Ambiguity and Accident Law*

Surajeet Chakravarty
Department of Economics
University of Exeter
Exeter, UK
S.Chakravarty@exeter.ac.uk

David Kelsey
Department of Economics
University of Exeter
Exeter, UK
D.Kelsey@exeter.ac.uk

11th August 2014.

Abstract

Environmental accidents often involve ambiguous risks, i.e. the relevant probabilities are unknown. This paper studies how liability rules are affected by ambiguity. The injurer and the victim choose a level of care, which is observable, and an unobservable action. Both actions may affect the size and/or likelihood of loss. We analyse the welfare implications of tort rules. First, we show that with ambiguity, negligence is likely to perform better than strict liability. Second, we propose a tort rule of negligence and punitive damages, which gives the efficient levels of both care and unobserved actions.

Address for Correspondence Surajeet Chakravarty, Department of Economics,
University of Exeter, Rennes Drive, Exeter EX4 4PU, UK.
e-mail, S.Chakravarty@exeter.ac.uk.

Keywords Tort, punitive damages, ambiguity, accident law, liability, environmental damage.

JEL Classification: D81, K13.

*Research supported in part by a Leverhulme Research Fellowship. We would like to thank Jayant Ganguli, Tigran Melkonyan, Joshua Teitelbaum, seminar audiences at Universities of Birmingham, Exeter, Liverpool, Warwick Business School, FUR, Newcastle, 2010, Royal Economic Society Conference, Cambridge, 2012, ESEM Malaga 2012, SAET Paris 2013, the referees and editor of this journal for helpful comments.

1 Introduction

1.1 Background

Many environmental disasters are the result of accidents, for instance the Union Carbide gas leak at Bhopal, the Exxon Valdez oil spill or the British Petroleum (BP) Deepwater Horizon oil spill. Such events are governed broadly by law of tort.¹ The aim of tort or liability law is to give agents an incentive to internalize the externalities caused by their actions.² A significant part of the literature has focused on the efficiency implications of different tort rules, like negligence and strict liability. The present paper studies how tort law is affected by ambiguity.

Risks with unknown probabilities are called *ambiguous*. We believe that agents may often perceive accidents as ambiguous, since they may not have sufficient information or time to assign precise probabilities. This is especially true of major accidents likely to cause environmental damage, since they are almost always rare events. Hence there may not be enough observations to base subjective probabilities on relative frequencies. Agents involved are likely to have poor information about the probability of disaster, since such accidents often have unique circumstances. Complexity can also make it hard to assign probabilities. Moreover the chance of an accident often depends on the behaviour of other people, something which is intrinsically hard to predict. These issues seem to be relevant to the BP oil spill. There was little previous experience of drilling at these depths and it involved new and highly advanced technology. The operation involved a complex interaction between a number of companies. Even now responsibility for the accident has not been allocated between them. In the present paper we study the implications of ambiguity for tort.

Our motivation for this analysis comes from environmental accidents, however ambiguity may also be a factor in other cases. Examples include firms using new machinery or procedures and road accidents in case of drivers who do not have enough experience of driving in certain roads or conditions.

The law and economics literature usually models agents as subjective expected utility (SEU from now on) maximizers, Savage (1954). We relax this assumption by allowing agents to perceive ambiguity, which we argue is relevant for many environmental accidents.³ Since ambiguity may cause agents to behave differently, it is desirable that tort rules be robust. We show that, negligence is likely to perform better than strict liability in the presence of ambiguity.

Ellsberg (1961) argued that individuals avoid risks with unknown probabilities. This has been confirmed by the subsequent experimental literature, see Camerer and Weber (1992). In case of unawareness of the probabilities, individuals may respond pessimistically by over-weighting bad events or optimistically by over-weighting good events compared to an individual who followed SEU. We shall model ambiguity using neo-additive preferences (explained below), which are introduced and axiomatized in Chateauneuf, Eichberger, and Grant (2007), (henceforth CEG).

1.2 Liability and Ambiguity

The present paper studies accident law in the presence of ambiguity. As usual we assume that it is clear who is the injurer and who is the victim. We shall adopt the convention that male pronouns e.g. he, him etc. refer to the injurer and female pronouns she, her etc. refer to the victim.

¹Tort law decides the liability in case of private harm due to act of negligence or lack of duty of care by the injurer. Environmental accidents can cause both public and private harm. For the purpose of our analysis we are going to ignore the difference between private and public harm. Victims are assumed to suffer damage and we are going to model them as a single representative agent.

²Calabresi (1970), Calcott and Hutton (2006), Posner (2007) and Shavell (1980).

³Shuman (1993) argues that we may not be able to provide a complete analysis of tort rules with the standard assumptions that agents are rational and are fully aware of the probability of an accident.

Teitelbaum (2007) has previously studied the impact of ambiguity in the context of unilateral accidents. In this model, the injurer may reduce the loss by taking precaution but the victim has no influence on the likelihood or amount of damage. We begin by extending Teitelbaum's analysis to the bilateral accident model, in which both agents are able to choose an observable action and an unobservable action.⁴ The observable action may be the investment in care, for example, safety standards on the drilling rig and the unobservable action could be quality of cement used in drilling. A consequence of this is that the injurer and the victim are playing a strategic game. We use theory of the impact of ambiguity in games from Eichberger and Kelsey (2014).

In the first place we study the implications of given liability rules. Tort laws should provide incentives so that agents to take into account externalities which they cause. The tort regimes which have been analysed extensively over the last few decades have been strict liability, negligence and strict liability with contributory negligence as a defence. Strict liability is when the injurer is liable for the damages from the accident independent of the precaution by either party. Negligence requires the injurer to be liable for the damages when he fails to take the level of care stipulated by the court. In the present paper we shall focus on these two rules.⁵

For strict liability, we show that if injurers and victims are ambiguity-averse they will provide more than optimal care. This arises because ambiguity causes them to overweight bad outcomes, which in this case means having an accident. The injurer's perceived liability is increased, so he invests more in care.⁶ Under negligence, ambiguity aversion will give him a strong incentive to provide the stipulated level of care since this will protect him against ambiguity. In contrast, if the injurer is ambiguity-seeking then he will under-weight the event where an accident occurs and the cost of the damage. Thus he may provide too little care. However this will only happen if the degree of ambiguity-seeking is high. Under negligence there is a discontinuous drop in the injurer's pay-off when care falls below the stipulated level. In addition the victim will provide less care below this point, since she is no longer liable. Thus there is a large drop in the injurer's pay-off when care falls below the stipulated level. Hence he will continue to provide the stipulated level of care unless he is very optimistic.

1.3 Efficient Tort Rules

After this we extend the previous literature by analysing the optimal tort rule when the agents can take actions which are not observable by the court. We find that one can implement efficient outcomes, even when this is not possible without ambiguity. If the chance of an accident is only affected by actions which the court can observe then it can control externalities by stipulating the levels of these actions.

However if the risk of an accident depends on unobservable actions, the situation is more complex. Assume that decreasing these actions is beneficial in the sense that it will reduce the size and/or likelihood of loss. In a model without ambiguity, Shavell (1987) argues that it is not possible to give both agents an incentive to choose the efficient action, while also balancing the budget.⁷ Strict liability would give the injurer an incentive to choose the correct action but would not give the victim any reward for decreasing her action. If the loss is divided in a fixed ratio $\beta : 1 - \beta$, $0 < \beta < 1$, between the injurer and the victim then neither will get the full marginal benefit of any reduction in damage brought about by their own efforts. Thus

⁴Observability here means that the action is observable and verifiable by a third party like a court of law.

⁵A similar analysis could be given for other tort rules such as strict liability with defence of contributory negligence.

⁶While over-investing in care may not sound obviously undesirable, it may cause firms to abandon some projects which are socially beneficial.

⁷There may be institutional reasons why the budget must be balanced. In addition, without budget balance, the court will no longer be impartial, since it will have a monetary interest in the outcome of the case.

both agents will expend insufficient effort to prevent an accident. It can be shown that using non-linear allocations of the damage does not help because again at least one agent must face less than the full marginal cost of any loss (s)he causes. Hence tort rules would fail to provide incentives to a potential injurer and the victims of the accident to undertake the correct amount of unobservable actions.

In the presence of ambiguity we show this no longer holds. The court can exploit ambiguity to ensure that both agents perceive that they pay the full marginal cost of damages. Moreover the rule which implements the optimum has a simple and fairly plausible form. Namely the victim bears the loss, unless the damage exceeds a certain threshold. Above this, the injurer bears the loss and pays a fine or punitive damages, F , to the victim. This works because ambiguity-aversion causes individuals to overweight bad outcomes. For the injurer, paying the fine is the worst outcome and the over-weighting will cause him to reduce the action. In contrast the victim perceives the worst outcome to be where the injurer chooses a low action and she receives no compensation for the loss. Hence the fine does not influence her choice. Thus provided the threshold and F are set correctly, both agents perceive themselves as paying the full marginal cost of their actions.

1.4 Literature Review

The literature on tort is extensive and old. Coase (1960) was one of the earliest papers to analyse how property rights can solve the problem of assigning incentives to internalize the social costs.⁸ Much of the analysis has concerned to what extent the injurer should be held liable; for example Landes and Posner (1987) and Shavell (1980). Shavell (1987) argues that a negligent injurer should bear the full cost of damages caused. He shows that strict liability and negligence both result in optimal care by the potential injurer. Moreover negligence will also give the victim an incentive to provide the efficient level of care.

