	ratio of life cycle expenditures to the initial investment
49	PWF	present worth factor
50	Q	heat gains of air (kJ)
51	q	heat flux (W/m^2)
52	q'	heat transfer rate per meter (W/m)
53	R_v	ratio of resale value at the end of analysis period to initial investment
54	r	cylindrical coordinate
55	r_1	radius of tunnel with insulation layer (m)
56	r_2	radius of tunnel without insulation layer (m)
57	r_3	radius of outer boundary (m)
58	T	temperature ($^{\circ}C$)
59	T_a	inlet air temperature ($^{\circ}C$)
60	T_0	original rock temperature ($^{\circ}C$)
61	T_1	insulation layer temperature ($^{\circ}C$)
62	T_2	rock temperature ($^{\circ}C$)
63	t	time (s)
64	ΔT_a	air temperature difference between inlet and outlet ($^{\circ}C$)
65	Y_0, Y_1	Bessel functions of the 2 ^{ed} kind of order 0 and 1
66	<i>Greek symbols</i>	

- 67 α thermal diffusivity (m^2/s)
- 68 α_1 thermal diffusivity of rock (m^2/s)
- 69 α_2 thermal diffusivity of insulation layer (m^2/s)
- 70 β, β_n eigenvalues
- 71 δ insulation thickness (m)
- 72 *subscript*
- 73 1 insulation layer
- 74 2 rock
- 75 *superscript*
- 76 (1) unsteady state solution
- 77 (2) steady state solution

78 1. Introduction

79 The increasing mineral production rate is essential to meet the growing demand of resources due
80 to the rapidly growing population and industrialization [1], which necessitates the extraction of the
81 resources from underground at greater depths. It was reported that the air temperature in the Anglo-
82 gold mine in South Africa was higher than 55 °C with the mining depth exceeding 3800 m. In China,
83 there were more than 74 coal mines with mining depth exceeding 900 m in 2015, and the original
84 rock temperature ranges between 35-45 °C. Recently, an increasing number of coal and metal mines
85 are facing the engineering challenge caused by hot and humid mining environment, which imposes
86 a negative impact on miners' health, mining facilities and productivity [2]. It is imperative for deep
87 mines to provide a suitable working environment through cost effective cooling measures.

88 Various cooling systems with improved overall efficiency and operation schemes with reduced
89 electricity cost have been adopted to achieve the required thermal environment in underground
90 spaces [3-5]. For instance, the operation scheme of the ice-cooling system was modified according
91 to the peak-valley electricity tariff to reduce the electricity cost [4]. The ground air cooler, chilled
92 water close to 0 °C and ice refrigeration system were used to explore an energy efficient solution
93 for deep mines [6]. Inrushing mine water was used as the source of cooling energy to control the
94 environment, which helped to reduce the energy consumption of the cooling system [7]. The
95 electrical energy consumed by cooling system accounted for more than 20% of total energy
96 consumed by the coal mine. The variable speed drive technology was used to reduce the overall
97 electricity demand, with a total energy consumption reduction of 33% [8, 9]. The thermal-hydraulic
98 characteristics of an integrated mine cooling system was investigated by using a simulation model
99 with a holistic view to assess the system's energy expenditure [10]. It was shown that the comfortable

100 and healthy environment conditions can be created without a significant increase in the capital cost.

101 Although above investigations indicate that these technologies can reduce the electricity cost, the
102 cooling system's investment and electricity consumption are considerably high for mines. This is
103 because a significant amount of heat transfers from tunnel surrounding rock to the airflow in deep
104 underground space, which leads to a large cooling load. The cooling load can be decreased through
105 thermal insulation of the building envelopes, leading to a reduced investment and electricity cost of
106 the cooling system for residential buildings [11-13]. Intuitively, thermal insulation is applicable to
107 reduce the overall energy consumption of cooling system in underground spaces [14]. Heat
108 transferred from the surrounding rock into underground space may account for more than 75% of
109 total cooling load in deep mines [15]. Therefore, thermal insulation presents the greatest potential
110 in minimizing the cooling requirement and reducing the electricity cost of cooling system [16]. Liu
111 et al. [17] studied the thermal insulation effects through various models based on one-dimensional
112 slab model with Dirichlet or Neumann boundary conditions. Many experimental investigations also
113 indicated that the outlet airflow temperature was obviously decreased when thermal insulation was
114 applied [18-21].

115 However, greater insulation thickness can increase the materials' costs. An economic assessment
116 should be carried out on the insulation design for a given underground tunnel to identify the
117 economic balance between the decrease in the electricity consumption and the increase in the
118 insulation cost, and such an approach has been broadly adopted in the industrial and residential
119 buildings [22-25]. Ozel et al. [26] investigated the optimum insulation thickness of building walls
120 by using life cycle cost method coupling with the consideration of the environmental impact analysis.
121 Motaghian et al. [27] reported that the optimum insulation arrangements should be determined by

122 employing multi-objective optimization. Dombayci [28] evaluated the environmental impact of the
123 optimal thickness of external wall insulation for buildings and found out that the energy expenditure
124 was reduced by 46.6%. More reduction in heating and cooling loads (86.63%) was achievable when
125 the optimal insulation thickness ranging between 3-5 cm was used to insulate external walls [29].
126 Kecebas et al. [30] and Erturk et al. [31-33] investigated the optimal insulation thickness of pipes
127 by using life cycle cost method, in which the effect of air gap was considered. The optimal insulation
128 thickness in similar engineering areas was also investigated by Huang et al. [34] using the life cycle
129 cost analysis. Adamczyk et al. [35] proposed that the investment of thermal insulation was beneficial
130 for ecological environment because the energy demand was reduced. Such improvements on the
131 thermal performance economically benefit the application due to the reduced electricity cost. The
132 capital cost associated with the use of thermal insulation can be recovered within a certain period of
133 time. Therefore, the payback period is usually used as a direct measure of the economic benefit of
134 an invested technical improvement [36]. An investment payback period ranged from 1 year to 2.3
135 years was achieved when the rock wool and polystyrene were used as the insulation material based
136 on the life cycle cost analysis [37]. A shorter payback period was also possible when different
137 configurations were conducted [38, 39].

138 Researchers have made a tremendous progress in studying the impact of thermal insulation on
139 building. However, the thermal insulation effect, including optimal insulation thickness, life cycle
140 saving and the payback period in deep mines is rarely investigated. Especially, the effect of various
141 rock temperatures on the optimal insulation thickness has not been reported, which indicates that a
142 further investigation on the economic benefit analysis on the thermal insulation effect is needed.
143 This paper is committed to investigating how the cooling load and the total cost are affected by

144 thermal insulation and understanding how to identify the optimal insulation thickness for a given
 145 underground tunnel. Firstly, the cylindrical heat conduction model with Robin boundary condition
 146 was solved by using the separation-of-variables method. Secondly, based on above analytic results,
 147 the effects of insulation thickness, thermal conductivities of insulation layer and rock, tunnel radius
 148 and convection coefficient on the heat flux reduction were investigated. Finally, we investigated the
 149 optimal insulation thickness, life cycle saving and payback period under various rock temperatures
 150 by employing the life cycle cost analysis. The flowchart of this study is shown in Fig. 1.

