Rotor UMP & Mechanical Response in HSPMSM in Typical Running Conditions

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Abstract—This paper presents a comprehensive analysis on the rotor unbalanced magnetic pull (UMP) and the consequent mechanical response in 6 typical running conditions in a 20000 rpm high speed permanent magnet synchronous motor (HSPMSM) through finite element analysis (FEA) and experiment. The 6 running cases include normal full-load, normal half-load, normal no-load, static air-gap eccentricity (SAGE) full-load, SAGE half-load, and SAGE no-load. The mechanical responses include the force-caused deformation, the radial vibration, and the dynamic harmonic response due to UMP. It shows that the increment of the load will increase the rotor deformation as well as vibration, and the SAGE case will induce a larger deformation to the rotor and cause a more intensive vibration than the normal condition when with the same load. The harmonic response of the rotor will obtain a max deformation amplitude at 3180Hz and 6080Hz.

Keywords—High speed permanent magnet synchronous motor (HSPMSM), unbalance magnetic pull (UMP), static air-gap eccentricity (SAGE), mechanical response

ABBREVIATION

HSPMSM	High speed permanent magnet synchronous motor
FEA	Finite element analysis
MMF	Magnetomotive force
PPUA	Permeance per unit area
MFD	Magnetic flux density
SAGE	Static air-gap eccentricity

NOMENCLATURE

g_0	radial air-gap length
μ_0	permeability of air and vacuum
Λ_0, Λ_s	Normal PPUA, PPUA component caused by SAGE
δ_s	Value of relatively SAGE
f	Electrical frequency of HSPMSM
ω	electrical angular frequency
α_m	mechanical circumferential angle
I_a	Rotor phase current
I_d, I_q	d-axis current, q-axis current
x_d, x_q	d-axis reactance, q-axis reactance
F_f	rotor MMF
F_ω, F_{a1}	Stator MMF of full-load & half-load

$F_{\delta r}, F_{\delta l}$	composite MMF of full-load & half-load
β_f, β_1	angle between composite MMF and rotor MMF
E_0	No-load electromotive force
ψ	internal power angle (angle between E_0 and I_a)
θ	Power angle (angle between E_0 and U)
φ	Power factor angle (angle between U and I_a)
B	MFD

I. INTRODUCTION

The HSPMSM, which is expected to be more available and meanwhile with less noise by both aircraft operators and aerospace industry, is a key component of the aircraft [1]. In addition to the aerospace field, HSPMSM is widely used in many engineering applications as well. To reduce the noises and vibrations of the rotor, typically, scholars developed magnetic bearings for less friction and energy loss [2]. During the design and manufacturing of the HSPMSM, researchers paid much attention to the electromagnetic properties as well as the cooling problems [2-6], while magnetic forces and vibrations have attracted much less attention. However, the severe UMP on the rotor will cause intense vibrations and noises [7], and also lead to over stresses to the rotor which may damage rotor components [8].

As the increment of the rotating speed and the capacity of the motor, the vibration problem in the high speed motors has become a critical issue [9]. Sumit Singhal et al. studied on the effects of laminated core on the rotor mode shapes, and found that the modal characteristics of the shaft are significantly different from the rotor core as it leads to stiffening of the system and also influences the location of nodes [10]. Chun Gan et al. investigated the effect of skewing the stator and/or rotor on the vibration reduction of the three-phase switched reluctance motors, and finally found that skewing the stator is more effective than skewing the rotor [11]. Sungyong Shin et al. proposed an improved two-step commutation control method to reduce the noise and vibration in switched reluctance motor [12]. Cristian Babetto et al. found that the shape of the rotor is pretty significant for the vibration control [13].

In this paper, a comprehend study including theoretical UMP analysis, finite element calculation, and experimental test for the numerical forces and mechanical responses, is carried out. The remainder of this paper is organized as follows. Section II analyzes the mechanics model of the rotor

as well as the exciting UMP acting on the rotor, while Section III carries out the finite element calculation and the experimental test on UMP and the mechanical responses. In Section IV, primary conclusions are drawn up and further work in next step is illustrated.

