Centrifuge modelling of tunnelling induced ground displacements: pressure and displacement control tunnels

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Abstract

Predicting the impact of tunnelling-induced ground movements on existing foundation systems or infrastructure is an important stage during tunnel design or risk assessment processes. Centrifuge modelling has been used extensively as a tool to study soil movements caused by tunnelling and their interactions with existing structures. In 2D plane-strain centrifuge models, tunnel volume loss can be simulated in a variety of ways, but is conventionally done using a fluid-filled flexible membrane or a rigid boundary mechanical model tunnel. The choice of model tunnel has an impact on the imparted tunnel boundary displacements and resulting ground deformations, yet a thorough quantitative evaluation of these effects has not been conducted. This paper aims to address this by contrasting plane-strain centrifuge test results from experiments using a flexible membrane model tunnel with those from a newly developed eccentric rigid boundary mechanical model tunnel. A quantitative

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assessment of surface and subsurface settlement trough characteristics as well as soil shear and volumetric response is provided. Results from numerical analyses using a hypoplastic constitutive model are also included, focusing on evaluating centrifuge spin-up effects and contrasting numerical outcomes with experimental data relating to settlement trough characteristics and soil shear/volumetric response. The outcomes of this paper should benefit future researchers considering which type of model tunnel to adopt when developing centrifuge tests related to 2D plane-strain tunnel modelling.

Keywords: Tunnelling, Centrifuge modelling, Greenfield settlements, Numerical modelling, Hypoplastic

1 Highlights

- 2 • Tunnelling induced ground movements for pressure and displacement
3 control tunnels.
- 4 • Centrifuge test results of greenfield tunnelling in sands.
- 5 • Calibration of hypoplastic constitutive model for numerical analysis.
- 6 • Soil volumetric and shear strain distribution due to tunnel volume loss.

7 1. Introduction

8 Tunnel construction in urban areas often occurs close to existing founda-
9 tion systems or infrastructure, such as pile groups, retaining structures, and
10 pipelines. Understanding the impact of tunnelling-induced ground move-
11 ments on these critical systems is an important stage of the tunnel design or
12 risk assessment processes.

13 Centrifuge modelling has been used extensively as a tool to study soil
14 movements caused by tunnelling because of its ability to reproduce full-
15 scale soil stress profiles within reduced scale models. Evaluating the ef-
16 fect of tunnelling-induced ground movements on structures is undoubtedly
17 a three-dimensional (3D) problem. Detailed simulation of the 3D interac-
18 tions between tunnel construction and existing structures can be achieved in
19 a centrifuge, for example the miniature boring machine from [Nomoto et al.](#)
20 [\(1999\)](#), or the staged de-pressurisation systems developed by [Ng et al. \(2013\)](#)
21 and [Gue and Elshafe \(2019\)](#). These methods add a level of complexity to
22 the modelling process and, in the case of the mechanical excavation systems,
23 require a significant amount of space within the centrifuge cradle. As a re-
24 sult, centrifuge models of the tunnelling process often simplify the problem
25 by considering the two-dimensional (2D) displacements (horizontal and verti-
26 cal) in the direction transverse to the longitudinal axis of the tunnel (i.e. the
27 plane-strain condition representing displacements a certain distance behind
28 the excavation face). Whilst this approach has limitations compared to the
29 real 3D scenario ([Franza et al., 2019b](#)), 2D centrifuge modelling has provided
30 valuable data to improve understanding of soil and soil-structure interaction
31 behaviour for tunnelling problems.

32 One of the early 2D plane-strain techniques used to simulate the effect of
 33 tunnel volume loss in centrifuge tests involved decreasing the internal pres-
 34 sure of a tunnel-shaped airbag ([Atkinson et al., 1975](#); [Potts, 1976](#); [Hagiwara
 35 et al., 1999](#)). A limitation of the pressurised airbag technique is that the com-
 36 pressibility of air makes it difficult to assess the magnitude of tunnel volume
 37 loss $V_{l,t}$ (i.e. the ground loss at the boundary of the tunnel). This limitation
 38 can be eliminated by replacing the air with water or oil (which may be as-
 39 sumed incompressible within the range of applied pressures). A concentric
 40 fluid-filled flexible membrane (FM) model tunnel has been used by several
 41 researchers ([Loganathan et al., 2000](#); [Vorster et al., 2005](#); [Jacobsz, 2003](#));
 42 the concentric system implies the flexible membrane is sealed onto a shaft
 43 located centrally within the membrane. For fluid-filled FM model tunnels,
 44 the deformed tunnel shape is somewhat unknown. As discussed in [Marshall
 45 and Franza \(2017\)](#) and [Ritter et al. \(2017\)](#), the shape of the tunnel lining is
 46 controlled by the equilibrium condition between the pressures from the soil
 47 and the fluid within the tunnel. Displacements around real shallow tunnels
 48 have been shown to take an eccentric shape, with greater displacements oc-
 49 curring at the tunnel crown compared to the base (invert) ([Loganathan and
 50 Poulos, 1998](#)). In order to encourage a more realistic profile of displacements,
 51 [Marshall \(2009\)](#); [Marshall et al. \(2012\)](#) modified the flexible membrane model
 52 tunnel to create an eccentric shape, where soil displacements at the invert of
 53 the tunnel were limited. The eccentric model tunnel system has been applied
 54 in several subsequent centrifuge studies related to tunnelling ([Farrell, 2010](#);
 55 [Williamson, 2014](#); [Zhou, 2015](#); [Franza, 2016](#); [Ritter et al., 2017](#)).

56 Another option for simulating 2D tunnel volume loss in the transverse

plane involves the use of a rigid boundary mechanical (RBM) model tunnel. For RBM model tunnels, the deformed tunnel shape, and by implication the soil displacements around the tunnel, are known. An RBM model tunnel was developed by [Katoh et al. \(1998\)](#) and has been used by several researchers to study soil-structure interaction problems ([Boonsiri and Takemura, 2015b,a](#)). This mechanical tunnel imposes concentric, uniform displacements around its circumference. [Shahin et al. \(2011\)](#) also developed a mechanical tunnelling device with 12 segments that move vertically downwards and converge towards the tunnel centre, achieving a fixed tunnel invert during tunnel volume loss, however this device was not used within a geotechnical centrifuge. To better replicate (in a centrifuge model) the real displacements that occur around shallow tunnels (according to [Loganathan and Poulos \(1998\)](#)), [Song et al. \(2018\)](#) developed a novel eccentric rigid boundary mechanical (eRBM) model tunnel that is able to induce an eccentric profile of soil displacements around the tunnel, predominately near the tunnel crown (results from tests using this model tunnel are presented in this paper).

[Marshall and Franza \(2017\)](#) showed, based on the data from [Boonsiri and Takemura \(2015b\)](#), that the set of equations provided by [Marshall et al. \(2012\)](#) for the prediction of settlement width parameters does not always provide a good fit to data obtained using a concentric RBM model tunnel. It was difficult to isolate a specific reason for the differences, since the variation of such factors as soil type, sample preparation method, and measurement errors between the different studies inevitably affected results. However, [Marshall and Franza \(2017\)](#) suggested that the different tunnel boundary conditions between the data sets (FM in [Marshall \(2009\)](#); [Franza \(2016\)](#);

82 [Zhou et al. \(2014\)](#) versus displacement control RBM lining in [Boonsiri and](#)
83 [Takemura \(2015b\)](#)) was probably the most influential parameter.

84 It is important to have an understanding of the implications of the choice
85 of model tunnel adopted for these types of centrifuge tests, however a thor-
86 ough evaluation of this has never been conducted. This paper aims to address
87 this shortcoming by investigating the effect of the type of centrifuge model
88 tunnel (i.e. flexible membrane FM or rigid boundary mechanical RBM) used
89 to simulate 2D tunnelling displacements. The paper provides details of the
90 newly developed eccentric rigid boundary mechanical eRBM model tunnel
91 (first presented in [Song et al. \(2018\)](#)) as well as data obtained using the
92 novel eRBM model tunnel for greenfield tunnelling scenarios. Experimen-
93 tal results are used to explore the effect of the different tunnel boundary
94 conditions on resulting ground deformations and shear/volumetric response.
95 Results from 2D finite element (FE) analyses using a hypoplastic constitu-
96 tive model are also presented. The FE analyses aimed to simulate the two
97 tunnel volume loss techniques applied in the centrifuge tests: (1) pressure
98 controlled (replicating FM model tunnels), and (2) eccentric displacement
99 controlled (replicating the eRBM model tunnel). Numerical results are used
100 to explore the effect of centrifuge spin-up on outcomes from the eRBM and
101 FM centrifuge tests, as well as evaluate the ability of the numerical model
102 to match centrifuge test results of ground settlements and shear/volumetric
103 response.