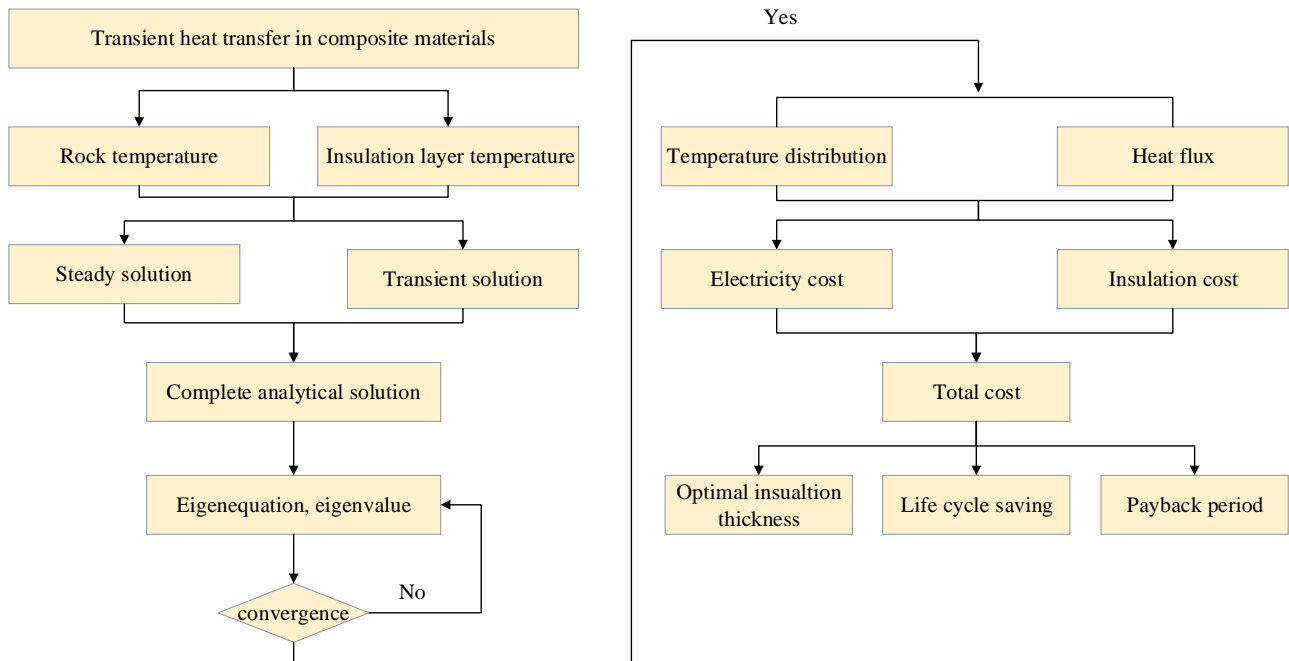


Fig. 1. Flowchart of this study.

153 2. Mathematical description

154 2.1. Heat transfer model

155 2.1.1. Basic equations

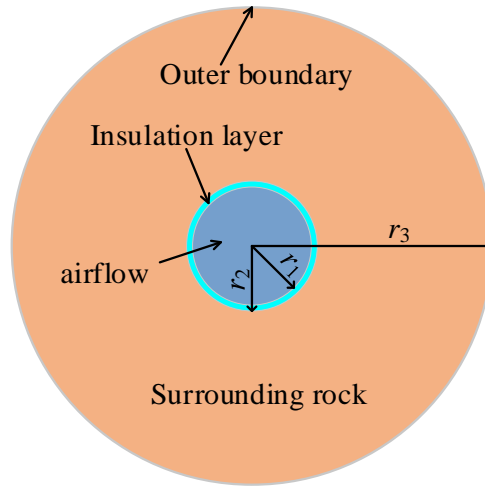
156 When the air flows through the underground tunnel, heat is dissipated from tunnel rock to the
 157 airflow driven by the temperature difference. To quantify this heat flux in a practical and reasonable
 158 way, the following assumptions are made to facilitate the analytic model [40, 41]: (1) The thermal-

159 physical parameters of tunnel rock and insulation layer are constant, homogeneous and isotropic. (2)
 160 The tunnel rock is assumed dry, which means that the mass transfer between rock and airflow is not
 161 present. (3) The contact resistance between rock and insulation layer is negligible. (4) The cross-
 162 section shape of underground tunnel is circular. Therefore, the schematic diagram of the engineering
 163 problem can be illustrated by Fig. 2, which simplifies the heat exchange between the tunnel rock
 164 and insulation layer into one-dimensional transient heat conduction process in the cylindrical
 165 coordinate.

166 The governing equation is described by Eq. (1) [42]:

$$167 \quad \frac{\partial T_i}{\partial t} = \alpha_i \left(\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r} \frac{\partial T_i}{\partial r} \right) \quad (1)$$

168 where T is temperature, t is time, r is cylindrical coordinate, α is thermal diffusivity. For the
 169 subscript, $i=1$ represents the insulation layer, and $i=2$ represents the rock.



170
 171 Fig. 2. Schematic of analytic model.

172 The initial and boundary conditions are given by Eq. (2) and Eq. (3):

$$173 \quad \begin{cases} T_1(r, 0) = T_0 \\ T_2(r, 0) = T_0 \end{cases} \quad (2)$$

174

$$\begin{cases} r = r_3 : T_2(r_3, t) = T_0 \\ r = r_1 : k_1 \frac{\partial T_1}{\partial r} = h(T_1 - T_a) \end{cases} \quad (3)$$

175 where T_0 is the initial temperature for both rock and insulation layer; r_1 is the radius of tunnel with
176 insulation layer; r_3 is the radius of outer boundary; T_a is air temperature; k is thermal conductivity.

177 Because the contact thermal resistance is negligible, the contact boundary condition is described
178 by Eq. (4):

179

$$r = r_2 : \begin{cases} T_1(r_2, t) = T_2(r_2, t) \\ k_1 \frac{\partial T_1(r_2, t)}{\partial r} = k_2 \frac{\partial T_2(r_2, t)}{\partial r} \end{cases} \quad (4)$$

180 where r_2 is radius of tunnel without insulation layer.

181 2.1.2. Analytic solution

182 The Eqs. (1)-(4) are solved by using the variable-separation-method [43]. The detailed process is
183 shown in the Appendix. The analytic solution is described by Eq. (5) and Eq. (6):

184

$$T_1^{(1)}(r, t) = \sum_{n=1}^{\infty} A_{1n} \left[J_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) + \frac{B_{1n}}{A_{1n}} Y_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) \right] \cdot \exp(-\beta_n^2 t) \quad (5)$$

185

$$T_2^{(1)}(r, t) = \sum_{n=1}^{\infty} A_{1n} \left[\frac{A_{2n}}{A_{1n}} J_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right) + \frac{B_{2n}}{A_{1n}} Y_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right) \right] \cdot \exp(-\beta_n^2 t) \quad (6)$$

186 The derivation of Eq. (5) and Eq. (6) can be obtained according to derivatives of *Bessel* function:

187

$$\frac{\partial T_1^{(1)}(r, t)}{\partial r} = - \sum_{n=1}^{\infty} A_{1n} \cdot \exp(-\beta_n^2 t) \frac{\beta_n}{\sqrt{\alpha_1}} \left[J_1\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) + B_{1n}' Y_1\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) \right] \quad (7)$$

188

$$\frac{\partial T_2^{(1)}(r, t)}{\partial r} = - \sum_{n=1}^{\infty} A_{1n} \cdot \exp(-\beta_n^2 t) \frac{\beta_n}{\sqrt{\alpha_2}} \left[A_{2n}' J_1\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right) + B_{2n}' Y_1\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right) \right] \quad (8)$$

189 where, $B_{1n}' = B_{1n} / A_{1n}$, $B_{2n}' = B_{2n} / A_{1n}$, $A_{2n}' = A_{2n} / A_{1n}$.

190 The boundary condition at $r=r_1$ is transformed to Eq. (9):

191
$$-k_1 \frac{\beta_n}{\sqrt{\alpha_1}} J_1\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) - h J_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_1\right) = B'_{1n} \left[h Y_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_1\right) + k_1 \frac{\beta_n}{\sqrt{\alpha_1}} Y_1\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) \right] \quad (9)$$

192 In addition, the boundary conditions at $r=r_2$ and $r=r_3$ are changed to:

193
$$\begin{cases} A'_{2n} J_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_3\right) + B'_{2n} Y_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_3\right) = 0 \\ A'_{2n} J_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_2\right) + B'_{2n} Y_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_2\right) = J_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_2\right) + B'_{1n} Y_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_2\right) \end{cases} \quad (10)$$

194 Then, the coefficients of A'_{2n}, B'_{2n} are computed and described by Eq. (11):

195
$$\begin{bmatrix} A'_{2n} \\ B'_{2n} \end{bmatrix} = \begin{bmatrix} J_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_3\right) & Y_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_3\right) \\ J_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_2\right) & Y_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_2\right) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ J_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_2\right) + B'_{1n} Y_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_2\right) \end{bmatrix} \quad (11)$$

196 Consequently, the eigenequation is obtained as:

197
$$\frac{k_1}{\sqrt{\alpha_1}} \cdot \begin{bmatrix} 1 & B'_{1n} \end{bmatrix} \cdot \begin{bmatrix} J_1\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_2\right) \\ Y_1\left(\frac{\beta_n}{\sqrt{\alpha_1}} r_2\right) \end{bmatrix} - \frac{k_2}{\sqrt{\alpha_2}} \cdot \begin{bmatrix} A'_{2n} & B'_{2n} \end{bmatrix} \cdot \begin{bmatrix} J_1\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_2\right) \\ Y_1\left(\frac{\beta_n}{\sqrt{\alpha_2}} r_2\right) \end{bmatrix} = 0 \quad (12)$$

198 The eigenequation is a transcendental equation, which can only be solved through numerical
 199 measure. The eigenvalues are calculated by employing MATLAB in this study, and the solution's
 200 convergence is also investigated. The analytic solution does not converge when the first 15
 201 eigenvalues are used. The analytic solution converges when the first 30 eigenvalues are employed,
 202 which is consistent with the solution of using the first 500 eigenvalues. Therefore, the first 30
 203 eigenvalues are used for the following computation.