II. THEORETICAL MODEL

A. Structure and Equivalent Mechanics Model of Rotor

The rotor is composed of the rotor core, the shaft, the permanent magnets, the sleeve, and the supporting part. Taking the prototype HSPMSM which will be studied in this paper as an example, the general structure of the rotor is indicated in Fig.1. The rotor core and the sleeve are the parts to hold the permanent magnets, while the supporting part is the component to hold the rotor and the shaft.

Since the physical structure of the rotor is a solid cylinder which is fixed by the bearings, the equivalent mechanics model of the rotor is a simply supported beam which can rotate about z axis, see Fig.1 (b). The exciting forces are the UMP F_x and F_y , as indicated in Fig.1 (c), where K_x and K_y are the elastic/stiffness coefficient in X- and Y- direction, respectively, D_x and D_y are the damping coefficients, and x_x and x_y are the movements. Usually they are written in matrix forms to formulate the kinetic equation, see Section D.

Fig. 1. Rotor structure and mechanics model: (a) picture of prototype rotor, (b) structure diagram, and (c) mechanics model

B. MFD in Typical Conditions

Due to different running conditions, MMF and PPUA will vary and therefore MFD will be changed, leading to different UMPs on the rotor. Given that most motors has static air-gap eccentricity (SAGE) at different extents, in this paper we investigate 6 typical running conditions, namely the normal full-load case, the normal half-load case, the normal no-load case, the SAGE full-load case, the SAGE half-load case, and the SAGE no-load case.

MMFs in different conditions can be illustrated as Fig.2, where F_f is the rotor MMF, F_a and F_{a1} are the stator MMF in full-load and half-load condition, respectively, and F_δ and $F_{\delta1}$ are the composite MMF of full-load and half-load, respectively.

Fig. 2. Space-time phasor diagram of HSPMSM

As indicated in Fig.2, F_{a1} is about half of F_a since in the half-load case the stator current is generally the half of the full-load current. Consequently, $F_{\delta1}$ is smaller than F_δ . In no-load condition, the stator current is very small (only to compensate the energy loss), and the composite MMF is almost equal to F_f which is even smaller than $F_{\delta1}$. Then the composite MMF in full-load, half-load, and no-load condition can be written as

$$F_\delta(\alpha_m, t) = \begin{cases} F_f \cos(\omega t - \alpha_m) & \dots \dots \dots \text{no-load} \\ F_{\delta r} \cos(\omega t - \alpha_m + \beta_r) & \dots \dots \dots \text{full-load} \\ F_{\delta1} \cos(\omega t - \alpha_m + \beta_1) & \dots \dots \dots \text{half-load} \end{cases} \quad (1)$$

where

$$\begin{cases} F_{\delta0} \approx F_f \\ F_{\delta r} = \sqrt{(F_r + F_a \sin \psi)^2 + (F_a \cos \psi)^2} \\ F_{\delta1} = \sqrt{(F_r + 0.5F_a \sin \psi)^2 + (0.5F_a \cos \psi)^2} \\ \beta_r = \arctan \frac{F_a \cos \psi}{F_r + F_a \sin \psi} \\ \beta_1 = \arctan \frac{0.5F_a \cos \psi}{F_r + 0.5F_a \sin \psi} \end{cases} \quad (2)$$

PPUA can be obtained through the reciprocal of the radial air-gap length

$$\Lambda(\alpha_m) = \frac{\mu_0}{g(\alpha_m)} \quad (3)$$

The radial air-gap length in normal condition and SAGE case is illustrated in Fig.3 and can be written as

$$g(\alpha_m) = \begin{cases} g_0 & \dots \dots \dots \text{normal condition} \\ g_0(1 - \delta_s \cos \alpha_m) & \dots \dots \dots \text{SAGE} \end{cases} \quad (4)$$

Feed Eq.(4) into (3) PPUA can be obtained

$$\Lambda(\alpha_m) = \begin{cases} \mu_0 / g_0 = \Lambda_0 & \dots \dots \dots \text{normal condition} \\ \frac{\mu_0}{g_0(1 - \delta_s \cos \alpha_m)} = \Lambda_0(1 + \delta_s \cos \alpha_m + \delta_s^2 \cos^2 \alpha_m + \dots) & (5) \\ \approx \Lambda_0(1 + \delta_s \cos \alpha_m) = \Lambda_0 + \Lambda_s \cos \alpha_m & \dots \dots \dots \text{SAGE} \end{cases}$$

where in the SAGE case it is expanded by Power Series. It is suggested that the PPUA amplitude in SAGE case will be larger than the normal one, because $\delta_s^2 \cos^2 \alpha_m = 0.5\delta_s^2(1 + \cos 2\alpha_m)$ and this lead to the DC component $\Lambda_0(1 + 0.5\delta_s^2)$ obviously larger than Λ_0 .