104 It is important to recognise that this paper does not attempt to make
105 any recommendation as to which type of model tunnel best replicates reality.
106 Each approach will have advantages and limitations, and neither gives a

complete representation of the real tunnelling scenario. The intention of this paper is to provide quantitative evidence of the implications of the use of the different model tunnel types, providing readers with the information they need to judge which approach best suits their needs.

2. Model tunnels

2.1. Flexible membrane (FM) model tunnel

The flexible membrane model tunnel has been used extensively within centrifuge tests (Loganathan et al., 2000; Vorster et al., 2005; Jacobsz, 2003). The model tunnel used in the tests presented in this paper was a version of the eccentric flexible membrane model tunnel first used by Marshall et al. (2012) and is described in detail in Zhou (2015); Zhou et al. (2014). The model tunnel is 90 mm in diameter and consists of an inner aluminium core encased within a 1 mm thick latex membrane. Water was used to fill the annular space between the inner core and the membrane, with tunnel volume loss achieved by extracting the water using an actuator controlled by the test operator.

2.2. Eccentric rigid boundary mechanical (eRBM) model tunnel

In a rigid boundary mechanical (RBM) model tunnel, the displacements required to replicate tunnel volume loss are induced by controlling the displacement of the tunnel boundary. The newly developed eccentric rigid boundary mechanical eRBM model tunnel is illustrated in Figure 1. This model tunnel enables non-uniform radial displacements around the tunnel and has a maximum diameter of 90 mm, a maximum displacement of 1.4 mm

130 at the tunnel crown, 0.9 mm at the springline, and zero displacement at the
 131 tunnel invert.

Figure 1: Newly developed eccentric rigid boundary mechanical (eRBM) model tunnel

132 The tunnel contains a single bi-directional screw shaft which is equipped
 133 with two flange nuts. Two hexagonal wedge-shaped shafts are fixed into
 134 the bi-directional ball screw flange nuts in opposite directions. Six tunnel
 135 segments representing the tunnel boundary are connected to the hexago-
 136 nal wedge-shaped shafts via six linear guide rails; these enable the relative
 137 movement between the segment and shaft along the wedge taper angle. The
 138 bi-directional screw shaft is driven by a stepper motor through a 1:10 ratio
 139 gearbox. Tunnel volume loss is achieved by rotating the bi-directional screw
 140 shaft, which causes the two hexagonal wedge-shaped shafts to move in op-

141 posite directions, causing the relative movement between the six individual
142 segments according to the different taper angles of the wedge-shaped shafts.
143 The wedge-shaped shaft has six surfaces, where the taper angle varies from 4°
144 at the tunnel crown to 0° at the tunnel invert. The six tunnel segments move
145 eccentrically towards the tunnel centreline and create the desired non-uniform
146 tunnel boundary displacement. Figure 2 shows the positions of the six tunnel
147 segments at the initial and final stages of testing. The tunnel volume loss can
148 be related to the horizontal displacement of the wedge-shaped shafts, which is
149 measured using a linear variable differential transformer (LVDT). In tests, the
150 computer controlled stepper motor was set to rotate by increments of 450° ,
151 which corresponds to 0.625 mm (1/8 of screw pitch) horizontal movement of
152 the wedge-shaped shafts and 0.18% of tunnel volume loss. The maximum
153 horizontal displacement of the wedge-shaped shafts is around 20 mm, giving
154 a maximum tunnel volume loss of 3.5%.

155 A gap is created between adjacent segments that make up the tunnel
156 circumference, as illustrated in Figure 2, creating a ‘step’ between segments
157 during tunnel volume loss. The maximum size of the ‘step’ (at maximum
158 tunnel volume loss) is 0.3 mm. The gaps are initially 2 mm (before tunnel
159 volume loss) and vary with tunnel volume loss at different rates for the various
160 gaps. Copper strips were placed along the gaps in the direction of the tunnel
161 length to create a smooth tunnel surface, as shown in Figure 2. The tunnel
162 was then sealed within two latex membranes to protect it from soil intrusion
163 during tests; this also ensured a smooth variation of soil deformation was
164 achieved across the gaps.

Figure 2: eRBM model tunnel deformed shape

165 3. Centrifuge modelling

166 This paper uses data from six centrifuge tests, as described in Table 1.
167 The flexible membrane (FM) test data were obtained from previous studies
168 (Xu et al., 2019; Zhou, 2015; Marshall, 2009), whereas the eRBM centrifuge
169 test data were newly acquired as part of the current study. A fine-grained
170 silica sand commonly known as Leighton Buzzard Fraction E sand was used
171 for all tests, which has a typical average diameter D_{50} of 0.14 mm and a
172 specific gravity G_s of 2.65. The sand has a maximum (e_{max}) and minimum
173 (e_{min}) void ratio of 1.01 and 0.61, respectively, and a coefficient of uniformity
174 C_u of 1.58 (Tan, 1990). Details of the preparation methodologies for the
175 FM tests can be found in the references provided in Table 1. This section
176 is mainly dedicated to describing the new eRBM centrifuge tests. In this
177 paper, dimensions of the models and results are described in model scale,
178 unless otherwise stated.

179 Note that the tests presented in Table 1 are not all consistent in terms
180 of tunnel size and centrifuge scaling factor. As such, soil displacements for a
181 given C/D_t ratio at a given tunnel volume loss will not be wholly consistent
182 (i.e. for the same magnitude of tunnel volume loss, a larger tunnel will incur
183 larger soil displacements than a smaller tunnel) and the levels of soil dilation
184 around deeper tunnels would be less than for shallower tunnels, which would
185 affect the characteristics of the settlement troughs above the tunnel. The
186 implications of this will be discussed in detail later in the paper.

187 The three eRBM centrifuge tests were carried out on the University of
188 Nottingham Centre for Geomechanics (NCG) 2 m radius, 50 g-tonne geotech-
189 nical centrifuge at an acceleration of 80 g. The model strongbox has inner

Table 1: Centrifuge test details in model scale [prototype scale in parentheses]

Test name	C (mm) [m]	D_t (mm) [m]	$\sigma_{v,t}$ (kPa) [same]	Reference
FM $C/D_t=1.3$	120 [8.160]	90 [6.12]	172	Xu et al. (2019)
FM $C/D_t=2.0$	180 [14.400]	90 [7.2]	275	Zhou (2015)
FM $C/D_t=2.4$	151 [11.325]	62 [4.65]	209	Marshall (2009)
eRBM $C/D_t=1.3$	117 [9.360]	90 [7.2]	198	this study
eRBM $C/D_t=2.0$	180 [14.400]	90 [7.2]	275	this study
eRBM $C/D_t=2.4$	216 [17.280]	90 [7.2]	319	this study

C = depth to tunnel crown; D_t = tunnel diameter; $\sigma_{v,t}$ = vertical stress at tunnel axis depth.

width $\times$ height dimensions of 700 mm $\times$ 400 mm (Figure 3) and is 150 mm wide. The box has an aluminium back wall and a transparent front acrylic wall that enables the acquisition of digital images of the soil during tests (used to measure sub-surface soil displacements using image analysis techniques). The model tunnel has a diameter of 90 mm and is located 75 mm above the strongbox base, giving a clear distance of 30 mm between the bottom of the tunnel and the strongbox base. The tunnel is fixed within the front and back walls of the centrifuge strongbox, preventing any movement of the tunnel ends during tests.

3.1. Sample preparation

Samples were created using the dry sand pouring technique. The tunnel was secured within the strongbox before sand pouring. To achieve a uniform sample around the model tunnel, the strong box was placed with the back wall facing downwards, allowing the sand to be poured in the direction of the tunnel longitudinal axis, with a temporary plate used to support the sand along the plane corresponding to the ground surface (consistent with

Figure 3: Centrifuge model

206 [Vorster \(2006\)](#); [Marshall \(2009\)](#); [Zhou \(2015\)](#); [Franza \(2016\)](#); [Farrell \(2010\)](#)).
207 The sand was prepared according to a methodology calibrated to achieve a
208 relative density (I_d) of 90%. After sand pouring, a thin layer of dyed sand was
209 placed uniformly on the top surface of the sample to increase the contrast and
210 help with the tracking of soil displacements using image analysis. The front
211 acrylic window was then bolted to the strong box, which was then rotated
212 to its upright position, and the temporary plate was removed, revealing the
213 ground surface. The gearbox and stepper motor were then assembled and
214 the LVDT placed at the front acrylic window to measure the movement of
215 one of the wedge-shaped shafts.

