A nonlinear perturbation study of a shoreface nourishment on a multiply barred beach

W.L. Chen,^{a,1,*}, N. Dodd^a

^a*Faculty of Engineering, University of Nottingham, Nottingham NG7 2RD, UK.*

Abstract

The 1D (cross-shore) nonlinear nourishment-as-a-perturbation model of *Chen and Dodd* (2019) is extended to include wave streaming and depth-varying mean return flow. This is shown to lead to physically consistent values for the migration rate of small amplitude bed perturbations in the whole nearshore region. The model is also extended to arbitrary wave incidence angle, and an alternative energy dissipation description is included. The model is applied to a background profile that includes multiple nearshore bars, and which also allows comparison with observations of nourishment development and migration. The chosen profile is that of the Terschelling shoreface nourishment. Over the initial nourishment region the linearised bed evolution equation predicts onshore small-amplitude bed perturbation migration rates in the troughs (where waves shoal) and offshore rates on the bars (where waves break). Sensitivity studies using the fully nonlinear model reveal wave height to be the primary controlling parameter for the resultant perturbed sediment flux. Perturbed sediment fluxes are onshore up to a threshold wave height; beyond this offshore fluxes predominate. Wave period and angle of incidence, and tide level are all secondary controlling factors, variation in which yields no change in perturbed sediment flux sign (for fixed wave height). Recorded wave and tide signals are used to drive the model to simulate six months of the initial nourishment evolution. It is shown that in calmer conditions the small amplitude migration rate of bed disturbances is a highly accurate predictor of the fully nonlinear migration rate (as described by the centre of mass of the nourishment); in storm periods the two predictors diverge. The observed overall migration rate is well described

*Corresponding author, wenlong.chen@cranfield.ac.uk

¹The present affiliation of the author is: School of Water, Energy and Environment, Cranfield University

by the model. The details of the profile evolution show a moderately good reproduction of those observed (Brier Skill Score = 0.52).

Keywords: nonlinear perturbation model, Terschelling nourishment, wave skewness, wave driven current, streaming effect

1. Introduction

Shoreface nourishments are adopted to mitigate the effect of erosion problems along coastlines (*Hamm et al.*, 2002). They are thought to be cheaper, more flexible in execution and more friendly to the environment than traditional hard coastal structures (*Capobianco et al.*, 2002). The method has been pioneered in the Netherlands. Since 1990, dozens of shoreface nourishments have been performed (*Hillen and Roelse*, 1995).

Understanding the temporal and spatial evolution of a shoreface nourishment is important, because it allows us to design them most effectively. Complex morphodynamical models are used by coastal engineers to predict this evolution, with reasonable success (e.g. *Grunnet et al.*, 2004). A disadvantage of using such a model is that they are computationally expensive to run. Furthermore, in complex models, the nourishment evolution is embedded in the dynamics of the total morphology, which thus must be disentangled from the "background" morphodynamics in order to understand it. Moreover, we are at present unable to predict those background morphodynamics because present-day complex models are unable to predict so-called beach recovery (*van Rijn et al.*, 2013), which is the gradual onshore sediment movement under prevailing milder wave conditions.

To circumvent these problems *Chen and Dodd* (2019) developed an approach that can be described as a "nourishment-as-a-perturbation" (hereafter NaaP) concept. In this approach they focus on the dynamics of the nourishment alone (i.e. the perturbation), and presented a 1D (cross-shore) nonlinear evolution model for the nourishment. In their study, on an idealised, plane beach, sediment flux is driven by wave skewness, wave asymmetry and mean current (return flow and tide), which are three of the main physical processes thought to be responsible for cross-shore sediment movement (*Roelvink and Reniers*, 2011). Because the nourishment alters the sea bed profile, the hydrodynamics and hence the sediment flux are also altered. The divergence of this perturbed sediment flux, i.e., the difference between the sediment fluxes with and without the

nourishment, then drives the evolution of the nourishment. The nourishment is additionally subject to down-slope diffusion under gravity. In this way, it is the difference between the sediment flux of the nourished and un-nourished beach that is of prime importance, rather than the absolute value, which obviates the need for accurate reproduction of beach recovery.

Thus, the dynamics of the nourishment are isolated, which potentially offers us a better understanding of their behaviour, as long as the background morphology does not evolve too drastically. A related method was applied by *De Swart and Calvete* (2003) to study the nonlinear evolution of human interventions on shoreface-connected sand ridges, by *van Leeuwen et al.* (2007) to study the linear evolution of a shoreface nourishment, and by *Roos et al.* (2008) to examine sand extractions. *Roos and Hulscher* (2003) also provided a general linear framework of looking at bed evolution as a perturbation problem. Model results of *Chen and Dodd* (2019) are consistent with observations, as well as prevailing theory on cross-shore sediment transport.

However, the study of *Chen and Dodd* (2019) was limited to an idealised, plane beach. For the concept to be useful, it must be valid for non-planar beaches, on which there may be multiple regions of breaking and shoaling, aside from the nourishment itself. Equally, it must be applicable to real beaches, on which the background profile is, notionally, evolving, at least for a limited duration.

We therefore present here a study, in which the nonlinear NaaP model of *Chen and Dodd* (2019) is applied to a real beach that possesses a multiple barred background profile on which a nourishment is placed. To do this we must also develop further the NaaP approach so as to include wave streaming, which is thought to play an important role in cross-shore sediment flux (*Fuhrman et al.*, 2009). Furthermore, the description of mean-current-driven cross-shore sediment flux of *Chen and Dodd* (2019) is depth-invariant, so that at deeper depths the mean near-bed sediment flux is likely to be overestimated. Therefore, we extend that approach so as to be more realistic. These two developments are also important because accurate sediment fluxes also provide accurate reproduction of nourishment kinematics, which allows us to consider nourishment migration speed separately from bed change.

For the application we choose the Terschelling nourishment. This field case is of a quasi-alongshore uniform background cross-shore profile that consists of a number of bars, on which an extended, shore-parallel nourishment was placed. To this end we also amend the approach of *Chen and Dodd* (2019) so as to utilise

an alternative wave energy description, and to include oblique wave incidence. In this application we seek to capture the nourishment morphodynamics and kinematics, rather than perform a model validation.

In the next section we present the NaaP model, with a focus on the new dynamics included. We then investigate the implications of these changes to the approach. Next we present a description of the Terschelling nourishment study case. We then present a sensitivity study of the site using the NaaP approach. Model simulations of the Terschelling nourishment are then presented and analysed. Finally, a discussion is presented, and conclusions are drawn.

2. Methodology

The (1D) nonlinear perturbation model for the cross-shore evolution of a shoreface nourishment proposed by *Chen and Dodd* (2019) is the basis for the present study. The model couples wave, tide and sediment dynamics, and focuses on the perturbed dynamics introduced by the nourishment. The reader is referred to *Chen and Dodd* (2019) for a full investigation of the earlier version of the model for a idealised nourishment on a plane beach.

Here we present the new model equations. These include the dynamical equations, and the equations parameterizing the processes.

2.1. The model equations

2.1.1. The evolution equation of the nourishment

The change in bed level with respect to the pre-nourishment (basic) alongshore-uniform profile, is denoted $b(x, t)$. It is determined from the sediment conservation equation

$$\frac{\partial b}{\partial t} + \frac{1}{1-p} \frac{\partial q'}{\partial x} - \gamma \frac{\partial^2 b}{\partial x^2} = 0, \quad (1)$$

where $p = 0.4$ is the porosity of the sediment. x is the cross-shore spatial, and t the time independent variable. γ is a morphological (downslope) diffusion coefficient. q' is the perturbed net cross-shore component of the sediment flux; here we consider arbitrary angle of wave incidence. Thus, q' is the difference between the total cross-shore component of the sediment flux (q) calculated with and without nourishment, i.e.,

$$\begin{aligned} q' &= q(E_w(x, t), E_r(x, t), \theta(x, t), z_{b,0}(x) + b(x, t)) \\ &- q(E_{w,0}(x), E_{r,0}(x), \theta_0(x), z_{b,0}(x)), \end{aligned} \quad (2)$$

Figure 1: Geometry and coordinate system, where symbol MSL indicates mean sea level.

where subscript $_{,0}$ refers to the un-nourished sea bed. The nourishment evolution is therefore purely cross-shore.

The perturbed sediment flux in (2) is therefore the difference between the fluxes that would result from two separate cross-shore simulations, one of the un-nourished cross-shore profile plus nourishment, and the other of the un-nourished profile only. Here, however, we assume that the un-nourished profile remains unchanged, either because it is a true equilibrium profile, or because it is evolving more slowly than the nourishment.