Fig. 3. Air-gap in (a) normal condition, (b) common SAGE case

Finally, MFD can be quickly obtained and written as

$$B(\alpha_m, t) = F_\delta(\alpha_m, t) \Lambda(\alpha_m)$$

$$= \begin{cases} F_f \Lambda_0 \cos(\omega t - \alpha_m) & \dots \text{normal no-load} \\ F_{\delta r} \Lambda_0 \cos(\omega t - \alpha_m + \beta_r) & \dots \text{normal full-load} \\ F_{\delta l} \Lambda_0 \cos(\omega t - \alpha_m + \beta_l) & \dots \text{normal half-load} \\ F_f (\Lambda_0 + \Lambda_s \cos \alpha_m) \cos(\omega t - \alpha_m) & \dots \text{SAGE no-load} \\ F_{\delta r} (\Lambda_0 + \Lambda_s \cos \alpha_m) \cos(\omega t - \alpha_m + \beta_r) & \dots \text{SAGE full-load} \\ F_{\delta l} (\Lambda_0 + \Lambda_s \cos \alpha_m) \cos(\omega t - \alpha_m + \beta_l) & \dots \text{SAGE half-load} \end{cases} \quad (6)$$

Qualitatively, according to Fig.2 and Eqs. (5) and (6), the MFD amplitude varies in the following orders

- No.1: SAGE full-load (the largest)
- No.2: normal full-load
- No.3: SAGE half-load
- No.4: normal half-load
- No.5: SAGE no-load
- No.6: normal no-load (the minimum)

C. UMP on Rotor in Typical Conditions

The rotor UMPs in X and Y direction can be calculated through

$$\begin{cases} F_x = LR \int_0^{2\pi} q(\alpha_m, t) \cos \alpha_m d\alpha_m \\ F_y = LR \int_0^{2\pi} q(\alpha_m, t) \sin \alpha_m d\alpha_m \\ q(\alpha_m, t) = \frac{B(\alpha_m, t)^2}{2\mu_0} \end{cases} \quad (7)$$

where B is the magnetic flux density (MFD) in the air-gap, t denotes the time, and α_m is the mechanical angle to indicate the circumferential position of the air-gap, as indicated in Fig. 2. MFD can be obtained by multiplying the magnetomotive force (MMF) by the permeance per unit area (PPUA).

Feed Eq.(6) into (7) it finally has

$$\begin{cases} F_x = \begin{cases} 0 & \dots \text{normal condition} \\ LRF_f^2 \pi [2\Lambda_0 \Lambda_s + \Lambda_0 \Lambda_s \cos(2\omega t - 2\beta)] / 4\mu_0 & \dots \text{SAGE full load} \\ LRF_\delta^2 \pi [2\Lambda_0 \Lambda_s + \Lambda_0 \Lambda_s \cos(2\omega t - 2\beta)] / 4\mu_0 & \dots \text{SAGE half load} \\ LRF_{\delta l}^2 \pi [2\Lambda_0 \Lambda_s + \Lambda_0 \Lambda_s \cos(2\omega t - 2\beta)] / 4\mu_0 & \dots \text{SAGE no load} \end{cases} \\ F_y = \begin{cases} 0 & \dots \text{normal condition} \\ LRF_f^2 \pi [\Lambda_0 \Lambda_s \sin(2\omega t - 2\beta)] / 4\mu_0 & \dots \text{SAGE full load} \\ LRF_\delta^2 \pi [\Lambda_0 \Lambda_s \sin(2\omega t - 2\beta)] / 4\mu_0 & \dots \text{SAGE half load} \\ LRF_{\delta l}^2 \pi [\Lambda_0 \Lambda_s \sin(2\omega t - 2\beta)] / 4\mu_0 & \dots \text{SAGE no load} \end{cases} \end{cases} \quad (8)$$

According to Eqs.(7) and (8), the UMP in normal condition is zero, while in SAGE cases the UMPs have the same amplitude rank order as MFD. As indicated in Eq.(8), ignoring the higher order harmonics, the UMP has only the DC component and the pulsating components at $2f$. However, if taking the higher MMF order harmonics (3rd, 5th, 7th, etc.) into account, the UMP on the rotor should have each even harmonic.

