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The small open economy in a generalized gravity model[☆]

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ABSTRACT

To provide sharp answers to basic questions in international trade, a standard approach is to focus on a small open economy (SOE). Whereas the classic tradition is to define a SOE as an economy that takes world prices as given, in the new trade literature it is defined instead as one that takes foreign-good prices and export demand schedules as given. We develop a gravity model that nests all its standard microfoundations and show how to take the limit so that an economy that becomes infinitesimally small behaves like a SOE. We then derive comparative statics and optimal policy for the SOE. Ignoring standard tax indeterminacies, optimal policy is characterized by export taxes and import tariffs equal to the (inverse) foreign demand and supply elasticities, respectively, and employment subsidies determined by the scale elasticity (under perfect competition) or markups (under monopolistic competition).

1. Introduction

How does an improvement in foreign productivity affect trade flows, prices, and wages? What are the welfare effects of import tariffs? What are optimal policies in open economies facing domestic distortions? To provide sharp answers to these and related questions in international trade, a standard approach has been to simplify the analysis by focusing on a small open economy (SOE). In the classical literature, a SOE is defined as an economy that takes world prices as given. In the context of modern trade theory, however, even infinitesimally small countries have pricing power, so a different conceptualization is needed.[1]

Flam and Helpman (1987) were the first to consider a SOE assumption in a new trade model, which they used to study the effects of various trade and industrial policies under monopolistic competition. Demidova and Rodríguez-Clare (2009) refined Flam and Helpman's definition of a SOE as one that takes foreign-good prices and export demand schedules as given, and further showed how

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[1] The fact that even infinitesimally small countries retain market power is why Gros (1987) finds that the optimal tariff in a Krugman (1980) setting does not converge to zero as the economy's size becomes infinitesimally small. Alvarez and Lucas (2007) reach the same conclusion in the context of the Ricardian model developed by Eaton and Kortum (2002) with productivity assumed proportional to country size.

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to extend the assumption to a setting with heterogeneous firms and selection à la Melitz (2003).[2] This modern version of the SOE assumption has now been used to study the comparative statics of trade-cost shocks (e.g., Demidova and Rodríguez-Clare, 2013), optimal trade policy (e.g., Demidova and Rodríguez-Clare, 2009; Haaland and Venables, 2016), and optimal industrial policy in open economies (e.g., Bartelme et al., 2021), among several different applications.

In this paper we revisit the SOE assumption in a generalized gravity model of trade that nests all the standard microfoundations that have been provided for such a model. We show how one can obtain the SOE as the limit in which an economy becomes infinitesimally small, although one must simultaneously let trade costs go to infinity to avoid awkward implications in the limit. The finding that this limit yields the SOE assumptions is important to formally link the results derived for the SOE to those derived in the standard case with large economies. We illustrate the usefulness of the SOE model by studying its implications for comparative statics and the optimal tariff.

Rather than limiting the analysis to a particular gravity microfoundation, as, for example, Demidova and Rodríguez-Clare (2009) do with the Melitz-Pareto model, we consider a general framework that nests the Armington and Eaton-Kortum models with external economies of scale (EES) and the Krugman and Melitz-Pareto models with nested preferences as in Kucheryavyy et al. (2023). We also allow for the fixed trade costs in the Melitz model to be in terms of labor in the source or destination country. This generality is possible by allowing for a positive scale elasticity and three different trade elasticities: one with respect to trade costs, one with respect to tariffs, and one with respect to wages. Specific models are obtained from particular combinations of these elasticities.[3] For example, the Krugman (1980) model corresponds to the case in which the three trade elasticities are the same and the scale elasticity is the inverse of this common trade elasticity. As another example, the Melitz-Pareto model with fixed trade costs paid in labor of the destination country is obtained by setting the scale elasticity equal to the inverse of the trade elasticity with respect to trade costs, and the trade elasticity with respect to trade costs equal to the one with respect to wages but lower than the one with respect to tariffs.

Simply letting an economy become infinitesimally small in such a framework implies that the domestic trade share tends to zero in the limit.[4] This not only makes it impossible to map the SOE to data, but it also leads to the awkward implication that the SOE would experience zero gains from optimal trade policy and infinite gains from trade.[5] To avoid this, we assume that inward and/or outward trade costs go to infinity. At one extreme, if the scale elasticity is zero — as in the Armington or Eaton-Kortum models with no EES — then we have the outward trade costs go to infinity; at the other extreme, if the scale elasticity is equal to the inverse of the trade elasticity — as in the standard Krugman and Melitz-Pareto models — then it is the inward trade costs that goes to infinity. Between these extremes, both outward and inward trade costs go to infinity at a rate determined by the scale and trade elasticity with respect to trade costs.

The equilibrium conditions in the single-sector SOE are simple and intuitive. As illustrated in Fig. 1, the equilibrium wage w is determined by the intersection of the downward sloping export demand curve $X(w) = D\left(AL^{\phi}\right)^{\epsilon} w^{-\rho}$ and the upward sloping import demand curve $M(w) = \frac{1-\lambda(w)}{1+(\bar{t}-1)\lambda(w)} wL$, with $\lambda(w) = \frac{\left(AL^{\phi}\right)^{\epsilon} w^{-\rho}}{\left(AL^{\phi}\right)^{\epsilon} w^{-\rho}+\bar{t}^{-\zeta}\mathcal{P}^{-\rho}}$ being the domestic trade share. Here D and $\mathcal{P}$ are exogenous parameters that capture the SOE's access to foreign markets on the export and import sides, respectively; A and L are productivity and labor endowment in the SOE; ϕ is the scale elasticity; ϵ, ρ, and ζ are the trade elasticity with respect to trade costs, wages, and tariffs, respectively; and $\bar{t}$ is (one plus) the SOE's import tariff. In turn, the gains from trade (equilibrium welfare divided by counterfactual autarky welfare) are given by

$$GT = \lambda^{-1/\epsilon}\left(\lambda + (1-\lambda)/\bar{t}\right)^{-\zeta/\epsilon}.$$

This collapses to the expression for gains from trade in Arkolakis et al. (2012) — henceforth ACR — if there are no tariffs ($\bar{t} = 1$). Even with tariffs ($\bar{t} > 1$), GT is decreasing in λ.

We can now use a simple graphical analysis to understand how different shocks affect the wage, trade flows, and welfare. An improvement in foreign productivity or a decline in inward trade costs would correspond to a decline in $\mathcal{P}$, leading to an upward shift in the M curve and a decline in the equilibrium wage. An increase in export demand corresponds to an increase in D, which leads to an upward shift in the X curve and an increase in the equilibrium wage. While the wage moves in opposite directions, in both cases there is an increase in imports (or exports) evaluated at international prices. This leads to a decline in the domestic trade share and an increase in the gains from trade.

Maximizing GT with respect to $\bar{t}$ yields the optimal tariff, which depends intuitively on the values of the different elasticities, as implied by our formula

$$\bar{t}^{*} - 1 = \frac{1}{(1+\rho)(\zeta/\rho) - 1}.$$

[2] In the Krugman (1980) model, Flam and Helpman's SOE takes as given the wage and the variety of goods in the rest of the world. The latter assumption implies that the export demand curve is fixed but not isoelastic. Demidova and Rodríguez-Clare (2009) instead take the wage and price index in the rest of the world as given, leading to an isoelastic export demand curve.

[3] We follow Demidova and Rodríguez-Clare (2009), Felbermayr et al. (2015), Haaland and Venables (2016) and Costinot et al. (2020) in modeling tariffs as demand shifters (in the terminology of Costinot and Rodríguez-Clare, 2014). This implies that the trade elasticity with respect to tariffs will differ from the one with respect to iceberg trade costs in the Melitz model, as was previously discussed in Costinot and Rodríguez-Clare (2014) and Felbermayr et al. (2015).

[4] See, for example, the analysis of a SOE in Alvarez and Lucas (2007), as well as the derivation of the optimal tariff for the Krugman (1980) model in Caliendo and Parro (2022) for the case in which the size of one of two countries goes to zero.

[5] Demidova and Rodríguez-Clare (2013) argue that they can obtain their SOE (with nonzero domestic trade share) in the Melitz-Pareto model by letting the economy become infinitesimally small, but their analysis is incorrect as the values of wage and productivity cutoffs in the limit were miscalculated due to wrongly assuming that the wage was strictly positive in the limit.

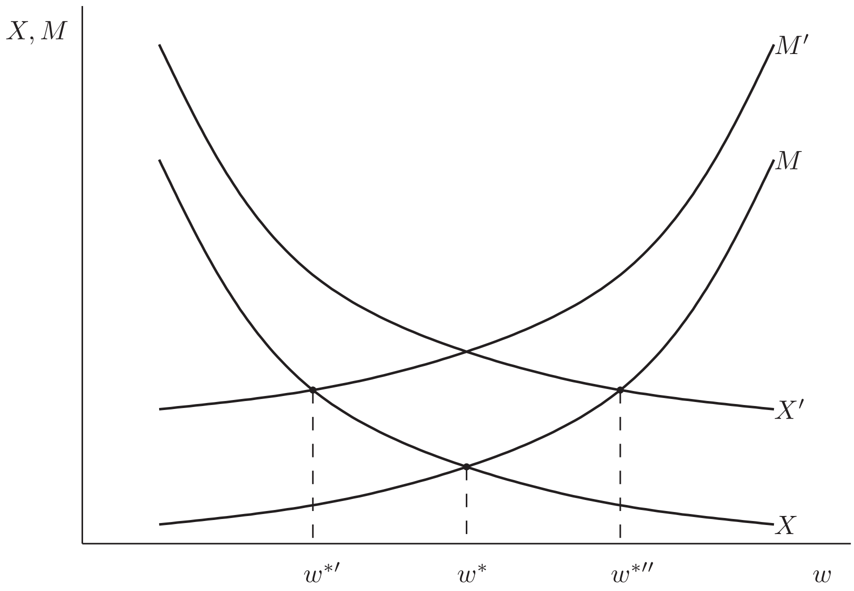

Fig. 1. Equilibrium conditions in the single-sector SOE.

Except for the Melitz-Pareto model with fixed trade costs paid in destination-country labor, all the microfoundations nested by our generalized gravity model entail $\zeta = \rho$ and so the optimal tariff is equal to the inverse of the trade elasticity with respect to wages, $\bar{t}^* - 1 = 1/\rho$. The Armington, Eaton-Kortum, and Krugman models (with or without EES, and with or without nested preferences) have $\rho = \varepsilon$ and so the optimal tariff is given by the inverse of the trade elasticity with respect to trade costs, as in Gros (1987) for the Krugman model and Alvarez and Lucas (2007) for the Eaton-Kortum model. In the Melitz-Pareto model with fixed trade costs paid in source-country labor we have $\rho > \varepsilon$. Thus, consistent with Demidova and Rodríguez-Clare (2009) and Costinot et al. (2020), the optimal tariff in this model is lower than the inverse of the trade elasticity with respect to trade costs, $\bar{t}^* - 1 = 1/\rho < 1/\varepsilon$. Finally, the Melitz-Pareto model with fixed trade costs paid in destination-country labor entails $\zeta > \rho = \varepsilon$, and hence, an optimal tariff even lower than the trade elasticity with respect to wages or trade costs, $\bar{t}^* - 1 < 1/\rho = 1/\varepsilon$.

How do these results differ in an environment with multiple sectors? First, we show that — like in the single-sector case — the multi-sector SOE in a generalized gravity model is obtained by letting the economy become infinitesimally small while sector-level trade costs grow to infinity at a rate determined by the trade and scale elasticities in their sector.[6]

Second, we perform comparative statics analysis in the case without taxes and subsidies, and show that the effects of changes in sector-specific foreign import supply or export demand parameters, denoted by $\mathcal{P}_k$ and D_k, have the same effect on the SOE wage as in the single-sector case. Namely, a increase in either $\mathcal{P}_k$ or D_k results in an increase in the SOE's wage. These changes are also causing labor to reallocate to sector k from all other sectors, which is accompanied by a fall in exports and domestic trade shares and a rise in imports in all sectors different from k. The effects of an increase in $\mathcal{P}_k$ or D_k on trade flows in sector k are nuanced. An increase in $\mathcal{P}_k$ results in a higher domestic trade share in sector k, but the effects on exports and imports in sector k are ambiguous. Similarly, an increase in D_k results in a rise in exports in sector k, but the effect on the domestic trade share and imports in sector k is ambiguous. In turn, all of these competing and ambiguous effects lead to an ambiguous effect on the SOE's welfare, consistent with the findings in the literature on the gains from trade in the presence of domestic distortions (e.g., Hagen (1958) and Święcki (2017)).

Finally, following Costinot et al. (2020), we show how to use a "micro-to-macro" approach to characterize the optimal policy in the multi-sector SOE. We find that the optimal policy is characterized by sector-level export taxes and import tariffs equal to the corresponding (inverse) foreign demand and supply elasticities, combined with sector-level employment subsidies determined by the sector's scale elasticity (in the perfect competition models) or markup (in the monopolistic competition models). Import tariffs are zero in all microfoundations except in the Melitz-Pareto model, where marketing fixed costs and selection lead to a negative supply elasticity, and hence, a negative import tariff (i.e., an import subsidy).[7]

[6] Here we consider all microfoundations mentioned above except the Melitz-Pareto model with fixed trade costs paid in labor of the destination country, as this poses some challenges that are left for future research. Moreover, the claims that we can make for the SOE being the limit as the size of the economy becomes infinitesimally small must be qualified in the presence of multiple sectors, as we discuss in Section 5.1.

[7] The SOE's optimal policy for the multi-sector Armington or Eaton-Kortum models extended to allow for external economies of scale is equivalent to that characterized in Bartelme et al. (2021), while the optimal policy in the generalized Krugman model is equivalent to that characterized in Lashkaripour and Lugovskyy (2023) except for the fact that they consider production subsidies instead of employment subsidies.