In (2) E_w is the wave energy density, E_r is the wave roller energy density, θ is the local wave angle of incidence relative to the shore normal, and $z_b = z_{b,0} + b$ is the total sea bed level: existing (un-nourished) profile ($z_{b,0}$) plus nourishment (b). Note that although mean water depth $h(x, t)$ does not appear explicitly in Eq. (2), $h \equiv -z_b$ because the perturbation in bed level (b) accounts for changes in depth, and we neglect wave set-up. Because our nourishment is on the shoreface, we expect set-up to be less important. We also neglect mean cross-shore tidal current. Throughout we take $\gamma = 3.5 \times 10^{-4} m^2 s^{-1}$, which is the same as that used in *Chen and Dodd (2019)*; *Larson and Hanson (2015)*. We adopt this simple diffusion model because we seek only to include this physical process at a realistic magnitude. Therefore, we do not pursue a more sophisticated representation.

The study geometry is encapsulated in Fig. 1, in which the background profile (in black), including any bars in that profile, is assumed not to evolve.

In the following we decompose q , the total net cross-shore sediment transport, into components from wave skewness, wave asymmetry, and a mean component:

$$q = q_{sk} + q_{as} + q_c \quad (3)$$

2.1.2. Velocity skewness, and asymmetry (acceleration skewness)

As waves shoal, their shape changes from sinusoidal to an increasingly skewed shape (long trough and more confined, exaggerated crest). After breaking, this shape progressively changes to become highly asymmetrical, i.e., pitched forward. Both skewness and asymmetry are thought to promote significant net onshore sediment transport, and so are considered here, and therefore are included in q and therefore q' . The skewness of wave velocity (S_{vel}) and acceleration (S_{acc}) are introduced to describe these changes; these are defined as:

$$S_{vel} = \frac{\langle u^3 \rangle}{\sigma^3(u)} \quad , \quad S_{acc} = \frac{\langle a^3 \rangle}{\sigma^3(a)}. \quad (4)$$

u denotes the intra-wave velocity; here the parameterized expression proposed by Ruessink *et al.* (2012) is used. $\langle . \rangle$ refers to a time average over the wave period (T). $a = \frac{\partial u}{\partial t}$ is the wave acceleration. $\sigma(.)$ is the standard deviation of the variable. Semi-analytical expressions of S_{vel} , S_{acc} , $\sigma(u)$ and $\sigma(a)$ are developed and presented in Chen and Dodd (2019), and they are used herein.

2.1.3. Sediment transport components

The net sediment transport due to the wave skewness and the wave asymmetry are given by

$$q_{sk} = K_s S_{vel} \sigma^3(u) \cos \theta \quad , \quad q_{as} = K_a S_{acc} \sigma(a) \cos \theta, \quad (5)$$

where q_{sk} (q_{as}) is the cross-shore component due to wave skewness (asymmetry). The mean current (u_c) driven cross-shore sediment flux component is given by the expression of Soulsby (1997)

$$q_c = A_{sb} u_c \cos \theta \left[\left(|u_c|^2 + \frac{0.018}{C_D} u_w^2 \right)^{1/2} - u_{cr} \right]^{2.4}. \quad (6)$$

$u_w = \frac{H}{2} \frac{gk}{\omega} \frac{\cosh kz_0}{\cosh kh}$ is the wave orbital velocity at the boundary layer edge z_0 (0.01 m), k is the linear wave number, $\omega = \frac{2\pi}{T}$ is the wave radial frequency, $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration, and H is wave height. $C_D = 0.005$ is the drag coefficient due to current alone. u_{cr} is the threshold current velocity; here we follow e.g. Roos *et al.* (2004) in setting $u_{cr} = 0$, because we wish to study nourishment dynamics. The coefficients, i.e., K_s , K_a and A_{sb} are the same as in Chen and Dodd (2019). $u_c \cos \theta$, the cross-shore component of the mean current, contains two components: near-bed flow derived by consideration of

157 mass balance, and near-bed mean flow due to streaming. These are considered
158 separately in § 2.3.

159 2.2. Wave energy balance

160 In the present study we consider both shoaling and refraction. Accordingly,
161 the wave energy density ($E_w = \frac{1}{8}\rho g H_{rms}^2$) balance in the cross-shore is:

$$162 \quad \frac{\partial(E_w c_g \cos \theta)}{\partial x} = -\mathcal{D}_w. \quad (7)$$

163 $\rho = 1027 \text{ kg/m}^3$ is the water density, H_{rms} the root mean square wave height,
164 and c_g is the group velocity from linear wave theory with representative wave
165 period T . Snell's law is used to determine $\theta(x)$ (*Dean and Dalrymple*, 1991).
166 Here, we use the energy dissipation expression $\mathcal{D}_w$, of *Baldock et al.* (1998).
167 This is

$$168 \quad \mathcal{D}_w = \frac{\rho g \omega}{8\pi} \exp \left[-\left(\frac{H_b}{H_{rms}} \right)^2 \right] (H_b^2 + H_{rms}^2), \quad (8)$$

169 H_b is the breaker height, and is given by (*Battjes and Janssen*, 1978),

$$170 \quad H_b = \frac{0.88}{k} \tanh \left(\frac{\gamma_b}{0.88} kh \right). \quad (9)$$

171 and where γ_b is the breaker coefficient, where (*Ruessink et al.*, 2003)

$$172 \quad \gamma_b = 0.76kh + 0.29. \quad (10)$$

173 By using this expression instead of that of *Van Leeuwen et al.* (2006) we lose
174 the convenience of shifting between random and monochromatic waves. On the
175 other hand, we gain the advantage of an expression that has been validated
176 under field conditions. The roller energy (E_r) equation is also extended to
177 consider refraction:

$$178 \quad \frac{\partial(E_r c \cos \theta)}{\partial x} = \mathcal{D}_w - \mathcal{D}_r, \quad (11)$$

179 where, $c = \frac{\omega}{k}$ is the phase velocity. The expression for roller energy dissipation
180 ($\mathcal{D}_r$) is given in *Chen and Dodd* (2019).

181 2.3. Mean current

182 If we consider mean current to be that that is time invariant at the timescale
183 of the surface gravity waves, then it has three main cross-shore contributions
184 here: tidal current, return flow (undertow) and streaming. We neglect the tidal

current, but make adjustments for tide through changes in free surface elevation. *Chen and Dodd* (2019) considered return flow only. Here we both improve the return flow description and introduce streaming into our sediment dynamics.

2.3.1. Streaming

The horizontal non-uniformity of the wave velocity field generates a mean bottom boundary layer current (streaming) in the direction of wave propagation (*Longuet-Higgins*, 1953). Streaming is thought to contribute primarily to mean onshore sediment transport (*Roelvink and Reniers*, 2011). *Longuet-Higgins* (1957) found that at the top of the boundary layer the induced wave averaged streaming velocity is

$$u_{str} = \frac{3}{16c} \left(\frac{\omega H_{rms}}{\tanh(kh)} \right)^2. \quad (12)$$

We make use of this equation herein.

2.3.2. Mean current based on mass conservation

Consideration of conservation of mean wave-induced mass flux over the water column leads to a mean return flow to balance the onshore net mass flux due to Stokes drift and roller (*Svendsen*, 2006). *Chen and Dodd* (2019) considered a depth-invariant mean return flow to balance the onshore flux. This leads to a probable overestimation of near-bed offshore-directed sediment flux in the offshore region. Here we make use of the method proposed by *Roelvink and Reniers* (2011), which is based on a so-called generalized Lagrangian mean approach, to calculate the return flow. This method is shown to give good results both inside and outside of the surf zone. In this approach, the mean current $u_L(z)$, z being the vertical coordinate (positive upward), is allowed to have a vertical profile,

$$u_L(z) = u_E(z) + u_{stokes}(z). \quad (13)$$

Here, $u_E(z)$ is the Eulerian velocity with a logarithmic vertical profile driven by surface and bottom shear stress, and $u_{stokes}(z)$ describes the vertical distribution of stokes drift (*Phillips*, 1977)

$$u_{stokes} = \frac{1}{8} (k H_{rms})^2 c \frac{\cosh(2k(z+h))}{\sinh^2(kh)} \cos \theta. \quad (14)$$

A detailed description of $u_L(z)$ is given in Appendix A.

215 2.4. Total sediment flux

216 We combine expressions for streaming and mass-conservation-induced flow
217 to get

$$218 \quad u_c \cos \theta = u_L|_{z=-h+0.1\text{m}} + u_{str} \cos \theta f_{damp}, \quad (15)$$

219 which thus consists of a near bottom wave current evaluated mean flow 0.1 m
220 above the sea bed from Eq. (13), and streaming in the onshore direction. Note
221 that a damping factor $f_{damp} = \exp(-D_w/3.0)$ is included which diminishes the
222 effect of streaming in the case of breaking waves. This is because the effect of
223 streaming is only relevant for non-breaking waves (*Roelvink and Reniers, 2011;*
224 *Kranenburg et al., 2012*).