216 *3.2. Instrumentation*

217 Two Canon Powershot G10 14.7 mega-pixel cameras were used to take
218 pictures of the soil behind the acrylic wall during tests. Two LED light strips
219 equipped with a diffusion plate were placed on either side of the strong box,
220 providing a uniform illumination to the acrylic wall. Surface and subsurface
221 soil movements were obtained using GeoPIV-RG ([Stanier et al., 2015](#)).

222 To assess the precision of the geoPIV-RG analysis based on the current
223 set up, two images were taken at elevated gravity with zero soil displace-
224 ments. For the analysis, 11844 subset patches were used, and the displace-
225 ment (in both horizontal and vertical directions) of each patch was calculated.
226 The horizontal displacement data provided a standard deviation of $4.6 \mu\text{m}$,
227 and vertical displacements provided a slightly higher standard deviation of
228 $6.4 \mu\text{m}$. This level of precision is similar to that reported by [Marshall and](#)
229 [Mair \(2011\)](#) for a similar assessment procedure.

230 The interface friction between the sand and acrylic window can affect

displacement measurements obtained by image analysis. The reliability of image-based soil deformation measurements in similar centrifuge tests has been studied by several researchers (Marshall et al., 2012; Elshafie et al., 2013; Ritter et al., 2017). Marshall et al. (2012) compared geoPIV displacements with LVDT measurements made within the middle of the box and demonstrated that the boundary friction did not have a significant effect on settlement trough shape. Therefore, the soil displacement data presented in this paper is based solely on image analysis measurements made at the soil-acrylic wall interface. It should also be mentioned that the comparisons of experimental data presented in this paper are all based on image analysis measurements obtained at an acrylic centrifuge container wall, hence any effects related to interface friction should be reasonably consistent across the data sets.

3.3. Testing procedure

The centrifuge package was spun to 80 g in stages of 10 g. Three stabilisation cycles were then performed for each test, whereby the model was spun from 80 g to 10 g and then back up to 80 g. By measuring the displacement at the ground surface during the stabilisation cycles, it was found that, for the type of soil used in these tests, three cycles was sufficient to obtain a consistent measurement of surface displacement between successive 10 g–80 g cycles. The stabilisation cycles help to achieve better consistency between tests by reducing localised, more highly-stressed zones (‘hung-up’ particles), thereby achieving a more uniformly stressed soil. After three stabilisation cycles, the tunnel volume loss process was started. Images were taken at key intervals throughout the tests (during centrifuge spin-up and tunnel volume

256 loss).

257 4. Numerical analyses

258 To obtain a better understanding of the relationship between soil move-
259 ments and the tunnel boundary condition (i.e. displacement or pressure
260 controlled), a series of two-dimensional numerical analyses was performed to
261 simulate eccentric displacement control (EDC) and flexible membrane (FM)
262 model tunnels. The ABAQUS (Hibbitt, 2002) finite element analysis (FEA)
263 software was used, along with the implementation of the hypoplastic consti-
264 tutive model developed by von Wolffersdorff (1996).

265 4.1. Finite element mesh and boundary conditions

266 Figure 4 shows the FE mesh for tests with a cover to diameter ratio
267 $C/D_t = 2$, which consists of 2040 elements and 6309 nodes. Taking advan-
268 tage of symmetry, only half of the model was simulated, using vertical roller
269 boundaries on the plane of symmetry through the tunnel centreline. Potts
270 and Zdravković (1999) indicated that for curved FE boundaries (i.e. the
271 tunnel boundary), higher order elements should be used, hence eight-node
272 quadratic plane strain elements (CPE8) were applied to the models. A fixed
273 boundary was used along the bottom of the mesh, a vertical roller boundary
274 was used on the right-side boundary (see Figure 4), and no constraints were
275 applied to the ground surface. The dimensions of the numerical model were
276 chosen to reduce boundary effects on the predicted displacements, hence they
277 do not match exactly with the dimensions within the centrifuge tests (which
278 are constrained by the size of the centrifuge cradle). It was also verified that
279 the effect of element size on displacement measurements was negligible.

Figure 4: Mesh of the numerical (FEA) analysis for $C/D_t = 2$

280 4.2. Numerical modelling details

281 The basic hypoplastic model was adopted for this study, which requires
 282 the following 8 parameters: critical state friction angle ϕ'_c , granular hard-
 283 ness h_s , fitting parameter n , minimum/maximum/critical void ratio at zero
 284 pressure $e_{d0}/e_{i0}/e_{c0}$, and $\alpha; \beta$ which govern the stiffness of the soil. The
 285 hypoplastic model parameters for Leighton Buzzard Fraction E sand were
 286 obtained and calibrated using oedometer and triaxial test data, as follows.

287 A critical state friction angle $\phi'_c = 31.4^\circ$ was used for the sand, based on an
 288 average value from five heap tests. [Herle and Gudehus \(1999\)](#) suggested that
 289 the initial void ratio in a proportional compression test (i.e. an oedometer
 290 test) with very loose sand can be considered as an appropriate estimate of
 291 e_{c0} since the sand would be poured in a similar way as a heap test, where the
 292 sample is close to critical state because of the large deformations. As shown
 293 by [Herle and Gudehus \(1999\)](#), h_s can be defined using:

$$h_s = 3p' \left(\frac{ne_p}{C_c} \right)^{1/n} \quad (1)$$

where C_c is the tangent compression index ($C_c = \Delta e / \Delta \ln p'$).

Using data from an oedometer test and considering two values of C_c at different magnitudes of mean effective stress p' (i.e. two points on an $e - \ln(p')$ curve; indicated below by subscripts 1 and 2), Equation 1 can be used to obtain an expression for n :

$$n = \frac{\ln(e_1 C_{c2} / e_2 C_{c1})}{\ln(p'_2 / p'_1)} \quad (2)$$

Using the above approach on data obtained from an oedometer test on Fraction E sand (see Figure 5), values of $e_{c0}=1.16$, $h_s = 1969$ MPa and $n = 0.447$ were obtained.

Figure 5: Oedometer test data for Leighton Buzzard Fraction E sand

The value of minimum void ratio at zero pressure e_{d0} can be determined

303 using (Herle and Gudehus, 1999):

$$e_{d0} = e_d \exp[(3p'/h_s)^n] \quad (3)$$

304 where the value of e_d mainly depends on the coefficient of uniformity C_u
305 and grain shape. Youd (1973) measured the value of e_d using a simple shear
306 test under a vertical pressure of 96 kPa, corresponding to $p' \approx 55$ kPa (based
307 on assumption of $K = 0.4$), and proposed a diagram relating e_d to grain
308 angularity and C_u . Based on the diagram of Youd (1973), a value of $e_d =$
309 0.615 was obtained. By substituting this value into Equation 3 (along with
310 previously determined parameter values), a value of $e_{d0} = 0.624$ was obtained.
311 As suggested by Herle and Gudehus (1999), the value of maximum void ratio
312 at zero pressure e_{i0} can be approximated as $e_{i0} \approx e_{c0} \times 1.2$, therefore, the
313 critical void ratio at zero pressure e_{i0} was determined as 1.392.

314 Two drained triaxial compression tests were conducted at an effective
315 confining pressure of 200 kPa using the same sand that was used in the cen-
316 trifuge tests, with the same relative density of $I_d = 90\%$. To calibrate the
317 parameters α and β , a single 3D cuboid element (C3D8) numerical analysis
318 was conducted in ABAQUS to replicate the triaxial tests. Figure 6 shows
319 that numerical simulation results using $\alpha = 0.065$ and $\beta = 0.3$ compare well
320 against both the triaxial volumetric data and the non-linear stress-strain re-
321 sponse of the soil. Table 2 summarises the Leighton Buzzard Fraction E sand
322 parameters adopted in this study for the basic hypoplastic model.