The literature has also focused on the issues of causation, and to what extent the actions by the injurer and the victim lead to the damages, Ben-Shahar (2000). In this paper we focus on the efficiency of liability rules and we will relate our results to the literature in the conclusion. The present paper aims to contribute to the recent literature in behavioural law and economics.⁹ Various scholars such as Jolls, Sunstein, and Thaler (1998) and Bar-Gill (2006) have argued that behavioural issues such as loss aversion or optimism have a role in designing law. An example of how psychological biases may change well accepted results, is that the Coase theorem may fail if the endowment effect creates biases in valuation, Kahneman, Knetsch, and Thaler (1990).¹⁰

The paper closest to ours is Teitelbaum (2007), which analyses the tort rules, when the injurer has ambiguous beliefs, see also Chappe and Giraud (2013) and Franzoni (2012). He specifically looks at the case when actions of the victim have no bearing on the outcome. The paper is concerned with the effect of ambiguity on the choices of the injurer and the implications for the efficient tort regime. We extend the analysis by modelling the interaction between the victim and the injurer, as a strategic game. This yields a new result. It can be possible to give the agents incentives to provide efficient actions, even when this is not possible without ambiguity.

⁸The economic of analysis of tort rules includes work by Brown (1973), Calabresi (1970), Cooter and Ulen (1999), Innes (1999), Landes and Posner (1987), Shavell (1987), and more recently Franzoni (2012) and Teitelbaum (2007).

⁹See Arlen (1998), Jolls, Sunstein, and Thaler (1998), Korobkin and Ulen (2000) and Parisi and Smith (2005). Chakravarty and Kelsey (2016) provides an explanation using ambiguity why strict liability may not apply in case of unexpected events. Treich (2010) discusses the impact and the policy implications of ambiguity aversion on the value of a statistical life.

¹⁰Some other papers which have analysed behavioural law and economics are Bigus (2006), Chorvat (2001), DeJoy (1989), Eide (2005), Faure (2008), Gigerenzer (2005) and Segal and Stein (2006).

Organisation of the paper In the next section we summarise the relevant literature. This is followed by a brief description of the model of ambiguity, which we shall be using in Section 2. In Section 3 we set up the model of accidents, first without and then with unobserved actions. The efficient tort rule is presented in Section 4 and Section 5 concludes. All proofs are grouped in the appendix.

2 Ambiguity

2.1 Neo-Additive Preferences

Ellsberg (1961) presented a thought experiment, which showed that individuals may not obey the SEU axioms when some or all probabilities are unknown. Much of the subsequent literature has assumed that individuals are averse to ambiguity. However, Ellsberg himself argues that his main message is not that individuals are uniformly ambiguity-averse but that ambiguity makes a difference.¹¹ The experimental evidence shows a similar pattern. The dominant mode of behaviour is ambiguity aversion, i.e. subjects tend to avoid risks where the probabilities are unknown, however in a minority of cases the opposite behaviour, ambiguity loving, is observed. Ambiguity loving is most common in choices involving losses and in the case of highly unlikely events, Kilka and Weber (2001).

Accidents, almost by definition, involve losses and many of them are unlikely events. Thus there is a strong case to allow for the possibility of ambiguity loving. We do this by using neo-additive preferences which are axiomatized by CEG. In this model, ambiguity has the effect of causing the best and worst outcomes of any given action to be over-weighted, (compared to SEU). The set of all states of nature is denoted by Ω and the set of outcomes is denoted by X . An act is a function $a : \Omega \rightarrow X$, which assigns outcomes to states. The CEG model represents ambiguity by allowing the decision-maker's beliefs to be non-additive. These beliefs are modelled as neo-additive capacities which are defined below.

Definition 2.1 *Let α, δ be real numbers such that $0 < \delta < 1, 0 < \alpha < 1$. A neo-additive capacity ν on Ω is defined by $\nu(A|\alpha, \delta, \pi) = \delta(1 - \alpha) + (1 - \delta)\pi(A)$, for $\emptyset \subsetneq A \subsetneq \Omega$, where π is an additive probability distribution on Ω .*

It is possible to define the expected value of a utility function with respect to a neo-additive capacity as a Choquet integral. Preferences which maximise expected utility with respect to a neo-additive capacity, may be represented by the following function defined on the space of acts:

$$V_i(a) = \delta(1 - \alpha)M_i(a) + \delta\alpha m_i(a) + (1 - \delta)\mathbf{E}_\pi u_i(a(s)), \quad (1)$$

where $\mathbf{E}_\pi u_i$ denotes the expected utility of u_i with respect to the (conventional) probability distribution π on Ω and $M_i(a) = \max_{s \in \Omega} u_i(a(s))$ and $m_i(a) = \min_{s \in \Omega} u_i(a(s))$. Thus the decision-maker maximises a convex combination of the maximum, the minimum and the average pay-off. This is a special case of Choquet Expected Utility (CEU), which was axiomatized by Schmeidler (1989).

One may interpret π as the decision-maker's belief. However it is an ambiguous belief. The decision-maker does not give it full weight in his/her preferences, which are also influenced by ambiguity measured by δ . He/she reacts to this ambiguity either in an pessimistic way by over-weighting bad outcomes or in an optimistic way by over-weighting good outcomes. Ambiguity attitude is captured by the parameter α , with higher values of α corresponding to more ambiguity aversion. For simplicity, we shall assume that the same α and δ apply to both agents. However

¹¹This is based on an unpublished lecture which Ellsberg gave at the Institute of Advanced Studies, Vienna, May 2010.

it would be straightforward to extend our analysis to the case where the agents have different degrees of ambiguity-aversion if there is a motivation for doing so.

Consider a given act a . Suppose that a yields its worst consequence in state $\underline{s}$, i.e. $a(\underline{s}) = m_i(a)$. Then according to equation (1) the decision-weight on $m_i(a)$ is $\delta\alpha + (1 - \delta)\pi(\underline{s})$. We say that the decision-maker is *pessimistic* (resp. *optimistic*) if this decision-weight is greater (resp. less) than the probability i.e. $\alpha > \pi(\underline{s})$ (resp. $\alpha < \pi(\underline{s})$). The extreme case of pessimism is $\alpha = 1$, in which case we say that the decision-maker is *ambiguity-averse*. Likewise we say that an individual is ambiguity-loving if $\alpha = 0$.

A possible criticism of these preferences is that they only allow the best and worst outcomes to be over-weighted. In many cases the worst outcome is a loss which would cause bankruptcy. However it is likely that individuals would also be concerned about potential losses which are very large but not bad enough to trigger bankruptcy. Despite this potential drawback, we believe this model is suitable for studying tort, since in an accident there are focal best and worst outcomes, namely no accident and being found liable respectively.

2.2 Games with Ambiguity

When there is the possibility of an accident, the pay-offs of the agents depend on the both their own action and that taken by the other party. Thus they are playing a strategic game. When there is likely to be ambiguity, Nash equilibrium cannot be applied since it does not allow players to have ambiguous beliefs. Instead we use a solution concept from Eichberger and Kelsey (2011), Eichberger and Kelsey (2014), which has been experimentally tested in Eichberger, Kelsey, and Schipper (2008). This theory allows beliefs about the behaviour of opponents to be ambiguous. This extends Dow and Werlang (1994) by allowing for the possibility of ambiguity seeking behaviour. Formally, we assume that each player maximizes his/her expected payoff with respect to an ambiguous belief. In equilibrium, beliefs have to be reasonable in the sense that each player “believes” that his/her opponents play best responses. To model this we require that the *support* of any given player’s beliefs contain only best responses of the other players. We define the support of a neo-additive capacity as follows:

Definition 2.2 *The support of the neo-additive capacity $\nu(\cdot|\alpha, \delta, \pi)$ is defined by $\text{supp } \nu = \text{supp } \pi$.*

As explained above, a neo-additive capacity is intended to represent a situation where the decision-maker’s belief is represented by the probability distribution π but (s)he is not fully confident in it. Given this it is plausible that the support of ν should coincide with that of π .¹² Denote by $R_i(\nu_{-i}) = \arg \max\{V_i(s_i, \nu_{-i}) \mid s_i \in S_i\}$ the best response correspondence of player i , given his/her beliefs are represented by the capacity ν_{-i} on S_{-i} . This enables us to present our definition of equilibrium.

Definition 2.3 *A pair of capacities $\nu^* = \langle \nu_1^*, \nu_2^* \rangle$ is an Equilibrium Under Ambiguity (EUA) if*

$$\emptyset \neq \text{supp } \nu_1^* \subseteq R_1(\nu_2^*) \text{ and } \emptyset \neq \text{supp } \nu_2^* \subseteq R_2(\nu_1^*).$$

Definition 2.3 requires the strategies in the support of a player’s equilibrium belief be best responses. However it is ambiguous whether the opponent plays best responses. As result, his/her best and worst possible plays are also taken into account when evaluating a strategy. Decision-relevant strategies outside the support can be interpreted as events a player views as unlikely but which, due to ambiguity cannot be completely ruled out.

¹²For a more detailed discussion and the relation of this definition to earlier support notions see Eichberger and Kelsey (2014).

3 Model of Liability

In this section we introduce our model of accidents. We analyse the solution and then show how it will be modified by ambiguity.

3.1 Set-up

There are two agents, an injurer and a victim, who are assumed to be risk-neutral in the sense that they do not have diminishing marginal utility of wealth.¹³ There is a third party, the court of law, which implements the liability rule. The injurer undertakes an activity which may cause harm or damage to the victim. The injurer (resp. victim) can take an observable action or care, x , (resp. y) at cost $a(x)$, (resp. $b(y)$). The care level of the injurer (resp. victim) may be any rational number of the form $\frac{k}{r}$ up to a maximum $\bar{x}$ (resp. $\bar{y}$), where r is a given non-negative integer.¹⁴ Both a and b are increasing, convex and satisfy $a(0) = b(0) = 0$. The agents believe that accidents occur with probability π . However this is an ambiguous belief in the sense that they do not have complete confidence in it. The model can be extended to the case where care affects the likelihood of an accident as well as the size of the damage.

An accident will cause damage $D(x, y)$, which is publicly observable and verifiable by all parties. We assume D is decreasing in x and y . For given y , (resp. x) $D(x, y)$ is convex in x (resp. y). This implies that there are diminishing marginal returns to both types of care. In addition, we assume that lack of care by the injurer causes more damage if the victim also supplies a low level of care. Formally we require $D(x, y)$ to display decreasing differences in $\langle x, y \rangle$, i.e. if $x_1 > x_2$, $D(x_1, y) - D(x_2, y)$ is an increasing function of y . This is different to many models of liability where it is assumed that care reduces the probability rather than the size of damage. In the context of ambiguity we believe it is more reasonable to assume that care reduces the size of damage. If an individual does not know the probability of loss how do they know the relationship between care and this unknown probability? However most of our results would have counterparts in a model where care affected the likelihood of loss.