225 The total net cross-shore sediment transport is therefore the summation of
226 the net sediment flux due to wave skewness, wave asymmetry and mean current:

$$227 \quad q = q_{sk} + q_{as} + q_c = q_{sk} + q_{as} + q_L + q_{str} \quad (16)$$

229 Note that q_L and q_{str} cannot be fully decoupled in Eq. (6); hereafter we refer
230 to q_c only, rather than q_L and q_{str} .

231 3. Nourishment kinematics

232 An analysis of the propagation of a small amplitude perturbation (nourish-
233 ment) via Eq. (2) yields (neglecting the diffusion term, *Chen and Dodd, 2019*)

$$234 \quad \frac{\partial b}{\partial t} + \frac{1}{1-p} \frac{\partial q'}{\partial b} \frac{\partial b}{\partial x} = -\frac{1}{1-p} \left\{ \frac{\partial q'}{\partial E'_w} \frac{\partial E'_w}{\partial x} + \frac{\partial q'}{\partial E'_r} \frac{\partial E'_r}{\partial x} + \frac{\partial q'}{\partial \theta'} \frac{\partial \theta'}{\partial x} \right\}. \quad (17)$$

236 Therefore

$$237 \quad c_n = \frac{1}{1-p} \frac{\partial q'}{\partial b} = -\frac{1}{1-p} \left\{ \frac{\partial q_{sk}}{\partial h} + \frac{\partial q_{as}}{\partial h} + \frac{\partial q_c}{\partial h} \right\} \quad (18)$$

238 where c_n is the intrinsic propagation speed of the perturbation.

239 In Fig. 2(a) we plot c_n as a function of cross-shore position on a plane beach
240 (slope 0.01) and offshore wave height. It can be seen that for small wave heights
241 and larger depths we always have positive c_n , indicating onshore propagation of
242 any small morphological perturbation as long as the forcing terms on the right
243 of Eq. (17) are negligible. These forcing terms are likely to be small prior to
244 breaking, but if we ignore the forcing terms for all depths and focus just on
245 the free kinematics shown in Fig. 2(a), we see that, starting from offshore (ex-
246 treme left), c_n becomes more positive as the waves shoal. This effect is partly

247 attributable to streaming, as can be seen by comparing Fig. 2(a) with 2(b), in
 248 which streaming is omitted. The rest is primarily due to skewness. Comparing
 249 Fig. 2(b) and Fig. 2(c) (the equivalent plot from *Chen and Dodd, 2019*), we see
 250 that the more accurate description of return flow implemented here is crucial
 251 to capture this behaviour. In the region of significant wave breaking the free
 252 kinematics are such that $c_n < 0$, indicating offshore propagation. Therefore, a
 253 point of convergence is likely to exist for $c_n \approx 0$; this point is located progres-
 254 sively further offshore with increasing wave height. c_n reaches a maximum of
 255 about 1.7×10^{-4} m/s (14.6 m/day).

256 Onshore propagation of a perturbation is not synonymous with onshore sed-
 257 iment movement, so positive c_n does not imply onshore movement of the sed-
 258 iment in a nourishment: the feeder effect (although it is likely to be at least
 259 partially true because the nourishment must be composed of some sediment to
 260 exist and propagate onshore). Regardless, the onshore propagation of the per-
 261 turbation (the shape) will result in the lee effect, in which waves dissipate their
 262 energy on the nourishment as it propagates into shallower water.

263 4. Terschelling nourishment

264 4.1. Study area and data

265 Terschelling is one of the chain of barrier islands along the northern Dutch
 266 coast that separate the North Sea from the Wadden Sea (Fig. 3). The me-
 267 dian grain size (D_{50}) varies from 150 μm to 220 μm (the sediment is coarser
 268 shoreward). To protect Terschelling from erosion, a total amount of 2.1 Mm^3
 269 ($D_{50} = 200 \mu\text{m}$) of sand was placed in the shoreface of Terschelling island from
 270 May to October 1993 (*Hoekstra et al., 1994*, NOURTEC project). The nour-
 271 ishment was placed in depths of 5 – 7 m, and has a longshore extent of about
 272 4.5 km (see black block denoted as ‘nourishment area’ in Fig. 3). Because our
 273 model is 1D (on/offshore) we are interested in the cross-shore movement of the
 274 nourishment. The elongated, quasi-shore-parallel geometry of the Terschelling
 275 nourishment therefore offers an opportunity to apply the model where we may
 276 expect it to describe the dominant dynamics. Although monitoring continued
 277 for a number of years, we focus on the first six months for our simulation, be-
 278 cause this means that we are within initial cross-shore adjustment region of the
 279 observed nourishment. This timescale is also short enough to limit the influence
 280 of the rhythmic bar system at the site, and also of alongshore sediment flux
 281 divergence, which are thought to contribute significantly over 3-year timescales

Figure 2: Comparison of intrinsic propagation speed (c_n , in m/s) calculated with: (a), description of total near bottom wave current $u_c \cos \theta$ from Eq. (15); (b), description of return flow $u_c \cos \theta$ from Eq. (15) but neglecting the u_{str} component; and (c), depth averaged return flow (Eq. (15) in *Chen and Dodd* (2019)). The plane beach (with slope of 0.01) is forced by normal incident waves (with $T = 6$ s). The dark thick curve highlights the zero contour of c_n . The thick red curve denotes the cross-shore position of the break point, identified here as the position of maximum wave height.

to onshore sediment movement (*Grunnet and Ruessink*, 2005). We discuss limitations in the Discussion section. Therefore, the central transect denoted by the shore normal black line in Fig. 3 is chosen. This transect is known as transect 17.000 in *Grunnet and Ruessink* (2005).

The sea bed profile of the study transect was measured both before and after the nourishment, see Fig. 4. The nourishment mostly filled in the trough between middle and outer bars. Most morphological change took place in this first six months after the nourishment (*Grunnet et al.*, 2004).

Figure 3: Location and study area. The transect denoted by the black solid line normal to the coast is the study transect. The figure is reproduced from Figure 1 in *Grunnet et al. (2004)*.

Figure 4: Measured cross-shore sea bed profiles of study transect.

Figure 5: Hydrodynamic conditions observed at -15 m offshore, showing: (a), offshore wave height H_{rms} ; (b), wave period T ; (c), wave incident angle θ (with respect to shore normal direction, negative value is measured in a clockwise direction); and (d), tide mean water level η . The grey horizontal dashed lines highlight the prevailing wave conditions observed at the site.

290 The hydrodynamic conditions from Nov 93 to May 94 are shown in Fig. 5
 291 (recorded at hourly intervals). As mentioned, tide is here accounted for by
 292 shifting the water surface. Also, offshore directed waves (i.e., $|\theta| > 90^\circ$) are not
 293 included in this study.

294 4.2. Numerical considerations

295 A uniform grid spacing $\Delta x = 1\text{m}$ in the cross-shore x direction is used. This
 296 was found to achieve convergence both hydrodynamically (convergence test for
 297 hydrodynamics only), and morphodynamically (convergence test for bed level).
 298 A so-called Euler-Heun method (Süli and Mayers, 2003), i.e., a predictor and
 299 corrector algorithm, is applied to update the nourishment Eq. (1). A 30 s
 300 morphodynamic time step is chosen which is the largest value that achieved

Figure 6: Sea bed profile showing basic profile (grey thick line) and nourishment (blue thin line).

301 convergence in bed evolution. The profile of April 1993 (see Fig. 4) was measured
 302 right before the implementation of the nourishment. We therefore take this un-
 303 nourished profile as the basic profile of our study, perturbations from which are
 304 described by Eq. (1): see Fig. 6, in which the initial nourishment ($b(x, t = 0)$)
 305 is the blue line. The pre-nourishment profile has three sandbars, referred to as
 306 the outer bar, middle bar and inner bar. Thus, $b(x, t = 0)$ is the sand placed in
 307 the trough between outer and middle bar (from $x = 1140$ to $x = 1600$ m).