323 The intention of the FE analyses was to simulate, as closely as possible,
324 the conditions within the various centrifuge tests, considering: (1) the effect
325 of increasing acceleration within the models from 1 g to 80 g, and (2) the
326 tunnel volume loss process. In relation to (1), two modelling approaches

Figure 6: Drained triaxial test data for Leighton Buzzard Fraction E sand and numerical (FEA) results using hypoplastic model (effective confining stress = 200 kPa)

Table 2: Adopted hypoplastic model parameters for Leighton Buzzard Fraction E sand

Parameter	Value	Source
Critical state friction angle ϕ'_c	31.4°	Heap test
Granular hardness h_s	1969 MPa	Oedometer test
Exponent n	0.447	Oedometer test
Minimum void ratio at zero pressure e_{d0}	0.624	Herle and Gudehus (1999)
Critical void ratio at zero pressure e_{c0}	1.16	Oedometer test
Maximum void ratio at zero pressure e_{i0}	1.392	Herle and Gudehus (1999)
Exponent α	0.065	Triaxial test
Exponent β	0.3	Triaxial test

327 were adopted: first, where the full 1 g to 80 g process was replicated, and
 328 second, where the 80 g stress conditions were imposed on the model as an
 329 initial condition (referred to as ‘80 g initially’). It was found that, when
 330 replicating the full 1 g to 80 g process, the numerical analyses were unstable
 331 (i.e. numerical convergence issues) for tunnel volume losses greater than
 332 $V_{l,t} = 1.8\%$. By replicating from 2 g to 80 g, numerical analyses could be
 333 achieved up to $V_{l,t} = 2\%$, hence results from models which replicated the
 334 stress increase from 2 g to 80 g are included in this paper (referred to as ‘2-80 g
 335 considered’). Comparison of results between ‘1-80 g’ and ‘2-80 g’ analyses at
 336 $V_{l,t} = 1.8\%$ showed that the difference in ground settlements were negligible.

337 In relation to (2), two tunnel volume loss simulation methodologies were
 338 undertaken: the eccentric displacement controlled (EDC) method ([Cheng
 339 et al., 2007](#)) to simulate the eccentric rigid boundary mechanical (eRBM)
 340 model tunnel, wherein the tunnel invert was fixed against translation to repli-
 341 cate the conditions of the eRBM centrifuge tests, and the pressure controlled

342 method (PCM) to simulate flexible membrane (FM) model tunnels.

343 The FE analysis simulation procedures for the ‘2-80 g considered’ models
344 can be summarised as follows. A geostatic step was conducted to impose a
345 predefined stress profile ($K_0 = 0.5$) within the soil elements under 2 g, after
346 which soil displacements were reset to zero (the maximum soil displacement
347 after the geostatic step was found to be 2×10^{-5} mm). The soil elements
348 inside the tunnel were then deactivated. For the EDC numerical analyses, a
349 tunnel boundary was used with a frictionless interface with the soil. Gravity
350 was then increased from 2 g to 80 g, where the rigid boundary of the model
351 tunnel was fixed in terms of both distortion (change in shape) and translation
352 (rigid body motion). For the PCM analyses, the internal tunnel pressure was
353 increased according to the g-level, accounting as well for the gradient of stress
354 due to the self-weight of water within the model tunnel. Upon reaching 80 g,
355 the simulation of tunnel volume loss was then conducted. For the EDC anal-
356 yses, a non-uniform displacement profile was imposed on the rigid boundary
357 elements by assuming zero displacement at the tunnel invert and maximum
358 displacements at the crown, with displacements oriented towards the tunnel
359 centre (identical to the eRBM model tunnel centrifuge tests; see deformed
360 tunnel shape in Figure 2). For PCM numerical analyses, the tunnel volume
361 loss was replicated by gradually decreasing the internal tunnel pressure.

362 For the ‘80 g initially’ numerical analyses, the geostatic step was con-
363 ducted for stress conditions corresponding to 80 g and $K_0 = 0.5$; all other
364 details were the same as the ‘2-80 g considered’ analyses.

365 5. Results

366 5.1. *Effect of model tunnel on initial conditions*

367 Before considering tunnel volume loss in detail, it is of interest to investi-
368 gate the effect of the increased self-weight in the centrifuge on the displace-
369 ments and stress state of the soil around the model tunnels (i.e. the effect
370 of centrifuge spin-up). For FM model tunnel experiments (centrifuge tests),
371 the internal tunnel pressure was set to an approximation of the vertical over-
372 burden pressure at the tunnel axis depth using a stand-pipe system. The
373 difference in self-weight and lateral earth pressure between the soil and wa-
374 ter ($K_0 = 1$ for water; K_0 assumed to be 0.5 for soil) will cause an imbalance
375 of stresses in the region around the model tunnel (also discussed by [Ritter](#)
376 [et al. \(2017\)](#)). PCM numerical analyses were conducted to investigate soil
377 deformations and ground stress changes as the soil self-weight was increased
378 during centrifuge spin-up from 2 g to 80 g. For the eRBM model tunnel, the
379 tunnel lining was fixed in place (displacement controlled), hence limited dis-
380 placement is expected for the soil close to the tunnel lining during centrifuge
381 spin-up.

382 Figure [7](#) shows the centrifuge and numerical (FEA) data for soil move-
383 ments around the tunnels during centrifuge spin-up. The vertical displace-
384 ments in Figure [7\(a\)](#) (positive displacement is downwards) show that the soil
385 moves downwards in the region above the tunnel crown for both FM and
386 eRBM centrifuge tests. For the EDC (eccentric displacement controlled) FE
387 analyses, since the tunnel boundary is fixed in place, the soil settlements
388 around the tunnel periphery are less than for the PCM numerical analyses
389 (note the different contour scale for this plot).

390 From the horizontal displacement data in Figure 7(b), despite the differ-
 391 ence in g-level increase (i.e. 1–80 g for centrifuge and 2–80 g for numerical
 392 FEA), both centrifuge and numerical (FEA) results for the FM/PCM tunnels
 393 show outwards soil movements close to the tunnel springline (negative dis-
 394 placement is to the left). This happens because the internal tunnel pressure
 395 at the tunnel springline is greater than the horizontal soil pressure, causing
 396 the tunnel boundary to push outwards at the springline until an increased
 397 passive earth pressure is achieved to balance the internal tunnel pressure.
 398 The numerical PCM results show greater horizontal soil movements than
 399 the FM centrifuge test (note the larger contour scale used for FEA PCM in
 400 Figure 7(b)), suggesting that the assumed K_0 for the soil may have been un-
 401 derestimated (K_0 assumed to be 0.5 for soil, consistent with [Marshall et al.](#)
 402 [\(2010\)](#); [Haji et al. \(2018\)](#)). For the eRBM centrifuge model tunnel, the overall
 403 shape does not change during centrifuge spin-up, but outwards movements
 404 of the soil in the region of the tunnel shoulders are observed from the FEA
 405 EDC results. The centrifuge horizontal displacements for the eRBM do not
 406 show a clear mechanism with increase in g-level.

407 To gain a better understanding of what happens within the soil around
 408 flexible membrane (FM) and eccentric rigid boundary mechanical (eRBM)
 409 model tunnels during gravity increase, as well as during the subsequent tunnel
 410 volume loss process, Figure 8 plots the numerical (FEA) results of horizontal
 411 and vertical stress paths of soil elements near the tunnel crown and springline
 412 for pressure controlled (PCM) and eccentric displacement controlled (EDC)
 413 simulations with $C/D_t = 1.3$ (positive stresses are compressive). The two
 414 modelling approaches are included: (1) the ‘2–80 g considered’ model and (2)

Figure 7: Soil movements around FM and eRBM model tunnels during centrifuge spin-up: (a) vertical displacements (positive downwards), (b) horizontal displacements (negative towards the left)

415 the ‘80 g initially’ model (the K_0 line with no tunnel is also shown in Figure 8).
 416 The simulation of tunnel volume loss up to $V_{l,t}=2\%$ was then performed for
 417 both models. For the solid circles in Figure 8(a), the stress states in the ‘2-
 418 80 g considered’ models vary according to the soil element locations (close to
 419 tunnel crown or springline), whereas for the solid circles in Figure 8(b) (‘80 g
 420 initially’ models), the stress states at 80 g all fall along the K_0 line. Prior to
 421 tunnel volume loss in Figure 8(b), there is an intermediate stage (solid circles)
 422 in both the EDC and PCM models. For the EDC model, the intermediate
 423 stage relates to the stress adjustments caused by the introduction of the
 424 frictionless rigid tunnel boundary element. For the PCM model, the stage
 425 is related to the stress imbalance caused by the different horizontal earth
 426 pressures within the sand and the water inside the model tunnel (as discussed
 427 by [Ritter et al. \(2017\)](#)); this is referred to below as ‘stress re-balance’. The
 428 intermediate stage is then followed by tunnel volume loss up to 2%, indicated
 429 by the open circles.