Let the expected utility of the injurer (resp. victim) under SEU be $u_i(x, y)$ (resp. $u_v(x, y)$). We normalise by assuming both agents will get utility zero in the event of no accident. The social welfare function is given by the sum of the expected utility of the injurer and the victim, $u_i(x, y) + u_v(x, y)$.¹⁵ Hence the social cost (or loss) from the accident is:

$$L(x, y) = \pi D(x, y) + a(x) + b(y).$$

We define the marginal effect of x loss by $ML_x(x, y) = L(x, y) - L(x - \frac{1}{r}, y)$. Marginal values of other variables will be defined analogously. We assume that there are diminishing marginal returns to care in the sense that the marginal losses, $ML_x(x, y)$ (resp. $ML_y(x, y)$) are increasing in x (resp. y). Optimal care levels x^* and y^* are given by the conditions $ML_x \leq 0, x \leq x^*$; $ML_x \geq 0, x \geq x^*$ and $ML_y \leq 0, y \leq y^*$; $ML_y \geq 0, y \geq y^*$.¹⁶ The optimal precaution or care levels, x^* and y^* , have been derived under the assumption of SEU. This could be justified by arguing the

¹³In other words the utility of wealth function is linear. This implies that the wealth of the agents will not have any impact on our results.

¹⁴Restricting strategies to be rational numbers ensures that the strategy spaces are finite. This enables us to apply the theory of games with ambiguity from Eichberger and Kelsey (2014), which assumes a finite strategy space.

¹⁵Note we ignore distributional issues. This is a standard assumption in the literature. There appears to be no a priori reason why the injurer should be more or less deserving than the victim. The assumption is best justified when both injurer and victim are firms owned by diversified shareholders. If distributional issues are relevant, for instance the victim is socially more deserving, then the social welfare function will need to be modified to take account of this.

¹⁶Note the second order condition is satisfied by concavity.

court should satisfy a higher standard of rationality than the injurer or victim.¹⁷ The problem here is to design incentives so that the agents choose efficient actions.

3.2 Liability without Ambiguity

To set a benchmark, we analyse two of the more common tort rules in the absence of ambiguity. First we consider strict liability, under which the injurer has to pay the damage no matter what levels of ex ante precaution the agents take. So the expected loss under strict liability is $-L_i^S(x, y) = -\pi D(x, y) - a(x)$ and $-L_v^S(x, y) = -b(y)$.¹⁸ For strict liability the injurer bears the full cost of the externality he causes. He will choose that care level where the marginal reduction in expected loss is equal to the marginal cost of care. In contrast the victim will choose to provide the lowest possible level of care, since care is costly and does not increase her pay-off.

The second liability rule is negligence, which requires the injurer to pay the loss if the care taken is less than a stipulated level, x^s , otherwise the victim bears the loss. This rule causes the injurer's pay-off to be discontinuous in his own strategy. In contrast the victim's pay-off is continuous in her own strategy but is discontinuous in the injurer's strategy. We assume that the court sets x^s at the efficient level without ambiguity, i.e. $x^s = x^*$. In case of negligence, the injurer provides the efficient level of care. This is because for lower levels of care he is fully liable for the damage. Hence for $x < x^*$ his marginal benefit of increasing care incorporates the externality on the victim, for a more detailed analysis see Shavell (1987). Given the injurer provides efficient care, the full cost falls on the victim, who internalises the externality resulting from her actions. As a result she chooses the efficient level of care y^* . Thus without ambiguity, negligence is better than strict liability since it gives both agents correct incentives.

3.3 Liability with Ambiguity

In this section we shall analyse the impact of ambiguity with given tort rules. We shall start by studying strict liability, followed by negligence. Throughout this section we shall assume that the agents perceive both the occurrence of the accident and each other's actions to be ambiguous. We assume that the injurer and the victim have the same perception of ambiguity, δ , and the same ambiguity attitude, α . Since each agent's pay-off depends on the actions chosen by the other, they are playing a strategic game. Thus we need to use theories of ambiguity in games. We use a solution concept for games with ambiguity from Eichberger and Kelsey (2014). The injurer and the victim have ambiguous beliefs regarding both the likelihood of an accident and the behaviour of the other agent. Recall with neo-additive preferences, ambiguity causes the agents to over-weight the best and worst outcomes. In all cases the best outcome is no accident and consequently no loss. We shall normalise the pay-off from no accident to zero. The worst outcome will depend on which liability rule is being used.

3.3.1 Concepts of Efficiency

In Section 3.1 we defined x^* and y^* to be the efficient levels of care in the absence of ambiguity. They remain efficient when there is ambiguity, in the sense that they control the negative externality between the injurer and the victim. We shall refer to these care levels as the *efficient care*

¹⁷In some cases the court will have access to information about a number of similar accidents which have occurred in the past. In contrast the agents may have never previously been involved in an accident of this kind. Thus the court can use the past data and experience and estimate the probability of accident π . Shavell (1987) provides an explanation how the court of law can estimate the probability. Nevertheless there is a case for extending the model to allow the court also to perceive ambiguity.

¹⁸Note that $u_i(x, y) = -L_i^s(x, y)$ and $u_v(x, y) = -L_v^s(x, y)$.

levels or actions. However there is a second kind of externality between the agents. If one agent creates ambiguity this may lower the other party's ex-ante utility due to ambiguity-aversion. We shall refer to the ex-ante utility losses due to ambiguity-aversion as the *subjective externality* between the agents. A situation in which social welfare is maximised taking into account ex ante losses due to ambiguity aversion we shall refer to as *fully efficient*. We define $\hat{x}$ and $\hat{y}$ to be the fully efficient levels of care.

We believe it is clear that the social planner should aim to reduce losses due to the physical externality. There may also be a case to require the social planner to control the subjective externality. However we anticipate that this second kind of intervention will be more controversial. We would note that full efficiency is possible in principle. The social planner could require the injurer and victim to provide the efficient care levels. The injurer is not liable for the damage and the social planner fully compensates the victim using funds raised by lump-sum taxation. Some other examples where full efficiency is possible are discussed in Kelsey and Spanjers (2004). However we note that, in practice, full efficiency will often not be achieved. The tort rules used in practice will, at best, induce the agents to perform the efficient actions. More typically they will only manage to achieve a Pareto improvement on the situation without legal intervention.

3.3.2 Strict Liability

Under strict liability the injurer is responsible for the loss. His largest possible loss from an accident is $D(x, 0)$, i.e., when the victim invests zero in care. The utility/loss from the best realization is $-a(x)$. Thus the (Choquet) expected utility of the injurer is:¹⁹

$$-\delta\alpha D(x, 0) - (1 - \delta)\pi D(x, y) - a(x).$$

His marginal benefit of increasing care is given by $-\delta\alpha\Delta_x D(x, 0) - (1 - \delta)\pi\Delta_x D(x, y)$.²⁰ The injurer puts decision-weight $\delta\alpha + (1 - \delta)\pi$ on the worst outcome. Thus the more ambiguity-averse he is (i.e. the greater is α) the greater the weight on the worst outcome. In addition due to decreasing differences, the marginal impact of his own care is greater in the worst case scenario i.e. $\Delta_x D(x, 0) < \Delta_x D(x, y)$. Both of these effects act to increase the perceived marginal benefit of care with ambiguity. Intuitively, ambiguity-aversion causes the injurer to overweight the possibility of an accident and hence increase his perceived marginal benefit of care.

Under strict liability, the victim's utility is decreasing in y . As a result she will always choose the lowest possible level of care. In contrast if the injurer is ambiguity-averse ($\alpha = 1$), he will provide a level of care which is more than that without ambiguity. This is reversed with ambiguity-loving. The following result states our main conclusions about strict liability with ambiguity.²¹

Proposition 1 *In equilibrium under strict liability, the victim always chooses the lowest possible level of care. The injurer's care is an increasing function of α . It is less (resp. more) than that without ambiguity as $\alpha < \pi$ (resp. $\alpha > \pi$).*

Strict liability is illustrated by the following example.

¹⁹In this equation the over weighting of the worst outcome can be plainly seen. The best outcome is no accident. This is also over-weighted. However the over-weighting is less obvious since the utility of no accident has been normalised to 0.

²⁰The difference operator is defined by $\Delta_x D(x, y) = D(x, y) - D(x - \frac{1}{r}, y)$. Differences of other variables are defined analogously.

²¹Theorem 3.1 from Eichberger and Kelsey (2014) shows that with parameters δ and α , if the game is such that the payoff of player i , $u_i(s_i, s_{-i})$, is increasing in s_{-i} , and has increasing differences in $\langle s_i, s_{-i} \rangle$ then an equilibrium in pure strategies will exist. (Here $s_i \in S_i$ is the strategy of player i , $s_{-i} \in S_{-i}$ is the strategy profile of the other players.) Under strict liability the pay-offs of each agent have increasing differences in (x, y) . Existence of equilibrium when the tort rule is negligence is proved in Proposition 4.

Example 1 Let $a(x) = x^2, b(y) = y^2, D(x, y) = 1 - x - y$, and let $0 \leq x + y \leq 1$. In the absence of ambiguity the efficient actions are, $x^* = \frac{\pi}{2}$ and $y^* = \frac{\pi}{2}$, where π is the probability with which the court believes that accidents occur. If the tort rule is strict liability then $x^s = \frac{\delta\alpha + \pi(1-\delta)}{2}$ and $y = 0$. For $\delta < 1, x^s \geq x^*$ as $\alpha \geq \pi$. This means that a pessimistic injurer will choose an excessive level of care.