308 4.3. Terschelling Nourishment kinematics

309 In Fig. 7 we again plot the intrinsic small amplitude nourishment propa-
 310 gation speed (c_n) at Terschelling for prevailing wave direction ($\theta = 11^\circ$) and
 311 period ($T = 7$ s). The picture is reminiscent of that in Fig. 2, except that the
 312 effect of the bars is apparent. These both cause breaking and restrict it to their
 313 vicinity, so we do not see the point of convergence moving offshore. As wave
 314 height increases, c_n becomes negative on the bars. This picture implies that
 315 a nourishment placed in between a pair of bars will stay there. This conclu-
 316 sion must be qualified by noting that it does not consider the forcing terms in
 317 Eq. (17) and only small amplitude perturbations. In the next section we look at
 318 the full, finite-amplitude dynamics. The other thing to note is that c_n is about
 319 an order of magnitude smaller than that in Fig. 2, with the maximum value
 320 $\approx 2.5 \times 10^{-5}$ m/s ≈ 2.16 m/day.

Figure 7: Sensitivity of c_n (in m/s) to offshore wave height for the Terschelling site (pre-nourishment). Wave period and incident angle are kept as 7 s and 11° , respectively. A positive (negative) c_n indicates onshore (offshore) propagation of the nourishment. Zero contour of c_n is highlighted by thick black dashed line. The representative wave condition used in section 4.4 is indicated with horizontal dashed line. White dots denote the location of break points and vertical red dashed lines indicate the shoreward and seaward end of nourishment.

Figure 8: Cross-shore hydrodynamics: (a) H_{rms} , (b) u_w , (c) u_c , (d) S_{vel} and (e) S_{acc} . Waves of representative value is imposed, i.e., $H_{rms} = 1.1$ m, $T = 7$ s and $\theta = 11^\circ$. Thick grey curves refer to hydrodynamics for the basic profile and the thin blue curves refer to that with nourished sea bed. Three vertical blue dashed lines denote the location of break points.

4.4. Effect of the finite-amplitude nourishment

We examine the effect of the Terschelling nourishment under moderate wave conditions: $H_{rms} = 1.1$ m, $T = 7$ s and $\theta = 11^\circ$ (Grunnet *et al.*, 2005). These wave heights represent those in between those of the storm events and the beach recovery (low wave height) durations; the period and angle of incidence are annual average values. The water level is set to be at mean tide ($\eta = 0$ m).

The basic profile (grey line in Fig. 8) shows the waves beginning to break on the middle bar ($x = 1608$ m), and then more pronounced breaking on the inner bar, and again near the shore (Fig. 8(a)). The nourishment induces some shoaling. The background u_c is onshore directed except when the waves are breaking. S_{vel} and S_{acc} behave as expected: skewness increases as the waves shoal on the bar, and decreases as they deshoal in the trough, and finally dimin-

Figure 9: Sediment flux of: (a), current-driven component q_c ; (b), wave-skewness-driven component q_{sk} ; (c), wave-asymmetry-driven component q_{as} ; and (d), total perturbed transport q' and (dashed line) b . Waves of representative value is imposed. Vertical dashed lines are as in Fig. 8. The thick grey (thin blue) curve represents the fluxes for the basic (nourished) profile.

ishes in very shallow water as the waves become very asymmetrical. As they do so S_{acc} achieves its maximum. The effect of the nourishment is to increase u_c , S_{vel} and S_{acc} , due to extra shoaling (Fig. 8(c-e)). The increase in u_c is due to an increase in both streaming velocity and near bottom wave current, both induced by the nourishment.

The consequence of the hydrodynamic changes in Fig. 8 can be seen on sediment fluxes in Fig. 9. Because S_{vel} , S_{acc} and u_c all increase due to the nourishment, q_{sk} , q_{as} and q_c increase (via Eq. (5) and Eq. (6)). Consequently, the nourishment induces an increase in all the sediment fluxes (see blue curves in Fig. 9), leading to an overall onshore (positive) sediment flux perturbation, q' , (see Fig. 9(d)), thus implying that placement of the nourishment increases

344 onshore supply of sediment at that location. The divergence of the positive q'
 345 will, overall, also lead to onshore propagation of the nourishment, because both
 346 b and q' are positive, with peaks roughly coinciding (see Fig. 9(d); also see *Chen*
 347 *and Dodd* (2019)).

348 In the next three subsections we examine the sensitivity of the perturbed
 349 sediment flux, q' , to variations in hydrodynamic parameters, keeping other hy-
 350 drodynamic parameters fixed. These parameter values can be interpreted as
 351 those that established the non-evolving background profile, or as different val-
 352 ues, but with the background profile evolving more slowly such that it can be
 353 considered invariant. In either case the resulting pattern in q' shows how the
 354 nourishment would start to evolve. The actual nourishment evolution is consid-
 355 ered later.

356 4.4.1. Sensitivity to wave height

357 We now consider the effect of differing wave height, keeping T and θ fixed.
 358 In Fig. 10 we show this in terms of perturbations in sediment transport rate
 359 induced by the nourishment: q'_c (mean), q'_{sk} (skewness driven), q'_{as} (asymmetry
 360 driven), and total q' . To help illustrate the effect we also indicate on Fig. 10
 361 three wave heights that we characterise as mild ($H_{rms} = 0.5$ m), moderate
 362 (as before), and stormy wave heights ($H_{rms} = 2.5$ m). In Fig. 10(d) we can
 363 see that for mild and moderate wave heights the nourishment causes a positive
 364 q' , and thus feeds the beach. This positive perturbation is primarily due to
 365 mean current and skewness. In contrast, for stormy waves we see a negative
 366 q' , primarily q_c , which thus transports sediment offshore. All three mechanisms
 367 (Fig. 10(a-c)) provide positive perturbations for mild and moderate conditions;
 368 for stormy waves q' reverses due to these bigger waves breaking on the nour-
 369 ishment. Under these latter conditions note the area immediately shoreward of
 370 nourishment. Here $q' > 0$ again, because the breaking occurring on the nour-
 371 ishment now no longer happens further shoreward. Therefore, the middle bar is
 372 now, relatively speaking, feeding the beach. This effect is in addition to the lee
 373 effect of the nourishment (i.e. pure reduction in wave heights shoreward). The
 374 cross-shore q' profile for mild and moderate wave heights is consistent with an
 375 onshore migration of the nourishment. The positive-negative-positive profile for
 376 stormy waves (initially positive on the extreme seaward side, where skewness
 377 out-competes mean current) leads to a generally offshore propagation.

Figure 10: Sensitivity of perturbed sediment flux to offshore waves height at mean tide ($\eta = 0$ m), showing: (a), current driven sediment flux perturbation (q'_c); (b), wave skewness driven sediment flux perturbation (q'_{sk}); (c), wave asymmetry driven sediment flux perturbation (q'_{as}); and (d), total perturbation q' . The wave period and incident angle are $T = 7$ s and $\theta = 11^\circ$. The zero contour line is highlighted with the black dashed line. The yellow contour lines show the wave energy dissipation ($\mathcal{D}_w$). The red vertical dashed lines indicate the shoreward and seaward end of initial nourishment. The horizontal dashed lines refer to mild ($H_{rms} = 0.5$ m), moderate ($H_{rms} = 1.1$ m) and stormy wave heights ($H_{rms} = 2.5$ m).

Figure 11: Sensitivity of perturbed sediment flux to offshore wave incidence angle at mean tide ($\eta = 0$ m), showing: (a), q'_c ; (b), q'_{sk} ; (c), q'_{as} ; and (d), q' . The wave period and incident angle are $H_{rms} = 1.1$ m and $T = 7$ s. The zero contour line is highlighted with the thick black dashed line. The horizontal black dashed line corresponds to the representative wave incidence $\theta = 11^\circ$. Red vertical dashed lines indicate the start and end position of initial nourishment. The yellow contour lines show the wave energy dissipation ($\mathcal{D}_w$).

378 4.4.2. Sensitivity to wave incident angle and period

379 We now hold $H_{rms} = 1.1$ m constant and vary θ and then T . The effect
 380 of θ on q' is presented in Fig. 11. For $\theta = 11^\circ$ (dashed line in Fig. 11) we
 381 see the results corresponding to those for $H_{rms} = 1.1$ m in Fig. 10. Note that
 382 the smaller colour bar range reveals the finer detail. As might be expected the
 383 effect mainly apparent is a reduction in the positive q' for larger angles, due
 384 to increased refraction yielding smaller (shoaling) wave heights, and therefore
 385 decreased skewness and onshore mean flow.

386 In Fig. 12 we show the sensitivity to wave period. For $T \approx 9$ s a peak in q'
 387 can be observed. This is due primarily to the corresponding peak in q'_{sk} . For
 388 this period (for this wave height) the nourishment can therefore be considered
 389 optimally active, yielding maximum enhanced onshore sediment movement and
 390 a general onshore migration of the nourishment. Note that a corresponding peak
 391 can be seen in Fig. 10(d), in which T is fixed and the "optimal" $H_{rms} \approx 1.4$ m.
 392 For longer periods asymmetry grows and skewness diminishes, as does mean
 393 current, and waves begin to break on the nourishment (≈ 1300 m), and also
 394 on the middle bar (≈ 1600 m). Therefore, for a broad range of periods and
 395 directions at moderate wave heights the nourishment is active and feeding sand
 396 onshore.