430 Considering the ‘80 g initially’ results in Figure 8(b), the PCM model
 431 stress state near the springline starts on the K_0 line (grey solid line). Hor-
 432 izontal stress then increases considerably and the vertical stress decreases
 433 slightly during the ‘stress re-balance’ process. The stress state at the solid
 434 grey circle is very close to the initial state of the ‘2-80 g considered’ model in
 435 Figure 8(a). The stress paths of the ‘2-80 g considered’ and the ‘80 g initially’
 436 models then follow near-identical paths during the tunnel volume loss process
 437 (proceeding to the open grey circles). For the stress paths of the PCM mod-
 438 els near the crown, the vertical stress state of the ‘2-80 g considered’ model in
 439 Figure 8(a) is slightly lower than the ‘80 g initially’ model (solid grey circle)

Figure 8: Vertical and horizontal stress paths for $C/D_t = 1.3$: (a) 2-80 g considered method, (b) 80 g initially method

in 8(b); the horizontal stresses are nearly the same. The stress paths during volume loss follow a similar path and ultimately (at 2% volume loss) reach the same location (grey open circle). For the EDC models, a clear offset between the stress states prior to tunnelling in the ‘2-80 g considered’ and ‘80 g initially’ models can be seen, but the shape of the stress paths during tunnel volume loss is similar.

The impact of the different modelling approaches on profiles of ground settlement, S_v , at three values of normalised depth, z/z_t , is provided in Figure 9 at a tunnel volume loss of $V_{l,t} \approx 2\%$. For the PCM model, as the value of internal tunnel pressure controls the volume loss, the magnitude of tunnel volume loss was calculated based on the displacement of the nodes along the tunnel lining.

In general, for a given type of model tunnel (i.e. EDC or PCM), the

453 shape of the settlement troughs between the ‘2-80 g considered’ and ‘80 g
454 initially’ models is similar. However, the magnitude of settlement for the
455 ‘2-80 g considered’ models is always slightly greater than the ‘80 g initially’
456 models (by up to about 15% at the surface). With an increase in depth, the
457 difference in settlement magnitude between the two modelling approaches
458 (i.e. ‘2-80 g considered’ versus ‘80 g initially’) becomes smaller. Comparing
459 results between EDC and PCM models, the PCM model provided a greater
460 settlement than the EDC for the soil close to the tunnel centreline ($x/D_t <$
461 1.5).

462 To summarise: (1) the PCM model tends to generate a ‘steeper’ dis-
463 placement profile with larger maximum settlement than the EDC model,
464 and (2) the magnitude of settlement for the ‘2-80g considered’ models is al-
465 ways greater than the ‘80 g initially’ models for both EDC and PCM models,
466 however this difference tends to decrease with depth.

467 5.2. *Effects on soil deformation mechanisms*

468 For a given tunnel volume loss, the shape and magnitude of soil dis-
469 placements are affected by the type (displacement/pressure controlled) and
470 displaced shape of the tunnel lining. For eccentric displacement controlled
471 model tunnels (EDC), the displaced tunnel shape is pre-defined, whereas for
472 flexible membrane FM model tunnels, the tunnel boundary shape is free to
473 deform according to the internal/external stresses applied to it. The effect
474 of these differences on resulting ground displacements are considered in this
475 section using both centrifuge test and numerical analysis data (based on the
476 ‘2-80g considered’ method presented in the previous section).

477 Figure 10 compares soil displacements from the eccentric rigid boundary

Figure 9: Settlement trough for $C/D_t = 1.3$ at $V_{l,t} \approx 2.0\%$: (a) $z/z_t=0$, (b) $z/z_t=0.27$, (c) $z/z_t=0.6$

478 mechanical eRBM model tunnel centrifuge test with those from a flexible
 479 membrane FM centrifuge test (Franza, 2016) for $C/D_t = 2$ at a tunnel vol-
 480 ume loss of $V_{l,t} = 2\%$; note that these two tests are identical except for
 481 the model tunnel type (refer to Table 1). For vertical displacements, the
 482 deformation patterns match well between the tests (eRBM and FM); how-
 483 ever, the eRBM settlement trough is slightly wider than the FM trough. For
 484 horizontal displacements, the FM model tunnel causes a zone of relatively
 485 high magnitude displacements above and slightly to the side of the tunnel
 486 crown, as well as a major zone of displacements at the ground surface with a
 487 maximum value similar to that near the tunnel. The zone of large horizontal
 488 displacements for the eRBM tunnel is located closer to the tunnel springline
 489 compared with the FM model tunnel. There is also a large zone of horizon-
 490 tal displacements at the ground surface for the eRBM tunnel, in a similar
 491 location but more widespread than for the FM tunnel; the magnitude of dis-
 492 placements at the ground surface is noted to be less than at the springline
 493 for the eRBM tunnel.

494 Figure 11 shows the numerical (FEA) results of vertical and horizontal
 495 soil displacements for simulations of the different model tunnels at $C/D_t = 2$.
 496 As previously mentioned, the pressure controlled method (PCM) was used
 497 to simulate the flexible membrane (FM) model tunnel, and the eccentric
 498 displacement controlled method (EDC) was used to replicate the eccentric
 499 rigid boundary mechanical (eRBM) model tunnel.

500 The vertical and horizontal soil displacements in Figure 11 using FEA
 501 PCM can be compared with the FM model tunnel centrifuge test data in
 502 Figure 10. The general pattern of vertical displacements from the FEA

Figure 10: Centrifuge test results of vertical and horizontal soil displacements for FM and eRBM model tunnels: $C/D_t = 2$ and $V_{l,t} = 2\%$

PCM simulation is similar to the FM model tunnel centrifuge test. The magnitude of vertical soil displacements above the tunnel crown in the FEA PCM is larger and more uniformly distributed in the region directly above the tunnel than in the FM centrifuge test. The dominant zone of horizontal displacements for the FEA PCM test is located adjacent to the tunnel spring-line and creates a larger influence zone than in the FM centrifuge test. The horizontal displacements in the FEA PCM test also show a greater influence zone at the soil surface, with a greater maximum magnitude compared with the FM centrifuge test.

The FEA EDC results in Figure 11 can be directly compared with the eRBM tunnel centrifuge results in Figure 10. The FEA EDC results give larger vertical settlements than the centrifuge test. Similar to the FEA PCM versus FM centrifuge results, the magnitude of vertical displacements for the soil located above the tunnel crown is larger and more uniformly distributed along the depth of the soil in the FEA EDC test compared to the eRBM centrifuge test. The horizontal displacements in the FEA EDC test show a similar pattern to the eRBM centrifuge test, but again displacements are greater and more dispersed in the FEA EDC test. For horizontal displacements, a more concentrated zone of major displacements is shown in the FEA EDC test than in the FEA PCM test.

5.3. Settlement troughs

Figure 12 presents the vertical settlement troughs at the surface ($z/z_t=0$) and subsurface ($z/z_t=0.5$) for centrifuge tests (refer to Table 1) and numerical (FEA) analyses at a tunnel volume loss of $V_{l,t} \approx 2\%$. Note that the FM centrifuge test and corresponding FEA PCM analysis for $C/D_t = 2.4$ have

Figure 11: Numerical (FEA) results of vertical and horizontal soil displacements for PCM, EDC, and CDC tunnels: $C/D_t = 2$ and $V_{l,t} = 2\%$ (PCM $V_{l,t} = 1.9\%$)

528 a tunnel diameter ($D_t = 62\text{ mm}$) that is smaller than other tests (where
 529 $D_t = 90\text{ mm}$); see Table 1. Whilst in dimensionless terms, the $C/D_t = 2.4$
 530 test should provide a direct comparison to the other tests, this comparison
 531 does not consider the fact that, for a given C/D_t ratio and tunnel volume
 532 loss, a larger tunnel will incur larger magnitude displacements than a smaller
 533 tunnel. In addition, for a given C/D_t , a larger tunnel will be deeper than a
 534 smaller tunnel, hence the stress conditions around the larger tunnel will be
 535 greater, impacting on the volumetric response of the soil during shear caused
 536 by tunnel volume loss (greater stresses resulting in less soil dilation). The
 537 impact of this issue will be discussed in relation to the obtained results.