204 The coefficients of $B'_{1n}, A'_{2n}, B'_{2n}$ are calculated by using the eigenvalues, and A'_{1n} is determined by
 205 the initial condition presented in Eq. (A10). Substituting the general solution into the initial condition,

206 Eq. (13) is obtained as:

$$207 \quad \begin{cases} T_1^{(1)}(r, 0) = \sum_{n=1}^{\infty} A_{1n} \left[J_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) + B_{1n}' Y_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) \right] = \sum_{n=1}^{\infty} A_{1n} \varphi_{1n}(r) \\ T_2^{(1)}(r, 0) = \sum_{n=1}^{\infty} A_{1n} \left[A_{2n}' J_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right) + B_{2n}' Y_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right) \right] = \sum_{n=1}^{\infty} A_{1n} \varphi_{2n}(r) \end{cases} \quad (13)$$

$$208 \quad \text{where, } \varphi_{1n}(r) = J_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right) + B_{1n}' Y_0\left(\frac{\beta_n}{\sqrt{\alpha_1}} r\right), \quad \varphi_{2n}(r) = A_{2n}' J_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right) + B_{2n}' Y_0\left(\frac{\beta_n}{\sqrt{\alpha_2}} r\right).$$

209 The orthogonal expression for $\varphi_{1n}(r)$ and $\varphi_{2n}(r)$ is described by Eq. (14):

$$210 \quad \sum_{i=1}^2 \frac{k_i}{\alpha_i} \int_{r_i}^{r_{i+1}} r \varphi_{1n}(r) \varphi_{2n}(r) dr = \begin{cases} 0 & m \neq n \\ N_n & m = n \end{cases} \quad (14)$$

$$211 \quad \text{where, } N_n = \sum_{i=1}^2 \frac{k_i}{\alpha_i} \int_{r_i}^{r_{i+1}} r \varphi_{in}^2(r) dr.$$

212 From Eq. (14), Eq. (15) can be obtained:

$$213 \quad \frac{k_1}{\alpha_1} \int_{r_1}^{r_2} T_1^{(1)}(r, 0) \cdot r \cdot \varphi_{1n}(r) dr + \frac{k_2}{\alpha_2} \int_{r_2}^{r_3} T_2^{(1)}(r, 0) \cdot r \cdot \varphi_{2n}(r) dr = A_{1n} \left[\frac{k_1}{\alpha_1} \int_{r_1}^{r_2} r \cdot \varphi_{1n}^2(r) dr + \frac{k_2}{\alpha_2} \int_{r_2}^{r_3} r \cdot \varphi_{2n}^2(r) dr \right] \quad (15)$$

214 The coefficient A_{1n} is shown as:

$$215 \quad A_{1n} = \frac{1}{N_n} \left[\frac{k_1}{\alpha_1} \int_{r_1}^{r_2} T_1^{(1)}(r, 0) \cdot r \cdot \varphi_{1n}(r) dr + \frac{k_2}{\alpha_2} \int_{r_2}^{r_3} T_2^{(1)}(r, 0) \cdot r \cdot \varphi_{2n}(r) dr \right] \quad (16)$$

216 2.2. Life cycle cost method

217 The economic benefit of thermal insulation is investigated by adopting the life cycle cost (LCC)
 218 analysis, which is a widely used method for evaluating all relevant costs over the life cycle time [22,
 219 30, 39, 44]. The main procedures of LCC analysis are described as follows: (1) The initial cost is
 220 calculated. (2) The operation cost is evaluated when the influences of inflation and interest rate are
 221 considered. (3) The minimum total cost is then determined to provide the optimal insulation thickness.
 222 (4) The payback period and the life cycle saving are obtained.

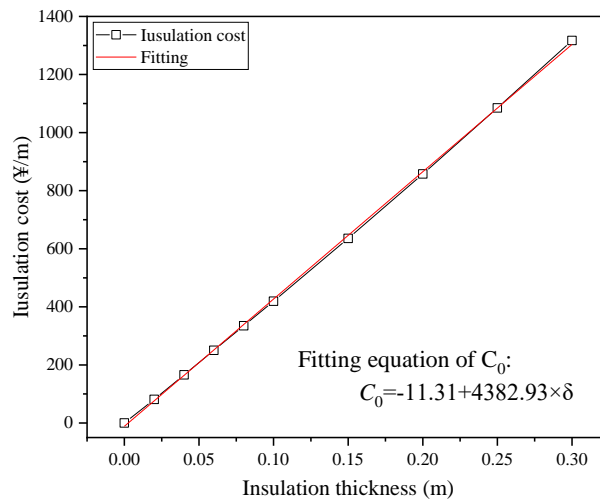
223 2.2.1. *Insulation cost*

224 The lightweight geopolymer concrete (LGC) with lower greenhouse gas emission is used as the
225 insulation material in this study. The LGC with thermal conductivity of 0.1 W/(mK) and dry density
226 of 450 kg/m³ was produced by using fly ash and alkali solution at room temperature [45]. The price
227 of insulation layer is about 325 ¥/m³ (¥: RMB). The insulation cost per tunnel length under various
228 insulation thicknesses is calculated.

229 Fig. 3 indicates that the insulation cost is approximately linearly increased when the insulation
230 thickness increases from 0 m to 0.3 m. For instance, the insulation cost increases from 81.25 ¥/m to
231 1317 ¥/m when the insulation thickness increases from 0.02 m to 0.3 m. The empirical correlation
232 between the insulation cost and the insulation thickness can be described by Eq. (17):

233
$$C_0 = -11.31 + 4382.93 \times \delta \tag{17}$$

234 where C_0 is the insulation cost per tunnel length. δ is the insulation thickness.



235
236 Fig. 3. The insulation cost versus thickness of insulation layer

237 2.2.2. *Annual electricity cost*

238 The annual electricity cost (C_1) is dependent on the total heat gain of the airflow. Due to the
239 unsteady process of the convective heat transfer, the annual heat gain (cooling load) is calculated by

240 the integration method:

$$241 \quad Q = m \cdot \int_0^{365 \times 24 \times 3600} c_p \cdot \Delta T_a dt \quad (18)$$

242 where Q is the heat gain of air, c_p is the specific heat capacity of air, m is the mass flow rate, ΔT_a is
243 the temperature difference between inlet and outlet air.

244 The economic evaluation can be carried out through the P_1 - P_2 economical method, which is shown
245 as follows [34]:

$$246 \quad P_1 = PWF(Ne, i, d) = \sum_{j=1}^{Ne} \frac{(1+i)^{j-1}}{(1+d)^j} = \begin{cases} \frac{1}{d-i} \left[1 - \left(\frac{1+i}{1+d} \right)^{Ne} \right], i \neq d \\ \frac{Ne}{1+i}, i = d \end{cases} \quad (19)$$

$$247 \quad P_2 = D + (1-D) \frac{PWF(N_{\min}, 0, d)}{PWF(N_L, 0, m')} + M_s \times PWF(Ne, i, d) - \frac{R_v}{(1+d)^{Ne}} \quad (20)$$

248 where P_1 is the ratio of life cycle cost to the first-year electricity cost. P_2 is the ratio of life cycle
249 expenditures to the initial investment. PWF is the present worth factor. Ne is the analysis period, year.
250 N_L is the term of loan, year. d is the market discount rate. i is the energy price rise rate. m' is the loan
251 interest rate. N_{\min} is the number of years over which mortgage payments contribute to the analysis
252 (usually the minimum of Ne and N_L). M_s is the ratio of the annual maintenance and operation cost to
253 initial investment. R_v is the ratio of resale value at the end of analysis period to initial investment. D
254 is the ratio of down payment to initial investment.

255 The annual electricity cost C_1 is calculated by Eq. (21):

$$256 \quad C_1 = \frac{Q}{3600 \times 250 \times COP} C_e \quad (21)$$

257 where COP is the coefficient of performance, C_e is the electricity price. All the parameters and their
258 values are listed in Table 1.

259

Table 1 Parameter for the life cycle cost analysis.