The force leading to an import subsidy in the Melitz model was pointed out by Demidova and Rodríguez-Clare (2009). However, this was dominated by the standard terms-of-trade force pushing for an export tax, and hence, the optimal policy in a single-sector environment is still an import tariff (or export tax). Haaland and Venables (2016) also characterize the optimal policy in an SOE, but restrict the analysis to an economy with two sectors: a Melitz-Pareto sector with fixed trade costs paid in the source country, and a homogeneous-good sector with constant or decreasing returns to labor. They show that the optimal policy entails an import subsidy, an employment subsidy, and an export tax in the Melitz-Pareto sector. In the working paper version of Costinot et al. (2020), the authors show in an extension that the same results hold without the SOE and Pareto assumptions, and with more general preferences. Our analysis with multiple sectors goes back to the SOE and Pareto assumptions in Haaland and Venables (2016), but is more general in that we dispense with the outside-good sector while using the micro-to-macro approach to connect the optimal tariff formula with more traditional concepts in the optimal taxation literature.

Our analysis is closely related to a contemporaneous paper by Caliendo and Feenstra (2024), in which they also study how to take a limit so as to achieve a SOE with a strictly positive domestic trade share, and study the optimal tariff in that limit economy. We highlight three differences between the two papers. First, we develop a generalized model that nests all standard microfoundations for the gravity model of trade (including the case in which fixed trade costs are paid in labor of the destination countries, which is absent in Caliendo and Feenstra (2024)) and then take the limit as one economy's size falls to zero, whereas Caliendo and Feenstra (2024) take the limit separately for each of the different microfoundations. We view our approach as having the benefit of simplicity and highlighting sufficient statistics that are common across all models, in the spirit of ACR and Kucheryavyy et al. (2023). Second, we develop a simple and intuitive graphical approach to comparative statics for the SOE, as illustrated in Fig. 1. Third, by introducing three different trade elasticities (i.e., with respect to trade costs, tariffs, and wages) we obtain a single generalized optimal tariff formula for all five microfoundations, including the Melitz-Pareto model with fixed trade costs paid in labor of destination countries.

There is a separate literature in international macroeconomics, starting with Gali and Monacelli (2005), that postulates a SOE to simplify the analysis of monetary policy in an open economy. Gali and Monacelli model a SOE as one of a continuum of countries trading differentiated goods and obtain a positive domestic trade share by separately allowing for domestic and foreign goods in the utility function. De Paoli (2009) drops the assumption of a continuum of countries and assumes instead that a SOE arises in the limit as a country becomes infinitesimally small, as we do in this paper. De Paoli assumes monopolistic competition with an exogenous measure of goods produced in each country proportional to its size, with Home and Foreign preferences shifting towards Home goods in such a way that in the limit both the wage and the domestic trade share in Home are positive and finite.

The rest of the paper is organized as follows. Section 2 presents the generalized gravity model, establishes that the equilibrium is unique, and describes how it nests the different microfoundations. Section 3 shows how to take the limit as one economy becomes infinitesimally small and describes the equilibrium of the resulting SOE. Section 4 studies comparative statics and the optimal tariff for the SOE. Section 5 extends the analysis to an economy with multiple sectors, and Section 6 derives the optimal policy result in the multi-sector SOE with the "micro-to-macro" approach developed in Costinot et al. (2020). Section 7 concludes.

2. A generalized single-sector gravity model

In this section we present a generalized single-sector and single-factor trade model exhibiting external economies of scale (EES) and satisfying a standard gravity equation. As shown in Section 2 of the Online Appendix, there are five different sets of microfoundations leading to the model equations that we present next: (i) an Armington model with technological EES; (ii) an Eaton-Kortum model with technological EES; (iii) a generalized Krugman model with nested CES preferences; (iv) a generalized Melitz-Pareto model with nested CES preferences and fixed trade costs paid in labor of source countries (the "Melitz-Pareto-source model"); and (v) a generalized Melitz-Pareto model with nested CES preferences and fixed trade costs paid in labor of destination countries (the "Melitz-Pareto-destination model"). The nested CES preferences in the last three models allow for a different elasticity of substitution between varieties produced within the same country and those produced across different countries. In turn, this allows the scale elasticity (defined below) to be different than the inverse of the trade elasticity.[8]

2.1. Gravity, price index, trade balance, and welfare

There are $N+1$ countries indexed by $i, j, l = 0, 1, \dots, N$. We let w_i and L_i denote the wage and labor endowment of i, A_i be a productivity shifter for i, τ_{ij} be the ad-valorem trade cost from i to j, and $\bar{t}_{ij}$ denote one plus the ad-valorem tariff that j imposes on imports from i. Without loss of generality, we set $\tau_{jj} = \bar{t}_{jj} = 1$. Trade shares $\lambda_{ij} \equiv X_{ij} / \sum_l X_{lj}$, where X_{ij} is j's expenditure on varieties from i, are given by

$$\lambda_{ij} = \frac{\bar{t}_{ij}^{-\zeta} \left[\tau_{ij} / \left(A_i L_i^{\phi}\right)\right]^{-\varepsilon} w_i^{-\rho}}{\sum_l \bar{t}_{lj}^{-\zeta} \left[\tau_{lj} / \left(A_l L_l^{\phi}\right)\right]^{-\varepsilon} w_l^{-\rho}} = \frac{\bar{t}_{ij}^{-\zeta} \left(\tau_{ij}/A_i\right)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l \bar{t}_{lj}^{-\zeta} \left(\tau_{lj}/A_l\right)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}, \tag{1}$$

where $\alpha \equiv \varepsilon\phi$. Parameter ε is the trade elasticity with respect to ad-valorem trade costs defined formally as $\varepsilon \equiv -\frac{\partial \ln(\lambda_{ij}/\lambda_{jj})}{\partial \ln \tau_{ij}}$; parameter ζ captures the trade elasticity with respect to tariffs, $\zeta \equiv -\frac{\partial \ln(\lambda_{ij}/\lambda_{jj})}{\partial \ln \bar{t}_{ij}}$; and parameter ρ is the trade elasticity with respect

[8] The analysis in this section follows closely the one in Kucheryavyy et al. (2023), but restricting it to the case of a single sector while extending it to allow for tariffs and the case with fixed trade costs paid in source labor.

to wages, $\rho \equiv -\frac{\partial \ln(\lambda_{ij}/\lambda_{jj})}{\partial \ln w_i}$. As discussed below, we need these three trade elasticities, along with the scale elasticity ϕ, to nest the five standard microfoundations for the gravity equation. We henceforth assume that $\varepsilon, \zeta, \rho > 0$.

Labor market clearing is given by

$$w_i L_i = \sum_j \Lambda_{ij} w_j L_j, \tag{2}$$

where

$$\Lambda_{ij} \equiv \frac{\lambda_{ij}/\bar{t}_{ij}}{\sum_l \lambda_{lj}/\bar{t}_{lj}} = \frac{\bar{t}_{ij}^{-(\zeta+1)} \left(\tau_{ij}/A_i\right)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l \bar{t}_{lj}^{-(\zeta+1)} \left(\tau_{lj}/A_l\right)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} \tag{3}$$

is the share of expenditure that j devotes to goods from i evaluated at pre-tariff import prices.

Turning to welfare, the price index in country j is given by

$$P_j = \delta_j w_j^{-\left(\frac{\rho}{\varepsilon}-1\right)} \left[\sum_i \frac{\lambda_{ij}/\bar{t}_{ij}}{L_j}\right]^{\frac{\zeta}{\varepsilon}-1} \left[\sum_i \bar{t}_{ij}^{-\zeta} \left(\tau_{ij}/A_i\right)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}\right]^{-\frac{1}{\varepsilon}}, \tag{4}$$

where δ_j is a model-specific constant defined for each of the five microfoundations in Section 2 of the Online Appendix. Combining this expression with (1) for $i = j$, the real wage is

$$\frac{w_j}{P_j} = \left[\frac{L_j}{\sum_i \lambda_{ij}/\bar{t}_{ij}}\right]^{\zeta/\varepsilon-1} \delta_j^{-1} A_j L_j^{\phi} \lambda_{jj}^{-1/\varepsilon}.$$

In Section 2.2 of the Online Appendix we show that welfare (i.e., real expenditure per capita) is given by

$$W_j \equiv \frac{w_j + T_j/L_j}{P_j} = L_j^{\zeta/\varepsilon-1} \left[\frac{1}{\sum_i \lambda_{ij}/\bar{t}_{ij}}\right]^{\zeta/\varepsilon} \delta_j^{-1} A_j L_j^{\phi} \lambda_{jj}^{-1/\varepsilon},$$

where T_j is j's tax revenue. Letting W_j^A denote welfare in the counterfactual corresponding to autarky and letting $\mathrm{GT}_j \equiv W_j/W_j^A$ denote the gains from trade, we then have

$$\mathrm{GT}_j = \lambda_{jj}^{-1/\varepsilon} \left[\sum_i \lambda_{ij}/\bar{t}_{ij}\right]^{-\zeta/\varepsilon}. \tag{5}$$

Taking labor of country N as the numeraire, $w_N \equiv 1$, an equilibrium is a wage vector $\boldsymbol{w} \equiv \left(w_0, w_1, \ldots, w_N\right)$ such that (2) holds for all i.

Proposition 1. *The equilibrium exists and is unique.*

The proof of Proposition 1 is provided in Section 3 of the Online Appendix and follows a standard logic. To establish existence, we demonstrate that all conditions outlined in Proposition 17.B.2 of Mas-Colell et al. (1995) are satisfied. For uniqueness, we show that the excess demand system exhibits the gross substitutes property.

2.2. Microfoundations

We finish this section by describing in Table 1 how the five different models map into the generalized model corresponding to the previous equations. Parameter η is the elasticity of substitution across varieties from different countries (applicable in all models except the Eaton-Kortum model), while σ is the elasticity of substitution across varieties from the same country (applicable in the Krugman and Melitz-Pareto models). Parameter γ is the technological scale elasticity in the Armington and Eaton-Kortum models. Parameter ϑ is the shape parameter of the Fréchet distribution in the Eaton-Kortum model while $\theta > \sigma - 1$ is the shape parameter of the Pareto distribution in the Melitz-Pareto model. Parameter ξ is given by $\xi \equiv \left[1 + \theta\left(\frac{1}{\eta-1} - \frac{1}{\sigma-1}\right)\right]^{-1}$.

Ad-valorem trade costs τ_{ij} are equal to the iceberg trade cost $\overline{\tau}_{ij}$ in all models except for Melitz-Pareto, where instead trade costs combine iceberg and fixed trade costs, $\tau_{ij} \equiv \left(f_{ij}/f_{jj}\right)^{1/(\sigma-1)-1/\theta} \overline{\tau}_{ij}$. The productivity shifter A_i equals productivity itself in the Armington model ($A_i = \overline{A}_i$), average productivity ($A_i = B_i^{1/\vartheta}$) in the Eaton-Kortum model, the common firm-level productivity adjusted by the effect of entry costs on varieties in the Krugman model ($A_i = \left(f_i^e\right)^{-1/(\sigma-1)} a_i$), and the lower bound of the support of the Pareto distribution adjusted by the effect of entry costs on average productivity of surviving firms in the Melitz model ($A_i = \left(f_i^e\right)^{-1/\theta} b_i$). Lastly, the constants δ^{EK}, δ^{K}, and δ_j^{M} are model-specific constants derived in Section 2 of the Online Appendix for each of the corresponding models.

The first three rows of Table 1 highlight several points. First, we have $\varepsilon = \zeta = \rho$ in the Armington, Eaton-Kortum, and Krugman models. Second, the trade elasticity with respect to tariffs ζ is larger than the trade elasticity with respect to trade costs ε in the Melitz-Pareto model. Third, the only difference between the two fixed cost specifications of the Melitz-Pareto model lies in the trade elasticity with respect to wages: $\varepsilon < \rho = \zeta$ in the Melitz-Pareto-source model, while $\varepsilon = \rho < \zeta$ in the Melitz-Pareto-destination model. The last point will create a difference in the SOE's optimal tariff between the two cases of the Melitz-Pareto model, as we will see in Section 4.2.

Table 1
Mapping the five different trade models into the general model.

Model	Armington-EES	EK-EES	Gen. Krugman	Gen. Melitz source	Gen. Melitz destination
ε	$\eta-1$	ϑ	$\eta-1$	$\theta\xi$	$\theta\xi$
ζ	$\eta-1$	ϑ	$\eta-1$	$\theta\xi\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]$	$\theta\xi\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]$
ρ	$\eta-1$	ϑ	$\eta-1$	$\theta\xi\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]$	$\theta\xi$
ϕ	γ	γ	$\frac{1}{\sigma-1}$	$\frac{1}{\theta}$	$\frac{1}{\theta}$
$\alpha\equiv\varepsilon\phi$	$(\eta-1)\gamma$	$\vartheta\gamma$	$\frac{\eta-1}{\sigma-1}$	ξ	ξ
τ_{ij}	$\overline{\tau}_{ij}$	$\overline{\tau}_{ij}$	$\overline{\tau}_{ij}$	$\left(f_{ij}/f_{jj}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\,\overline{\tau}_{ij}$	$\left(f_{ij}/f_{jj}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\,\overline{\tau}_{ij}$
A_i	$\overline{A}_i$	$B_i^{\frac{1}{\vartheta}}$	$\left(f_i^e\right)^{-\frac{1}{\sigma-1}}a_i$	$\left(f_i^e\right)^{-\frac{1}{\theta}}b_i$	$\left(f_i^e\right)^{-\frac{1}{\theta}}b_i$
δ_j	1	δ^{EK}	δ^{K}	δ_j^{M}	δ_j^{M}

In the fourth row of Table 1 we see that in the standard Krugman and Melitz-Pareto models, $\phi = 1/\varepsilon$ and hence $\alpha = 1$. However, in the generalized Krugman and Melitz-Pareto models with nested CES preferences we may have $\alpha \neq 1$, as in the Armington and Eaton-Kortum models with EES (see Kucheryavyy et al., 2023).