538 In general, for $C/D_t = 1.3$ and 2.0 (identical tunnel diameter and similar
 539 stress levels between FM and eRBM centrifuge tests), at the given tunnel
 540 volume loss of $V_{l,t} \approx 2\%$, the FM and eRBM centrifuge tests show a similar
 541 settlement shape and magnitude at the surface. In addition, the subsurface
 542 settlements for both tests are greater than the surface settlement for the soil
 543 close to the tunnel centreline; with increase in horizontal distance from the
 544 tunnel x/D_t , subsurface settlement becomes less than at the surface. For
 545 $C/D_t = 2.4$, because the diameter of the FM model tunnel is less than the
 546 eRBM model tunnel, for the given value of $V_{l,t}$, the FM model tunnel shows
 547 smaller settlements than the eRBM model tunnel. As for $C/D_t = 1.3$ and
 548 2.0 , the shape of the FM and eRBM settlement troughs are similar. The
 549 settlement trough shape characteristics are considered later in Figure 18.

550 For the numerical (FEA) results in Figure 12 (b), as mentioned earlier, the
 551 FEA PCM analyses are intended to simulate the FM model tunnel centrifuge
 552 tests, and the FEA EDC analyses are intended to replicate the eRBM model

553 tunnel centrifuge tests. For $C/D_t = 1.3$ and 2.0 , the FEA PCM settlements
 554 are always slightly higher than the corresponding FEA EDC model. As with
 555 the centrifuge tests, subsurface settlements close to the tunnel centreline
 556 are greater than the surface soil settlement, but becomes less than surface
 557 settlement with an increase in horizontal distance from the tunnel x/D_t . The
 558 FEA PCM settlements are generally less than FEA EDC for $C/D_t = 2.4$,
 559 for the same reason previously mentioned for the centrifuge tests (due to the
 560 smaller tunnel size in the FEA PCM $C/D_t = 2.4$ analysis). To compare PCM
 561 against EDC numerical results for a consistent tunnel size, an additional
 562 FEA PCM analysis was conducted for $C/D_t = 2.4$ with a tunnel diameter
 563 $D_t = 90$ mm (labelled in Figure 12 (b)). Comparison of the FEA PCM results
 564 with $D_t = 90$ mm against FEA EDC for $C/D_t = 2.4$ show consistent trends
 565 with those previously described for $C/D_t = 1.3$ and 2.0 .

566 [Franza and Marshall \(2018\)](#) suggested (based on FM centrifuge test data)
 567 that a localised zone of large settlements concentrated around the tunnel
 568 periphery is indicative of the boundary of an arch; these localised zones can
 569 be discerned in Figure 10 for both FM and eRBM centrifuge tests. These
 570 localised zones are not observed in the numerical model results in Figure 11,
 571 where settlements propagate upwards more gradually.

572 The numerical (FEA) outcomes in Figure 12 generally over-predict settle-
 573 ments compared to the centrifuge tests. This over-prediction is mainly due
 574 to the fact that the numerical analyses were not able to accurately replicate
 575 the localised zone of vertical settlement close to the tunnel crown, with tun-
 576 nel boundary displacements therefore propagating through the soil towards
 577 the surface more in the numerical analyses than in the centrifuge tests. As

Figure 12: Settlement trough data: (a) centrifuge and (b) numerical (FEA) for $V_{i,t} \approx 2\%$

suggested by [Franza et al. \(2019a\)](#), for dense sand, the arching mechanism has an influential impact, and the soil tends to dilate below the developed arch. However, in the numerical simulations, soil dilation was not accurately replicated, which affected the ability of the numerical models to replicate soil displacements.

5.4. Relationship between soil and tunnel volume loss

Figure 13 shows the relationship between surface soil volume loss ($V_{l,s,surf}$; calculated as the integration of surface settlement data) and tunnel volume loss ($V_{l,t}$; obtained directly from the displacements or volume changes of the model tunnels) for both eRBM and FM centrifuge tests. Note that for an undrained case (zero volumetric strain), $V_{l,s,surf} = V_{l,t}$. As discussed in [Marshall et al. \(2012\)](#) and [Franza et al. \(2019a\)](#), for dry sand (or drained soil conditions), soil volume loss will differ from tunnel volume loss due to the dilative/contractive nature of the soil, which depends on the relative density of the soil, magnitude of the shear strain, and the confining pressures during shear. At a given soil depth, if the value of $V_{l,s,surf}$ is lower than $V_{l,t}$, the soil below that level has undergone an overall dilative response.

For the FM model tunnel with $C/D_t = 1.3$ in Figure 13, $V_{l,s,surf}$ is always less than $V_{l,t}$, indicating that the soil below the surface experienced an overall dilative response, whereas for $C/D_t = 2.0$ and 2.4 , the value of $V_{l,s,surf}$ is greater than $V_{l,t}$ for tunnel volume losses lower than $V_{l,t} \approx 1.8\%$ and 2.3% , respectively, indicating an overall contractive response. These trends are consistent with the expected effect of C/D_t (for constant tunnel size), where the soil above shallower tunnels is more dilative because of the lower levels of confining stress. The trends of the data for the eRBM model tunnel follow

603 those of the FM model tunnel, however the eRBM values of $V_{l,s, surf}$ are
 604 consistently lower than the FM values, indicating more dilative conditions in
 605 the eRBM tests. This suggests that the magnitude of shear strains around
 606 the tunnel circumference in the eRBM model tunnel tests were greater than
 607 in the FM model tunnel tests. Despite the fact that, because of the different
 608 tunnels sizes (see Table 1), the $C/D_t = 2.4$ eRBM tunnel was deeper than
 609 the FM tunnel (which would tend to make the eRBM soil more contractive),
 610 the eRBM $C/D_t = 2.4$ results in Figure 13 indicate a more dilative response
 611 than the equivalent FM test; this occurs because the larger eRBM tunnel
 612 causes higher levels of shear strain than the smaller FM tunnel, resulting in
 613 a more dilatant volumetric soil response for the eRBM tunnel (this aspect is
 614 discussed in more detail later).

Figure 13: Variation of surface soil volume loss with tunnel volume loss for centrifuge tests

615 The accumulated soil response indicated by $V_{l,s}$ averages out localised
 616 volumetric responses within the soil beneath the calculated level; the soil

617 may experience localised contraction at a certain depth yet $V_{l,s}$ could still
 618 indicate an overall dilative response. To investigate this feature, Figure 14
 619 plots soil volume loss ($V_{l,s}$) with depth at a tunnel volume loss of $V_{l,t} \approx 2.0\%$
 620 for both centrifuge and numerical tests. The vertical dashed lines indicate
 621 a constant-volume condition. For the centrifuge tests, $V_{l,s}$ values are always
 622 less than $V_{l,t} \approx 2.0\%$, apart from the FM $C/D_t = 2.4$ test, where the cumu-
 623 lative soil response close to the surface ($z/z_t < 0.1$) gives a soil volume loss
 624 slightly greater than 2.0% . The value of $V_{l,s}$ for the centrifuge tests reduces
 625 approximately linearly with depth (z/z_t) and shows a maximum value at the
 626 soil surface and a minimum value close to the tunnel crown. The soil vol-
 627 ume loss at a given depth ratio (z/z_t) also differs with C/D_t ratio, where a
 628 greater C/D_t gives a higher soil volume loss (due to higher overall confining
 629 stresses, as mentioned earlier). For centrifuge tests with $C/D_t = 1.3$ (both
 630 FM and eRBM), the soil close to the tunnel crown ($z/z_t \approx 0.7$) shows a value
 631 of $V_{l,s}$ that is approximately 50% of $V_{l,t}$. These data indicate that the soil
 632 experienced considerable localised dilation below $z/z_t \approx 0.7$. In addition,
 633 with a decrease in z/z_t (moving away from the tunnel towards the surface),
 634 the value of $V_{l,s}$ tends to increase, suggesting that the soil above $z/z_t = 0.7$
 635 underwent overall contraction.