397 4.4.3. Sensitivity to mean water level (tide)

398 The evolution of the nourishment is also affected by tide. Here, the effect
 399 of tide is accounted for by shifting the water surface (thus changing total water
 400 depth). At Terschelling, tides are semi-diurnal and meso-tidal. Tidal ranges are
 401 about 1.2 m (Neap) and 2.8 m (Spring), respectively.

402 For $H_{rms} = 1.1$ m the sensitivity of q' to tide is shown in Fig. 13. Onshore
 403 sediment flux ($q' > 0$) is induced for a wide range of water levels (Fig. 13(d)).
 404 The dominant processes are skewness-driven and mean q' , which mostly rein-
 405 force each other, but which on lower tides are opposed, because of wave breaking
 406 on low tides under only moderate wave heights.

407 4.5. Nourishment evolution

408 Up till now, we have examined the linear kinematics of the Terschelling nour-
 409 ishment (Fig. 7), the perturbed finite amplitude sediment dynamics (Fig. 8–9),
 410 and the sensitivity of those dynamics to the wave and tide conditions (Fig. 10–
 411 13). In these we used the initial nourishment (Nov. 1993) to examine its re-
 412 sponse. We now start from that initial condition, and allow it to evolve over

Figure 12: Sensitivity of perturbed sediment flux to wave period at mean tide ($\eta = 0$ m), showing: (a), q'_c ; (b), q'_{sk} ; (c), q'_{as} ; and (d), q' . The wave period and incident angle are kept as $H_{rms} = 1.1$ m and $\theta = 11^\circ$. The zero contour line is highlighted with the thick black dashed line. The horizontal black dashed line corresponds to representative wave period $T = 7$ s. Red vertical dashed lines indicate the shoreward and seaward end of initial nourishment. The yellow contour lines show the wave energy dissipation ($\mathcal{D}_w$).

Figure 13: Sensitivity of perturbed sediment flux to tide elevation, showing: (a), q'_c ; (b), q'_{sk} ; (c), q'_{as} ; and (d), q' . The wave period, incident angle and period are kept as $H_{rms} = 1.1$ m, $\theta = 11^\circ$ and $T = 7$ s. The zero contour line is highlighted with the thick black dashed line. The horizontal black dashed line corresponds to mean tide. Red vertical dashed lines indicate the shoreward and seaward end of initial nourishment. The yellow contour lines show the wave energy dissipation ($\mathcal{D}_w$).

413 ~ 6 months of hourly observed hydrodynamical data (Fig. 5). The resulting
 414 evolution of the nourishment is shown in Fig. 14.

415 Fig. 14(a) shows the variation of q' in time and space in the vicinity of
 416 the initial nourishment location. Initially (Nov 1993) non-zero q' is limited to
 417 the region of the initial nourishment. Gradually the region of $|q'| > 0$ extends
 418 shoreward (there is very little deviation in q' seaward of the initial nourishment,
 419 which is therefore not shown). q' is mostly positive (i.e. onshore sediment
 420 transport due to the nourishment) for most of the six months in the initial
 421 nourishment region; any negative durations correspond to larger wave heights,
 422 consistent with the sensitivity study. The only prominent negative q' regions
 423 occur in the trough between the middle and inner bar—therefore shoreward of the
 424 initial nourishment—during energetic wave conditions. This is due to increased
 425 wave energy dissipation on the nourishment leading to less wave energy being
 426 available for onshore sediment movement in the trough, resulting in relative
 427 offshore sediment flux. Over the last two months, when wave heights diminish,
 428 we can see almost entirely onshore perturbed sediment flux. During the last two
 429 months the most intense perturbed sediment dynamics gradually shifts over into
 430 the region shoreward of the initial nourishment region, indicative of nourishment
 431 migration.

432 In Fig. 14(b) we plot the sediment flux divergence. Because differentiation
 433 is an un-smoothing operation this plot shows more small scale fluctuations, but
 434 the overall picture is of positive divergence (erosion) in the initial nourishment
 435 region, and negative divergence (deposition) in the region immediately shore-
 436 ward. This also indicates a migration of the nourishment shoreward. Although
 437 sediment diffusion is excluded from this picture, each q' has been influenced by
 438 diffusion previously, so this effect is present too.

439 All physical processes are included in Fig. 14(c), which shows the evolution
 440 of the perturbation. We can see its onshore movement and relatively static
 441 seaward boundary, along with a gradual diffusion.

In Fig. 15 we plot the path of the centre of mass ($x_M(t)$) of the simulated
 nourishment (black line), where:

$$x_M(t) = \frac{\int_0^{2400\text{m}} b(x, t) x \, dx}{\int_0^{2400\text{m}} b(x, t) \, dx} \quad (19)$$

(following *Roos et al.*, 2008, who examined a pit). Although there are fluctu-
 ations in the migration rate, a fairly steady onshore average migration can be

Figure 14: The evolution of: (a), sediment transport perturbation (q'); (b), cross-shore gradient of q' ($\frac{\partial q'}{\partial x}$); (c), nourishment (b); and (d), offshore wave heights (H_{rms}). The vertical thick black dashed lines refer to the seaward and shoreward end of the initial nourishment. The horizontal dashed lines in (c) indicate the time of sea bed observations.

observed (black line). There are two notable events where offshore migration can clearly be seen: January 10 and March 27. These days are notable for (January 10) very high wave height ($H_{rms} \approx 4.9$ m) and long period ($T_p \approx 15.5$ s) waves; and (March 27) fairly high wave height ($H_{rms} \approx 3.2$ m) and quite long period ($T_p \approx 11.5$ s) waves (see Fig. 5 or Fig. 14(d)). The mean migration rate

$$\overline{c_M} = \frac{1}{\tau} \int_{t_0}^{t_0+\tau} \frac{d}{dt} x_M(t) dt \approx 7.02 \text{m/month} \approx 0.234 \text{m/day} \quad (20)$$

Figure 15: The migration of the nourishment as measured by $x_M(t)$ (black line), the centre of mass of the simulated nourishment; and $x_n(t)$ (blue line), the path calculated using the spatially averaged linear migration rate, $c_n(t)$. Crosses denote the observed centre of mass of the nourishment, $x_{M,obs}$, found using the pre-nourishment profile as a reference bathymetry, at the two times at which subsequent measured cross-shore bathymetries exist. For the final $x_{M,obs}$ value the calculation is as in Eq. (19), but for the intermediate value the integration is to 1900 m only.

where τ is the duration of the simulation, and t_0 corresponds to the start of the simulation showed in Fig. 14, i.e. Nov. 1993. We can compare this to the average linear c_n from Fig. 7. If we restrict the average to the initial nourishment region, and time average according to observed H_{rms} , we get

$$\overline{c_n} = \frac{1}{\tau} \int_{t_0}^{t_0+\tau} c_n(t) dt = \frac{1}{\tau} \int_{t_0}^{t_0+\tau} \int_{1140m}^{1600m} c_n(x, H_{rms}(t)) dx dt \approx 0.272m/day. \quad (21)$$

This value is fairly similar to $\overline{c_M}$. In Fig. 15 (blue line) we plot the corresponding nourishment position using $c_n(t)$:

$$x_n(t) = x_M(t_0) + \int_{t_0}^{t_0+t} c_n(t') dt'. \quad (22)$$

442 It is immediately apparent that $x_n(t)$ is a very good approximation to $x_M(t)$ up
 443 until the first aforementioned offshore propagation event. Thereafter the two
 444 positions diverge to some extent. Prior to the January 10th event the x_n and
 445 x_M evolutions are very close, implying little overall bed change has taken place.
 446 After the event the details of the two time developments no longer correspond
 447 so closely.

448 It is instructive to examine this a little further. In Fig. 16 we show the
 449 effect of storms and calm periods during the simulation (a storm being defined
 450 here as an event for which $H_{rms} > 2$ m continuously for at least 3 hours). In

Figure 16: (a) Total wave energy entering the initial nourishment region during a storm (black crosses): $\int_{t_b}^{t_e} E c_g \cos \theta(x = 1140\text{m}, t) dt$, where t_b (t_e) is time when H_{rms} first exceeds (subsequently falls below) the threshold; and during a calm period (red crosses, t_b and t_e are reversed). (b) Volume of bed change per m alongshore in the initial nourishment region due to a storm (black asterisks): $\int_{1140\text{m}}^{1600\text{m}} |b(x, t_e) - b(x, t_b)| dx$; and during a calm period (red asterisks, t_b and t_e are reversed). A storm is defined here as an event for which $H_{rms} > 2$ m continuously for at least 3 hours, and calm periods are the intervals in between the storms).