Turning to welfare, since $\varepsilon = \zeta$ in the Armington-EES, Eaton-Kortum-EES, and generalized Krugman models, while $\zeta > \varepsilon$ in the Melitz-Pareto model, expression (5) implies that in the presence of tariffs, gains from trade given ε are higher in the Melitz-Pareto model than in the other models. This is because the tariff revenue transferred to the representative household increases j's total expenditure, which increases the mass of entrants surviving in j's domestic market, thereby lowering its price index. Formally, we can rewrite (5) as

$$\mathrm{GT}_j = \frac{\lambda_{jj}^{-1/\varepsilon}}{\sum_i \lambda_{ij}/\bar{t}_{ij}}\left(\frac{1}{\sum_i \lambda_{ij}/\bar{t}_{ij}}\right)^{\zeta/\varepsilon-1}.$$

The term $1/\sum_i\left(\lambda_{ij}/\bar{t}_{ij}\right)$ captures the ratio of total expenditure to wage income in country j, and is referred to as the *tariff multiplier* in Felbermayr et al. (2015). As long as $\bar{t}_{ij} > 1$ for all $i \neq j$ in country j, implying that there are tariff revenues, the tariff multiplier will be strictly higher than one, $1/\sum_i\left(\lambda_{ij}/\bar{t}_{ij}\right) > 1$. This implies that the second term in the expression above is larger than 1 in the Melitz-Pareto model since it has $\zeta > \varepsilon$, leading to larger gains from trade in this model relative to the others, conditional on the same data and ε.

3. Small open economy

We now use the generalized gravity model described in the previous section to obtain a well-behaved equilibrium with a SOE. Suppose that country 0's labor is expressed as $L_0 \equiv n\widetilde{L}_0$, where $\widetilde{L}_0$ is constant. We will explore conditions under which, as $n \to 0$, country 0 becomes a SOE in the limit.

3.1. A first look

To understand potential problems of an equilibrium with a SOE and get an idea of how to fix them, we first consider two popular examples with two countries ($i, j, l = 0, 1$). Our numeraire assumption now entails $w_1 = 1$.

Example 1. Consider the Armington or Eaton-Kortum model without EES (i.e., $\alpha = 0$). In this case, Eq. (2) reduces to

$$\underbrace{\frac{A_0^{\varepsilon}\bar{t}_{01}^{-1-\zeta}\tau_{01}^{-\varepsilon}w_0^{-\rho}}{A_0^{\varepsilon}\bar{t}_{01}^{-1-\zeta}\tau_{01}^{-\varepsilon}w_0^{-\rho}+A_1^{\varepsilon}}}_{=\Lambda_{01}} L_1 = \underbrace{\frac{A_1^{\varepsilon}\bar{t}_{10}^{-1-\zeta}\tau_{10}^{-\varepsilon}}{A_0^{\varepsilon}w_0^{-\rho}+A_1^{\varepsilon}\bar{t}_{10}^{-1-\zeta}\tau_{10}^{-\varepsilon}}}_{=\Lambda_{10}} w_0 n\widetilde{L}_0. \tag{6}$$

This is country 0's trade balance condition, with exports on the left-hand side and imports on the right-hand side, both evaluated at pre-tariff import prices. One can show that it must be the case that $w_0 \to \infty$ as $n \to 0$. The reason for this is that country 0's wage is the only variable that can adjust to ensure equality between country 0's exports and shrinking imports.[9] This requires country 0's production becoming more costly as n decreases.

[9] To see this formally, suppose that there is a bounded subsequence of equilibrium wages w_0 corresponding to some sequence of $n \to 0$. Then for this subsequence of wages, country 0's imports go to zero while exports are bounded away from zero. Consequently, the trade balance condition cannot hold for all sufficiently small n, leading to a contradiction.

One way to resolve the issue with infinite wages is to follow Alvarez and Lucas (2007) and assume that country 0's productivity shifter A_0 is proportional to $n^{1/\epsilon}$, $A \equiv n^{1/\epsilon}\tilde{A}_0$, where $\tilde{A}_0$ is constant. Substituting this into (6), multiplying both sides by $1/n$, and taking $n \to 0$, we obtain $\left(\widetilde{A}_0\bar{t}_{01}^{-1-\zeta}\tau_{01}^{-\epsilon}\widetilde{w}_0^{-\rho}/A_1^{\epsilon}\right)L_1 = \widetilde{w}_0\widetilde{L}_0$, which is solved for a unique $\widetilde{w}_0 \in (0,\infty)$. Intuitively, productivity, and hence, exports shrink to zero as the country becomes infinitesimally small without the need for the wage to increase to infinity. One unfortunate implication, however, is that country 0 stops buying anything domestically, $\Lambda_{00} \to 0$, which is not empirically relevant.

We propose another way to resolve the issue of $w_0 \to \infty$ as $n \to 0$. We can make exports shrink by making them increasingly costly as $n \to \infty$. Formally, assume that $\tau_{01} \equiv n^{-1/\epsilon}\widetilde{\tau}_{01}$, where $\widetilde{\tau}_{01}$ is constant. Substituting this into (6), multiplying both sides by $1/n$, and taking $n \to 0$, country 0's trade balance condition reduces to

$$\frac{A_0^{\epsilon}\bar{t}_{01}^{-1-\zeta}\widetilde{\tau}_{01}^{-\epsilon}\widetilde{w}_0^{-\rho}}{A_1^{\epsilon}}L_1 = \frac{A_1^{\epsilon}\bar{t}_{10}^{-1-\zeta}\tau_{10}^{-\epsilon}}{A_0^{\epsilon}\widetilde{w}_0^{-\rho} + A_1^{\epsilon}\bar{t}_{10}^{-1-\zeta}\tau_{10}^{-\epsilon}}\widetilde{w}_0\widetilde{L}_0.$$

This equation determines a unique equilibrium wage $\widetilde{w}_0 \in (0,\infty)$, while also having $\Lambda_{00} \in (0,1)$. Intuitively, as country 0's labor force shrinks it also becomes increasingly costly for country 0 to export to the rest of the world, and this prevents the wage from shooting off to infinity without leading to its domestic trade share shrinking to zero.

Example 2. Consider the standard Krugman or Melitz-Pareto model (i.e., $\alpha = 1$). In this case, when multiplied on both sides by $1/n$, Eq. (2) leads to

$$\underbrace{\frac{A_0^{\epsilon}\bar{t}_{01}^{-1-\zeta}\tau_{01}^{-\epsilon}w_0^{-\rho}n\widetilde{L}_0}{A_0^{\epsilon}\bar{t}_{01}^{-1-\zeta}\tau_{01}^{-\epsilon}w_0^{-\rho}n\widetilde{L}_0 + A_1^{\epsilon}L_1}}_{=\Lambda_{01}}\cdot\frac{L_1}{n} = \underbrace{\frac{A_1^{\epsilon}\bar{t}_{10}^{-1-\zeta}\tau_{10}^{-\epsilon}L_1}{A_0^{\epsilon}w_0^{-\rho}n\widetilde{L}_0 + A_1^{\epsilon}\bar{t}_{10}^{-1-\zeta}\tau_{10}^{-\epsilon}L_1}}_{=\Lambda_{10}}w_0\widetilde{L}_0. \tag{7}$$

One can show that, as n approaches zero, the corresponding sequence of wages remains bounded away from zero and from above.[10] The problem is that, as n approaches zero, Λ_{10} approaches one and Λ_{00} approaches zero. This is similar to what happens in the Alvarez and Lucas (2007) approach described above except that here productivity decreases endogenously through the external economies of scale in the Krugman and Melitz-Pareto models.

To counteract the effects of decreasing productivity and shrinking exports, we propose making imports increasingly costly as n decreases. Formally, assume that $\tau_{10} \equiv n^{-1/\epsilon}\widetilde{\tau}_{10}$, where $\widetilde{\tau}_{10}$ is constant. Substituting this into (7) and taking $n \to 0$, country 0's trade balance becomes

$$\frac{A_0^{\epsilon}\bar{t}_{01}^{-1-\zeta}\tau_{01}^{-\epsilon}\widetilde{w}_0^{-\rho}\widetilde{L}_0}{A_1^{\epsilon}L_1}L_1 = \frac{A_1^{\epsilon}\bar{t}_{10}^{-1-\zeta}\widetilde{\tau}_{10}^{-\epsilon}L_1}{A_0^{\epsilon}\widetilde{w}_0^{-\rho}\widetilde{L}_0 + A_1^{\epsilon}\bar{t}_{10}^{-1-\zeta}\widetilde{\tau}_{10}^{-\epsilon}L_1}\widetilde{w}_0\widetilde{L}_0.$$

Note that we have eliminated n in all terms of Λ_{10}. Since the above equation is solved for a unique equilibrium wage $\widetilde{w}_0 \in (0,\infty)$, we again obtain $\Lambda_{00} \in (0,1)$.

3.2. General case

The two examples presented in Section 3.1 reveal that by adjusting ad-valorem trade costs appropriately, we can obtain a well-behaved equilibrium with a SOE where the domestic expenditure share is positive (in contrast to, for example, Alvarez and Lucas, 2007). Generalizing this idea to $N+1$ countries is straightforward, while extending it to a general α is slightly less so.

The general case entails $\alpha \in [0,1]$ with Examples 1 and 2 from Section 3.1 representing the extremes with $\alpha = 0$ and $\alpha = 1$, respectively.[11] For a given trade elasticity, higher values of α are associated with stronger EES, and so moving from Example 1 to Example 2 involves increasing the strength of EES. The presence of any level of EES ($\alpha > 0$) implies that productivity falls as country 0 becomes small, with productivity falling faster for stronger EES (larger α). This declining productivity reduces exports of country 0, and so makes it less necessary to increase outward trade costs as $n \to 0$. On the other hand, this fall in productivity implies the need to increase inward trade costs so that the domestic trade share does not fall to zero as $n \to 0$. This leads to the following adjustments in trade costs with elasticities that depend on α:

$$L_0 \equiv n\widetilde{L}_0,$$
$$\tau_{0j} \equiv n^{-(1-\alpha)/\epsilon}\widetilde{\tau}_{0j}, \quad j = 1,\dots,N,$$

[10] To see this formally, suppose that for some sequence of $n \to 0$, there exists a subsequence of equilibrium wages w_0 converging to ∞. Then for this subsequence of wages, the right hand-side of (7) converges to infinity while the left-hand side converges to zero, resulting in a contradiction. Thus, it must be the case that w_0 is bounded as $n \to 0$. One can similarly verify that there cannot be any subsequence of wages converging to 0 as $n \to 0$.

[11] All the results in this section remain valid with $\alpha < 0$ or $\alpha > 1$. The case with $\alpha < 0$ would arise if there are external diseconomies of scale ($\phi < 0$) or if there is a fixed factor leading to decreasing returns to labor in the Armington or EK microfoundations. One could also have $\alpha < 0$ in the Krugman model with the Benassy (1996) correction for love of variety or in the Melitz model with congestion effects leading entry costs to increase with the number of firms serving the market as in Bhattarai and Kucheryavyy (2024). The case with $\alpha > 1$ would arise if economies of scale are "strong" in the Armington or EK microfoundations, or if the elasticity of substitution across domestic varieties is lower than the elasticity of substitution across country-level aggregates, $\sigma < \eta$, in the Krugman and Melitz microfoundations.

$$\tau_{i0} \equiv n^{-\alpha/\varepsilon}\widetilde{\tau}_{i0}, \quad i = 1,\ldots,N.$$

In the case of $\alpha \in [0,1]$, these adjustments imply that both the *value* of exports and imports of country 0 decrease at the same rate as n due to increases in export costs, τ_{0j}, as well as due to diminishing productivity of country 0 in the presence of EES. At the same time, any potential collapse in the domestic trade share resulting from diminishing productivity in the presence of EES is counteracted by an equivalent increase in the costs of imports, τ_{i0}.[12]

With the above adjustments, the equilibrium system of equations given by (2) can be written as

$$w_i L_i = \Lambda_{i0} w_0 n \widetilde{L}_0 + \sum_{j=1}^{N} \Lambda_{ij} w_j L_j, \quad i = 1,\ldots,N, \tag{8}$$

$$w_0 \widetilde{L}_0 = \Lambda_{00} w_0 \widetilde{L}_0 + \sum_{j=1}^{N} \left(\Lambda_{0j}/n\right) w_j L_j, \tag{9}$$

Note that (9) is obtained by multiplying (2) for $i = 0$ by $1/n$.[13] In contrast to Λ_{0j}, which approaches zero as n approaches zero, Λ_{0j}/n stays positive even if n approaches zero.