636 For greater tunnel depths (greater mean effective stress), the soil will
 637 tend to have a more contractive (less dilative) response. The rate of change
 638 of contractive response with depth is indicated by the slope of $V_{l,s}$ with depth.
 639 The slope for $C/D_t = 1.3$ for the eRBM centrifuge model tunnel is less than
 640 for $C/D_t = 2.0$ and 2.4 , indicating that, for a given shear strain (same
 641 tunnel volume loss and tunnel size), the soil in the $C/D_t = 1.3$ tests is less

642 contractive than the soil in the tests with greater tunnel depths ($C/D_t = 2.0$
643 and 2.4), as is expected.

644 The centrifuge results show that, for a given value of C/D_t , the soil
645 volume losses are similar throughout the depth of the soil, with values from
646 the FM tests being marginally larger than the eRBM tests.

Figure 14: Variation of $V_{l,s}$ with depth at $V_{l,t} \approx 2.0\%$: (a) centrifuge tests; (b) numerical (FEA) simulations

647 For the numerical (FEA) analyses, both pressure control (PCM) and ec-
648 centric displacement control (EDC) methods show soil volume loss is greater
649 than tunnel volume loss across the full range of depth. Consistent with the
650 centrifuge test results, higher C/D_t ratios give greater values of $V_{l,s}$. Close
651 to the tunnel crown ($z/z_t \approx 0.7$), unlike the centrifuge tests, the value of
652 $V_{l,s}$ is greater than 2.0%, indicating that the soil below this level underwent
653 contraction, with soil contraction continuing up to the surface. Similar to

centrifuge test results, but to a much more significant degree, at a given depth z/z_t , the PCM soil volume losses $V_{l,s}$ are greater than the EDC values (explaining the larger settlements for the PCM results in Figure 12b). The slope of the $V_{l,s}$ data with depth (indicating the rate of change of contractive response with depth) increases with C/D_t ratio, similar to the centrifuge test results.

As was done for Figure 12, result from the additional $C/D_t = 2.4$ FEA PCM test with $D_t = 90$ mm is presented in Figure 14(b). The slope of $V_{l,s}$ with depth is similar for the two PCM analyses for $C/D_t = 2.4$ in Figure 14(b) ($D_t = 62$ mm from Table 1 and $D_t = 90$ mm for the additional analysis labelled in the figure). The notable difference between these two analyses is the soil volume loss near the tunnel crown ($z/z_t \approx 0.7$), where $V_{l,s}$ for the analysis with $D_t = 90$ mm is less than the analysis with $D_t = 62$ mm, indicating that the soil response for $D_t = 90$ mm was more dilative than for $D_t = 62$ mm. If we consider the mean stresses in the two tests, the analysis with $D_t = 90$ mm will have a deeper tunnel than for $D_t = 62$ mm (given the same C/D_t), hence the overall stress conditions will be higher for the $D_t = 90$ mm analysis, which would imply that the soil should be more contractive, yet this is not what occurred (note that, applying the relative dilatancy concept from Bolton (1986), the relative dilatancy index is reduced from 4.35 for the shallower tunnel to 3.97 for the deeper tunnel, i.e. only a 9% reduction). The other aspect that can explain the outcome where $V_{l,s}$ for $D_t = 90$ mm is less than for $D_t = 62$ mm (i.e. more dilative for the larger tunnel) is the magnitude of shear strain around the tunnels combined with the volumetric response of the soil (illustrated in Figure 6 for triaxial

679 conditions). The smaller tunnel will incur lower levels of shear strain than
 680 the larger tunnel (this was verified from the FEA analyses), hence the soil
 681 near the tunnel for the $D_t = 62$ mm will be more contractive than the larger
 682 tunnel, resulting in the higher levels of soil volume loss at $z/z_t \approx 0.7$ in
 683 Figure 14(b).

684 Figure 14 indicates that the numerical analyses did not accurately sim-
 685 ulate the soil volumetric behaviour close to the tunnel circumference; below
 686 $z/z_t \approx 0.7$ (close to the tunnel crown), a significant soil dilative response
 687 is observed for the centrifuge test data, whereas a contractive response is
 688 obtained in the numerical results. Above $z/z_t \approx 0.7$, both centrifuge and
 689 numerical results indicate an overall contractive response, though the rate
 690 of change of contraction with depth is greater in the FE results than in the
 691 centrifuge.

692 Figure 15 shows the spatial distribution of volumetric and shear strains
 693 from the eRBM and FM centrifuge tests for $C/D_t = 2.0$ at a tunnel vol-
 694 ume loss of $V_{l,t} = 2.0\%$ (which have the same size of tunnel; see Table 1).
 695 Both model tunnels indicate concentrated zones of shear and negative vol-
 696 umetric (dilation is negative) strain close to the tunnel boundary; near the
 697 crown/shoulder for the FM test and closer to the tunnel springline for the
 698 eRBM test. The magnitudes of the volumetric strains for the eRBM test
 699 are considerably larger than the FM test (the maximum dilative volumetric
 700 strain for the eRBM test is approximately nine times higher than the FM
 701 test). The magnitudes of shear strain for eRBM test are also significantly
 702 higher than the FM test. These results confirm the conclusions made ear-
 703 lier (i.e. eRBM values of $V_{l,s,surf}$ are consistently lower than the FM values,

704 indicating more dilative conditions in the eRBM test). In Figure 15, the
 705 highlighted regions of dilating soil are very localised around the tunnel (and
 706 below about $z/z_t = 0.7$); the majority of the soil is contractive, which ex-
 707 plains the trends in Figure 14a (i.e. an increase in soil volume loss with a
 decrease in depth, suggesting contractive soil behaviour above $z/z_t \approx 0.7$).

Figure 15: Contours of shear and volumetric strain for centrifuge tests with $C/D_t = 2.0$ at $V_{l,t} = 2.0\%$

708

709 Figure 16 shows the shear and volumetric strain results for the corre-
 710 sponding numerical (FEA) analyses (PCM and EDC). The zone of concen-

711 trated shear strain is located at the tunnel shoulders for both methods, with
 712 similar magnitude of shear strains as well. There is a region of dilating soil
 713 close to the tunnel (highlighted in Figure 16), which is noted to be smaller
 714 in size and lower in magnitude than in the centrifuge tests, demonstrating
 715 why the numerical model under-predicted the overall dilative soil volume loss
 716 results in the centrifuge tests in Figure 14.

Figure 16: Contours of shear and volumetric strain for numerical (FEA) test with $C/D_t = 2.0$ at $V_{l,t} \approx 2.0\%$

717 Figure 17 summarises the key features from the shear and volumetric

718 strain distributions of the centrifuge tests. For a given tunnel volume loss,
 719 the eRBM model tunnel causes higher magnitude shear strains than the FM
 720 model tunnel, resulting in a more dilative volumetric response in the eRBM
 721 test than the FM test. The zone of dilating soil for the FM model tunnel
 722 is located above the tunnel crown, whereas for the eRBM model tunnel, the
 723 zone is located above the tunnel springline. The soil located outside the
 724 dilative area undergoes contractive soil behaviour, hence soil volume loss
 725 increases as depth decreases.

Figure 17: Illustration of soil strains due to tunnel volume loss based on the observations in Figure 15

726 5.5. Effect of model tunnel on settlement trough shape characteristics

727 To investigate settlement trough shape characteristics, modified Gaus-
 728 sian curves (Vorster et al., 2005) were fitted to the settlement data using
 729 the approach of Marshall et al. (2012). Marshall et al. (2012) indicated that
 730 the conventional Gaussian curve does not always provide a good fit to set-
 731 tlement data for tunnels in sand, which was substantiated by several other

732 studies (Vorster et al., 2005; Marshall et al., 2012; Zhou et al., 2014; Franza
733 and Marshall, 2018; Franza et al., 2019a). The goodness of fit (based on
734 the coefficient of determination R^2) of the modified Gaussian curve to ver-
735 tical displacements is shown in Figure S1 of the supplemental data. Results
736 demonstrate that the modified Gaussian curve provided a very good fit to
737 the data, with a slight reduction in the goodness of fit to the eRBM data
738 with depth, especially below $z/z_t=0.6$. For the numerical analyses, the mod-
739 ified Gaussian curve performed very well for both model types. Analyses
740 presented next are based on the obtained curve fitting results using the mod-
741 ified Gaussian curve.