Parameters	Values
Interest rate (i)	8%
Lifetime (Ne)	10 Years
Market discount rate (d)	6%
Coefficient of Performance (COP)	2.8
Electricity price (C_e)	1 ¥/kWh

260 The heat gain of airflow from the tunnel with a length of 250 m, wind speed of 1 m/s, original
 261 surrounding rock temperature ranging from 35 °C to 45 °C, inlet air temperature of 20 °C and tunnel
 262 radius of 2 m is calculated after the tunnel is ventilated for 12 months. The heat gain of airflow and
 263 the annual electricity cost of the cooling system with different insulation thicknesses are listed in
 264 [Table 2](#). It is observed that the heat gain of airflow and the annual electricity cost per meter decrease
 265 from 1844 MJ/m to 897 MJ/m and from 183 ¥/m to 89 ¥/m respectively when the insulation thickness
 266 increases from 0 to 0.3 m.

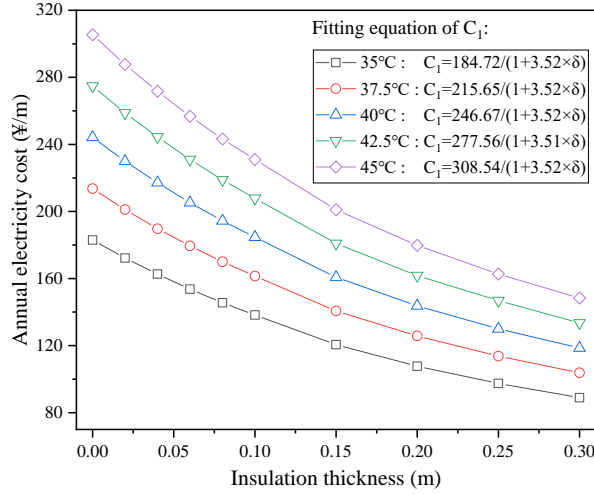
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Table 2 Annual electricity cost versus insulation thickness.

Insulation thickness (m)	0	0.04	0.08	0.1	0.15	0.2	0.25	0.3
Heat gains (MJ/m)	1844	1640	1467	1394	1216	1086	983	897
Annual electricity cost (¥/m)	183.0	162.7	145.6	138.3	120.6	107.7	97.5	89.0

268 [Fig. 4](#) shows the influence of rock temperature on the annual electricity cost under various
 269 insulation thickness. The annual electricity cost is observed to be decreased with the increased
 270 insulation thickness. It is known that the heat transfer can be significantly reduced by a larger

271 insulation thickness, thereby giving a reduced cooling load, which then leads to a lower annual
 272 electricity cost. The empirical correlations between the annual electricity cost and the insulation
 273 thickness under various rock temperatures ranging from 35 °C to 45 °C are also presented in Fig. 4.



274
 275 Fig. 4. Annual electricity cost under various original rock temperature.

276 2.2.3. Total cost, life cycle saving and payback period

277 The total cost (C_t) consists of the electricity cost over the life time and the insulation cost, which
 278 can be described by Eq. (22):

$$279 \quad C_t = P_1 C_1 + P_2 C_0 \quad (22)$$

280 Life cycle saving (LCS) is the difference between the saved energy cost over the life cycle time
 281 and the insulation cost, which can be calculated by Eq. (23):

$$282 \quad LCS = P_1 \Delta C_1 - P_2 C_0 \quad (23)$$

283 where ΔC_1 is the electricity cost difference with and without the insulation layer.

284 By setting Eq. (23) to zero, the payback period N can be calculated:

$$285 \quad N = \begin{cases} \frac{\ln[1 - ((P_2 \times C_0) / (\Delta C_1)) \times (d - i)]}{\ln((1 + i) / (1 + d))}, & i \neq d \\ ((P_2 \times C) / (\Delta C_1)) \times (1 + i), & i = d \end{cases} \quad (24)$$

286 3. Results and discussion

287 *3.1. Verification of analytic solution*

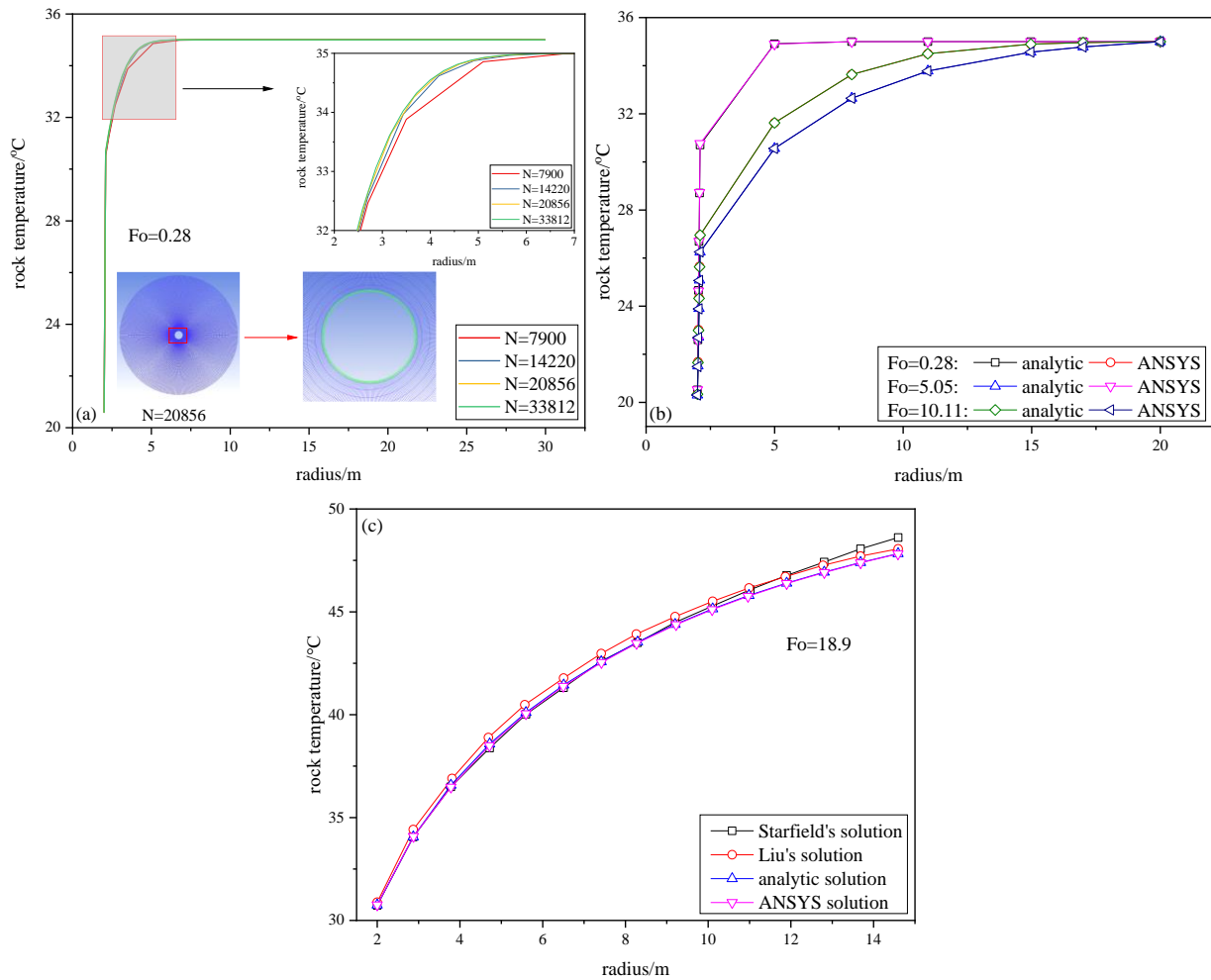
288 A numerical model is simulated by ANSYS Fluent 18.0 to verify the analytic solution. The
289 quadrilateral mesh with high quality is created through ICEM CFD (Integrated Computer Engineering
290 and Manufacturing code for Computational Fluid Dynamics). To decrease the total mesh number and
291 shorten the computing time, uniform grid is employed in the insulation layer and nonuniform grid is
292 used in surrounding rock region, respectively.

293 The key thermal properties and boundary conditions are listed in [Table 3](#). The thermal properties
294 of tunnel and ventilation parameters are obtained from a coal mine in Xuzhou, China. The properties
295 of an insulation material are provided by a literature [\[45\]](#). The thermal penetration length is defined
296 as the depth at which the dimensionless temperature $(T - T_a)/(T_0 - T_a)$ inside the surrounding rock
297 reduces to 0.99 based on the boundary layer theory [\[46\]](#). To eliminate the boundary effect, the outer
298 boundary radius should be larger than the thermal penetration length [\[18, 40\]](#). The analytic analysis
299 indicated that the outer boundary radius in [Table 3](#) is larger than the thermal penetration depth, which
300 is not discussed in this paper. The grid independence investigation is conducted on this numerical
301 model. Four grids with mesh numbers of 7900, 14220, 20856 and 33812 are generated, respectively.
302 The rock temperature distribution under different meshing are compared when the tunnel is ventilated
303 for about ten days ($Fo=0.28$), as shown in [Fig. 5 \(a\)](#). It is seen that the calculated rock temperature
304 does not vary when the grid number increased to 20856. To minimise the computational time without
305 compromising the calculation accuracy, the mesh with the grid number of 20856 is employed for the
306 following investigation.