The following proposition characterizes an equilibrium with a SOE as a limit of equilibria of the gravity model introduced Section 2:

Proposition 2. *For any $n > 0$, let $\left(w_0^{(n)}, w_1^{(n)}, \ldots, w_N^{(n)}\right)$ be the wage vector that solves the system of Eqs. (8)–(9). As $n \to 0$ $\left(w_0^{(n)}, w_1^{(n)}, \ldots, w_N^{(n)}\right)$ converges to a finite positive vector $\left(w_0^{(0)}, w_1^{(0)}, \ldots, w_N^{(0)}\right)$ such that*

1. *$\left(w_1^{(0)}, \ldots, w_N^{(0)}\right)$ solves (2) for all $i, j, l = 1,\ldots,N$ not including country 0;*
2. *Given $\left(w_1^{(0)}, \ldots, w_N^{(0)}\right)$, $w_0^{(0)}$ solves equation*

$$\left[\sum_{j=1}^{N} \frac{A_0^{\varepsilon} \bar{t}_{0j}^{-1-\zeta} \widetilde{\tau}_{0j}^{-\varepsilon} \widetilde{L}_0^{\alpha} w_j^{(0)} L_j}{\sum_{i=1}^{N} A_i^{\varepsilon} \bar{t}_{ij}^{-1-\zeta} \tau_{ij}^{-\varepsilon} \left[w_i^{(0)}\right]^{-\rho} L_i^{\alpha}}\right] w_0^{-\rho} = \frac{\sum_{i=1}^{N} A_i^{\varepsilon} \bar{t}_{i0}^{-1-\zeta} \widetilde{\tau}_{i0}^{-\varepsilon} \left[w_i^{(0)}\right]^{-\rho} L_i^{\alpha}}{A_0^{\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{i=1}^{N} A_i^{\varepsilon} \bar{t}_{i0}^{-1-\zeta} \widetilde{\tau}_{i0}^{-\varepsilon} \left[w_i^{(0)}\right]^{-\rho} L_i^{\alpha}} w_0 \widetilde{L}_0. \tag{10}$$

The proof of Proposition 2 is provided in Section 4 of the Online Appendix. The proof is relatively straightforward but tedious. While we intuitively expect the limit of the sequence of equilibrium wages $\left(w_0^{(n)}, w_1^{(n)}, \ldots, w_N^{(n)}\right)$ to solve the equilibrium system of equations obtained from (8)–(9) in the limit as $n \to 0$, this convergence is not guaranteed. Generally, the solutions of a sequence of equations may not necessarily converge to a solution of the equation obtained in the limit, even if this equation is uniformly continuous.

We prove Proposition 2 in three steps. First, we formally show that the sequence of equilibrium wages $\left(w_0^{(n)}, w_1^{(n)}, \ldots, w_N^{(n)}\right)$ is bounded from above, then we show that it is bounded away from zero, and finally, we show that all converging subsequences of this sequence have the same limit that solves the equilibrium system of equations obtained from (8)–(9) in the limit as $n \to 0$. The limit of (8)–(9) as $n \to 0$ gives the two sets of equations described in Proposition 2: the first set of equations is given by (2) reduced to countries $1,\ldots,N$, and the second set of equations consists of Eq. (9) only.

Proposition 2 has two important implications. First, from $w_0^{(0)} \in (0,\infty)$, the SOE has a positive domestic expenditure share (evaluated at pre-tariff import prices) $\Lambda_{00}^{(0)} \equiv \lim_{n\to 0} \Lambda_{00}^{(n)} \in (0,1)$. This allows for applications of our model with the SOE in quantitative analysis using actual production and trade data. Second, all variables within a group of large countries 1 to N are determined independently of country 0. This satisfies Demidova and Rodríguez-Clare (2013, p. 269) SOE assumptions that the rest of the world's cutoff productivity for domestic sales, mass of entrants, income, and price index are independent of variables related to the SOE.

Having established the equilibrium of the SOE as a limit of the generalized gravity model of Section 2, we now turn to examining the properties of this limit. To do so, we treat the wages of countries 1 through N as given and focus on Eq. (10). For brevity of exposition, we dispense with the superscript "(0)" denoting the limit, and drop the tildes and the country 0 subscript. Consequently, for instance, the wage $w_0^{(0)}$ becomes w, labor allocation $\tilde{L}_0$ becomes L, and import tariffs $\bar{t}_{i0}$ become $\bar{t}_i$. Additionally, we streamline notation by letting $p_i \equiv \left[\tilde{\tau}_{i0}/\left(A_i L_i^{\phi}\right)\right]^{\varepsilon/\rho} w_i$ to denote a shifter of the SOE's price of imports from country i, and by introducing

$$D \equiv \sum_{j=1}^{N} \frac{\bar{t}_{0j}^{-1-\zeta} \widetilde{\tau}_{0j}^{-\varepsilon} w_j L_j}{\sum_{i=1}^{N} A_i^{\varepsilon} \bar{t}_{ij}^{-(\zeta+1)} \tau_{ij}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}$$

to denote the market access abroad, which is exogenous to the SOE. With this notation, the expenditure shares (at domestic prices) devoted by the SOE to itself and to imports from country i simplify, respectively, to

$$\lambda(w) = \frac{A^{\varepsilon} L^{\alpha} w^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{l=1}^{N} \bar{t}_l^{-\zeta} p_l^{-\rho}}$$

[12] These adjustments also work if $\alpha < 0$ or $\alpha > 1$, with the interpretation changing in obvious ways.

[13] See Appendix A for a more detailed description of the equilibrium system of equations.

and

$$\lambda_i(w) = \frac{\bar{t}_i^{-\zeta} p_i^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{l=1}^{N} \bar{t}_l^{-\zeta} p_l^{-\rho}},$$

and Eq. (10) reduces to

$$X(w) = M(w), \tag{11}$$

with exports $X(w) = DA^{\varepsilon} L^{\alpha} w^{-\rho}$ on the left-hand side, and imports

$$M(w) = \frac{\sum_{i=1}^{N} \lambda_i(w)/\bar{t}_i}{1 - \sum_{i=1}^{N} \left(1 - 1/\bar{t}_i\right) \lambda_i(w)} wL$$

on the right-hand side.[14]

Turning to welfare, we use $\mathrm{GT} \equiv W/W^A$ introduced in Section 2.2 as the SOE's welfare measure,

$$\mathrm{GT} \equiv W/W^A \equiv \lambda^{-1/\varepsilon} \left(\lambda + \sum_{i=1}^{N} \lambda_i/\bar{t}_i\right)^{-\zeta/\varepsilon}. \tag{12}$$

Without tariffs we would have $\lambda + \sum_{i=1}^{N} \lambda_i/\bar{t}_i = \lambda + \sum_{i=1}^{N} \lambda_i = 1$ and so the previous expression leads directly to the ACR formula, $\mathrm{GT}'/\mathrm{GT} = \left(\lambda'/\lambda\right)^{-1/\varepsilon}$. With tariffs, however, the SOE's welfare depends not only on λ but also on λ_i and $\bar{t}_i$ through the tariff multiplier. The relative change in the SOE's welfare in this case is

$$\mathrm{GT}'/\mathrm{GT} = \left(\lambda'/\lambda\right)^{-1/\varepsilon} \left[\Lambda\left(\lambda'/\lambda\right) + \sum_{i=1}^{N} \Lambda_i \left(\lambda_i'/\lambda_i\right) / \left(\bar{t}_i'/\bar{t}_i\right)\right]^{-\zeta/\varepsilon}.$$

Thus, to obtain GT'/GT, we need to know relative changes in λ, λ_i, and $\bar{t}_i$, as well as the initial values of $\Lambda = \lambda \Big/ \left[\lambda + \sum_{l=1}^{N} \lambda_l/\bar{t}_l\right]$ and $\Lambda_i = \left(\lambda_i/\bar{t}_i\right) \Big/ \left[\lambda + \sum_{l=1}^{N} \lambda_l/\bar{t}_l\right]$, and the two trade elasticities ε and ζ.

4. Comparative statics and optimal policy in the SOE

4.1. Comparative statics

Fig. 1 illustrates how the SOE's equilibrium wage is determined. Curve X is downward sloping while curve M is upward sloping, leading to a unique equilibrium wage w^* at which the SOE achieves trade balance. Now imagine a shock that improves the SOE's market access to the rest of the world, as captured by an increase in D. This shifts curve X up to curve X', leading to an increase in the equilibrium wage from w^* to $w^{*\prime\prime}$. On the other hand, a shock that improves the SOE's access to foreign goods, as captured by a decline in p_i for some i, shifts curve M up to curve M', resulting in a decrease in the equilibrium wage from w^* to $w^{*\prime}$. Intuitively, an increase in D generates a trade surplus for the SOE, and a higher wage is needed to restore the trade balance. Conversely, a decline in p_i results in a trade deficit, and the trade balance is restored through a reduced wage. In both cases the shock leads to an increase in trade.

The improvement in market access to the rest of the world clearly increases welfare. This follows immediately from the fact that the increase in the wage leads to an increase in $\lambda_i(w)$ for all i, leading to a decline in both λ and

$$\lambda + \sum_{i=1}^{N} \lambda_i/\bar{t}_i = 1 - \sum_{i=1}^{N} \left(1 - 1/\bar{t}_i\right) \lambda_i.$$

The decline in p_i for some i also increases welfare if there is no variation in tariffs across source countries, $\bar{t}_i = \bar{t}$. In this case GT simplifies to

$$\mathrm{GT} = \lambda^{-1/\varepsilon} \left[\lambda + (1-\lambda)/\bar{t}\right]^{-\zeta/\varepsilon}.$$

Since this is decreasing in λ, all we need to show is that λ falls with the shock. Now, if $\bar{t}_i = \bar{t}$ then we have

$$M(w) = \frac{1-\lambda}{1 + \left(\bar{t} - 1\right)\lambda} wL$$

(see footnote 14). Since the shock leads to a decrease in w, the fact that $M(w)$ increases then necessarily implies that λ falls.[15] In contrast, if $\bar{t}_i$ varies across countries, a decline in p_i for some i could lead to a fall in welfare. This is because such a shock could exacerbate the tariff-induced misallocation in the SOE's expenditures across different origins.

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[14] If we impose $\bar{t}_i = \bar{t}$ for all i, define $\mathcal{P} \equiv \left(\sum_{i=1}^{N} p_i^{-\rho}\right)^{-1/\rho}$, and use $\lambda(w) = 1 - \sum_{i=1}^{N} \lambda_i(w)$, then we get $\lambda(w) = \frac{\left(AL^{\phi}\right)^{\varepsilon} w^{-\rho}}{\left(AL^{\phi}\right)^{\varepsilon} w^{-\rho} + \bar{t}^{-\zeta} \mathcal{P}^{-\rho}}$ and $M(w) = \frac{1-\lambda(w)}{1+\left(\bar{t}-1\right)\lambda(w)} wL$, as in the Introduction.

[15] Since $\widetilde{\tau}_{i0}$ (which is part of p_i) includes both variable and fixed trade costs in the Melitz-Pareto model, the analysis above can be applied to shocks in both types of trade costs in that model. This generalizes the results in Demidova and Rodríguez-Clare (2013), who studied the case with no differences between elasticities of substitution within/across countries (i.e., $\eta = \sigma$) and no tariffs (i.e., $\bar{t}_i = 1$ for all i).

Table 2
The SOE's optimal tariff for the five microfoundations.

Model	Armington-EES	EK-EES	Gen. Krugman	Gen. Melitz source	Gen. Melitz destination
ε	$\eta-1$	ϑ	$\eta-1$	$\theta\xi$	$\theta\xi$
ζ	$\eta-1$	ϑ	$\eta-1$	$\theta\xi\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]$	$\theta\xi\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]$
ρ	$\eta-1$	ϑ	$\eta-1$	$\theta\xi\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]$	$\theta\xi$
$\bar{t}^*-1$	$\frac{1}{\eta-1}$	$\frac{1}{\vartheta}$	$\frac{1}{\eta-1}$	$\frac{1}{\theta\xi\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]}$	$\frac{1}{(1+\theta\xi)\left[1+\frac{1}{\sigma-1}-\frac{1}{\theta}\right]-1}$

4.2. Optimal policy

From (11) we obtain (see Appendix C)

$$\frac{\partial \ln w}{\partial \ln \bar{t}_i} = \frac{\zeta+1}{\Delta}\cdot\frac{\Lambda}{1-\Lambda}\Lambda_i > 0, \tag{13}$$

where $\Delta \equiv 1+\rho(1+\Lambda) > 1$. Intuitively, as with a shock that improves the SOE's access to foreign goods (i.e., a decline in p_i), a decrease in $\bar{t}_i$ leads to a trade deficit, so a lower wage is needed to restore trade balance. Differentiating (12) and using (13), we obtain (see Appendix C)

$$\frac{\partial \ln \mathrm{GT}}{\partial \ln \bar{t}_i} = \underbrace{\frac{\rho}{\varepsilon}\left[1-\lambda+\zeta(\Lambda-\lambda)\right]\frac{\zeta+1}{\Delta}\cdot\frac{\Lambda}{1-\Lambda}\Lambda_i}_{\text{terms-of-trade effect}}\underbrace{-\frac{\zeta}{\varepsilon}(\zeta+1)\left(\lambda_i-\Lambda_i\right)}_{\text{direct effect}}. \tag{14}$$

This highlights the well-known tradeoff associated with a tariff: a higher tariff reduces welfare by discouraging imports (the direct effect) but generates gains by increasing the wage (the terms-of-trade effect).[16] Equating $\frac{\partial \ln \mathrm{GT}}{\partial \ln \bar{t}_i}$ to zero yields the optimal tariff, which is the same across all source countries.

Proposition 3. *The SOE's optimal tariff is the same for all source countries, $\bar{t}_i^* = \bar{t}^*$, with $\bar{t}^*$ given by*

$$\bar{t}^* - 1 = \frac{1}{(1+\rho)\frac{\zeta}{\rho}-1}. \tag{15}$$

The proof of Proposition 3 is provided in Appendix C. Table 2 shows what the SOE's optimal tariff formula in (15) implies for each of the five different microfoundations from Section 2.2. In the Armington-EES, Eaton-Kortum-EES, generalized Krugman, and generalized Melitz-Pareto-source model, we have $\zeta = \rho$, so the SOE's optimal tariff is the inverse of ρ, the trade elasticity with respect to wages. However, since in the first three models we have $\varepsilon = \rho$ while in the latter model we have $\varepsilon < \rho$, then treating ε as a parameter from the data, and so the same regardless of the model, would imply that the optimal tariff is lower in the Melitz-Pareto-source model than in the Armington-EES, Eaton-Kortum-EES, and generalized Krugman models. Demidova and Rodríguez-Clare (2009) and Costinot et al. (2020) explain that this is because of the decline in the import price index associated with the increase in the variety of available foreign goods caused by higher overall imports. This weakens the terms-of-trade gains from the tariff, leading to a lower optimal tariff.