742 When applying a modified Gaussian curve, as described by Marshall et al.
743 (2012), the shape of the settlement trough can be described by three points:
744 S_{max} , $S_v(i_g) \approx 0.606 S_{max}$, and $0.5S_v(i_g) \approx 0.303 S_{max}$, where i_g is the offset
745 from the tunnel centreline to the inflexion point of the standard Gaussian
746 curve. The offset of the second and third points ($S_v(i_g)$ and $0.5S_v(i_g)$) are
747 referred to as x^* and x^{**} , respectively. These offsets can be defined using
748 $x^* = K^*(z_t - z)$ and $x^{**} = K^{**}(z_t - z)$, where K^* and K^{**} are trough
749 width parameters. Figure 18 presents the trough width parameters K^* and
750 K^{**} with soil depth for both centrifuge and numerical (FEA) analyses at a
751 tunnel volume loss of $V_{l,t} \approx 2.0\%$. Considering first the centrifuge data, the
752 FM model tunnel centrifuge tests give lower trough width parameters than
753 the eRBM model tunnel tests, and the FM K^* values do not increase with
754 depth as quickly as they do for the eRBM model tunnel. With an increase
755 in C/D_t ratio, the difference in K^* and K^{**} between FM and eRBM model
756 tunnels decreases; at $C/D_t = 2.4$, the values of K^* and K^{**} are very close for

757 $z/z_t \leq 0.6$ (despite the fact that the FM and eRBM tunnels for $C/D_t = 2.4$
758 differed in size; see Table 1).

759 Comparing centrifuge data to numerical (FEA) results, Figure 18 shows
760 that the trough width parameters for the centrifuge tests are always lower
761 than the numerical values, and the difference between centrifuge and nu-
762 merical values is relatively consistent across the range of depth and C/D_t
763 ratios. Whilst the numerical results of settlement trough parameters are
764 clearly shifted (increased) compared to the centrifuge data, the trends with
765 depth match relatively well. Results from the additional FEA PCM test for
766 $C/D_t = 2.4$ with $D_t = 90$ mm are also presented in Figure 18 (note that
767 the FEA PCM analysis referred to in the legend relates to $D_t = 62$ mm as it
768 replicated the FM centrifuge test from Table 1). The FEA PCM $D_t = 90$ mm
769 results match the FEA EDC for $C/D_t = 2.4$, consistent with $C/D_t = 2.0$.
770 The numerical results support the conclusion from the centrifuge test data in
771 that, as C/D_t increases, the difference between EDC (eRBM) and PCM (FM)
772 decreases. In addition, comparing PCM $D_t = 90$ mm with PCM $D_t = 62$ mm
773 shows that, for the given tunnel volume loss, the through width parameters
774 decrease with the increase in tunnel diameter. This outcome is a result of
775 the effect of the higher magnitude shear strains that occur around the larger
776 tunnel on the volumetric soil response, as discussed previously.

777 Figure 18 demonstrates that, for both FM and eRBM model tunnels, the
778 numerical analyses predict a wider settlement trough than observed exper-
779 imentally, which can be attributed to the ability of the constitutive model
780 to describe the real stress-strain behaviour of the soil within the centrifuge
781 tests. Figure 16 demonstrated that the volumetric strain behaviour of the

Figure 18: K^* and K^{**} with depth for centrifuge and numerical (FEA) tests at $V_{l,t} \approx 2.0\%$

soil close to the tunnel boundary, in particular the high level of dilation, was not accurately simulated in the numerical analyses, which will affect the settlement trough shape.

The constitutive model adopted within the numerical analyses presented here was the basic hypoplastic model, where the stress path dependent soil stiffness is not considered. The soil elements close to the tunnel boundary will undergo a change in the direction of principal stresses during tunnel volume loss, and the soil elements close to the tunnel springline will experience a stress release process. The stress paths during tunnel volume loss are in a different direction to those during the centrifuge acceleration stage (see Figure 8). Therefore, the soil stiffness predicted by the FE analyses will not be entirely representative of the soil behaviour in the centrifuge tests.

6. Conclusions

Two types of model tunnel are commonly adopted to simulate 2D tunnel volume loss in plane strain centrifuge tests: flexible membrane (FM) and rigid boundary mechanical (RBM). This paper provided an in-depth assessment of the effect of the different model tunnel boundary conditions on resulting ground deformations. Finite element analyses were conducted using both pressure and displacement control methods to compare against the centrifuge test data. The paper does not give a recommendation as to which type best replicates reality, as each has advantages and limitations; instead the paper aims to provide readers with quantitative data which will allow them to make an informed decision regarding their choice of model tunnel. The following conclusions can be drawn from the presented results:

- 806 • Centrifuge spin-up has an effect on the initial conditions of the soil
807 during centrifuge tests, which is affected by the type of model tunnel. In
808 a FM model tunnel, the internal tunnel pressure at the tunnel springline
809 is greater than the horizontal soil pressure, and the soil at the springline
810 is pushed outwards during spin-up. For an eRBM model tunnel, the
811 overall shape of the tunnel boundary does not change during spin-up,
812 however some settlement was measured due to the increased self-weight
813 of the tunnel.

- 814 • Two numerical analysis approaches were studied: (1) the ‘2-80 g consid-
815 ered’ model which simulated the (nearly) complete centrifuge spin-up
816 process, and (2) a simplified ‘80 g initially’ model, where the initial
817 stress state at 80 g was imposed. The magnitude of tunnelling re-
818 lated settlements for the ‘2-80 g considered’ models was always slightly
819 greater than the ‘80 g initially’ models for both tunnel types (displace-
820 ment and pressure controlled).

- 821 • Modified Gaussian curves were fitted to the settlement data from the
822 centrifuge and numerical tests. The eRBM model tunnel always pro-
823 vided a wider settlement trough than the FM model tunnel, and set-
824 tlement troughs from the numerical analyses were considerably wider
825 than the centrifuge test data.

- 826 • Centrifuge test data demonstrated that localised zones of high shear
827 strain and soil dilation occur around the model tunnels, though the
828 location of this zone differs between the two model tunnel types. The
829 eRBM generally caused higher levels of shear strain and more dilative

830 soil response than the FM model tunnel, which has an impact on the
831 relationship between soil and tunnel volume loss. Numerical analyses
832 were not able to accurately replicate this zone of localised dilation,
833 hence soil volume loss in the numerical analyses was much more con-
834 tractive than in the centrifuge tests. This issue was a main cause of
835 the discrepancy between centrifuge and numerical results of soil volume
836 loss and settlement trough width.

7. NOTATION

A_t	Tunnel cross-sectional area
C	Depth of cover above the tunnel
C_u	Coefficient of uniformity
D_t	Diameter of the tunnel
D_{50}	Average size of the soil particle
e_{c0}	Critical void ratio at zero pressure
e_{d0}	Minimum void ratio at zero pressure
e_d	Void ratio, depends on C_u
e_{i0}	Maximum void ratio at zero pressure
e_{max}	Maximum void ratio
e_{min}	Minimum void ratio
e_{p0}	Initial void ratio of compression test at zero pressure
G_s	Specific gravity
h_s	Granular hardness
i or i_s	Horizontal distance between tunnel and inflection point
i_g	Horizontal distance between tunnel and inflection point of greenfield settlements
I_d	Relative density
K	Trough width parameter
K_0	Static state lateral earth pressure coefficient
K^*	Trough width parameter based on x^*
K^{**}	Trough width parameter based on x^{**}
n	Controls curve fitting parameter
q	Deviator stress
R^2	Coefficient of determination

S_v	Settlement of soil
S_{max}	Maximum soil settlement
$V_{l,s}$	Soil volume loss
$V_{l,s,surf}$	Soil volume loss at surface
$V_{l,t}$	Tunnel volume loss
x	Horizontal distance from the tunnel centre line
x^*	Horizontal distance of $S_v(i_g)$
x^{**}	Horizontal distance of $0.5S_v(i_g)$
z_t	Depth of the tunnel axis
z	Depth measure from soil surface
α	Governs the peak friction angle of the soil
β	Governs the soil stiffness
γ	Shear strain
ϵ_{vol}	Volumetric strain
ϕ'_c	Critical state friction angle
σ_v	Vertical stress
$\sigma_{v,t}$	Vertical stress at tunnel depth
σ_h	Horizontal stress

840 **8. Supplemental data**

Figure S1: Goodness of fit of modified Gaussian curves to (a) centrifuge, and (b) numerical (FEA) data

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