Table 3 Key thermal properties and boundary conditions.

Parameters	Value
Original rock temperature, T_0	35 °C
Thermal conductivity of rock (TCR), k_1	2.4 W/(mK)
Thermal diffusivity of rock, α_1	$1.3 \times 10^{-6} \text{ m}^2/\text{s}$
Tunnel radius, r_1	2 m
Convection coefficient, h	20 W/(m ² K)
Inlet air temperature, T_a	20 °C
Thermal conductivity of insulation layer (TCIL), k_2	0.1 W/(mK)
Thermal diffusivity of insulation layer, α_2	$0.4 \times 10^{-6} \text{ m}^2/\text{s}$
Outer boundary radius, r_3	30 m
Insulation thickness, δ ($\delta=r_2-r_1$)	0.1 m

308 The temperature distribution calculated from the analytic solution when $Fo=0.28$, $Fo=5.05$ and
309 $Fo=10.11$ are compared with that given by Fluent, as shown in Fig. 5 (b). The tunnel surface
310 temperature difference between analytic and Fluent is 0.18 °C, 0.23 °C and 0.03 °C when $Fo=0.28$,
311 5.05 and 10.11 respectively, which shows that the analytic results are in good agreement with the
312 numerical results. Starfield et al. [47] and Liu et al. [17] computed rock temperature when the tunnel
313 rock is dry. The rock temperature is also solved through the analytic solution in this study and the
314 comparison of rock temperature distribution is presented in Fig. 5 (c). The tunnel surface temperature
315 of Starfield's, Liu's, analytic and ANSYS solution is 30.73 °C, 30.89 °C, 30.75 °C and 30.75 °C
316 respectively when $Fo=18.9$. The maximum relative error is lower than 0.46%. Therefore, the validity
317 of the analytic solution in this study can be proved.



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Fig. 5. Model validation: (a) grid independence verification, analytic solution versus (b) ANSYS and (c) literatures' results.

3.2. Thermal insulation effect

3.2.1. Rock temperature

The rock temperature distributions in the cross section of the tunnel without and with insulation layer are investigated, as shown in Fig. 6. As can be seen from Fig. 6 (a), the rock temperature considerably decreases near the tunnel wall due to the cooling effect caused by the airflow with lower temperature. Such cooling effect weakens as it moves further away from the tunnel wall. The rock temperature stays almost at a constant level when the distance is above a certain threshold, which is the length of the outer boundary that needs to be determined in the analytic model. The rock

330 temperature is expected to decrease rapidly at the initial stage due to the transient heat transfer process.
331 After a period of time, the reducing rate of the rock temperature decreases gradually. For instance,
332 the tunnel surface temperature decreases from 35 °C to 20.90 °C when Fo increases from 0 to 0.84,
333 with a temperature difference of 14.10 °C. In contrast, the rock temperature difference only reduces
334 by 0.83 °C when Fo increases from 0.84 to 10.11. This is because that the temperature difference
335 between airflow and tunnel is reduced by the increased Fo number. Similar rock temperature
336 distribution is achieved by Wang et al. [40, 48] through experimental and analytic investigations.

337 Fig. 6 (b) presents the rock temperature distribution of tunnel with insulation layer. It is seen that
338 the temperature gradient in insulation layer is significantly larger than that in surrounding rock. This
339 is because that the TCIL is smaller than that of surrounding rock. For instance, the temperature
340 gradient in insulation layer is 101.79 °C/m, in comparison with that of 8.47 °C/m in surrounding rock
341 at Fo=0.28. The temperature gradient in the insulation layer also decreases when the Fo number
342 increases, which is caused by the reduced temperature difference between airflow and tunnel. As we
343 can also see from Fig. 6, the temperature gradient near the tunnel wall is large, which suggests that
344 the closer the distance to the tunnel wall, the higher the heat flux is [49, 50]. Therefore, an excellent
345 thermal insulation performance can be achieved when the insulation layer is installed on the tunnel
346 wall [42].

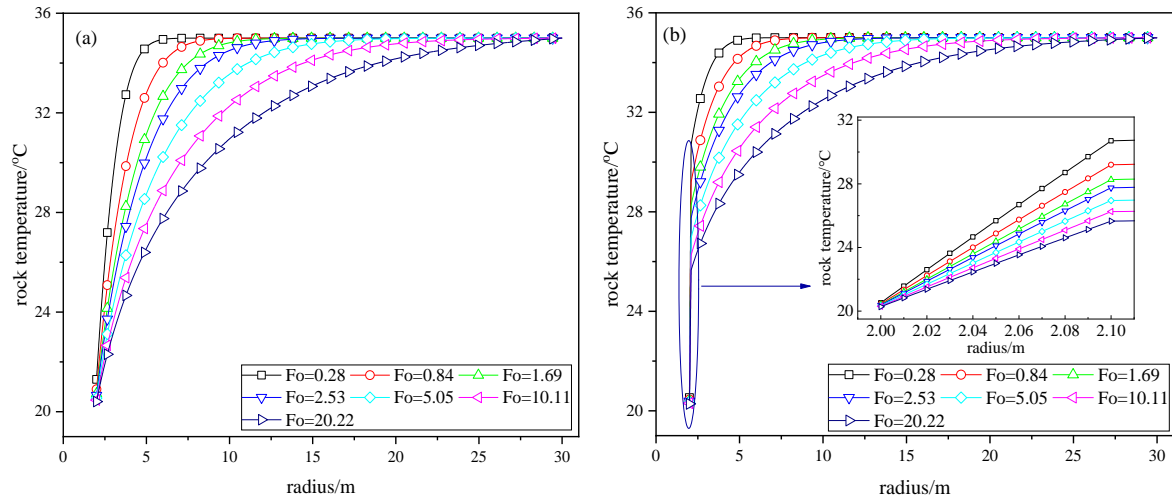


Fig. 6. Rock temperature of tunnel (a) without and (b) with insulation layer.

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349 To verify the thermal insulation effect, the surplus temperature is monitored during the unsteady
 350 heat transfer process. The surplus temperature is defined by the temperature difference between tunnel
 351 wall and airflow, which can be used to compute the heat convection between tunnel and airflow [42].
 352 The variation of surplus temperature with Fo is listed in Table 4, which shows that the surplus
 353 temperature of original tunnel is always higher than that of insulated tunnel. Although the surplus
 354 temperature's ratio decreases gradually when the Fo increases, the heat convection of original tunnel
 355 is still about 1.43 times higher than that of insulated tunnel when Fo=20.22. It indicates that thermal
 356 insulation can reduce about 30% of total heat convection when the tunnel is ventilated for two years.
 357 Therefore, the thermal insulation can considerably reduce heat convection between tunnel and airflow,
 358 which contributes to the decreased cooling load.

359

Table 4 Surplus temperature versus Fo number.

Fo	0.28	0.84	1.69	2.53	5.05	10.11	20.22
Original tunnel (°C)	1.30	0.90	0.73	0.65	0.55	0.47	0.40
Insulated tunnel (°C)	0.52	0.45	0.40	0.38	0.34	0.30	0.28

360 3.2.2. Heat flux reduction

361 Because the thermal insulation effect is dependent on the parameters related to insulation layer,
362 tunnel and ventilation conditions, it is necessary to investigate the impact of insulation thickness,
363 TCIL, TCR, tunnel radius and convection coefficient on thermal insulation effect. A series of
364 calculations of heat flux of the tunnel with/without insulation layer have been conducted under
365 various parameters. Therefore, the effects of these various parameters on heat flux reduction are
366 presented. Because the moisture leads to an increased TCIL and a coupled heat and mass transfer, the
367 thermal insulation effect can be considerably reduced by moisture transfer [45]. Therefore, it may not
368 be feasible for wet tunnel to control underground environment by using thermal insulation, which is
369 not discussed in this study.