If $D(x, y)$ is additively separable (as in this example) then in equilibrium without ambiguity the injurer will choose the efficient level of care. Otherwise he will choose a higher level of care to compensate for the fact the victim provides an inefficiently low level of care.

3.3.3 Negligence

Next we analyse negligence. Let x^s denote the level of care which the court stipulates that the injurer must provide to avoid liability. If the injurer bears the liability (for not investing x^s) then the largest possible damage from the accident is $D(x, 0)$, i.e., when the victim invests zero in care. The utility from the best realization is $-a(x)$. Under negligence, the injurer will have the following (Choquet) expected utility:

$$-\delta\alpha D(x, 0) - (1 - \delta)\pi(x, y)D(x, y) - a(x) \text{ if } x < x^s, \quad = -a(x) \text{ if } x \geq x^s.$$

For the victim the worst utility is $-b(y) - D(\bar{x}, y)$. This arises when the injurer provides just enough care to make the victim liable. The highest utility is $-b(y)$. Hence her (Choquet) expected utility is:

$$-\delta\alpha D(\bar{x}, y) - (1 - \delta)\pi(x, y)D(x, y) - b(y) \text{ if } x \geq x^s, \quad = -b(y) \text{ if } x < x^s.$$

We assume that there is no ambiguity for the court regarding the measurement of x^s .

Consider first the case where both agents are ambiguity-averse, ($\alpha = 1$). Assume that $x^s = x^*$, the ex-post efficient action. The injurer will select the stipulated level of care, x^* , since this will completely protect him from ambiguity. The victim will overweight the worst outcome, which is an accident. Given the injurer provides stipulated care, the victim will have to pay the loss. The victim will then have full incentive to provide care. As a result she will choose the fully efficient level of care. This is greater than the level an ambiguity-neutral individual would choose. The victim selects a higher level of care to protect herself from ambiguity.

Secondly suppose that agents are mildly ambiguity-seeking. In this case, the injurer will continue to provide stipulated care due to the discontinuity in pay-off at x^s . Optimism will decrease the victim's marginal benefit and hence she will choose less than the efficient level of care. The final possibility is that the injurer has a high degree of ambiguity preference which causes him to prefer the ambiguous risk of an accident to providing the stipulated level of care. In this case the victim will provide only minimal care since she will not be liable. This analysis is summarised in the following proposition.

Proposition 2 Assuming the court stipulates that the injurer take the efficient action, x^* , there are three types of equilibria under negligence:

1. if the agents are ambiguity averse, ($\alpha = 1$) the injurer takes stipulated care and victim will provide a level of care $\hat{y}$ greater than the ex-post efficient level y^* ;
2. if the agents have a low degree of ambiguity seeking, the injurer takes stipulated care and victim will under-provide care, $y < y^*$;
3. if ambiguity seeking is sufficiently strong the injurer will take less than stipulated care $x < x^*$ and the victim will choose $y = 0$.

It would also be possible to stipulate that the injurer provide a higher level of care.²² However due to decreasing differences, the victim would respond by reducing care. Hence this would have an ambiguous effect on welfare. In the case of unilateral accidents full efficiency could be achieved. More generally, if the injurer's action were more important than that of the victim, then it would be desirable to stipulate a higher level of care, provided it were not greater than the fully efficient level. Negligence is illustrated by the following example.

Example 2 Let the cost function be $a(x) = x^2$, $b(y) = y^2$, and let $D(x, y) = 1 - x - y$. The efficient actions are: $x^* = \frac{\pi}{2}$ and $y^* = \frac{\pi}{2}$. Then if the agents are ambiguity-averse, the injurer will take efficient care and the victim will provide excessive care. If the agents are ambiguity-loving then in most cases the injurer will provide the efficient level of care and the victim will under-provide care. If the level of ambiguity is very high, the injurer will assume liability and under-provide care as a result the victim will take no care.

Ambiguity-Aversion Let $\alpha = 1$ and $\delta > 0$, then if the injurer takes stipulated care his cost is $\frac{\pi^2}{4}$. If he provides a lower level of care his (Choquet) expected loss is $-\delta(1 - x) - (1 - \delta)\pi(1 - x) - x^2$, (taking into account the fact the victim will respond with $y = 0$). This is maximised where $x = \frac{\delta + \pi + \delta\pi}{2}$. Since this is greater than the efficient action $x = \frac{\pi}{2}$, the injurer will always take stipulated care. Hence the victim will receive no compensation. Her expected loss is, $-\delta(1 - \frac{\pi}{2} - y) - (1 - \delta)\pi(1 - \frac{\pi}{2} - y) - y^2$ this is maximised by choosing $y = \frac{\delta(1 - \pi) + \pi}{2} > \frac{\pi}{2}$. Thus ambiguity-aversion causes the victim to be cautious and over-provide care.

Ambiguity-Loving In this case, $\alpha = 0$ and $\delta > 0$. The injurer's loss function is $(1 - \delta)\pi(1 - x - y) + x^2$. This is minimised when $x = \underline{x} = \frac{(1 - \delta)\pi}{2}$. So the injurer faces the choice between providing stipulated care at cost $\frac{\pi^2}{4}$ and providing a lower level of care $\underline{x}$, which yields a (Choquet) expected loss of $(1 - \delta)\pi(1 - \frac{(1 - \delta)\pi}{2}) + \frac{(1 - \delta)^2\pi^2}{4}$.²³ So the efficient action will be taken if $(1 - \delta) \left[\pi(1 - \frac{(1 - \delta)\pi}{2}) + \frac{(1 - \delta)\pi^2}{4} \right] > \frac{\pi^2}{4} \Leftrightarrow (1 - \delta)[4 - (1 - \delta)\pi] > \pi$. This inequality always holds provided that δ is sufficiently small. The injurer will take efficient care if the degree of ambiguity δ is below a critical level $\bar{\delta}$, which is given by, $\bar{\delta} = 1 - \frac{2 - \sqrt{4 - \pi^2}}{\pi}$. This is a decreasing function of π . The table below shows values of $\bar{\delta}$ for some sample values of π .

π	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\bar{\delta}$	1	0.9373	0.8730	0.8054	0.7321

As can be seen, the injurer will only fail to provide efficient care for very high levels of ambiguity. Hence we believe that this possibility is not likely to be important in practice.

Under negligence we find that if the injurer is sufficiently optimistic, we move away from optimality towards under-provision of care. However if he is slightly optimistic then he will provide efficient care. This is due to two factors. Firstly there is the discontinuity in the injurer's payoff function at $\hat{x}$. Secondly if he fails to provide stipulated care, the victim will respond by discontinuously reducing her level of care. Both together imply that there is a large marginal penalty for reducing care below the stipulated level. Negligence, therefore, is more robust to ambiguity seeking than strict liability. This may be important since, as discussed earlier, accidents share many of the features of situations where ambiguity seeking is commonly observed. So negligence is more likely to result in optimal care if we take into account the possibility of ambiguity. Negligence is the most frequently used tort rule under common law in Britain and United States. Robustness to ambiguity may provide one reason for this.

²²As can be seen from the proof the court could stipulate any level of care up to the best response to zero care by the victim. If the D function is additively separable this would correspond to the fully efficient level of care.

²³The formula takes into account the fact that the victim's best response to $\underline{x}$ is $y = 0$.

3.4 Unobservable Actions

We now extend the model by assuming that the damage can also be affected by unobservable actions undertaken by the agents. One example of such an action is the activity level in Shavell (1987). However there are other possibilities, for instance it could be the quality of cement used in drilling in the BP oil well, or the use being made of the nuclear reactor at Windscale, UK.²⁴

The injurer (resp. victim) can choose an unobservable action s (resp. t), which gives him (resp. her) benefit $\phi(s)$ (resp. $\psi(t)$). The action level of the injurer (resp. victim) may be any rational number of the form $\frac{k}{r}$ from the interval $[0, \bar{s}]$ (resp. $[0, \bar{t}]$), where r is a given non-negative integer. The functions ϕ and ψ are increasing and concave. The damage *increases* with the unobservable action. This is represented by a damage function, $\tilde{D}(x, y, s, t)$, which is increasing in s and t .²⁵ We assume that D is supermodular in $-x, -y, s, t$.²⁶ This implies that the effect of lack of care is worse if the unobservable action is high. The cost for the injurer (resp. victim) for undertaking care x (resp. y) and unobservable action s (resp. t) is $sa(x)$ (resp. $tb(y)$).²⁷ The social surplus is given by

$$S(x, y, s, t) = \phi(s) + \psi(t) - \pi \tilde{D}(x, y, s, t) - sa(x) - tb(y).$$

Let x^*, y^*, s^* and t^* denote the observable and unobservable actions which maximise this expression. We shall call these *efficient actions*, since they control the negative externality between the agents.

3.4.1 Strict Liability

Under strict liability, the injurer and the victim, respectively, have the following (Choquet) expected utility:

$$u_i^S(x, y, s, t) = \phi(s) - \alpha \delta \tilde{D}(x, 0, s, \bar{t}) - (1 - \delta) \pi \tilde{D}(x, y, s, t) - sa(x),$$

$$u_v^S(x, y, s, t) = \psi(t) - tb(y).$$

The analysis of the choice of the observable action is similar to that in Section 3.3. For the injurer, ambiguity will increase the marginal benefit of the observable action and reduce that of the unobservable action. Thus one would expect ambiguity to cause an ambiguity-averse injurer to choose higher observable and lower unobservable actions. Since the victim is not liable, she chooses the lowest possible care level and an excessive unobservable action. The following result states this formally.

Proposition 3 *Assume that the tort rule is strict liability.*

1. *If both agents are ambiguity-averse, ($\alpha = 1$) then the injurer chooses more than the efficient observable action, x^* , and the victim chooses $y = 0$; the injurer will choose a lower*

²⁴The BP oil spill occurred at one of the deepest oil wells in history, Judith Evans, 2010, BP ‘pressured rig disaster workers to drill faster’. http://www.timesonline.co.uk/tol/news/world/us_and_americas/article7128842.ece Evans. Similarly the Windscale nuclear accident in Cumbria, UK, the reactor was being used to build material for a H-bomb for which it had not been constructed, see Windscale: A nuclear disaster. <http://news.bbc.co.uk/1/hi/sci/tech/7030281.stm> Dwyer (2007).