The generalized Melitz-Pareto-destination model is the exception to the rule that the optimal tariff is the inverse of the trade elasticity with respect to wages. Mechanically, this happens because the trade elasticity with respect to tariffs is larger than the one with respect to wages, $\zeta > \rho$, and this leads to an optimal tariff that is lower than the inverse of ρ, $\bar{t}^* - 1 < 1/\rho$ in (15). More specifically, since $\zeta > \rho$ then $\bar{t}^* - 1 < 1/\rho$, implying that the optimal tariff in the standard Melitz-Pareto model is smaller if fixed trade costs are paid in destination countries rather than in source countries (assuming we equalize ρ across the two Melitz-Pareto models). To understand this result, note that if fixed trade costs are paid in destination countries then imports are associated with a higher demand for labor in the importing country. Thus, a higher tariff lowers labor demand and counteracts the improvement in the terms of trade arising from the standard channels. Since the terms-of-trade gains from the tariff are smaller, the result is a smaller optimal tariff.

Finally, as discussed in Demidova and Rodríguez-Clare (2009), the effect of a tariff can be equally achieved by an export tax (a direct expression of Lerner symmetry) or a subsidy to consumption of domestic varieties.

5. Multiple sectors

In this and the next sections we extend our analysis to multiple sectors, indexed by $k = 1,\ldots,K$. The structure within each sector is exactly as in the previous sections, and labor can move freely across sectors. Upper-tier preferences are Cobb–Douglas,

[16] The sign of the direct effect is non-positive if and only if $\lambda_i \geq \Lambda_i$. It could be positive when $\lambda_i < \Lambda_i$, which is true if $\bar{t}_l$ for some $l \neq i$ is so high. In that case, increasing $\bar{t}_i$ toward $\bar{t}_l$ could increase GT through the direct effect. This is consistent with the literature on tariff uniformity (e.g., Hatta, 1977; Michael et al., 1993).

with the share of expenditure in country j that is allocated to sector k denoted by $\beta_{j,k}$. We first introduce a model that nests all microfoundations considered above except for the Melitz-Pareto-destination model, which presents challenges that go beyond the scope of this paper.[17] We then show how to take the limit to obtain the SOE, and end by deriving the optimal policy.

A multi-sector generalized gravity model of trade nesting Armington or Eaton-Kortum with sector EES, generalized Krugman, and generalized Melitz-Pareto-source model, entails sector-level trade shares given by

$$\lambda_{ij,k} = \frac{\bar{t}_{ij,k}^{-\zeta_k} \left(\tau_{ij,k}/A_{i,k}\right)^{-\varepsilon_k} w_i^{-\zeta_k} L_{i,k}^{\alpha_k}}{\sum_l \bar{t}_{lj,k}^{-\zeta_k} \left(\tau_{lj,k}/A_{l,k}\right)^{-\varepsilon_k} w_l^{-\zeta_k} L_{l,k}^{\alpha_k}}. \tag{16}$$

All parameters and variables with subscript k are simply the multi-sector versions of their single-sector counterparts. Their mappings across different microfoundations are given by the same Table 1 as in the single-sector case, with the caveat that all parameters listed in Table 1 require an additional subscript k.[18]

To achieve the first best, we allow for three policy variables: $\bar{t}_{ij,k}^m$, $\bar{t}_{ij,k}^x$, and $\bar{s}_{j,k}$. Here $\bar{t}_{ij,k}^m \equiv 1 + t_{ij,k}^m$ is one plus country j's ad valorem import tariff from country i in sector k; $\bar{t}_{ij,k}^x \equiv 1 - t_{ij,k}^x$ is one minus country i's ad valorem export tax to country j in sector k; and $\bar{s}_{i,k} \equiv 1 - s_{i,k}$ is one minus country i's ad valorem employment subsidy in sector k.[19] Within each sector these policies can be aggregated as $\bar{t}_{ij,k} \equiv \bar{t}_{ij,k}^m \bar{s}_{i,k}/\bar{t}_{ij,k}^x$.

In contrast to the single-sector case, where we allowed $\alpha \in [0,1]$, here we restrict α_k to lie within the range $[0,1)$ for all k. This restriction helps us avoid dealing with corner allocations, where $L_{i,k} = 0$ for some i and k, that can arise if $\alpha_k = 1$.[20] With our assumption $\alpha_k \in [0,1)$, the goods market clearing in sector k of country i entails

$$w_i L_{i,k} = \sum_j \left(1/\bar{t}_{ij,k}\right) \lambda_{ij,k} \beta_{j,k} X_j, \tag{17}$$

where $X_j \equiv w_j L_j + T_j$ is the total expenditure in country j, with tax revenues given by

$$T_j = \sum_k \left\{ \sum_i \left[\frac{\bar{t}_{ij,k}^m - 1}{\bar{t}_{ij,k}^m} \lambda_{ij,k} \beta_{j,k} X_j + \frac{1 - \bar{t}_{ji,k}^x}{\bar{t}_{ji,k}^m} \lambda_{ji,k} \beta_{i,k} X_i \right] - (1 - \bar{s}_{j,k}) w_j L_{j,k} \right\}. \tag{18}$$

Given the total labor demand in sector k of country i, $L_{i,k}$, the labor market clearing in country i entails

$$L_i = \sum_k L_{i,k}. \tag{19}$$

The consumer price of goods that country j buys from country i in sector k is given by

$$P_{ij,k} = \delta_{j,k} \cdot X_{ij,k}^{1-\frac{\zeta_k}{\varepsilon_k}} \cdot \frac{\bar{t}_{ij,k}^{\frac{\zeta_k}{\varepsilon_k}} \tau_{ij,k} w_i^{\frac{\zeta_k}{\varepsilon_k}}}{A_{i,k} L_{i,k}^{\phi_k}}, \tag{20}$$

where $X_{ij,k} \equiv \lambda_{ij,k} X_{j,k}$.[21] The consumer price index in sector k of country j is given by

$$P_{j,k} = \left[\sum_i P_{ij,k}^{\frac{1}{1-(1+\zeta_k)/\varepsilon_k}} \right]^{1-(1+\zeta_k)/\varepsilon_k}. \tag{21}$$

In the Armington, Eaton-Kortum, and Krugman models, we have $\zeta_k = \varepsilon_k$ and so the price index has the familiar expression $P_{j,k} = \left[\sum_i P_{ij,k}^{-\varepsilon_k}\right]^{-1/\varepsilon_k}$. In the Melitz-Pareto-source model, the bilateral consumption aggregates are combined by a CES aggregator with the elasticity of substitution η_k into a country-level consumption aggregate, and the corresponding price index is $P_{j,k} = \left(\sum_i P_{ij,k}^{1-\eta_k}\right)^{\frac{1}{1-\eta_k}}$. Given that $\eta_k - 1 = \frac{1}{(1+\zeta_k)/\varepsilon_k - 1}$ (see Table 1), we get expression (21) for $P_{j,k}$. Furthermore, for the Melitz-Pareto-source model, we

[17] See Section 1 of the Online Appendix for the details on the microfoundations. In the Melitz-Pareto-destination model, country i's total labor demand in sector k includes a component resulting from the fixed marketing costs, that is proportional to country i's sectoral imports. In the presence of import tariffs, export taxes, and employment subsidies, these policy instruments directly influence this component, thereby rendering the expression for the labor market clearing condition generally incompatible with (19). However, in the absence of policy instruments, the Melitz-Pareto-destination model can be used as a microfoundation of the multi-sector gravity model, as explained in Kucheryavyy et al. (2023).

[18] We set the trade elasticity with respect to wages as ζ_k. This is without loss of generality because all microfoundations considered here entail $\zeta_k = \rho_k$.

[19] See Section 1 of the Online Appendix for more details on these policy instruments.

[20] See Kucheryavyy et al. (2023) for details. Also, strictly speaking, as discussed by Kucheryavyy et al. (2023), the multi-sector version of the Eaton-Kortum model with EES is special. For any $\alpha_k > 0$, this model supports both an interior equilibrium, where $L_{i,k} > 0$ for all i and k, and various corner equilibria, where $L_{i,k} = 0$ for some set of pairs (i,k), provided that the labor market clearing conditions are satisfied. However, these corner equilibria cannot be reached by simply letting $L_{i,k} \to 0$, so we ignore this issue here.

[21] In the Armington, Eaton-Kortum, and Krugman models, we have $\zeta_k = \varepsilon_k$, yielding the familiar expression $P_{ij,k} = \delta_{j,k} \cdot \left(\bar{t}_{ij,k}\tau_{ij,k} w_i\right) / \left(A_{i,k} L_{i,k}^{\phi_k}\right)$ (see expressions (1), (3), and (6) in Sections 1.1-1.3 of the Online Appendix). For the Melitz-Pareto-source model, see the derivations in Section 1.4 of the Online Appendix leading to expression (19) for $P_{ij,k}$ therein.

can use $X_{ij,k} = \left(P_{ij,k}/P_{j,k}\right)^{\eta_k - 1} \beta_{j,k} X_j$ in (20) and (21) to derive an alternative expression for $P_{j,k}$,

$$P_{j,k} = \delta_{j,k} \cdot \left(\beta_{j,k} X_j\right)^{1-\frac{\zeta_k}{\varepsilon_k}} \left[\sum_i \bar{t}_{ij,k}^{-\zeta_k} \left(\tau_{ij,k}/A_{i,k}\right)^{-\varepsilon_k} w_i^{-\zeta_k} L_{i,k}^{\alpha_k}\right]^{-\frac{1}{\varepsilon_k}}, \tag{22}$$

which is also valid for the Armington, Eaton-Kortum, and Krugman models.

5.1. The SOE with multiple sectors

As in the single-sector case, we assume that inward and outward trade costs in each sector increase at a particular rate as country 0 becomes small, so that this country becomes a properly defined SOE in the limit. Formally, we let

$$\begin{aligned} L_0 &\equiv n\widetilde{L}_0, \\ \tau_{0j,k} &\equiv n^{-(1-\alpha_k)/\varepsilon_k}\widetilde{\tau}_{0j,k}, \quad j = 1, \dots, N, \\ \tau_{i0,k} &\equiv n^{-\alpha_k/\varepsilon_k}\widetilde{\tau}_{i0,k}, \quad i = 1, \dots, N, \end{aligned}$$

and let $n \to 0$ while keeping the variables with tilde constant.[22] Intuitively, we think of country 0 in the limit economy as the SOE, with the other N countries as the rest of the world. However, establishing this rigorously as we did in Proposition 1 for the single-sector model presents some challenges here because we lack a comprehensive characterization of the conditions under which the multi-sector generalized gravity model has a unique equilibrium. A full analysis is beyond the scope of this paper, but we can offer the following limited result. Consider an equilibrium of the world economy without country 0, suppose that this is equilibrium is the limit of a sequence of equilibria with $n \to 0$, and further suppose that the SOE has a unique equilibrium for that rest-of-the-world outcome. Then that SOE equilibrium must be the limit of a sequence of equilibria with $n \to 0$. If the rest of the world consists of one or two countries, and no country has taxes, then the equilibrium uniqueness results in Kucheryavyy et al. (2023) can be used to establish the desired result, namely, the SOE equilibrium is unique and is the limit of a sequence of world equilibria as $n \to 0$.

As in the single-sector case, in formulating the multi-sector SOE's system of equilibrium conditions, we remove the tildes and the index of country 0. As an example, in this notation, the SOE's total labor endowment is denoted by L, wage is denoted by w, net tax revenues are denoted by T, and the total expenditure is given by $X = wL + T$. Also, the SOE's amount of labor allocated to sector k is denoted by L_k. In addition to this, we denote by $D_{j,k}$ the market access abroad, and by $p_{i,k}^m$ a shifter of the SOE's price of imports from country i in sector k. Both $D_{j,k}$ and $p_{i,k}^m$ are taken by the SOE as exogenously given.[23]

Then the SOE's expenditure share on goods produced in sector k of country i is

$$\lambda_{i,k} = \frac{\left[\bar{t}_{i,k}^m p_{i,k}^m\right]^{-\zeta_k}}{A_k^{\varepsilon_k} \bar{s}_k^{-\zeta_k} w^{-\zeta_k} L_k^{\alpha_k} + \sum_{l=1}^N \left[\bar{t}_{l,k}^m p_{l,k}^m\right]^{-\zeta_k}},$$

and the SOE's expenditure share on domestic goods in sector k is $\lambda_k = 1 - \sum_{i=1}^N \lambda_{i,k}$. The SOE's net tariff revenue can be written as

$$T = \sum_{k=1}^K \left(\sum_{i=1}^N \frac{\bar{t}_{i,k}^m - 1}{\bar{t}_{i,k}^m} \lambda_{i,k} \beta_k X + \sum_{j=1}^N \left(1 - \bar{t}_{j,k}^x\right) \left(\bar{s}_k / \bar{t}_{j,k}^x\right)^{-\zeta_k} A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} D_{j,k} - \left(1 - \bar{s}_k\right) w L_k \right).$$

The SOE's system of equilibrium conditions consists of the goods and labor market clearing conditions,

$$w L_k = \left(\lambda_k / \bar{s}_k\right) \beta_k X + \sum_{j=1}^N \left(\bar{s}_k / \bar{t}_{j,k}^x\right)^{-\zeta_k - 1} A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} D_{j,k}, \quad k = 1, \dots, K; \tag{23}$$

$$\sum_{k=1}^K L_k = L.$$

Also, the SOE's sector-k price index is given by

$$P_k = \delta_k \cdot \left(\beta_k X\right)^{1-\frac{\zeta_k}{\varepsilon_k}} \left[\bar{s}_k^{-\zeta_k} A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} + \sum_{i=1}^N \left(\bar{t}_{i,k}^m p_{i,k}^m\right)^{-\zeta_k}\right]^{-\frac{1}{\varepsilon_k}}, \tag{24}$$

the SOE's price index is $P = \prod_{k=1}^K \left(P_k/\beta_k\right)^{\beta_k}$, and the SOE's welfare is $W = (X/L)/P$.