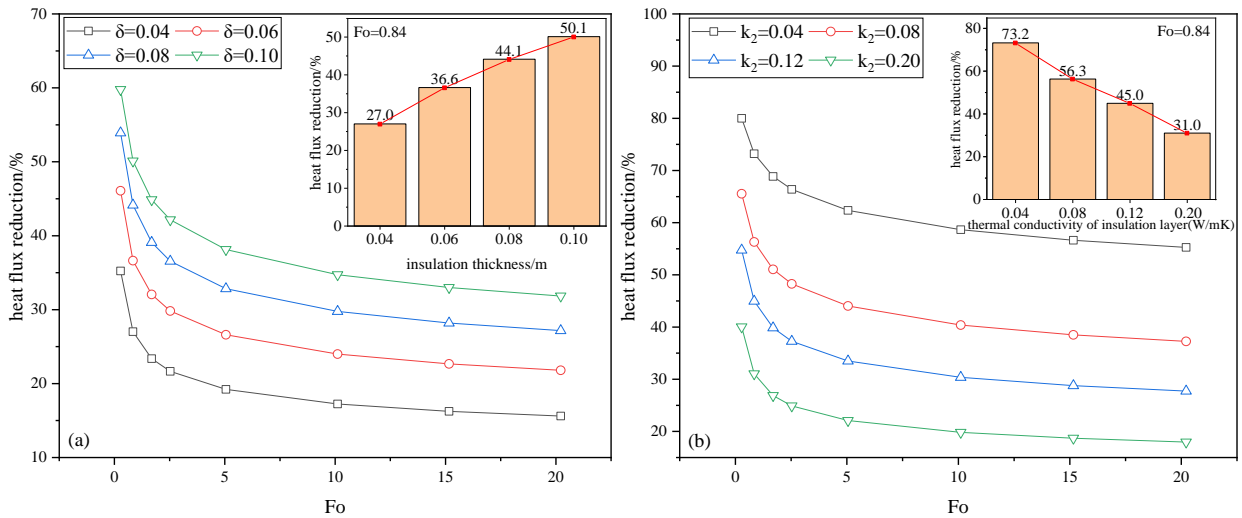
370 Fig. 7 shows the changes of heat flux reduction achieved by various parameters, such as insulation
371 thickness, TCIL, TCR, tunnel radius and convection coefficients. The impact of insulation thickness
372 on heat flux reduction is shown in Fig. 7 (a). Taking the profile under $Fo=0.84$ as an example, it is
373 found that the heat flux reduction increases when a thicker insulation is used, which indicates that
374 larger insulation thickness can weaken the heat transfer from tunnel rock to airflow. For instance, the
375 heat flux reduction increases from 27.0% to 50.1% when the insulation thickness increases from 0.04
376 m to 0.1 m. However, larger insulation thickness implicates an increased insulation cost. A balance
377 needs to be achieved among them to achieve the best cost effectiveness, which is discussed in more
378 details in section 3.3. Fig. 7 (b) shows that the heat flux reduction increases when the TCIL decreases.
379 Zhou et al. [51] also found that the heat flux reduction non-linearly decreased with the increased
380 TCIL, which is similar with the variation presented in Fig. 7 (b). This is because a lower TCIL can

381 result in a larger thermal resistance [42]. As the heat flux reduction is strongly dependent on the TCIL,
382 preference should be given to the insulation layer with a lower thermal conductivity. Zhang et al. [52]
383 also indicates that a smaller TCIL can lead to a lower heat flux and a better thermal insulation effect.

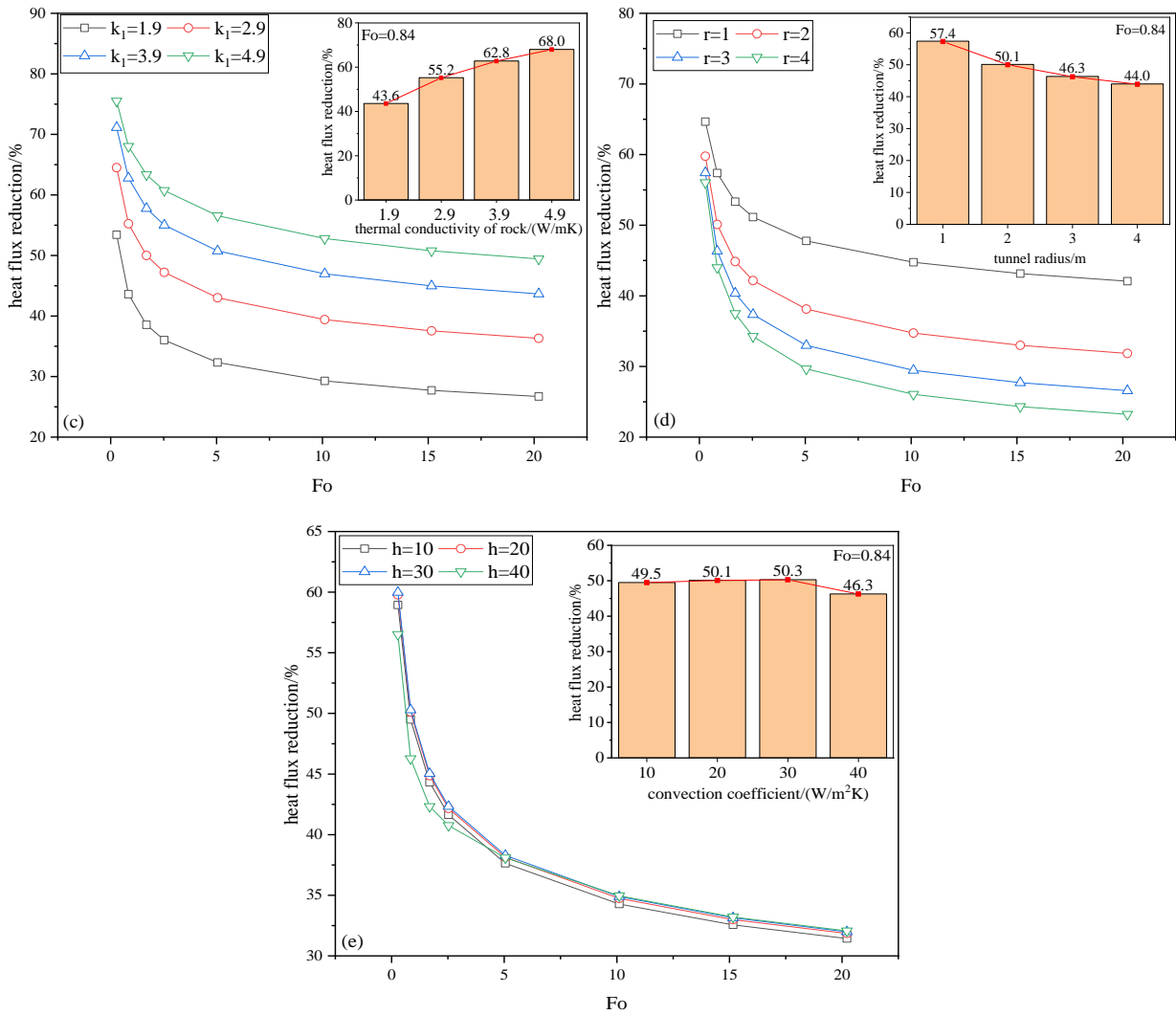
384 Fig. 7 (c) illustrates the impact of TCR on the heat flux reduction with insulation thickness of 0.1
385 m and TCIR of 0.1 W/(mK). It is seen that an increase in the TCR contributes to an enhancement in
386 heat flux reduction, suggesting that the thermal insulation effect is significant when the TCR is high.
387 For instance, the heat flux reduction increases from 43.6% to 68.0% when the TCR is increased from
388 1.9 W/(mK) to 4.9 W/(mK). Zhu [19] indicated that an increase in TCR (thermal diffusivity) could
389 lead to a quicker decrease in rock temperature, which is consistent with a larger heat flux reduction.
390 It is known that a higher TCR is equivalent to a smaller thermal resistance of surrounding rock. When
391 the thermal resistance of the surrounding rock is coupled with that of the insulation layer, the smaller
392 the thermal resistance of the surrounding rock, the larger the increasing rate of total thermal resistance
393 is. Therefore, one can expect that a higher TCR coupled with the insulation layer can lead to a
394 significant thermal insulation effect. The influence of the tunnel radius on the heat flux reduction with
395 an insulation thickness of 0.1 m and TCIR of 0.1 W/(mK) is shown in Fig. 7 (d), from which we can
396 find that the heat flux reduction decreases when the tunnel radius increases. This indicates that the
397 thermal insulation effect is significant when the tunnel radius is smaller. The reason is that the total
398 thermal resistance is increased by the decreased tunnel radius. Compared with the tunnel with a small
399 radius, a thicker insulation layer (or smaller TCIL) can achieve the same thermal insulation effect for
400 those tunnels with a large radius.