451 Fig. 16(a) the energy entering the initial nourishment region during each storm
 452 or calm duration is plotted. Unsurprisingly, largest energy inputs are during
 453 the extended calm periods (low energy densities for long durations). The next
 454 largest inputs are the large storms, the two largest (January 10 and March 27),
 455 corresponding to the aforementioned offshore migrating x_M events, possessing
 456 the largest energies. In Fig. 16(b) we show the corresponding net bed change
 457 (absolute value), and there is a rough correspondence between large total energy
 458 inputs and net bed change. If we look back at Fig. 15 we see that it is during
 459 the storm events that the nonlinear migration rate (as indicated by $x_M(t)$)
 460 departs from the linear, small amplitude one (as indicated by $x_n(t)$), typically
 461 in an offshore movement of x_M . This offshore movement of x_M is consistent
 462 with offshore sediment transport and migration for higher wave energy density.
 463 However, the $\bar{c}_n(t)$ depends only on the small amplitude $c_n(x, t)$ in the initial
 464 nourishment region, and regardless of H_{rms} , this is likely to remain positive (see
 465 Fig. 7), hence the divergence during these events.

466 In Fig. 17 we can see the simulated bed change (dashed lines) which are
 467 compared with field observations (solid lines). The most general point to note
 468 is that sediment is eroded on the seaward side of the nourishment and deposited
 469 onto the middle bar and locations further shoreward, consistent with Fig. 14,
 470 and that this is also what is observed in the field. Closer inspection reveals re-
 471 gions of lesser and greater discrepancy. Seaward of the initial nourishment both
 472 observations and simulations reveal little change, and in the offshore portion of
 473 the initial nourishment there is similarly reasonably close correspondence be-
 474 tween simulations and observations. Some significant discrepancies emerge in
 475 the vicinity of the middle bar. Although the simulated peaks do grow, as ob-
 476 served, at both Dec 93 and May 94 the simulated peak remains almost at the
 477 original middle bar location, whereas the observed one migrates. The filling in
 478 of the trough between middle and inner bars is reproduced by the model, but
 479 it is slower than in reality. Finally, on the seaward side of the inner bar erosion
 480 is evident by May 94, which is mostly not reproduced by the model. Note that
 481 the observed Dec 93 profile is curtailed at $x \approx 1900$ m, because shoreward of
 482 that the data is unreliable.

483 The model skill is evaluated quantitatively through a "Brier skill score" type
 484 parameter (*van Rijn et al.*, 2003; *Fernández-Mora et al.*, 2015) which is defined
 485 as:

$$486 \quad S = 1 - \frac{\int_{x_1}^{x_2} (z_{b,sim}(x) - z_{b,obs})^2 dx}{\int_{x_1}^{x_2} (z_{b,obs}(x) - z_{b,0})^2 dx}, \quad (23)$$

Figure 17: Comparison of the simulated and observed bed levels for the Terschelling nourishment, during (Dec 93) and at the end of (May 94) the simulation. (a) background profile + nourishment. (b) nourishment only (in the case of the observations, the initial background profile is subtracted).

where $z_{b,sim}$ and $z_{b,obs}$ are simulated and observed bed level in May 1994. $x_1 = 1000$ m and $x_2 = 2000$ m. $S = 1.0$ indicates a perfect match between simulation and observation. A skill of $S = 0.52$ is achieved by this model.

Finally, in Fig. 15 we also plot the centre of mass of the observed morphological evolution, with respect to the Nov 93 pre-nourishment profile. There are only two estimates, based on the two measured profiles. We see a significantly faster onshore migration based on the Dec 93 observation than that simulated or based on the linear kinematics, but similar observed and simulated migration by May 94.

5. Discussion

5.1. Linear and nonlinear kinematics

The migration rates $c_n(t)$ and $c_M(t)$ are apparently closely related. This implies that for relatively low energy conditions, which predominate over extended durations, we may use linear theory to accurately predict an average location for a shoreface nourishment without simulating the nourishment evolution itself. The results shown here are merely for an average $\bar{c}_n$ calculated over the extent of the initial nourishment. Since c_n can be calculated for the whole profile the approach is extendable if the nourishment extent is similarly modified (once again using $c_n(x, t)$). Storm conditions, however, are likely to induce a qualitative change, because the presence of the nourishment transforms wave conditions during the storm, which is what we observe. However, it is straightforward to recalculate c_n with a proposed nourishment in place, and thereby arrive at an estimate that is valid for storm waves, until the nourishment propagates away from the initial placement region.

Linked to this is the underlying change in the original bathymetry. A large enough change will render the NaaP approach invalid. Observations of transects from Terschelling seem to point to the pre-nourishment cross-shore profile being recovered after 6 years (*Grunnet and Ruessink*, 2005). This implies that the background profile is persistent, notwithstanding Summer / Winter variations and / or oblique bar migration. We also use the second moment of the nourishment to estimate the diffusion coefficient as

$$\gamma_{M,obs} \approx \frac{\int \Delta b(x - x_M)^2 dx}{2\Delta t \int b dx} \quad (24)$$

511 We obtain $\gamma_{M,obs} \approx 1.8 \times 10^{-3} m^2/s$ for the change from the start of the nour-
 512 ishment to Dec 93. This result gives an indication that our chosen γ is smaller
 513 than that observed. However, other numerical experiments considering a range
 514 from 3.5×10^{-4} to $8 \times 10^{-4} m^2/s$ (not shown) indicate little difference to results
 515 in the upper range of these values, which implies that using the larger value
 516 here probably would not make a significant difference to the simulations. From
 517 Dec 93 to May 94, in contrast, we get -2.0×10^{-3} . This anti-dissipative result
 518 is perhaps not surprising considering the change in the observed profile in be-
 519 tween these dates, and seems to provide evidence that the background profile
 520 has evolved by this date, whether in a purely 1D sense, or in 2D (see also § 5.2.)
 521 We also note that, although further work is required to evaluate for what du-
 522 ration the approach can remain viable in general, the deviation of c_n and c_M
 523 would seem to be a necessary condition.

524 The estimates of $x_{M,obs}$ of Fig. 15 show close (May 94) and poor (Dec 93)
 525 correspondence with the model results (both x_M and x_n). It should be noted
 526 that the profile of Dec 93 is only used up to 1900m. This is because shoreward
 527 of this point the profile exhibits a straight line segment, which seems likely to
 528 correspond to missing data. Including this section moves $x_{M,obs}$ offshore of its
 529 initial value, which seems to support the decision not to include this section.
 530 Excluding from the calculation of $x_{M,obs}$ for May 1994 the portion of the domain
 531 $x > 1900$ m also yields a larger $x_{M,obs}$ value, consistent with the over-predicted
 532 value of $x_{M,obs}$ observed for December 1993 in Fig. 15.

533 5.2. Model limitations

534 As noted earlier, we seek to capture the nourishment morphodynamics rather
 535 than attempt a model validation. Nonetheless, it is instructive to discuss the
 536 possible reasons for observed model-data discrepancies.

537 We do not consider here the effects of wave set-up. This effect is likely to be
 538 limited in the cross-shore, because we consider a shoreface nourishment and so
 539 exclude the typical shoreline maximum in set-up. There are other mechanisms
 540 for onshore sediment transport that we do not consider. The only two that
 541 could have an impact for the Terschelling case are sediment mobilisation by
 542 the injection of breaking-induced turbulence (e.g. Zhou *et al.*, 2017) and phase-
 543 related effects (e.g. Ruessink *et al.*, 2009). The significance of these cannot
 544 be discarded, but the effects are not likely to dominate those that are included.
 545 Setting $u_{cr} = 0$ will also have an effect. But we note that if we use the expression
 546 for u_{cr} advocated by Soulsby (1997) this threshold is exceeded for about 91%

of all points in the model (x, t) domain, indicating that setting $u_{cr} = 0$ is a reasonable approximation.

The down-slope effect of gravity on sediment is accounted for through a linear diffusion term with a constant coefficient. The presence of wave stirring of different intensities (see e.g. *Larson and Hanson, 2015*) at different points along the cross-shore transect means that this is likely to be an oversimplification. Increased diffusion, particularly in regions of wave breaking, would be likely to result in some of the missing infilling of the inner trough and a reduction in the middle bar elevation, both more consistent with observation. But agreement in the outer trough would seem likely to be worse.

This model study also does not consider the effect of the difference in grain size between the nourishment and the native sand (*Guillen and Hoekstra, 1997*). This is because only the bed-load component of the Soulsby-van Rijn formula is included, and thresholds of movement omitted.