5.2. Comparative statics

In this section, we discuss how the SOE's wage, trade flows, and welfare respond to changes in foreign market access and foreign-good prices. One challenge here is that, in the presence of taxes in the SOE, comparative statics become difficult to characterize

[22] See Appendix A for the description of the system of equilibrium conditions with the transformed labor endowment, trade costs, and model variables.

[23] See Appendix B for the formal definitions.

because of the differences between consumer and producer prices and nontrivial income effects of net tax revenues. Consequently, we limit our analysis to the case without taxes in the SOE.

Without taxes, the SOE's goods market clearing condition (23) becomes

$$wL_k = \lambda_k \beta_k wL + E_k,$$

with the SOE's expenditure share on domestic goods in sector k given by

$$\lambda_k = \frac{A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k}}{A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} + \mathcal{P}_k^{-\zeta_k}},$$

and where $E_k \equiv A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} D_k$ are SOE's exports to the rest of the world in sector k, and $D_k \equiv \sum_{i=1}^{N} D_{i,k}$ and $\mathcal{P}_k \equiv \left(\sum_{i=1}^{N} p_{i,k}^{-\zeta_k}\right)^{-1/\zeta_k}$. As we formally show in Section 5 of the Online Appendix, an increase in D_k or $\mathcal{P}_k$ results in an increase in the SOE's wage and a reallocation of labor from all sectors $s \neq k$ to sector k, which is accompanied by a fall in exports and domestic trade shares and a rise in imports in all sectors $s \neq k$.

The effects of an increase in D_k or $\mathcal{P}_k$ on trade flows in sector k are nuanced. An increase in D_k raises the SOE's exports in sector k, but the effects on λ_k and sector-k imports are ambiguous due to competing forces. On the one hand, increased wages result in a decrease in the domestic expenditure share and an increase in imports. On the other hand, increased employment in sector k raises productivity due to economies of scale, offsetting the negative effects stemming from higher wages. Similarly, an increase in $\mathcal{P}_k$ leads to an increase in the SOE's domestic trade share in sector k, but the effects on sector-k imports and exports are ambiguous.

Turning to welfare, the presence of sector-varying production externalities in the Armington and Eaton-Kortum models, and markups in Krugman and Melitz models, implies that there are domestic distortions, and hence improvements in foreign market access or declines in foreign-good prices do not necessarily increase welfare — see, for instance, Hagen (1958) and Święcki (2017). Of course, as one can show by appealing to the envelope theorem, if the SOE implements the optimal policy then such foreign shocks would increase its welfare.

5.3. Optimal policy

We can derive the optimal policy for the SOE via maximization of $W = (w + T/L)/P$ w.r.t. sector-level tariffs, export taxes, and employment subsidies. For simplicity, here we assume that tariffs and export taxes are uniform across partner countries (we drop this assumption in Section 6). As shown in Section 6 of the Online Appendix, this leads to the following result:

Proposition 4. *With multiple sectors, the SOE's optimal policy is given by*

$$\bar{t}_k^m = \bar{t}\frac{\varepsilon_k}{\zeta_k}, \quad \bar{t}_k^x = \bar{t}\frac{\varepsilon_k}{1+\zeta_k}, \quad \bar{s}_k = \bar{s}\frac{\varepsilon_k}{\alpha_k+\zeta_k},$$

where $\bar{t}$ and $\bar{s}$ are shifters of the level of taxes (tariffs and export taxes) and employment subsidies across sectors and are undetermined.

How does this compare to the result in Proposition 3 for the single-sector model? In contrast to Proposition 3, here we allow for both tariffs and export taxes, leading to the indeterminacy in levels captured by $\bar{t}$. This indeterminacy is a direct consequence of Lerner symmetry, which implies that what matters is $\bar{t}^m/\bar{t}^x$. Proposition 4 indicates that $\frac{\bar{t}^m}{\bar{t}^x} = \frac{1+\zeta}{\zeta}$, as in Proposition 3 for $\rho = \zeta$ (which holds in all models considered here, excluding the Melitz-Pareto-destination model). In addition, $\bar{s}$ is undetermined because a common employment subsidy across sectors has no effect given our assumption that the labor supply is perfectly inelastic. Accordingly, in the single-sector model in Sections 2–4 we implicitly set $\bar{s} = 1$ without loss of generality.[24,25]

Instead of providing some intuition for these results here, we consider in the next section an alternative approach to the derivation of optimal policy that will more naturally reveal the forces at play.

6. A micro-to-macro approach to optimal policy

We now provide an alternative and more intuitive derivation of the result in Proposition 4 following the "micro-to-macro" representation developed for the Melitz model in Costinot et al. (2020), applied here to the generalized gravity model of the SOE.

Let P_k^d be the consumer price index of domestic varieties in sector k and Q_k^d be the corresponding quantity index, let $P_{i,k}^m$ be the pre-tariff consumer price index for varieties imported from country i and $Q_{i,k}^m$ be the corresponding quantity index, and let $P_{j,k}^x$ be the pre-export tax producer price index of varieties that the SOE sells to country j and $Q_{j,k}^x$ be the corresponding quantity index.[26] As an example, in the Armington model P_k^d would simply be the price of the domestic variety in sector k, $P_{i,k}^m$ would be

[24] These two sources of indeterminacy are discussed in Bartelme et al. (2021).

[25] At the end of Section 4 we stated that the effects of a tariff could be equivalently achieved with a subsidy to consumption of domestic varieties. With multiple sectors we could certainly use sector-level subsidies to consumption of domestic varieties instead of employment subsidies, but we would then still need sector-level import tariffs and export taxes. If the subsidy $\bar{s}\frac{\varepsilon_k}{\alpha_k+\zeta_k}$ were applied to consumption rather than employment, the export tax would be correspondingly smaller. This adjustment is needed to internalize the EES on production destined to foreign markets (in the Armington and EK microfoundations) or because producers are already charging markups on their exports (in the Krugman and Melitz microfoundations).

[26] In Appendix D.1, we show how the SOE's equilibrium system of equations can be expressed in terms of these prices and quantities.

the international price index of sector-k goods imported by the SOE from country i, and $P^x_{j,k}$ would be the international price index of sector-k goods produced by the SOE and imported by country j, with the corresponding quantity indices being just the actual quantities of the respective goods. Using these definitions, net tax revenues can be written as

$$T = \sum_{k,i}\left(\bar{t}^m_{i,k}-1\right)P^m_{i,k}Q^m_{i,k} + \sum_{k,j}\left(1-\bar{t}^x_{j,k}\right)P^x_{j,k}Q^x_{j,k} - \sum_k\left(1-\bar{s}_k\right)wL_k,$$

while profits are

$$\Pi = \sum_k P^d_kQ^d_k + \sum_{k,j}\bar{t}^x_{j,k}P^x_{j,k}Q^x_{j,k} - \sum_k \bar{s}_kwL_k.$$

Following the approach in Bartelme et al. (2021), we let the social planner choose taxes and subsidies as well as quantities Q^d_k, $Q^m_{i,k}$, and $Q^x_{j,k}$ so as to maximize welfare $W = (w + \Pi/L + T/L)/P$.[27]

Totally differentiating welfare, we get (see Appendix D.2)

$$d\ln W \propto \sum_k\left(P^d_k - w\frac{\partial L_k}{\partial Q^d_k}\right)dQ^d_k + \sum_k\sum_j\left[\left(1-\epsilon^x_{j,k}\right)P^x_{j,k} - w\frac{\partial L_k}{\partial Q^x_{j,k}}\right]dQ^x_{j,k}$$
$$+\sum_k\sum_i P^m_{i,k}\left(\bar{t}^m_{i,k}-1-\epsilon^m_{i,k}\right)dQ^m_{i,k}.$$

Here $\epsilon^x_{j,k} \equiv -\frac{\partial\ln P^x_{j,k}}{\partial\ln Q^x_{j,k}}$ is the inverse elasticity of country $j's$ demand from the SOE in sector k and $\epsilon^m_{i,k} \equiv \frac{\partial\ln P^m_{i,k}}{\partial\ln Q^m_{i,k}}$ is the inverse elasticity of country $i's$ supply to the SOE in sector k. In turn, derivatives $\frac{\partial L_k}{\partial Q^d_k}$ and $\frac{\partial L_k}{\partial Q^x_k}$ come from a function $L_k : \mathbb{R}^{1+N} \mapsto \mathbb{R}$ capturing the total labor cost associated with domestic and foreign sales. This function is given by (see Appendix D.1)

$$L_k\left(Q^d_k, Q^x_{1,k},\dots,Q^x_{N,k}\right) = \left[\left(\frac{\delta_kQ^d_k}{A_k}\right)^{\varepsilon_k/\zeta_k} + \sum_{j=1}^N\left(\frac{\delta_{j,k}\tau^x_{j,k}Q^x_{j,k}}{A_k}\right)^{\varepsilon_k/\zeta_k}\right]^{\zeta_k/(\zeta_k+\alpha_k)}. \tag{25}$$

Under the optimal policy, any feasible variation in $\left\{Q^d_k, Q^d_{j,k}, Q^m_{i,k}\right\}$ (i.e., any variation that respects labor market clearing) gives $d\ln W = 0$. As shown in Appendix D.3, this implies

$$P^d_k = \bar{s}w\frac{\partial L_k}{\partial Q^d_k},\quad \bar{t}\left(1-\epsilon^x_{j,k}\right)P^x_{j,k} = \bar{s}w\frac{\partial L_k}{\partial Q^x_{j,k}},\quad \bar{t}^m_{i,k} = \bar{t}\left(1+\epsilon^m_{i,k}\right), \tag{26}$$

for some tariff and subsidy shifters $\bar{t}$ and $\bar{s}$. These are familiar optimality conditions: the domestic price must be proportional to marginal cost, the marginal export revenue must be proportional to the marginal cost, and the tariff must be proportional to one plus the inverse elasticity of foreign supply.

To arrive at the optimal policies, the next step is to use the equilibrium conditions for aggregate price indices P^d_k and $P^x_{j,k}$. Combining expressions (20) (after transforming them into the SOE notation) and (25) we can show that (see Appendix D.4)

$$P^d_k = \left(\frac{\alpha_k+\zeta_k}{\varepsilon_k}\right)\bar{s}_kw\frac{\partial L_k}{\partial Q^d_k},\quad \bar{t}^x_{j,k}P^x_{j,k} = \left(\frac{\alpha_k+\zeta_k}{\varepsilon_k}\right)\bar{s}_kw\frac{\partial L_k}{\partial Q^x_{j,k}}. \tag{27}$$

In the Armington and EK models $\frac{\alpha_k+\zeta_k}{\varepsilon_k} = 1+\gamma_k$, capturing the gap between social and private marginal costs arising from EES, while in the Krugman and Melitz models $\frac{\alpha_k+\zeta_k}{\varepsilon_k} = \frac{\sigma_k}{\sigma_k-1}$, capturing the gap between the price and social marginal cost arising from the markup. Combining these expressions with (26), we obtain

$$\bar{s}_k = \bar{s}\frac{\varepsilon_k}{\alpha_k+\zeta_k},\quad \bar{t}^k_{j,k} = \bar{t}\left(1-\epsilon^x_{j,k}\right),\quad \bar{t}^m_{i,k} = \bar{t}\left(1+\epsilon^m_{i,k}\right). \tag{28}$$

The final step is to derive the elasticities $\epsilon^x_{j,k} \equiv -\frac{\partial\ln P^x_{j,k}}{\partial\ln Q^x_{j,k}}$ and $\epsilon^m_{i,k} \equiv \frac{\partial\ln P^m_{i,k}}{\partial\ln Q^m_{i,k}}$. The elasticity $\epsilon^x_{j,k} \equiv -\frac{\partial\ln P^x_{j,k}}{\partial\ln Q^x_{j,k}}$ captures how the export price changes with a higher export quantity and is read off from country j's demand curve. The inverse CES demand of country j in industry k is given by $P^x_{j,k} = \left[Q^x_{j,k}\right]^{-\frac{1+\zeta_k-\varepsilon_k}{1+\zeta_k}}\overline{D}^x_{j,k}$, where $\overline{D}^x_{j,k}$ is the export demand shifter (see Appendix D.1). This implies that $\epsilon^x_{j,k} = \frac{1+\zeta_k-\varepsilon_k}{1+\zeta_k}$.[28] Similarly, the elasticity $\epsilon^m_{i,k} \equiv \frac{\partial\ln P^m_{i,k}}{\partial\ln Q^m_{i,k}}$ captures how the import price changes with a higher import quantity and

[27] In the approach we follow below, we first totally differentiate W, taking as given the behavior of domestic consumers as well as foreign demand for the SOE's goods and foreign supply of goods to the SOE. We then consider the behavior of producers, as implied by the equilibrium conditions for the SOE's producer prices, to arrive at first-order conditions for tariffs, employment subsidies and export taxes.