401 Fig. 7 (e) shows the impact of convection coefficient on the heat flux reduction. It can be observed
402 that the heat flux reduction is not strongly dependent on the convection coefficient. For instance, the

403 heat flux reduction only increases from 46.3% to 50.3% when the convection coefficient increases
 404 from 10 W/(m²K) to 40 W/(m²K). The heat transfer can be enhanced by increased convection
 405 coefficient. However, the decreased tunnel surface temperature also leads to a reduced temperature
 406 difference between tunnel and airflow. This indicates that the increase in the convection coefficient
 407 is offset by the decrease in the temperature difference. Therefore, the thermal insulation effect does
 408 not significantly depend on the convection coefficient. Zhang [18] experimentally proved that the
 409 dimensionless rock temperature was not strongly dependent on the Bi number (convection
 410 coefficient), which is consistent with the results showed in Fig. 7 (e). As we can see from the profiles
 411 shown in Fig. 7, all these effects of insulation thickness, TCIL, TCR, tunnel radius and convection
 412 coefficient on the heat flux reduction are non-linear.



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416 Fig. 7. Heat flux reduction versus (a) insulation thickness, (b) TCIL, (c) TCR, (d) tunnel radius, (e) convection
 417 coefficient.

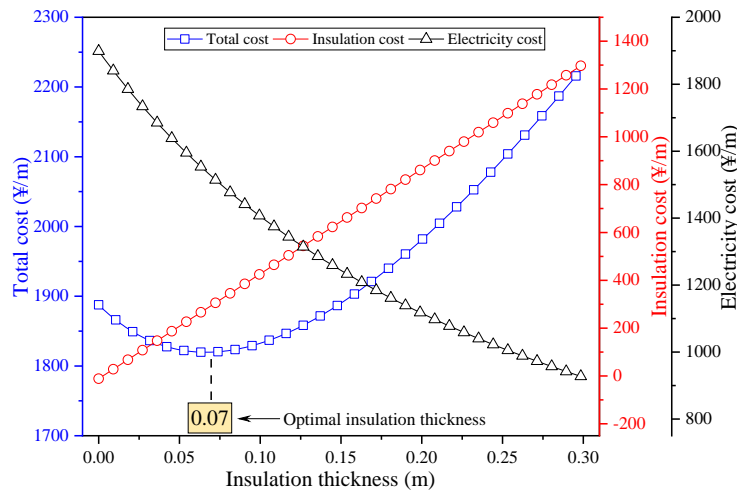
418 *3.3. Economic benefit analysis*

419 *3.3.1. Total cost*

420 The insulation cost, electricity cost and the total cost under different insulation thicknesses are
 421 calculated based on the life cycle cost method. The effect of insulation thickness on the total cost is
 422 presented in Fig. 8. It is seen from Fig. 8 that the total cost firstly decreases and then increases when
 423 the insulation thickness increases. For instance, the total cost decreases from 1887 ¥/m to 1820 ¥/m
 424 when the insulation thickness increases from 0 m to 0.07 m. The total cost begins to increase when

425 the insulation thickness is larger than 0.07 m. It is known that the optimum insulation thickness should
 426 be determined against the lowest total cost [53]. Therefore, the optimal insulation thickness is 0.07 m
 427 when the rock temperature is 35 °C, with the lowest total cost of 1820 ¥/m.

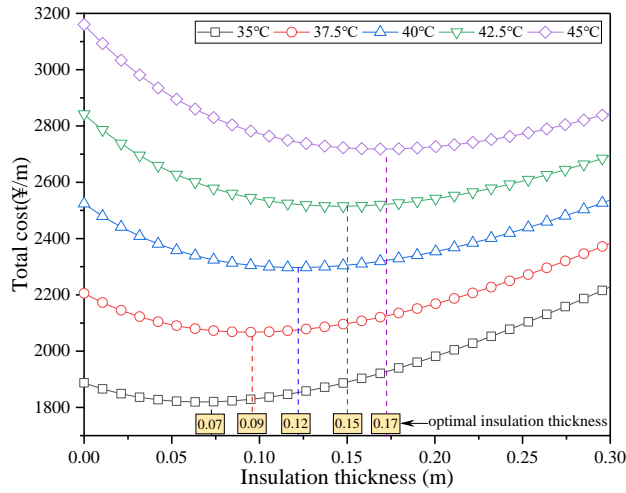
428 It is also found from Fig. 8 that the electricity cost decreases slowly with the increased insulation
 429 thickness. The reason for this phenomenon may attributed to the fact that the thermal conductivity of
 430 insulation layer is not small enough to reduce more heat dissipation from tunnel to the airflow. One
 431 can expect that the insulation layer with lower thermal conductivity can lead to a remarkable reduction
 432 in the electricity cost. In addition, the thermal insulation effect for the tunnel with higher rock
 433 temperature may be more significant, which can be proved through the results shown in Fig. 4.



434 Fig. 8. Total cost versus the insulation thickness (rock temperature=35 °C).

435
 436 In order to investigate the influence of rock temperature on the optimal insulation thickness, the
 437 total costs under various rock temperatures are also evaluated and shown in Fig. 9. It is found that an
 438 increase in the rock temperature can lead to an increase in the total cost. Under the same insulation
 439 thickness, the larger the rock temperature, the higher the total cost is. In addition, the optimal
 440 insulation thickness is observed to be increased when the rock temperature increases. For instance,
 441 the optimal insulation thickness increases from 0.07 m to 0.17m when the rock temperature increases

442 from 35 °C to 45 °C. This may be attributed to the fact that a larger insulation thickness is needed to
 443 reduce the cooling load for the case with a higher rock temperature.



444

445

Fig. 9. Total cost versus the insulation thickness (various rock temperatures)

446 *3.3.2. life cycle saving and payback period*

447 The effect of the rock temperature on the life cycle saving is investigated and presented in Fig. 10.

448 It is seen from Fig. 10 (a) that the life cycle saving firstly increases and then decreases when the

449 insulation thickness increases, which indicates that there is a maximum life cycle saving for various

450 cases with different rock temperatures. As shown in Fig. 10 (b), the maximum life cycle savings per

451 tunnel length are 61.2 ¥/m, 128.2 ¥/m, 213.2 ¥/m, 310.1 ¥/m and 421.8 ¥/m when the rock temperature

452 is 35 °C, 37.5 °C, 40 °C, 42.5 °C and 45 °C, respectively. In addition, it is also observed from Fig. 10

453 that the optimal insulation thicknesses corresponding to the maximum life cycle saving under various

454 rock temperatures are consistent with the optimal insulation thickness illustrated in Fig. 9.

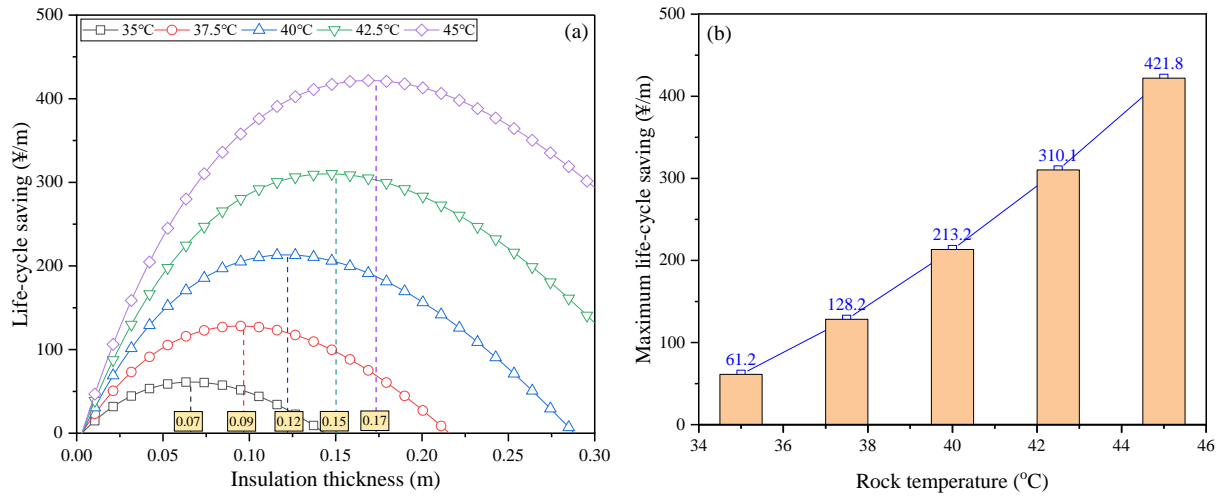


Fig. 10. Life cycle saving versus (a) insulation thickness, (b) rock temperature.