The assumption probably most likely to have an impact on the model-data comparison is that of alongshore uniformity. According to *Spanhoff et al. (1997)*, the yearly averaged longshore transport rate in the nearshore zone of Terschelling is about $1 \text{ Mm}^3/\text{year}$, so, although the site is chosen because of its quasi-alongshore-uniform geometry (see Fig. 3) it would, nonetheless, not take a large alongshore divergence in radiation stresses to generate alongshore sediment flux gradients. *van Duin et al. (2004)* also observed bar straightening post-nourishment at Egmond-aan-Zee.

By taking the difference between pre- and post-nourished sea bed elevation and integrating over the cross-shore profile, we can examine the change in sediment volume (per m alongshore) over time. Immediately after the nourishment, this volume is $532.6 \text{ m}^3/\text{m}$ in excess of the pre-nourishment profile of October 1993. By December 1993, this value is reduced to $474.8 \text{ m}^3/\text{m}$. By May 1994, it has dropped to $320.4 \text{ m}^3/\text{m}$. Because the simulation is limited to 6 months it seems likely that a significant proportion of this loss is due to alongshore sediment flux gradients. Moreover, most of the difference between modelled and observed profiles constitutes an over-prediction of bed level, which is consistent with this explanation.

6. Conclusion

The inclusion of wave streaming and depth-varying mean return flow into bed evolution equation yields physically consistent migration speeds for small

582 bed perturbations: onshore in the shoaling zone and offshore where breaking
 583 occurs. Onshore migration is fastest where waves feel the bottom most promi-
 584 nently, prior to breaking. The equivalent small amplitude migration rates for
 585 the initial nourishment region of the Terschelling profile therefore shown on-
 586 shore migrations in the trough region and offshore migrations on the bars that
 587 delineate the initial nourishment. Using the resulting 1D (cross-shore) nonlinear
 588 nourishment-as-a-perturbation (NaaP) concept, also including arbitrary angle
 589 of incidence and field validated energy dissipation description, sensitivity studies
 590 are carried on for the Terschelling nourished profile. It is found that wave height
 591 is the primary controlling parameter for the resultant perturbed sediment flux.
 592 Perturbed sediment fluxes are onshore up to a wave height threshold of about
 593 1.5 m, beyond which offshore perturbed sediment fluxes predominate. Wave
 594 period, tide level and wave angle are all secondary effects, variation in which
 595 yields no change in perturbed sediment flux direction for fixed wave height.
 596 Wave period is the most significant effect of these.

597 The first six months of the Terschelling nourishment evolution are repro-
 598 duced by the fully nonlinear NaaP model. Recorded wave and tide signals are
 599 used to drive the model. It is shown that in calmer conditions a mean small
 600 amplitude migration rate of bed disturbances is a highly accurate predictor of
 601 the fully nonlinear migration rate (as described by the centre of mass of the
 602 nourishment). In storm periods the two predictors diverge, but still remain rea-
 603 sonably close. The reason for this is that the introduction of the nourishment
 604 yields wave breaking over the nourishment (formerly the trough) during storm
 605 conditions, whereas storm waves break on the bars in the original profile. The
 606 observed overall migration rate, again measured by the centre of mass of the
 607 nourishment, is well described by the fully nonlinear model. The correspon-
 608 dence between linear small amplitude and nonlinear migration rates offers the
 609 possibility of low-cost accurate predictions of short-to-medium term nourish-
 610 ment evolution.

611 The modelled profile evolution shows a moderately good reproduction of
 612 the observed one (Brier Skill Score = 0.52). The inclusion of the streaming
 613 and the more accurate description of the mean flow in the model improved the
 614 Brier Skill Score from 0.46 to 0.52. Discrepancies between the simulated and
 615 observed results are thought primarily to result from the 1D nature of the model.
 616 Significant 2D effects are observed at the site, albeit most notably over a longer
 617 timescale (*Grunnet and Ruessink*, 2005). Loss of mass in the cross-shore profile
 618 seems to corroborate this.

619 Acknowledgements

620 The support of the UK Engineering and Physical Sciences Research Council
621 (EPSRC) under the MORPHINE project (grant EP/N007379/1) and of the
622 University of Nottingham is gratefully acknowledged. We are grateful to prof.
623 dr. Gerben Ruessink for providing the Terschelling data. We are also grateful
624 to the suggestions of two anonymous reviewers, particularly with regard to the
625 suggestion to look at the second moment.

626 References

- 627 Baldock, T., P. Holmes, S. Bunker, and P. V. Weert, Cross-shore hydrodynamics
628 within an unsaturated surf zone, *Coast. Eng.*, *34*(3), 173 – 196, doi:10.1016/
629 S0378-3839(98)00017-9, 1998.
- 630 Battjes, J., and J. Janssen, Energy loss and set-up due to breaking of random
631 waves, *Coast. Eng. Proc.*, *1*(16), 32, doi:10.9753/icce.v16.32, 1978.
- 632 Capobianco, M., H. Hanson, M. Larson, H. Steetzel, M. J. F. Stive, Y. Chatelus,
633 S. Aarninkhof, and T. Karambas, Nourishment design and evaluation: ap-
634 plicability of model concepts, *Coast. Eng.*, *47*(2), 113 – 135, doi:10.1016/
635 S0378-3839(02)00123-0, 2002.
- 636 Chen, W., and N. Dodd, An idealised study for the evolution of a shoreface
637 nourishment, *Cont. Shelf Res.*, *178*, 15 – 26, doi:10.1016/j.csr.2019.03.010,
638 2019.
- 639 De Swart, H. E., and D. Calvete, Non-linear response of shoreface-connected
640 sand ridges to interventions, *Ocean Dynamics*, *53*(3), 270–277, doi:10.1007/
641 S10236-003-0044-9, 2003.
- 642 Dean, R. G., and R. A. Dalrymple, *Water Wave Mechanics for Engineers and*
643 *Scientists*, WORLD SCIENTIFIC, doi:10.1142/1232, 1991.
- 644 Fernández-Mora, A., D. Calvete, A. Falqués, and H. E. de Swart, Onshore
645 sandbar migration in the surf zone: New insights into the wave-induced
646 sediment transport mechanisms, *Geophys. Res. Let.*, *42*(8), 2869–2877, doi:
647 10.1002/2014GL063004, 2015.
- 648 Fuhrman, D. R., J. Fredsoe, and B. M. Sumer, Bed slope effects on turbulent
649 wave boundary layers: 2. comparison with skewness, asymmetry, and other
650 effects, *J. Geophys. Res.*, *114*, C03,025, 2009.

- 651 Grunnet, N. M., and B. Ruessink, Morphodynamic response of nearshore bars
652 to a shoreface nourishment, *Coast. Eng.*, *52*(2), 119 – 137, doi:10.1016/j.
653 coastaleng.2004.09.006, 2005.
- 654 Grunnet, N. M., D.-J. R. Walstra, and B. Ruessink, Process-based modelling
655 of a shoreface nourishment, *Coast. Eng.*, *51*(7), 581 – 607, doi:10.1016/j.
656 coastaleng.2004.07.016, 2004.
- 657 Grunnet, N. M., B. G. Ruessink, and D. Walstra, The influence of tides, wind
658 and waves on the redistribution of nourished sediment at terschelling, *Coastal*
659 *Eng.*, *52*, 617–631, 2005.
- 660 Guillen, J., and P. Hoekstra, Sediment distribution in the nearshore zone: grain
661 size evolution in response to shoreface nourishment (Island of Terschelling,
662 The Netherlands, *Estuarine, Coastal and Shelf Science*, *45*, 639–652, 1997.
- 663 Hamm, L., M. Capobianco, H. H. Dette, A. Lechuga, R. Spanhoff, and M. J. F.
664 Stive, A summary of european experience with shore nourishment, *Coast.*
665 *Eng.*, *47*(2), 237 – 264, doi:10.1016/S0378-3839(02)00127-8, 2002.
- 666 Hillen, R., and P. Roelse, Dynamic preservation of the coastline in the nether-
667 lands, *J. Coast. Conserv.*, *1*(1), 17–28, doi:10.2307/25098187, 1995.
- 668 Hoekstra, P., K. T. Houwman, A. kroon, P. V. Vessem, and B. G. Ruessink,
669 The NOUTEC experiment of Terschelling: process-oriented monitoring of a
670 shoreface nourishment (1993-1996), in *Proc. 1st Int. Conf. on Coastal Dy-*
671 *namics*, edited by A. Arcilla, M. J. F. Stive, and N. C. Kraus, pp. 402–416,
672 ASCE, 1994.
- 673 Kranenburg, W. M., J. S. Ribberink, R. E. Uittenbogaard, and S. J. M. H.
674 Hulscher, Net currents in the wave bottom boundary layer: On waveshape
675 streaming and progressive wave streaming, *J. Geophys. Res. : Earth Surf.*,
676 *117*(F3), doi:10.1029/2011JF002070, 2012.
- 677 Larson, M., and H. Hanson, Model of the evolution of mounds placed in the
678 nearshore, *Journal of Integrated coastal zone management*, *15*(1), 21 – 33,
679 doi:10.5894/rgci530, 2015.
- 680 Longuet-Higgins, M. S., Mass transport in water waves., *Trans. Royal Soc.*
681 *London Ser. A.*, *245*, 535–581, 1953.