[28] This elasticity can also be inferred from (21), which implies $\frac{\partial\ln Q_{ij,k}}{\partial\ln P_{ij,k}} = \frac{1}{1-(1+\zeta_k)/\varepsilon_k} - 1 = -\frac{1+\zeta_k}{1+\zeta_k-\varepsilon_k}$. Here the partial derivative takes everything in country j as given, as needed to obtain an elasticity of country $j's$ import demand for sector k exports from the SOE.

is read off from country i's supply curve. Combining $X_{ij,k} = P_{ij,k}Q_{ij,k}$ and (20) we get $\epsilon^m_{i,k} = \varepsilon_k/\zeta_k - 1$.[29] Using these results for elasticities $\epsilon^x_{j,k}$ and $\epsilon^m_{i,k}$ in the expressions for tariffs and export taxes in (28) finally yields

$$\bar{s}_k = \bar{s}\frac{\varepsilon_k}{\alpha_k + \zeta_k}, \quad \bar{t}^x_{j,k} = \bar{t}\frac{\varepsilon_k}{1+\zeta_k}, \quad \bar{t}^m_{i,k} = \bar{t}\frac{\varepsilon_k}{\zeta_k},$$

as in Proposition 4.

It is instructive to revisit the first-order conditions above separately for each of the four microfoundations we have considered. From (25) specialized to the Armington and Eaton-Kortum models, we get

$$\frac{\partial L_k}{\partial Q^d_k} = \frac{1}{1+\gamma_k}\cdot\frac{L_k}{Q^d_k + \sum_j \tau^x_{j,k}Q^x_{j,k}} \quad \text{and} \quad \frac{\partial L_k}{\partial Q^x_{j,k}} = \frac{1}{1+\gamma_k}\cdot\frac{L_k\tau^x_{j,k}}{Q^d_k + \sum_j \tau^x_{j,k}Q^x_{j,k}}.$$

However, since economies of scale are external to the firms, prices satisfy

$$P^d_k = \bar{s}_k w\frac{L_k}{Q^d_k + \sum_j \tau^x_{j,k}Q^x_{j,k}} \quad \text{and} \quad \bar{t}^x_{j,k}P^x_{j,k} = \bar{s}_k w\frac{L_k\tau^x_{j,k}}{Q^d_k + \sum_j \tau^x_{j,k}Q^x_{j,k}}.$$

We also have that import prices are fixed, and hence, $\epsilon^m_{i,k} = 0$ for all i,k, while $\epsilon^x_{j,k} = \frac{1}{1+\varepsilon_k}$, and thus, the optimal policy satisfies

$$\bar{s}_k = \frac{1}{1+\gamma_k}, \quad \bar{t}^x_{j,k} = \frac{\varepsilon_k}{1+\varepsilon_k}, \quad \bar{t}^m_{i,k} = 1.$$

The employment subsidy makes firms internalize the external economies of scale in each sector, the export tax makes firms internalize the terms-of-trade externalities, and there are no tariffs because the foreign supply curve is flat in all sectors.

Next, consider the generalized Krugman model. Here firms charge the monopolistic competition markup over marginal cost and hence

$$P^d_k = \frac{\sigma_k}{\sigma_k - 1}\bar{s}_k w\frac{\partial L_k}{\partial Q^d_k} \quad \text{and} \quad \bar{t}^x_{j,k}P^x_{j,k} = \frac{\sigma_k}{\sigma_k - 1}\bar{s}_k w\frac{\partial L_k}{\partial Q^x_{j,k}}.$$

We also have that import prices are fixed, and hence, $\epsilon^m_{i,k} = 0$ for all i,k, while $\epsilon^x_{j,k} = \frac{1}{1+\varepsilon_k}$. This implies that the optimal policy satisfies

$$\bar{s}_k = \frac{\sigma_k - 1}{\sigma_k}, \quad \bar{t}^x_{j,k} = \frac{\varepsilon_k}{1+\varepsilon_k} = \frac{\eta_k - 1}{\eta_k}, \quad \bar{t}^m_{i,k} = 1.$$

The employment subsidy removes the markup distortion. Because of that, firms would not be charging the monopoly price on exports, so the export tax equal to the foreign demand elasticity is needed to get firms to exploit their monopoly power. Finally, as in the Argminton and Eaton-Kortum models, there are no tariffs because the foreign supply curve is flat in all sectors.

Lastly, the pricing equations in the generalized Melitz-Pareto model are the same as in the Krugman model, but foreign supply and demand elasticities are different: $\epsilon^m_{i,k} = \varepsilon_k/\zeta_k - 1 < 0$ and $\epsilon^x_{j,k} = \frac{1+\zeta_k-\varepsilon_k}{1+\zeta_k} \neq \frac{1}{1+\varepsilon_k}$. Therefore, the optimal policy satisfies

$$\bar{s}_k = \frac{\sigma_k - 1}{\sigma_k}, \quad \bar{t}^x_{j,k} = \frac{\varepsilon_k}{1+\zeta_k} = \frac{\eta_k - 1}{\eta_k}, \quad \bar{t}^m_{i,k} = \frac{\varepsilon_k}{\zeta_k} = \frac{1}{1 + 1/(\sigma_k - 1) - 1/\theta_k} < 1,$$

where the inequality follows from the standard assumption $\theta_k > \sigma_k - 1$. There is an import subsidy to deal with the fact that the foreign supply curve is downward sloping.

7. Conclusion

Basic questions in the field of international economics can be more easily addressed by considering a small open economy. We have derived the equations characterizing the equilibrium of such an economy in a generalized gravity model as the limit in which the economy becomes infinitesimally small, provided we simultaneously let trade costs go to infinity at a rate determined by the magnitude of the scale and trade elasticities. These equilibrium equations lead to a simple graphical analysis that can be used to study comparative statics, and the optimal tariff can be derived by differentiation of a simple function giving welfare in terms of the tariff. The comparative statics results show how the SOE's wage, trade flows, and welfare are affected by foreign shocks, while our optimal tariff formula highlights the role of the trade elasticities with respect to wages and tariffs.

The results extend naturally to an environment with multiple sectors, with some exceptions and qualifications. The optimal policy entails export taxes and import tariffs to deal with terms-of-trade externalities and employment subsidies to deal with domestic distortions associated with economies of scale and markups. Import tariffs are zero in all microfoundations except in the Melitz-Pareto model, where marketing fixed costs and selection lead to a negative supply elasticity and import subsidies. Turning to comparative statics, improvements in foreign-market access or declines in foreign-good prices still lead to increases in trade flows, but the effects on trade and welfare are nuanced as a result of scale economies and domestic distortions.

[29] In all models except Melitz-Pareto, we have $\zeta_k = \varepsilon_k$, which implies $\epsilon^m_{i,k} = 0$ and $\epsilon^x_{j,k} = 1/\left(1+\zeta_k\right)$. In contrast, in the Melitz-Pareto model, $\zeta_k > \varepsilon_k$, and so country i's supply curve to the SOE is downward sloping, $\epsilon^m_{i,k} < 0$, reflecting increasing returns arising from the fixed marketing cost and Melitz selection. Since this also applies to the SOE's supply curve to any foreign market, the inverse elasticity of demand in any foreign market for the SOE's exports will be higher than $\left(1 + \frac{d\ln P_{j,k}Q_{j,k}}{d\ln \bar{t}^x_{j,k}}\right)^{-1} = (1+\zeta_k)^{-1}$ for any j, where the equality follows from the definition of ζ_k.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Transformed system of equilibrium conditions

In this appendix, we apply the labor and trade cost adjustments outlined in Sections 3 and 5 to derive the transformed system of equilibrium equations for the generalized gravity model. We first focus on the multi-sector version before transitioning to the single-sector version.

A.1. Multiple sectors

In the multi-sector version of the gravity model, the labor and trade costs adjustment are given by

$$\begin{aligned}
&L_0 \equiv n\widetilde{L}_0;\\
&\tau_{0j,k} \equiv n^{-(1-\alpha_k)/\varepsilon_k}\widetilde{\tau}_{0j,k}, \quad j=1,\dots,N,\ k=1,\dots,K;\\
&\tau_{i0,k} \equiv n^{-\alpha_k/\varepsilon_k}\widetilde{\tau}_{i0,k}, \quad i=1,\dots,N,\ k=1,\dots,K.
\end{aligned}$$

Defining $\widetilde{L}_{0,k} \equiv L_{0,k}/n$, trade shares (16) can be written as

$$\lambda_{ij,k} = \frac{A_{i,k}^{\varepsilon_k}\bar{t}_{ij,k}^{-\zeta_k}\tau_{ij,k}^{-\varepsilon_k}w_i^{-\zeta_k}L_{i,k}^{\alpha_k}}{nA_{0,k}^{\varepsilon_k}\bar{t}_{0j,k}^{-\zeta_k}\widetilde{\tau}_{0j,k}^{-\varepsilon_k}w_0^{-\zeta_k}\widetilde{L}_{0,k}^{\alpha_k} + \sum_{l=1}^{N}A_{l,k}^{\varepsilon_k}\bar{t}_{lj,k}^{-\zeta_k}\tau_{lj,k}^{-\varepsilon_k}w_l^{-\zeta_k}L_{l,k}^{\alpha_k}}, \quad i,j=1,\dots,N, \quad k=1,\dots,K; \tag{A.1}$$

$$\frac{\lambda_{0j,k}}{n} = \frac{A_{0,k}^{\varepsilon_k}\bar{t}_{0j,k}^{-\zeta_k}\widetilde{\tau}_{0j,k}^{-\varepsilon_k}w_0^{-\zeta_k}\widetilde{L}_{0,k}^{\alpha_k}}{nA_{0,k}^{\varepsilon_k}\bar{t}_{0j,k}^{-\zeta_k}\widetilde{\tau}_{0j,k}^{-\varepsilon_k}w_0^{-\zeta_k}\widetilde{L}_{0,k}^{\alpha_k} + \sum_{l=1}^{N}A_{l,k}^{\varepsilon_k}\bar{t}_{lj,k}^{-\zeta_k}\tau_{lj,k}^{-\varepsilon_k}w_l^{-\zeta_k}L_{l,k}^{\alpha_k}}, \quad j=1,\dots,N, \quad k=1,\dots,K; \tag{A.2}$$

$$\lambda_{i0,k} = \frac{A_{i,k}^{\varepsilon_k}\bar{t}_{i0,k}^{-\zeta_k}\widetilde{\tau}_{i0,k}^{-\varepsilon_k}w_i^{-\zeta_k}L_{i,k}^{\alpha_k}}{A_{0,k}^{\varepsilon_k}\bar{s}_{0,k}^{-\zeta_k}w_0^{-\zeta_k}\widetilde{L}_{0,k}^{\alpha_k} + \sum_{l=1}^{N}A_{l,k}^{\varepsilon_k}\bar{t}_{l0,k}^{-\zeta_k}\widetilde{\tau}_{l0,k}^{-\varepsilon_k}w_l^{-\zeta_k}L_{l,k}^{\alpha_k}}, \quad i=1,\dots,N, \quad k=1,\dots,K; \tag{A.3}$$

$$\lambda_{00,k} = \frac{A_{0,k}^{\varepsilon_k}\bar{s}_{0,k}^{-\zeta_k}w_0^{-\zeta_k}\widetilde{L}_{0,k}^{\alpha_k}}{A_{0,k}^{\varepsilon_k}\bar{s}_{0,k}^{-\zeta_k}w_0^{-\zeta_k}\widetilde{L}_{0,k}^{\alpha_k} + \sum_{l=1}^{N}A_{l,k}^{\varepsilon_k}\bar{t}_{l0,k}^{-\zeta_k}\widetilde{\tau}_{l0,k}^{-\varepsilon_k}w_l^{-\zeta_k}L_{l,k}^{\alpha_k}}, \quad k=1,\dots,K. \tag{A.4}$$

The sectoral goods market clearing conditions (17) can be written as

$$w_iL_{i,k} = \frac{n\lambda_{i0,k}}{\bar{t}_{i0,k}}\beta_{0,k}\left(w_0\widetilde{L}_0 + \widetilde{T}_0\right) + \sum_{j=1}^{N}\frac{\lambda_{ij,k}}{\bar{t}_{ij,k}}\beta_{j,k}\left(w_jL_j + T_j\right), \quad i=1,\dots,N, \quad k=1,\dots,K; \tag{A.5}$$

$$w_0\widetilde{L}_{0,k} = \frac{\lambda_{00,k}}{\bar{s}_{0,k}}\beta_{0,k}\left(w_0\widetilde{L}_0 + \widetilde{T}_0\right) + \sum_{j=1}^{N}\frac{\lambda_{0j,k}/n}{\bar{t}_{0j,k}}\beta_{j,k}\left(w_jL_j + T_j\right), \quad k=1,\dots,K; \tag{A.6}$$

where

$$T_j = \sum_{k=1}^{K}\left\{n\widetilde{T}_{0j,k} + \sum_{i=1}^{N}T_{ij,k} - \left(1-\bar{s}_{j,k}\right)w_jL_{j,k}\right\}, \quad j=1,\dots,N; \tag{A.7}$$

$$\widetilde{T}_0 \equiv \frac{T_0}{n} = \sum_{k=1}^{K}\left\{\sum_{i=1}^{N}\widetilde{T}_{i0,k} - \left(1-\bar{s}_{0,k}\right)w_0\widetilde{L}_{0,k}\right\}, \tag{A.8}$$

with

$$T_{ij,k} \equiv \frac{\bar{t}_{ij,k}^{m}-1}{\bar{t}_{ij,k}^{m}}\lambda_{ij,k}\beta_{j,k}\left(w_jL_j+T_j\right) + \frac{1-\bar{t}_{ji,k}^{x}}{\bar{t}_{ji,k}^{m}}\lambda_{ji,k}\beta_{i,k}\left(w_iL_i+T_i\right), \quad i,j=1,\dots,N, \quad k=1,\dots,K;$$

$$\widetilde{T}_{0j,k} \equiv \frac{\bar{t}^m_{0j,k}-1}{\bar{t}^m_{0j,k}} \cdot \frac{\lambda_{0j,k}}{n}\beta_{j,k}\left(w_jL_j+T_j\right) + \frac{1-\bar{t}^x_{j0,k}}{\bar{t}^m_{j0,k}}\lambda_{j0,k}\beta_{0,k}\left(w_0\widetilde{L}_0+\widetilde{T}_0\right),$$
$$j=1,\ldots,N, \quad k=1,\ldots,K;$$
$$\widetilde{T}_{i0,k} \equiv \frac{\bar{t}^m_{i0,k}-1}{\bar{t}^m_{i0,k}}\lambda_{i0,k}\beta_{0,k}\left(w_0\widetilde{L}_0+\widetilde{T}_0\right) + \frac{1-\bar{t}^x_{0i,k}}{\bar{t}^m_{0i,k}} \cdot \frac{\lambda_{0i,k}}{n}\beta_{i,k}\left(w_iL_i+T_i\right),$$
$$i=1,\ldots,N, \quad k=1,\ldots,K.$$