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The optimal insulation thickness, maximum life cycle saving and payback period are listed in Table

458

5. It is found that the optimal insulation thickness and maximum life cycle saving increase from 0.07

459

m and 61.2 ¥/m to 0.17 m and 421.8 ¥/m when the rock temperature increases from 35 °C to 45 °C.

460

Interestingly, a shorter payback period is achieved by the increased rock temperature. For instance,

461

the payback period reduces from 9.96 years to 4.91 years when the rock temperature increases from

462

35 °C to 45 °C.

463

Table 5 Optimal insulation thickness, maximum life cycle saving and payback period under various rock

464

temperatures.

Rock temperature (°C)	Optimal insulation thickness	Maximum life cycle saving (¥/m)	Payback period (year)
	(m)		
35°C	0.07	61.2	9.96
37.5°C	0.09	128.2	8.3
40°C	0.12	213.2	6.85

42.5°C	0.15	310.1	5.72
45°C	0.17	421.8	4.91

465 **4. Conclusions**

466 The transient heat conduction model of insulated underground tunnel is solved by using the
467 separation-of-variable method in this study. The rock temperature and heat flux reduction achieved
468 by applying thermal insulation are investigated. In addition, the optimal thickness, life cycle saving
469 and payback period are discussed based on the life cycle cost method. The main findings are as
470 follows:

471 (1) The temperature gradient in the insulation layer is considerably larger than that in surrounding
472 rock. The surplus temperature of original tunnel is 2.5-1.43 times higher than that of tunnel with
473 insulation layer when Fo increases from 0.28 to 20.22.

474 (2) An increase in the insulation thickness and TCR results in an increase in the heat flux reduction.
475 A decrease in TCIL and tunnel radius have a positive impact on the heat flux reduction. The heat flux
476 reduction is not strongly dependent on the convection coefficient.

477 (3) The total cost firstly decreases and then increases when the insulation thickness increases. The
478 optimal insulation thickness and the maximum life cycle saving increase from 0.07 m to 0.17 m and
479 from 61.2 ¥/m to 421.8 ¥/m, the payback period reduces from 9.96 year to 4.91 years respectively
480 when the rock temperature increases from 35 °C to 45 °C.

481 This study presents a preliminary investigation on how the thermal insulation can be used to save
482 the cooling energy and overall system cost in the mining environment in an effective and economic
483 way. More factors, including CO₂ emission and environmental conditions, need to be accounted for
484 in further investigations to gain an in-depth understanding on how to effectively minimise the overall

485 energy consumption in the mining industry, thereby contributing to the carbon neutral target.

486 **Acknowledgements**

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489 anonymous reviewers for their constructive suggestions on this study.

490 **Appendix:**

491 The Eqs. (1)-(4) can be solved by using the variable-separation-method. According to the
492 superposition principle, the solution for above equations can be expressed by Eq. (A1):

$$493 \begin{cases} T_1(r,t) = T_1^{(1)} + T_1^{(2)} \\ T_2(r,t) = T_2^{(1)} + T_2^{(2)} \end{cases} \quad (A1)$$

494 where the superscript '(1)' represents the unsteady state solution, and the superscript '(2)' represents
495 the steady-state solution.

496 For the steady-state solution, the basic equations are:

$$497 \frac{\partial^2 T_2^{(2)}}{\partial r^2} + \frac{1}{r} \frac{\partial T_2^{(2)}}{\partial r} = 0 \quad (A2)$$

$$498 \frac{\partial^2 T_1^{(2)}}{\partial r^2} + \frac{1}{r} \frac{\partial T_1^{(2)}}{\partial r} = 0 \quad (A3)$$

$$499 T_2^{(2)}(r_3, t) = T_0 \quad (A4)$$

$$500 r = r_1 : \quad k_1 \frac{\partial T_1^{(2)}}{\partial r} = h(T_1^{(2)} - T_a) \quad (A5a)$$

$$501 r = r_2 : \quad \begin{cases} T_1^{(2)}(r_2, t) = T_2^{(2)}(r_2, t) \\ k_1 \frac{\partial T_1^{(2)}(r_2, t)}{\partial r} = k_2 \frac{\partial T_2^{(2)}(r_2, t)}{\partial r} \end{cases} \quad (A5b)$$

502 The general solutions for Eq. (A2) and Eq. (A3) are given by Eq. (A6) and Eq. (A7):

$$503 T_1^{(2)}(r) = C_1 \ln r + D_1 \quad (A6)$$

504
$$T_2^{(2)}(r) = C_2 \ln r + D_2 \quad (\text{A7})$$

505 Eq. (A6) and Eq. (A7) are substituted into Eq. (A4) and Eq. (A5), the coefficients C_1, C_2, D_1, D_2
 506 can be obtained by using Eq. (A8):

507
$$\begin{cases} C_1 = (T_0 - T_a) \left[\ln r_2 + \frac{k_1}{hr_1} - \ln r_1 - \frac{k_1}{k_2} (\ln r_2 - \ln r_3) \right]^{-1} \\ C_2 = \frac{k_1}{k_2} C_1 \\ D_1 = \frac{k_1}{hr_1} C_1 - (C_1 \ln r_1 - T_a) \\ D_2 = T_0 - \frac{k_1}{k_2} C_1 \ln r_3 \end{cases} \quad (\text{A8})$$

508 For the unsteady state solution, the governing equation and the contact boundary condition are also
 509 described by Eq. (1) and Eq. (4). The initial condition and boundary condition are given by Eq. (A9)
 510 and Eq. (A10):

511
$$\begin{cases} T_2^{(1)}(r_3, t) = 0 \\ k_1 \frac{\partial T_1^{(1)}}{\partial r} \Big|_{r=r_1} = h T_1^{(1)} \Big|_{r=r_1} \end{cases} \quad (\text{A9})$$

512
$$\begin{cases} T_1^{(1)}(r, 0) = T_0 - T_1^{(2)}(r) \\ T_2^{(1)}(r, 0) = T_0 - T_2^{(2)}(r) \end{cases} \quad (\text{A10})$$

513 The unsteady state equation can be solved by using the separation-of-variables method [40, 43],
 514 and the following form of temperature is adopted for both insulation layer and surrounding rock
 515 regions (subscripts and superscripts are omitted):

516
$$T(r, t) = R(r) \cdot \Gamma(t) \quad (\text{A11})$$

517 Substituting Eq. (A11) into the governing equation, two ordinary differential equations can be
 518 obtained in the form of Eq. (A12) and Eq. (A13):

519
$$\Gamma'(t) + \alpha \beta^2 \Gamma(t) = 0 \quad (\text{A12})$$

520
$$r^2 R''(r) + rR'(r) + \beta^2 r^2 R(r) = 0 \quad (\text{A13})$$

521 where β is the positive eigenvalue.

522 The fundamental solution of Eq. (A12) is given by Eq. (A14):

523
$$\Gamma(t) = \exp(-\alpha\beta^2 t) \quad (\text{A14})$$

524 The general solution of Eq. (A13) can be expressed by Eq. (A15):

525
$$R(r) = A_1 J_0(\beta r) + B_1 Y_0(\beta r) \quad (\text{A15})$$

526 Therefore, the general form of the unsteady state solution is derived and described by Eq. (A16):

527
$$T_i^{(1)}(r, t) = \sum_{n=1}^{\infty} [A_{in} J_0(\beta_{in} r) + B_{in} Y_0(\beta_{in} r)] \cdot \exp(-\alpha_i \beta_{in}^2 t) \quad (\text{A16})$$

528 The coefficients of A_{in} , B_{in} , β_{in} can be solved from the initial condition and boundary conditions
529 and the contact boundary condition.

530 From the contact boundary condition, Eq. (A17) can be obtained:

531
$$\alpha_1 \beta_{1n}^2 = \alpha_2 \beta_{2n}^2 = \beta_n^2 \quad (\text{A17})$$

532 Substituting Eq. (A17) into Eq. (A16), it is obtained as:

533
$$T_i^{(1)}(r, t) = \sum_{n=1}^{\infty} \left[A_{in} J_0\left(\frac{\beta_n}{\sqrt{\alpha_i}} r\right) + B_{in} Y_0\left(\frac{\beta_n}{\sqrt{\alpha_i}} r\right) \right] \cdot \exp(-\beta_n^2 t) \quad (\text{A18})$$

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