²⁵We assume that damage increases with the unobservable action so that it can be interpreted as an activity level, if desired.

²⁶Supermodularity is the multi-dimensional equivalent of strategic complementarity. See Topkis (1998) for a formal account.

²⁷In this paper we have tried to keep our model close to that of Shavell (1987). Hence the cost function is multiplicative. However we believe that this assumption is not crucial and similar results hold for all supermodular functions.

unobservable action than optimal, but the victim will choose an unobservable action above the efficient level.

2. If both agents are ambiguity-loving i.e. $\alpha = 0$, then the injurer chooses less than the efficient observable action, x^* , the victim chooses $y = 0$; both the injurer and the victim will choose an unobservable action above the efficient level.

3.4.2 Negligence

We maintain the earlier assumption that the court stipulates the ex-ante efficient level of care x^* . Under negligence, the injurer and victim respectively have following (Choquet) expected utilities:

$$\begin{aligned} U_i^N(x, y) &= \phi(s) - \alpha\delta\tilde{D}(x, 0, s, \bar{t}) - (1 - \delta)\pi\tilde{D}(x, y, s, t) - sa(x), \text{ if } x < x^*; \\ &= \phi(s) - sa(x) \text{ if } x \geq x^*; \end{aligned}$$

$$\begin{aligned} U_v^N(x, y) &= \psi(t) - \alpha\delta\tilde{D}(x^*, y, \bar{s}, t) - (1 - \delta)\pi\tilde{D}(x, y, s, t) - tb(y), \text{ if } x \geq x^*; \\ &= \psi(t) - tb(y) \text{ if } x < x^*. \end{aligned}$$

First we establish existence of equilibrium. This is not completely straightforward, since under negligence the injurer's pay-off function is neither continuous nor supermodular. Hence standard existence results do not apply directly. The following result establishes that an EUA exists when the liability rule is negligence. This holds for any stipulated care level which the court may choose to implement.

Proposition 4 *Assume that the tort rule is negligence and the court stipulates that the injurer provide care level, x^s . Then an equilibrium with ambiguity exists for any given $\alpha, \delta, 0 \leq \alpha \leq 1, 0 \leq \delta \leq 1$.*

The analysis regarding the observable action is similar to that in Section 3.3.3. Assume that the court stipulates that the injurer take the efficient action. First suppose that the agents are ambiguity-averse, in which case the injurer will choose $x = x^*$, since this completely protects him from ambiguity. So the injurer will choose $s > s^*$ since he will no longer be liable. The victim will choose $t < t^*$, as she over-weights the damage in her decision-making. Secondly assume the agents have a low degree of ambiguity loving. Then due to the discontinuity, the injurer will continue to choose the stipulated level of the observable action. The victim will choose an excessively high unobservable action, since ambiguity loving increases her marginal benefit. Finally consider the case where the agents are sufficiently ambiguity loving. Then the injurer will ignore the risk of liability and hence provide $x < x^*$. Such an injurer will select $s < s^*$, since he will be facing liability. The victim will choose $t > t^*$ since the injurer is liable. So negligence will do quite poorly in terms of social welfare if the injurer is highly ambiguity loving. These arguments are summarised in the following proposition.

Proposition 5 *Assume that the court stipulates that the injurer take the efficient action, x^* , there are three types of equilibria under negligence.*

1. If the agents are ambiguity averse, the injurer takes the stipulated observable action, x^* , and victim will choose an observable action which is too low. The unobservable action of the injurer (resp. victim) will be above (resp. below) the efficient level.
2. If the agents have a low degree of ambiguity seeking, the injurer takes the stipulated observable action, x^* , and victim will under-provide y . The unobservable action of the injurer (resp. victim) will be above (resp. below) the efficient level.

3. *If ambiguity seeking is sufficiently strong, the injurer will take less than the stipulated level of the observable action, $x < x^*$ and the the victim will choose $y = 0$. The victim's unobservable action will be above the efficient level.*

In case 3 we cannot tell the effect on the injurer's unobservable action without further information. Ambiguity loving tends to increase the perceived marginal benefit of the action. However the fact the injurer will be liable provides a countervailing incentive to reduce it. It is not possible to tell which effect is stronger without further information.

4 Optimal Tort Rules

In this section we show that when there is ambiguity there is a tort rule which yields efficient choice of both actions even when this is not possible without ambiguity. Consider a rule which imposes liability on the injurer if he fails to choose the stipulated observable action or the realized damage is above a certain threshold, otherwise the victim is liable. If the damage is above the threshold, the injurer must pay for the damage plus a fine F . The fine is paid to the victim and may be interpreted as punitive damages. Otherwise if the injurer chooses the stipulated action, the victim is liable. The threshold is chosen so that is clear that *both* injurer and victim, have taken inefficient actions. As a result the fine is an ambiguous risk for the injurer, since it depends on the action of the other agent. However the injurer can protect himself from ambiguity by choosing the efficient unobservable action.

Throughout this section we shall assume that both agents are ambiguity averse. First, the victim will be liable for all his losses if the injurer takes the stipulated observable action and the size of the damage is below a threshold. Let us define the loss incurred due to the accident as $L \equiv \tilde{D}(x, y, s, t)$ and let the threshold damage be $\hat{L}$. If $x < \hat{x}$ and there is an accident the injurer faces a liability equivalent to the loss.²⁸

If the loss is beyond the threshold, $L \geq \hat{L}$, then the injurer faces a liability equivalent to the loss and a punitive fine $F > 0$ even if he has taken stipulated care. Recall the fine, is paid by the injurer to the victim. It is perceived as a bad outcome by the injurer. However the victim perceives the fine as a good outcome. As a result it gets a positive weight when the injurer makes a decision but zero weight when the victim makes a decision. (Since the fine is not paid in equilibrium.) Thus the equilibrium is sustained by an ambiguous punishment which would be triggered if one agent deviated.²⁹ If the size of F is chosen correctly, it can induce both, injurer and victim, to choose the efficient action.

So the injurer knows the expected utility he will face if $x < \hat{x}$, is $\phi(s) - \delta \tilde{D}(x, 0, s, \bar{t}) - (1 - \delta) \pi [\tilde{D}(x, y, s, t)] - sa(x)$ and if the loss observed is $L \geq \hat{L}$ then his expected utility will be $\phi(s) - \delta [\tilde{D}(x, 0, s, \bar{t}) + F] - (1 - \delta) \pi [\tilde{D}(x, y, s, t) + F] - sa(x)$. Since the injurer will face liability for the loss if $x < \hat{x}$, he will choose $\hat{x}$. This will be true since the injurer is pessimistic and the expected cost from providing $\hat{x}$ will always be lower than the cost of bearing the liability. Punitive damages are only triggered if both agents choose excessive unobservable actions. If the injurer is ambiguity-averse he will want to protect himself against the risk of punitive damages which he can do by choosing $s = \hat{s}$.

Given the injurer chooses $\hat{x}$ and $\hat{s}$, the victim will bear all the losses unless the threshold is exceeded, which only occurs out of equilibrium. So for $L < \hat{L}$, the marginal benefit from

²⁸ Recall that $\hat{x}$ is defined to be the ex-ante efficient value of x .

²⁹ Battigalli, Cerreia-Vioglio, Maccheroni, and Marinacci (2012) also have a model in which an equilibrium is sustained by ambiguous beliefs off the equilibrium path.

increasing the unobservable action will be,

$$\Delta\psi(t) - \delta\Delta_t\tilde{D}(\hat{x}, y, \bar{s}, t) - (1 - \delta)\pi\Delta_t\tilde{D}(\hat{x}, y, s, t) - \frac{b(y)}{r}.$$

The proposal is illustrated by the following example in which the agents only have one unobservable action each to choose.

Example 3 Let $s \in \{\sigma_1, \sigma_2, \sigma_3\}$ and $t \in \{\tau_1, \tau_2, \tau_3\}$ and let the optimal actions be σ_2 and τ_2 , respectively. The damage is $\tilde{D}(s, t)$, and $\hat{l}$ is the threshold. Assume that without ambiguity aversion, the injurer and the victim would choose σ_3 and τ_3 respectively and assume $w(\tau_1) = 0$ and $u(\sigma_1) = 0$. Let $F > 0$. Choose $\hat{l}$ such that $\hat{l} = \tilde{D}(\sigma_3, \tau_3)$. The injurer will face liability if damage is greater than $\hat{l}$. Since the following hold true

$$\begin{aligned}\phi(\sigma_2) - \sigma_2 a(x) &\geq \phi(\sigma_3) - \delta \left[\tilde{D}(\sigma_3, \tau_3) + F \right] - (1 - \delta)\pi \left[\tilde{D}(\sigma_3, t) + F \right] - \sigma_3 a(x) \\ \phi(\sigma_2) - \sigma_2 a(x) &\geq -\sigma_1 a(x),\end{aligned}$$

where $t \in \{\tau_1, \tau_2, \tau_3\}$. So the injurer will choose σ_2 . Given this, the victim will choose τ_2 since:

$$\begin{aligned}\psi(\tau_2) - \delta\tilde{D}(\sigma_3, \tau_2) - (1 - \delta)\pi \left[\tilde{D}(s, \tau_2) \right] - \tau_2 b(y) \\ \geq \psi(\tau) - \delta\tilde{D}(\sigma_3, \tau) - (1 - \delta)\pi \left[\tilde{D}(s, \tau) \right] - \tau b(y), \quad \text{for } \tau = \tau_1, \tau_3.\end{aligned}$$

The fine causes the injurer to choose the efficient action. Given this, the victim will not be compensated for any loss hence she will also choose the efficient action.