D. Mechanical Response Analysis

As indicated in Fig. 1(c), the vibration in X direction and Y direction can be treated as the periodic movement in these two directions, respectively. The kinetic equation for the rotor can be written as

$$\begin{cases} M\ddot{x}_x(t) + D_x\dot{x}_x(t) + K_x x_x(t) = F_x(\alpha_m, t) \\ M\ddot{x}_y(t) + D_y\dot{x}_y(t) + K_y x_y(t) = F_y(\alpha_m, t) \end{cases} \quad (9)$$

where M is the mass matrix of the rotor, D_x and D_y are the damping matrix in X direction and Y direction, respectively, K_x and K_y are the stiffness matrix in X direction and Y direction, respectively, $F_x(\alpha_m, t)$ and $F_y(\alpha_m, t)$ are the exciting load matrices, i.e., the UMP in X direction and Y direction, respectively, and $x_x(t)$ and $x_y(t)$ are the displacement response matrix of the structural mass points in X direction and Y direction, respectively.

According to the correlation principle between the exciting force and the mechanical response, the vibration should have the same frequencies as the exciting forces. For example, if the pulsating frequency of the exciting force is $2f$, then, the rotor/rotor will vibrate at $2f$ as well.

If taking the higher-order MMF harmonics into account in addition to the 1st harmonic indicated in Eq.(2), the rotor vibrations should have each even harmonics, with the 2nd harmonic as the primary component. Comparing the formulas for each condition in Eq.(8), it can be found that in the SAGE cases, the amplitude of the 2nd-harmonic UMP will be increased as the load rises. As the increment of SAGE, Λ_s will be increased, and the 2nd vibration amplitude will also be enlarged.

III. FINITE ELEMENT CALCULATION AND EXPERIMENT

A. FEA and Experiment Setup

The FEA and the experimental work are taken on a 20000 rpm HSPMSM which is indicated in Fig.4 (a), in the University of Nottingham. The primary parameters of the prototype machine are shown in Table I.

Fig. 4. FEA model of the prototype HSPMSM: (a) prototype HSPMSM test rig, (b) Halbach Array of PM poles, and (c) 3D model of the rotor.

TABLE I. PARAMETERS OF STUDIED OBJECT

Parameters	Value	Parameters	Value
Rated power	50kW	pole-pairs	$p=3$
Conductor per slot	8	Winding layers	2
Fundamental frequency	$f=1000\text{Hz}$	Stator outer diameter	163mm
stator slots	36	Stator inner diameter	96mm
pitch	5	Rated phase current	150A
Rotor diameter	92mm	Rated rotating speed	$n=20000\text{rpm}$
Radial air-gap length	$g_0=2\text{mm}$	Parallel branches	1

The PM poles work in the form of Halbach Array, as indicated in Fig.4 (b). The stator winding is loaded with the rated current, the half-rated current, and almost no current to simulate the full-load, half-load, and no-load condition, respectively. During FEA calculation the rotor is offset along X axis and 40% SAGE is simulated. The magnetic pull obtained in the electromagnetics module is then imported into the structural module to get the mechanical response.

During the experiment the velocity sensors are fixed on both the motor and the upright fixing plate (near the bearing). The HSPMSM is excited to rotate from 0rpm to the rated rotating speed and then slow down again to 0rpm, to test both the vibration amplitude and the harmonic response.

B. Results and Discussion

UMPs and force density on rotor due to different running conditions are indicated in Fig.5 and Fig.6, while the deformations which reflect the vibration amplitudes are shown in Fig.7, and the harmonic response spectrum is indicated in Fig.8.

As indicated in Fig.5, the SAGE full load case has the maximum UMP, while the SAGE half load and the SAGE no load cases have the 2nd and the 3rd UMP, respectively. In the three normal cases the UMP is nearly zero. However, in reality it is very hard for a motor to strictly keep the air-gap symmetric, i.e., there is more or less a SAGE. Consequently, rotor UMP is actually rare to be zero.