Longuet-Higgins, M. S., The mechanics of the boundary-layer near the bottom in a progressive wave. appendix to Russell, R.C.H. and Osorio, J.D.C.: An experiment investigation of drift profiles in a closed channel., in *Proc. 6th Int. Conf. Coast. Eng. Miami, FL.*, pp. 184–193, 1957.

Phillips, O. M., *The Dynamics of the Upper Ocean*, second ed., Cambridge Univ. Press, Cambridge, 1977.

Roelvink, D., and A. Reniers, *A Guide to Modeling Coastal Morphology*, WORLD SCIENTIFIC, doi:10.1142/7712, 2011.

Roos, P. C., and S. J. M. H. Hulscher, Large-scale seabed dynamics in off-shore morphology: Modeling human intervention, *Rev. Geophys.*, *41*(2), 1010, doi:10.1029/2002RG000120, 2003.

Roos, P. C., S. J. M. H. Hulscher, M. A. F. Knaapen, and R. M. J. van Damme, The cross-sectional shape of tidal sand banks: modeling and observations, *J. Geophys. Res.*, *F02003*, 2004.

Roos, P. C., S. J. M. H. Hulscher, and H. J. de Vriend, Modelling the morphodynamic impact of offshore sandpit geometries, *Coastal Eng.*, *55*, 704–715, 2008.

Ruessink, B., D. Walstra, and H. Southgate, Calibration and verification of a parametric wave model on barred beaches, *Coast. Eng.*, *48*(3), 139 – 149, doi:10.1016/S0378-3839(03)00023-1, 2003.

Ruessink, B. G., T. J. J. van den Berg, and L. C. van Rijn, Modeling sediment transport beneath skewed asymmetric waves above a plane bed, *J. Geophys. Res.*, *114*(C11021), doi:10.1029/2009JC005416, 2009.

Ruessink, B. G., G. Ramaekers, and L. C. van Rijn, On the parameterization of the free-stream non-linear wave orbital motion in nearshore morphodynamic models, *Coast. Eng.*, *65*, 56 – 63, doi:10.1016/j.coastaleng.2012.03.006, 2012.

Soulsby, R., *Dynamics of marine sands, a manual for practical applications*, 249 pp., Thomas Telford, 1997.

Spanhoff, R., E. J. Biegel, J. V. Graaff, and P. Hoekstra, Shoreface Nourishment at Terschelling, The Netherlands: Feeder berm or breaker berm?, in *Proc. 3rd Int. Conf. on Coastal Dynamics*, edited by E. B. Thornton, pp. 863–872, ASCE, 1997.

- 714 Süli, E., and D. F. Mayers, *An Introduction to Numerical Analysis*, Cambridge
715 University Press, doi:10.1017/CBO9780511801181, 2003.
- 716 Svendsen, I. A., *Introduction to nearshore hydrodynamics*, 722 pp., World Sci-
717 entific, 2006.
- 718 van Duin, M. J. P., N. R. Wiersma, D. J. R. Walstra, L. C. van Rijn, and
719 M. J. F. Stive, Nourishing the shoreface: observations and hindcasting of the
720 Egmond case, The Netherlands, *Coast. Eng.*, *51*(8), 813 – 837, doi:10.1016/
721 j.coastaleng.2004.07.011, coastal Morphodynamic Modeling, 2004.
- 722 Van Leeuwen, S. M., N. Dodd, D. Calvete, and A. Falqués, Physics of nearshore
723 bed pattern formation under regular or random waves, *J. Geophys. Res.*,
724 *111*(F01023), doi:10.1029/2005JF000,360, 2006.
- 725 van Leeuwen, S. M., N. Dodd, D. Calvete, and A. Falqués, Linear evolution
726 of a shoreface nourishment, *Coast. Eng.*, *54*(5), 417 – 431, doi:10.1016/j.
727 coastaleng.2006.11.006, 2007.
- 728 van Rijn, L., D. Walstra, B. Grasmeijer, J. Sutherland, S. Pan, and J. Sierra,
729 The predictability of cross-shore bed evolution of sandy beaches at the time
730 scale of storms and seasons using process-based profile models, *Coast. Eng.*,
731 *47*(3), 295 – 327, doi:10.1016/S0378-3839(02)00120-5, 2003.
- 732 van Rijn, L. C., J. S. Ribberink, J. van der Werf, and D. J. R. Walstra, Coastal
733 sediment dynamics: recent advances and future research needs, *Journal of*
734 *Hydraulic Research*, *51*(5), 475–493, 2013.
- 735 Zhou, Z., T.-J. Hsu, D. Cox, and X. Liu, Large-eddy simulation of wave-breaking
736 induced turbulent coherent structures and suspended sediment transport on
737 a barred beach, *J. Geophys. Res.*, *122*, 207–235, 2017.

738 **Appendix A. wave driven current profile**

Following *Roelvink and Reniers* (2011), we assume a linear distribution of cross-shore shear stress ($\tau_x(z)$) in vertical direction (z , positive upward),

$$\tau_x(z) = \tau_{sx} + \frac{\tau_{sx} - \tau_{bx}}{hz}, \quad (\text{A.1})$$

where τ_{sx} and τ_{bx} are the surface and bottom shear stress, forced by roller dissipation, which is given by:

$$\tau_{sx} = \frac{\mathcal{D}_r}{c} \cos \theta. \quad (\text{A.2})$$

The velocity and shear stress satisfy

$$\mu_v \frac{\partial u_E}{\partial z} = \frac{\tau_x}{\rho}, \quad (\text{A.3})$$

where u_E is the Eulerian velocity. The vertically eddy viscosity μ_v has a parabolic profile

$$\mu_v = -k_a v_* z \frac{z+h}{h}, \quad (\text{A.4})$$

with k_a being the von Karman constant. v_* is the shear velocity, consisting of a constant part and a breaking dependent part, i.e., $v_* = \frac{\mu}{Z_0 k_a} + \sqrt{\frac{\tau_{sx}}{\rho}}$ with $\mu = 0.025 \text{ m}^2 \text{ s}^{-1}$ being the constant viscosity of water and $Z_0 = 1 \text{ m}$ a vertical length scale. Therefore, u_E can be written as

$$u_E = -\frac{1}{\rho k_a v_*} \left(\tau_{sx} \ln \frac{z}{z_0 - h} - \tau_{bx} \ln \frac{h+z}{z_0} \right), \quad (\text{A.5})$$

739 using the bed boundary condition, i.e., $u_E = 0$ at $z = -h + z_0$ (z_0 is the bed
740 roughness).

The vertical integrated current should balance the onshore flux ($u_{onshore} = \frac{E_w + 2E_r}{\rho h c}$) due to stokes drift and roller. This leads to

$$-\frac{1}{\rho k_a v_*} \left(\tau_{bx} \left(1 - \ln \frac{h}{z_0} \right) - \tau_{sx} \right) + u_{onshore} = 0, \quad (\text{A.6})$$

and furthermore,

$$\tau_{bx} = \frac{\tau_{sx} + \rho k_a v_* u_{onshore}}{1 - \ln(h/z_0)}. \quad (\text{A.7})$$

741 Now we can substitute the expression of bottom shear stress into Eq. (A.5) to
742 obtain the vertical profile of u_E . The u_L is then simply the superposition of u_E
743 and u_{stokes} . The flow velocity evaluated at 0.1 m above the bed is used for the
744 calculation of q_c . As shown in Fig. A.18, the near bed current calculated with
745 Eq. (13) has a positive value in the shoaling zone and is negative in the surf
746 zone.

Figure A.18: Vertical distribution of u_L (black solid curve) at: (a), $x = 1100$ m, shoal zone; and (b), $x = 1970$ m, surf zone. Wave of representative characteristics (as in §4.4) and basic sea bed profile are used to obtain this plot. The vertical blue solid line represents the depth averaged return flow (calculated with Eq.15 in *Chen and Dodd* (2019)). The red circle denotes the evaluated near bed current for the calculation of q_c .