The following result shows formally that this liability rule can achieve efficiency

Proposition 6 If the agents are ambiguity-averse, the fully efficient observable and unobservable actions may be implemented by the following tort rule:

1. the victim is liable for the loss, if L is below an appropriately set threshold $\hat{L}$ and the injurer provides stipulated care,
2. the injurer is liable for the loss $L \leq \hat{L}$ if he fails to invest in the stipulated care,
3. the injurer is liable for the loss L and punitive damages F (paid to the victim) if the loss is $L > \hat{L}$.

So while the negligence rule gives the optimal observable action, a punitive fine borne by the injurer in case of excessive damage results in the efficient unobservable actions.³⁰ A high enough fine levied on the injurer will restrict his unobservable action level while the negligence rule will induce him to take the stipulated observable action. Given that the injurer chooses the efficient actions, all the cost will fall on the victim. This will internalise the externality, hence the victim will also choose efficient actions.

These issues are relevant for environmental accidents like the Gulf of Mexico oil spill, in which case the liability rule would apply as follows. If the damage is not substantial, then the liability is borne by the victims as long as BP takes stipulated care. However if the damage is extremely large, then BP would not only bear the liability but also pay a substantial penalty

³⁰ A related point is made in Kelsey and Spanjers (2004), who show that ambiguity may increase the set of outcomes which can be implemented in a context of team production. They show that with non-additive beliefs, it is possible to make both parties receive the full marginal benefit of their actions.

to the victim. It can be argued that the amount BP paid to the local businesses is in excess of their true losses.³¹ So it is not implausible that there might be a punitive component to the payment. Further, even if the fine and the threshold is not set optimally, it may still be possible to make social welfare higher than in the case without ambiguity. It may be worth noting that the BP oil spill was caused by the multiple failures of the drilling technology and the equipment to contain oil spills, (blow out preventer etc.). As in our model, punitive damages were triggered by the actions of two or more agents.³²

This result is related to the punitive damages literature in law and economics, Cooter (1982). Polinsky and Shavell (1998) argue that punitive damages should be used if the injurer may escape full liability for the damage they have caused. Proposition 6, implies that for standard damages, negligence rules will protect the injurer from liability. In addition a punitive damage, F , borne by the injurer in case of excess damages will deter him and provide incentives to choose the efficient actions.

There are some potential disadvantages of this proposal. Firstly the size of the fine depends on the injurer's degree of ambiguity-aversion. It may not be completely straight-forward for the court to determine this.³³ However in many cases involving uncertainty, courts already need to determine the values of subjective variables such as the risk premium or the monetary equivalent of loss of life or limb. Hence using our proposal is not different in principle to using other liability rules when there is uncertainty. While the size of the optimal fine may be difficult for the regulator or courts to compute precisely, even a fine which is too small will be a Pareto improvement over the status quo. However a large fine may lead to the injurer closing the business or not carrying out any activity.

This section shows that if agents are ambiguity-averse, ambiguous punishments may be used to induce them to choose the efficient actions. One possible objection is that the evidence suggests that individuals are not uniformly ambiguity-averse but also at times exhibit ambiguity-seeking behaviour, Kilka and Weber (2001). In fact this reinforces our point. With ambiguity-loving individuals it may be possible to implement efficient actions by offering them ambiguous *rewards*. As in the previous case, this will work even when efficiency is not possible with non-ambiguous incentive schemes.

5 Conclusion

In an analysis of tort rules Shavell (1987) shows negligence and strict liability provide incentives to choose efficient actions if the stipulated level of care is set correctly. The present paper extends this by analysing the impact of ambiguity in bilateral accidents. We find that under strict liability, ambiguity causes the injurer to restrict the unobservable action. While the level may not be fully optimal, there may still be an improvement over the situation without ambiguity. For negligence, if the injurer is pessimistic then he will invest optimally in care but will choose an unobservable action which is too high. The victim will over-provide care but will also increase the unobservable action. Interestingly if the injurer is optimistic and does under-provide care, then the victim will choose the lowest possible observable action and also increase the unobservable action. Thus with highly optimistic injurers, negligence does not seem to work well, while with pessimism negligence does better in terms of social welfare. In contrast if there is no ambiguity, Shavell (1980) shows that negligence gives efficiency for a greater range of cases.

³¹In the litigation after the Exxon Valdez oil spill the punitive damage imposed on Exxon was around five billion dollars, which on appeal the US Supreme Court reduced to half a billion dollars. (Exxon Shipping Co. v. Baker, 554 U.S. 471 (2008))

³²Note that this analysis can be extended to the case of multiple victims using the approach of Kelsey and Spanjers (2004).

³³Note that Kilka and Weber (2001) do estimate degrees of ambiguity-aversion in their experiments.

Shavell (1987) argues that we cannot get optimal levels of unobservable actions by the agents. In the present paper we show that when there is ambiguity, it is possible to design a liability rule where the two agents undertake optimal actions both observable and unobservable. This is achieved by a rule which couples negligence and punitive damages in the form of an ambiguous threat of a large fine. This can explain the use of punitive damages in tort as a deterrence mechanism. The results also agree with the analysis of Polinsky and Shavell (1998) that punitive damages should be used if the injurer is not liable to the full extent of the damage caused by the accident. We have argued in particular this can be relevant for accidents which cause environmental damage.

Here we have questioned how behavioural issues may affect the analysis of liability rules. Much of the law and economics literature assumes decision makers follow the Savage axioms. However the arguments of Ellsberg (1961) suggest that individuals deviate from SEU by avoiding situations with unknown probabilities. The law should deal with actual behaviour and not idealised rational decisions. Hence we need to study how legal rules are affected by behavioural issues. Allowing for the possibility of ambiguity may be specifically relevant in case of tort. Accidents occur too infrequently to estimate subjective probabilities from relative frequencies. We find that the standard result that strict liability and negligence are efficient is no longer true but may be differently effective depending on ambiguity attitudes of the agents.³⁴ Instead we can show that a negligence rule coupled with punitive damages for excessive loss can provide the right incentives. Analysing legal rules by using a decision making model other than SEU may throw more light on their effectiveness.

There are a number of directions in which this research could be extended. Firstly one could study how other areas of law are affected by ambiguity. Our proposed liability rule works because ambiguity causes the two agents to attach different decision weights to the same event. This could be important in contract law. Criminal law is another possible application. Ambiguous punishments could make legal sanctions more effective.

Secondly one could investigate the implications of other recent developments in decision theory for tort law. A promising direction would be to study the implications of unawareness. Unawareness refers to situations where, individuals do not know all the possible consequences of their actions. This may be relevant for tort, since there are many circumstances in which potential injurers may be unaware that their actions have the capacity to cause damage to victim's property. In particular this may arise when the injurer is using a new technology.

References

- ARLEN, J. (1998): "The Future of Behavioral Economics and the Law," *Vanderbilt Law Review*, 55, 1765.
- BAR-GILL, O. (2006): "The Evolution and Persistence of Optimism in Litigation," *Journal of Law, Economics, and Organization*, 22, 490–507.
- BATTIGALLI, P., S. CERREIA-VIOGLIO, F. MACCHERONI, AND M. MARINACCI (2012): "Self-confirming equilibrium and model uncertainty," *working paper*, *Universita Bocconi*.
- BEN-SHAHAR, O. (2000): "Causation and Foreseeability," in *Encyclopedia of Law and Economics*, ed. by B. Bouckaert, and G. D. Geest. Edward Elgar.

³⁴Using smooth ambiguity model Klibanoff, Marinacci, and Mukerji (2005), most of our results will stand, apart from the robustness of the negligence rule and the optimal tort rule in Section 4, which are driven by the specific model of ambiguity we use.

- BIGUS, J. (2006): “Tort Liability and Probability Weighting Function According to Prospect Theory,” in *Sixteenth annual meeting of the American Law and Economics Association*, Berkeley, California.
- BROWN, J. P. (1973): “Toward an Economic Theory of Liability,” *Journal of Legal Studies*, 2, 323–349.
- CALABRESI, G. (1970): *The Cost of Accidents: A Legal and Economic Analysis*. Yale University Press, New Haven.
- CALCOTT, P., AND S. HUTTON (2006): “The Choice of a Liability Regime when there is a Regulatory Gatekeeper,” *Journal of Environmental Economics and Management*, 51, 153–164.
- CAMERER, C. F., AND M. WEBER (1992): “Recent Developments in Modelling Preferences: Uncertainty and Ambiguity,” *Journal of Risk and Uncertainty*, 5, 325–370.
- CHAKRAVARTY, S., AND D. KELSEY (2016): “The Act of God Clause,” *Journal of Public Economic Theory*, forthcoming.
- CHAPPE, N., AND R. GIRAUD (2013): “Confidence Optimism and Litigation, A Litigation Model under Ambiguity,” *working paper*, *University of Franche-Comté*.
- CHATEAUNEUF, A., J. EICHBERGER, AND S. GRANT (2007): “Choice under Uncertainty with the Best and Worst in Mind: NEO-Additive Capacities,” *Journal of Economic Theory*, 137, 538–567.
- CHORVAT, T. R. (2001): “Ambiguity and Income Taxation,” *Cardozo Law Review*, 23, 617–651.
- COASE, R. H. (1960): “The Problem of Social Cost,” *Journal of Law and Economics*, 3, 1–44.
- COOTER, R. (1982): “Economic Analysis of Punitive Damages,” *Southern California Law Review*, 56, 79–102.
- COOTER, R., AND T. ULEN (1999): *Law and Economics*. Addison Wesley, 3rd edn.
- DEJOY, D. M. (1989): “The Optimism Bias and Traffic Accident Risk Perception,” *Accident Analysis and Prevention*, 21, 333–40.
- DOW, J., AND S. R. C. WERLANG (1994): “Nash Equilibrium under Uncertainty: Breaking Down Backward Induction,” *Journal of Economic Theory*, 64, 305–324.
- EICHBERGER, J., AND D. KELSEY (2011): “Are the Treasures of Game Theory Ambiguous?,” *Economic Theory*, 48, 313–339.
- (2014): “Optimism and Pessimism in Games,” *International Economic Review*, 55, 483–505.
- EICHBERGER, J., D. KELSEY, AND B. SCHIPPER (2008): “Granny Versus Game Theorist: Ambiguity in Experimental Games,” *Theory and Decision*, 64, 333–362.
- EIDE, E. (2005): “Accident Liability with Rank Dependent Expected Utility,” *working paper* *University of Oslo*.
- ELLSBERG, D. (1961): “Risk, Ambiguity and the Savage Axioms,” *Quarterly Journal of Economics*, 75, 643–669.