Force densities on rotor for the 6 typical conditions indicated in Fig.6 suggest that the density rank order is the same as that of the MFD. In no load condition, the force density has only the radial component, while in loaded cases the force density will have both a radial and a tangential component at the same time. In addition, comparing Fig.5 with Fig.6 it can be inferred that the force density in the radial direction for the 3 normal conditions should be symmetrically distributed so that the general UMP can be zero. However, the occurrence of SAGE will break this kind of symmetry.

Fig. 5. UMPs on rotor in 6 typical running conditions (a) UMP in X direction, (b) UMP in Y direction

Rotor deformations due to the UMP and rotation shown in Fig.7 generally indicate that the deformation amplitudes rank at the same order as the MFD. The SAGE full load case will cause the max deformation, while the normal no load condition brings in minimum deformation. On one hand, the increment of the load will increase the rotor deformation. On the other hand, the SAGE case will induce a larger deformation to the rotor than the normal condition when with the same load.

The harmonic response calculation result indicated in Fig.8 shows that the rotor vibration will have the most intensified amplitudes at 3180Hz and 6080Hz. However, the rated working frequency of the prototype HSPMSM is 1000Hz, which is far away from these two frequencies. Consequently, the prototype HSPMSM can work safely within the affordable vibration.

Actual vibrations according to the varied rotating speed (different frequency, as indicated in Fig.9) for normal no-load, normal half-load, and normal full-load cases are indicated in Fig.10 (since the prototype HSPMSM is not able to simulate SAGE currently due to the condition limit, we have not yet obtained the vibration response in SAGE cases). Fig.10 is actually the harmonic response result.

Fig. 6. UMP density on rotor in 6 typical conditions: (a) normal full load, (b) normal half load, (c) normal no load, (d) SAGE full load, (e) SAGE half load, and (f) SAGE no load.

Fig. 7. Mechanical deformation due to UMP in 6 typical conditions: (a) normal full load, (b) normal half load, (c) normal no load, (d) SAGE full load, (e) SAGE half load, and (f) SAGE no load.

Fig. 8. Harmonic response of rotor

It is suggested from Figs. 9 and 10 that the vibration will obtain the maximum amplitudes when speeding up to the

rated rotating speed and slowing down from the rated speed (at about 18000rpm to 19500rpm). Moreover, it still shows that the full load case will have the max vibration amplitude, while the no-load case has the minimum. This conclusion is in consistent with the previously theoretical and FEA results. Since the rated rotating speed is 20000rpm (1000Hz) which is less than 3180Hz and 6080Hz, we do not get the full-band harmonic response. But it still confirms the security and validates the dynamic characteristic quality of the prototype HSPMSM since the maximum vibration amplitude is small (less than 4mm/s).

Fig. 9. Real-time rotating speed

Fig. 10. Test vibration

IV. CONCLUSION

This paper studies on the UMP and the mechanical response on the rotor in 6 different typical running conditions, namely normal full-load, normal half-load, normal no-load, SAGE full-load, SAGE half-load, and SAGE no-load, in HSPMSM. Detailed UMP formulas, which can be employed for a quick force calculation and assessment, are deduced for the 6 running cases, respectively. It shows that in the SAGE full-load case the UMP and the mechanical response will have the peak value, while in the normal no-load condition the value will be the minimum.

In this paper the mechanical response including rotor deformation, radial vibration and harmonic response, is studied by FEA and experiment as well. It shows that the increment of the load will increase the rotor deformation and vibration. In addition, the SAGE case will induce a larger deformation to the rotor and cause a more intensive vibration than the normal condition when with the same load. The harmonic response analysis suggests that the rotor will obtain the max deformation amplitude at 3180Hz and 6080Hz which are far away from the rotor's working frequency 1000Hz. The prototype HSPMSM can work safely due to its qualified dynamic characteristics.

Since it is very hard to strictly keep the rotor centered with respect to the stator, each motor should more or less have a SAGE. Bearing wearing, stator core deformation, radial rotor offset, etc., can all lead to a typical SAGE case and consequently the rotor UMP and vibration will be significantly intensified. From this point of view, special care should be taken to check whether the bearings and the stator core are in good fit.

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