- FAURE, M. G. (2008): “Calabresi and Behavioural Tort Law and Economics,” *Erasmus Law Review*, 1, 75–102.
- FRANZONI, L. (2012): “Optimal Liability Design under Ambiguity,” *working paper*, University of Bologna.
- GIGERENZER, G. (2005): “Is the Mind Irrational or Ecologically Rational?,” in *The Law and Economics of Irrational Behavior*, ed. by F. Parisi, and V. L. Smith, pp. 37–67, Stanford, California. Stanford University Press.
- INNES, R. (1999): “Optimal liability with stochastic harms, judgement-proof injurers, and asymmetric information,” *International Review of Law and Economics*, 19, 181–203.
- JOLLS, C., C. SUNSTEIN, AND R. THALER (1998): “Behavioral Economic Analysis of Redistributive Legal Rules,” *Stanford Law Review*, 50, 1471–1550.
- KAHNEMAN, D., J. L. KNETSCH, AND R. THALER (1990): “Experimental Tests of the Endowment Effect and the Coase Theorem,” *Journal of Political Economy*, 98, 1325–1340.
- KELSEY, D., AND W. SPANJERS (2004): “Ambiguity in Partnerships,” *Economic Journal*, 114, 528–546.
- KILKA, M., AND M. WEBER (2001): “What Determines the Shape of the Probability Weighting Function under Uncertainty?,” *Management Science*, 47, 1712–1726.
- KLIBANOFF, P., M. MARINACCI, AND S. MUKERJI (2005): “A Smooth Model of Decision Making under Ambiguity,” *Econometrica*, 73, 1849–1892.
- KOROBKIN, R., AND T. ULEN (2000): “Law and Behavioural Science: Removing the Rationality Assumption from Law and Economics,” *California Law Review*, 88, 1051–1144.
- LANDES, W. M., AND R. A. POSNER (1987): *The Economic Structure of Tort Law*. Harvard University Press, Cambridge, Mass.
- MILGROM, P., AND J. ROBERTS (1990): “Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities,” *Econometrica*, 58, 1255–1277.
- PARISI, F., AND V. L. SMITH (2005): *The Law and Economics of Irrational Behavior*. Stanford University Press., Stanford, California.
- POLINSKY, A. M., AND S. SHAPELL (1998): “Punitive Damages: An Economic Analysis,” *Harvard Law Review*, 111, 869–962.
- POSNER, R. A. (2007): *Economic Analysis of Law*. Aspen Law and Business, 7th ed. edn.
- SAVAGE, L. J. (1954): *Foundations of Statistics*. Wiley, New York.
- SCHMEIDLER, D. (1989): “Subjective Probability and Expected Utility without Additivity,” *Econometrica*, 57, 571–587.
- SEGAL, U., AND A. STEIN (2006): “Ambiguity Aversion and the Criminal Process,” *Notre Dame Law Review*, 81, 1495–1551.
- SHAPELL, S. (1980): “Strict Liability versus Negligence,” *Journal of Legal Studies*, 9, 1–25.
- (1987): *Economic Analysis of Accident Law*. Harvard University Press, Cambridge, Mass.

- SHUMAN, D. W. (1993): “The Psychology of Deterrence in Tort Law,” *Kansas Law Review*, pp. 115–913.
- TEITELBAUM, J. (2007): “A Unilateral Accident Model under Ambiguity,” *Journal of Legal Studies*, 36, 431–477.
- TOPKIS, D. M. (1998): *Supermodularity and Complementarity*. Princeton University Press, New Jersey.
- TREICH, N. (2010): “The Value of a Statistical Life under Ambiguity Aversion.,” *Journal of Environmental Economics and Management*, 59, 15–26.

A Appendix

This appendix contains the proofs of our results.

A.1 Observable Actions

Proof of Proposition 1: Since the victim never pays for the loss and care is costly, it is a dominant strategy for her to provide zero care. This remains true with ambiguity because neo-additive preferences respect dominance. Thus the support of the injurer's equilibrium belief must be $y = 0$. Given this the injurer's (Choquet) expected utility is:

$$-L_i^S(x, y) = -\delta\alpha D(x, 0) - (1 - \delta)\pi D(x, 0) - a(x).$$

The marginal damage with ambiguity is $(\delta\alpha + (1 - \delta)\pi) MD_x(x, 0)$, which is increasing in α . Since marginal cost is unchanged by ambiguity, the injurer's level of care will also be increasing in α . This is greater than/less than the marginal damage without ambiguity as $\alpha \gtrless \pi$. The injurer will choose a care level which is greater than/less than that without ambiguity as $\alpha \gtrless \pi$. ■

Proof of Proposition 2: Providing stipulated care is enough to protect the injurer from liability and hence also ambiguity. Since care is costly, the injurer will never provide more than stipulated care. Hence the injurer must choose between stipulated care and a lower level of care.

Assume first that both agents are ambiguity-averse. We shall prove by contradiction that there does not exist an equilibrium in which the injurer provides less than stipulated care. Suppose if possible that such an equilibrium exists. Then the victim's best response is to provide zero care since she is not liable. The injurer's best response to zero care is to provide more than the efficient level of care. (This is similar to the argument in the proof of Proposition 1.) However this contradicts the original hypothesis that there is an equilibrium in which the injurer chooses less than stipulated care.

Thus the injurer will neither provide more nor less than stipulated care. The only remaining possibility is that the injurer provides stipulated care. Now consider the victim. If she believes the injurer will provide the stipulated level of care x^* , then her Choquet expected utility is $-\delta D(x^*, y) - (1 - \delta)\pi D(x^*, y) - b(y)$. Compared to the situation without ambiguity, the marginal damage has been raised from $\pi \Delta_y D(x^*, y)$ to $(\delta + (1 - \delta)\pi) \Delta_y D(x^*, y)$. Since the marginal cost of care is unchanged, the victim will choose to provide more than the efficient level of care. This establishes that $\langle x^*, \hat{y} \rangle$ are equilibrium care levels when the agents are ambiguity-averse. A similar analysis applies whenever the agents are pessimistic.

In the case where the agents have a low level of optimism, we claim that $\langle x^*, \tilde{y} \rangle$ is an equilibrium, where $\tilde{y} < y^*$. Firstly consider the victim. If she believes that the injurer will choose the stipulated action, x^* , her (Choquet) expected utility will be:

$$-\delta\alpha D(x^*, y) - (1 - \delta)\pi D(x^*, y) - b(y).$$

Denote the value of y which maximises this expression by $\tilde{y}(\alpha)$. It is a monotonic decreasing function of α .

Secondly consider the injurer. If he provides stipulated care, his (Choquet) expected pay-off will be $-a(x^*)$. If the injurer provides less than stipulated care, the victim will provide zero care since she will not be liable. Thus the injurer's expected utility will be

$$-\delta\alpha D(x, 0) - (1 - \delta)\pi D(x, 0) - a(x).$$

Let $\tilde{x}(\alpha)$ denote the value of x which maximises this expression. Then the injurer should choose

whichever of x^* and $\tilde{x}$ gives the highest expected pay-off. We have already established that x^* is a best response when $\alpha = 1$. By continuity it will remain a best response when α is in a neighbourhood of 1.

When the injurer is more optimistic, i.e. α is smaller, $\tilde{x}(\alpha)$ will be the best response and hence the injurer will assume liability. In this equilibrium the victim provides zero care. ■

A.2 Unobservable Actions

We now analyse the model from Section 3.4, where the agents may take some actions which the court cannot observe. The marginal social benefit from changes in x, y, s and t , MS_x, MS_y, MS_s and MS_t respectively are defined by:

$$\begin{aligned} MS_x &= -s\Delta a(x) - \pi\Delta_x\tilde{D}(x, y, s, t), \\ MS_y &= -t\Delta b(y) - \pi\Delta_y\tilde{D}(x, y, s, t), \\ MS_s &= \Delta\phi(s) - \pi\Delta_s\tilde{D}(x, y, s, t) - \frac{1}{r}a(x) = 0, \\ MS_t &= \Delta\psi(t) - \pi\Delta_t\tilde{D}(x, y, s, t) - \frac{1}{r}b(y) = 0. \end{aligned}$$

The efficient actions are (x^*, y^*, s^*, t^*) , which are given by the conditions: $MS_x \geq 0, x \leq x^*$; $MS_x \leq 0, x \geq x^*$; etc.³⁵ Full efficiency can be defined analogously.

Proof of Proposition 3 : As in the proof of Proposition 1, it is a dominant strategy for the victim to provide zero care, and to select the value of the non-observable action which maximises her private utility. Denote this by t' . This is characterised by the first order condition $MB_t^v = \psi(t') - \psi(t' - \frac{1}{r}) = 0$, (recall $b(0) = 0$).

Hence the support of the injurer's belief is $\langle 0, t' \rangle$. For him the worst outcome is that an accident occurs and the victim has taken the lowest possible observable action and the highest non-observable action. Given this, his (Choquet) expected utility is:

$$\phi(s) - \delta\alpha\tilde{D}(x, 0, s, \bar{t}) - (1 - \delta)\pi\tilde{D}(x, 0, s, t') - a(x).$$

The marginal damage with ambiguity is $\delta\alpha\Delta_x\tilde{D}(x, 0, s, \bar{t}) + (1 - \delta)\pi\Delta_x\tilde{D}(x, 0, s, t')$. This is increasing in α since $\Delta_x\tilde{D}(x, 0, s, \bar{t}) > \Delta_x\tilde{D}(x, 0, s, t')$ by increasing differences. Since marginal cost is unchanged by ambiguity, the injurer's level of care will also be increasing in α . If $\alpha > \pi$, then marginal damage is greater with ambiguity. Thus the injurer will choose a care level which is greater than that without ambiguity in this case.

The marginal damage from increasing s is $\delta\alpha\Delta_s\tilde{D}(x, 0, s, \bar{t}) + (1 - \delta)\pi\Delta_s\tilde{D}(x, 0, s, t')$. By similar reasoning, if $\alpha > \pi$, this is higher than without ambiguity. The marginal cost of a higher non-observable action is $\frac{a(x)}{r}$. If $\alpha > \pi$, this will be greater than the marginal cost without ambiguity because x is larger in equilibrium. The marginal benefit of s is not affected by ambiguity. However both the direct cost of s and the potential liability are greater with ambiguity. Hence the injurer will choose a lower non-observable action.

If $\alpha = 0$ then, if there is ambiguity, the marginal damage from increasing x is $(1 - \delta)\pi\Delta_x\tilde{D}(x, 0, s, t')$, which is less than that without ambiguity. Hence the injurer will choose a lower level of care with ambiguity. Similarly the marginal damage associated with the non-observable action $(1 - \delta)\pi\Delta_s\tilde{D}(x, 0, s, t')$ is less. The marginal cost, $\frac{a(x)}{r}$ is lower because x is lower and the marginal benefit is unchanged. This implies that the injurer will choose a higher non-observable action with ambiguity. ■

Proof of Proposition 4 Let $\hat{\Gamma}$ denote a modified game in which the injurer's care level is

³⁵Here, as in previous sections, x^* and y^* , x^* , y^* , s^* and t^* are derived under the assumption of SEU.

fixed at $x = x^s$, the other strategies and pay-offs are unchanged. Since this game is supermodular, it has an equilibrium, which will in general depend on α .³⁶ Let s', y', t' denote the equilibrium values of s, y , and t in this subgame. Let $\tilde{\Gamma}$ denote the game which would arise if the tort rule was strict liability. The game $\tilde{\Gamma}$ is also supermodular and hence has an equilibrium $\langle x, s, y, t \rangle = \langle \tilde{x}, \tilde{s}, \tilde{y}, \tilde{t} \rangle$.

The injurer will either choose care level x^s if his (Choquet) expected pay-off from $\langle x^s, s', y', t' \rangle$ is greater than that from $\langle \tilde{x}, \tilde{s}, \tilde{y}, \tilde{t} \rangle$ or choose $\tilde{x}$ otherwise. It is straightforward to show that the one he chooses is an EUA. ■

Proof of Proposition 5: First consider the case where both agents are ambiguity averse. By similar reasoning to that used in the proof of Proposition 2 we may show that ambiguity-aversion will cause the injurer to choose the stipulated level of care x^* . Consequently he will not be liable and will hence choose the level of unobservable action, s'' , where marginal benefit is equal to marginal cost, $MB_s^i(s'') = \frac{a(x^*)}{r}$. This will be above the efficient level since it does not take account of the externality.

Ambiguity-aversion will induce the victim to be cautious and choose care (resp. unobservable action) above (resp. below) the efficient levels. This is reinforced by the fact that s is too high and supermodularity of the $\tilde{D}$ function, which increases the marginal benefit of y and reduces that of t .

Now suppose that the agents are not ambiguity averse, i.e. $0 \leq \alpha < 1$. Then, as in the proof of Proposition 2, the injurer will need to make a discrete choice between providing efficient care x^* and providing a lower level of care $x(\alpha)$.

For low levels of optimism (i.e. α close to 1) the injurer will continue to choose x^* in equilibrium. This is due to the discontinuity in the injurer's pay-off under negligence, which penalises care levels below x^* disproportionately. There is an equilibrium $\langle x^*, \tilde{y}, s'', \tilde{t} \rangle$, where $\tilde{y} < y^*$, $\tilde{t} < t^*$ and s'' satisfies $MB_s^i(s'') = \frac{a(x^*)}{r}$.³⁷ First consider the victim. If she believes that the injurer will choose actions, x^* and s'' her (Choquet) expected utility will be:

$$-\delta\alpha\tilde{D}(x^*, y, s'', t) - (1 - \delta)\pi\tilde{D}(x^*, y, s'', t) - b(y). \quad (2)$$

Denote the values of y and t which maximise this expression by $\tilde{y}(\alpha)$ and $\tilde{t}(\alpha)$. They are both decreasing in α .

Now consider the injurer. If he provides stipulated care his (Choquet) expected pay-off will be

$$\phi(s'') - a(x^*). \quad (3)$$

If the injurer provides less than stipulated care, the victim will provide zero care since she will not be liable. Thus the injurer's expected utility will be

$$\phi(s) - \delta\alpha\tilde{D}(x, 0, s, t'') - (1 - \delta)\pi\tilde{D}(x, 0, s, t'') - a(x), \quad (4)$$

where t'' is defined by $MB_t^v(t'') = 0$. Let $\hat{x}(\alpha)$ and $\hat{s}(\alpha)$ denote the values of x and s which maximise this expression. Then the injurer should choose either $\langle x^*, s'' \rangle$ or $\langle \hat{x}(\alpha), \hat{s}(\alpha) \rangle$ depending on whether equation (2) is greater or less than (4). We have already established that $\langle x^*, s'' \rangle$ is a best response when $\alpha = 1$. By continuity it will remain a best response when α is in a neighbourhood of 1.

When the injurer is more optimistic, i.e. α is smaller, $\tilde{x}(\alpha) < x^*$ and $\tilde{s}(\alpha)$ will be the best response and hence the injurer will assume liability. In this equilibrium, the victim provides zero care and chooses the unobservable action $t = t''$. ■

³⁶See Milgrom and Roberts (1990) and Topkis (1998).

³⁷If $\tilde{D}$ is not separable this will be different from the s'' in Proposition 4.

A.3 Optimal Tort Rules

Proof of Proposition 6: The pay-offs of the victim and injurer are respectively:

$$\begin{aligned} \psi(t) - \delta \tilde{D}(\hat{x}, y, \bar{s}, t) - (1 - \delta)\pi \tilde{D}(x, y, s, t) - tb(y), & \text{ if } x \geq \hat{x} \text{ and } L < \hat{L}; \\ \psi(t) - tb(y), & \text{ if } x < \hat{x}; \\ \psi(t) - tb(y) + F, & \text{ if } L \geq \hat{L}; \end{aligned}$$

$$\begin{aligned} \phi(s) - sa(x), & \text{ if } x \geq \hat{x} \text{ and } L < \hat{L}, \\ \phi(s) - \delta \tilde{D}(x, 0, s, \bar{t}) - (1 - \delta)\pi \tilde{D}(x, y, s, t) - sa(x), & \text{ if } x < \hat{x}, \text{ and } L < \hat{L}; \\ \phi(s) - \delta [\tilde{D}(x, 0, s, \bar{t}) - F] - (1 - \delta)\pi \tilde{D}(x, y, s, t) - sa(x), & \text{ if } s > \hat{s}, t \leq \hat{t}; \\ \phi(s) - \delta [\tilde{D}(x, 0, s, \bar{t}) - F] - (1 - \delta)\pi [\tilde{D}(x, y, s, t) - F] - sa(x), & \text{ if } s > \hat{s}, t > \hat{t}. \end{aligned}$$

The injurer will set $x = \hat{x}$ if

$$\phi(s) - sa(\hat{x}) \leq \phi(s) - \delta [\tilde{D}(x, 0, s, \bar{t})] - (1 - \delta)\pi [\tilde{D}(x, y, s, t)] - sa(x), \forall x \neq \hat{x}.$$

So a sufficiently pessimistic injurer will always find it beneficial to invest in care levels $\hat{x}$. If the damage level is too high, $L \geq \hat{L}$, then the (Choquet) expected utility includes the fine F . Hence

$$\phi(s) - \delta [\tilde{D}(x, 0, s, \bar{t}) + F] - (1 - \delta)\pi [\tilde{D}(x, y, s, t) + F] - sa(x).$$

Provided F is set such that the marginal cost, is equal to MB_s at the efficient level of $\hat{s}$, then the level of s will be optimal. Given that injurer is going to invest $\hat{x}$, the victim has an expected cost

$$\psi(t) - \delta \tilde{D}(\hat{x}, y, \bar{s}, t) - (1 - \delta)\pi \tilde{D}(x, y, s, t) - tb(y).$$

Assume that the ambiguity perceived by the victim is not too high. So her marginal benefit of t and y are respectively,

$$\begin{aligned} t : \Delta\psi(t) - \delta \Delta_t \tilde{D}(\hat{x}, y, \bar{s}, t) - (1 - \delta)\pi \Delta_t \tilde{D}(x, y, s, t) - \frac{b(y)}{r}, & \text{ if } L < \hat{L}, \\ \Delta\psi(t) - \frac{b(y)}{r} & \text{ otherwise.} \end{aligned}$$

$$\begin{aligned} y : -\delta \Delta_y \tilde{D}(\hat{x}, y, \bar{s}, t) - (1 - \delta)\pi \Delta_y \tilde{D}(x, y, s, t) - t\Delta b(y), & \text{ if } L < \hat{L}, \\ -t\Delta b(y) & \text{ otherwise.} \end{aligned}$$

The injurer will choose care and unobservable action levels such that the size of the damage is $L < \hat{L}$.

Given that the injurer chooses the stipulated observable action, we now check if he will undertake the optimal level of unobservable action. The victim will bear all the losses unless the threshold is exceeded. However this is a best case scenario, which only occurs out of equilibrium. Since the victim is assumed to be ambiguity averse the fine F does not enter her pay-off. So the victim will maximise

$$-\delta \tilde{D}(\hat{x}, y, \hat{s}, t) - (1 - \delta)\pi \tilde{D}(\hat{x}, y, \hat{s}, t) - tb(y).$$

This differs from the ex-ante social welfare by $-sa(x)$, which is a lump sum from the victim's point of view. It follows that the victim will choose the fully efficient values of y and t . ■