The labor market clearing conditions (19) can be written as

$$L_i = \sum_{k=1}^{K} L_{i,k}, \quad i=1,\ldots,N; \tag{A.9}$$
$$\widetilde{L}_0 = \sum_{k=1}^{K}\widetilde{L}_{0,k}. \tag{A.10}$$

From (20), the adjusted bilateral price indices associated with country 0 can be written as

$$\widetilde{P}_{0j,k} \equiv n^{\frac{\zeta_k+1-\varepsilon_k}{\varepsilon_k}} P_{0j,k} = \left(X_{0j,k}/n\right)^{1-\frac{\zeta_k}{\varepsilon_k}} \frac{\left(\bar{s}_{0,k}\bar{t}^m_{0j,k}/\bar{t}^x_{0j,k}\right)^{\frac{\zeta_k}{\varepsilon_k}} w_0^{\frac{\zeta_k}{\varepsilon_k}}}{A_{0,k}\widetilde{L}_{0,k}^{\alpha_k/\varepsilon_k}} \left[p^x_{j,k}\right]^{\frac{\zeta_k}{\varepsilon_k}}, \tag{A.11}$$
$$j=1,\ldots,N, \; k=1,\ldots,K;$$
$$\widetilde{P}_{i0,k} \equiv n^{\frac{\zeta_k+\alpha_k-\varepsilon_k}{\varepsilon_k}} P_{i0,k} = \delta_{0,k}\cdot\left(X_{i0,k}/n\right)^{1-\frac{\zeta_k}{\varepsilon_k}}\left(\bar{t}^m_{i0,k}p^m_{i,k}\right)^{\frac{\zeta_k}{\varepsilon_k}}, \tag{A.12}$$
$$i=1,\ldots,N, \; k=1,\ldots,K;$$
$$\widetilde{P}_{00,k} \equiv n^{\frac{\zeta_k+\alpha_k-\varepsilon_k}{\varepsilon_k}} P_{00,k} = \delta_{0,k}\cdot\left(X_{00,k}/n\right)^{1-\frac{\zeta_k}{\varepsilon_k}} \frac{\bar{s}_{0,k}^{\frac{\zeta_k}{\varepsilon_k}} w_0^{\frac{\zeta_k}{\varepsilon_k}}}{A_{0,k}\widetilde{L}_{0,k}^{\alpha_k/\varepsilon_k}}, \tag{A.13}$$
$$k=1,\ldots,K;$$

where

$$p^x_{j,k} \equiv \delta_{j,k}^{\varepsilon_k/\zeta_k}\widetilde{\tau}_{0j,k}^{\varepsilon_k/\zeta_k}, \tag{A.14}$$
$$p^m_{i,k} \equiv \left(\bar{s}_{i,k}/\bar{t}^x_{i0,k}\right)\left[\widetilde{\tau}_{i0,k}/\left(A_{i,k}L_{i,k}^{\phi_k}\right)\right]^{\varepsilon_k/\zeta_k} w_i. \tag{A.15}$$

Here, we can use $X_{0j,k}/n = \left(\lambda_{0j,k}/n\right)\beta_{j,k}\left(w_jL_j+T_j\right)$ for $j=1,\ldots,N$, and $X_{i0,k}/n = \lambda_{i0,k}\beta_{0,k}\left(w_0\widetilde{L}_0+\widetilde{T}_0\right)$ for $i=0,1,\ldots,N$.

Next, let us define adjusted bilateral quantity indices associated with adjusted bilateral price indices $\widetilde{P}_{0j,k}$, $\widetilde{P}_{i0,k}$, and $\widetilde{P}_{00,k}$ as

$$\widetilde{Q}_{0j,k} \equiv \frac{X_{0j,k}/n}{\widetilde{P}_{0j,k}}, \qquad j=1,\ldots,N, \; k=1,\ldots,K; \tag{A.16}$$
$$\widetilde{Q}_{i0,k} \equiv \frac{X_{i0,k}/n}{\widetilde{P}_{i0,k}}, \qquad i=1,\ldots,N, \; k=1,\ldots,K; \tag{A.17}$$
$$\widetilde{Q}_{00,k} \equiv \frac{X_{00,k}/n}{\widetilde{P}_{00,k}}, \qquad k=1,\ldots,K. \tag{A.18}$$

Substituting the above definitions of $\widetilde{Q}_{0j,k}$, $\widetilde{Q}_{i0,k}$, and $\widetilde{Q}_{00,k}$ into (A.11)–(A.13), and solving for $\widetilde{P}_{0j,k}$, $\widetilde{P}_{i0,k}$, and $\widetilde{P}_{00,k}$, we get

$$\widetilde{P}_{0j,k} = \widetilde{Q}_{0j,k}^{\frac{\varepsilon_k}{\zeta_k}-1} \frac{\left(\bar{s}_{0,k}\bar{t}^m_{0j,k}/\bar{t}^x_{0j,k}\right)w_0}{A_{0,k}^{\varepsilon_k/\zeta_k}\widetilde{L}_{0,k}^{\alpha_k/\zeta_k}} p^x_{j,k}, \qquad j=1,\ldots,N, \; k=1,\ldots,K; \tag{A.19}$$
$$\widetilde{P}_{i0,k} = \delta_{0,k}^{\frac{\varepsilon_k}{\zeta_k}} \cdot \widetilde{Q}_{i0,k}^{\frac{\varepsilon_k}{\zeta_k}-1}\bar{t}^m_{i0,k}p^m_{i,k}, \qquad i=1,\ldots,N, \; k=1,\ldots,K; \tag{A.20}$$
$$\widetilde{P}_{00,k} = \delta_{0,k}^{\frac{\varepsilon_k}{\zeta_k}} \cdot \widetilde{Q}_{00,k}^{\frac{\varepsilon_k}{\zeta_k}-1} \frac{\bar{s}_{0,k}w_0}{A_{0,k}^{\varepsilon_k/\zeta_k}\widetilde{L}_{0,k}^{\alpha_k/\zeta_k}}, \qquad k=1,\ldots,K. \tag{A.21}$$

Using the definitions of $\widetilde{P}_{0j,k}$, $\widetilde{P}_{i0,k}$, $\widetilde{P}_{00,k}$, $\widetilde{Q}_{0j,k}$, $\widetilde{Q}_{i0,k}$, and $\widetilde{Q}_{00,k}$, we can write the goods market clearing condition (A.6) in sector k of country 0 alternatively as

$$\bar{s}_{0,k}w_0\widetilde{L}_{0,k} = \widetilde{P}_{00,k}\widetilde{Q}_{00,k} + \sum_{j=1}^{N}\bar{t}^x_{0j,k}\widetilde{P}_{0j,k}\widetilde{Q}_{0j,k}/\bar{t}^m_{0j,k}, \tag{A.22}$$

and expression (A.8) for the adjusted net tax revenue of country 0 as

$$\widetilde{T}_0 = \sum_{k=1}^{K} \left\{ \sum_{i=1}^{N} \frac{\bar{t}^m_{i0,k} - 1}{\bar{t}^m_{i0,k}} \widetilde{P}_{i0,k} \widetilde{Q}_{i0,k} + \sum_{j=1}^{N} \frac{1 - \bar{t}^x_{0j,k}}{\bar{t}^m_{0j,k}} \widetilde{P}_{0j,k} \widetilde{Q}_{0j,k} - \left(1 - \bar{s}_{0,k}\right) w_0 \widetilde{L}_{0,k} \right\}. \tag{A.23}$$

Substituting expressions (A.19) and (A.21) for $\widetilde{P}_{0j,k}$ and $\widetilde{P}_{00,k}$ into the above expression for the goods market clearing condition, and after doing some algebra, we get

$$\widetilde{L}^{1+\alpha_k/\zeta_k}_{0,k} = \delta^{\frac{\varepsilon_k}{\zeta_k}}_{0,k} A^{-\frac{\varepsilon_k}{\zeta_k}}_{0,k} \widetilde{Q}^{\frac{\varepsilon_k}{\zeta_k}}_{00,k} + \sum_{j=1}^{N} A^{-\frac{\varepsilon_k}{\zeta_k}}_{0,k} \widetilde{Q}^{\frac{\varepsilon_k}{\zeta_k}}_{0j,k} p^x_{j,k}. \tag{A.24}$$

In Section 6 and Appendix D, we also use the inverse foreign demand curve from the point of view of the SOE. To get that inverse demand curve, we use the expression

$$Q_{0j,k} = P^{\frac{1}{1-(1+\zeta_k)/\varepsilon_k}-1}_{0j,k} P^{-\frac{1}{1-(1+\zeta_k)/\varepsilon_k}}_{j,k} X_{j,k}, \tag{A.25}$$

where $Q_{0j,k}$ is the quantity of country-0-sector-k good demanded by country j. In the Armington, Eaton-Kortum, and Krugman models, we have $\zeta_k = \varepsilon_k$, and expression (A.25) turns into $Q_{0j,k} = P^{-\varepsilon_k-1}_{0j,k} P^{\varepsilon_k}_{j,k} X_{j,k}$. In the Melitz-Pareto-source model, (A.25) is $Q_{0j,k} = P^{-\eta_k}_{0j,k} P^{\eta_k-1}_{j,k} X_{j,k}$. Thus, in all microfoundations that we consider here, expression (A.25) has a familiar form.

Solving for $P_{0j,k}$ from (A.25), we get the inverse demand curve

$$P_{0j,k} = Q^{-\frac{1+\zeta_k-\varepsilon_k}{1+\zeta_k}}_{0j,k} D^x_{j,k},$$

where

$$D^x_{j,k} \equiv P^{\frac{\varepsilon_k}{1+\zeta_k}}_{j,k} X^{1-\frac{\varepsilon_k}{1+\zeta_k}}_{j,k}. \tag{A.26}$$

The transformed version of this expression is

$$\widetilde{P}_{0j,k} = \widetilde{Q}^{-\frac{1+\zeta_k-\varepsilon_k}{1+\zeta_k}}_{0j,k} D^x_{j,k}. \tag{A.27}$$

Finally, country-0's adjusted price index in sector k can be written as

$$\widetilde{P}_{0,k} \equiv n^{\frac{\zeta_k+\alpha_k-\varepsilon_k}{\varepsilon_k}} P_{0,k} = \left[\widetilde{P}^{\frac{1}{1-(1+\zeta_k)/\varepsilon_k}}_{00,k} + \sum_{i=1}^{N} \widetilde{P}^{\frac{1}{1-(1+\zeta_k)/\varepsilon_k}}_{i0,k} \right]^{1-(1+\zeta_k)/\varepsilon_k},$$

or alternatively as

$$\widetilde{P}_{0,k} = \delta_{0,k} \cdot \left(\beta_{0,k} X_0 / n\right)^{1-\frac{\zeta_k}{\varepsilon_k}} \left[\bar{s}^{-\zeta_k}_{0,k} A^{\varepsilon_k}_{0,k} w_0^{-\zeta_k} \tilde{L}^{\alpha_k}_{0,k} + \sum_{i=1}^{N} \left(\bar{t}^m_{i0,k} p^m_{i,k} \right)^{-\zeta_k} \right]^{-\frac{1}{\varepsilon_k}}. \tag{A.28}$$

The adjusted SOE country-level consumer price index can be written as

$$\widetilde{P}_0 \equiv n^{\sum_{k=1}^{K} \frac{\beta_{0,k}(\zeta_k+\alpha_k-\varepsilon_k)}{\varepsilon_k}} P_0 = \prod_{k=1}^{K} \left(\widetilde{P}_{0,k} / \beta_{0,k} \right)^{\beta_{0,k}}. \tag{A.29}$$

Then, given the welfare in country 0 in term of unadjusted variables, $W_0 \equiv x_0 w_0 / P_0$, the adjusted welfare can be written as

$$\widetilde{W}_0 \equiv n^{-\sum_{k=1}^{K} \frac{\beta_k(\zeta_k+\alpha_k-\varepsilon_k)}{\varepsilon_k}} W_0 = \frac{\widetilde{X}_0 / \tilde{L}_0}{\widetilde{P}_0}, \tag{A.30}$$

where $\widetilde{X}_0 \equiv X_0 / n = w_0 \widetilde{L}_0 + \widetilde{T}_0$.

A.2. Single sector

In order to get single-sector versions of expressions (A.1)–(A.4) for trade shares, replace labor demand $L_{i,k}$ with labor endowment L_i, remove the sector index k, and substitute $\bar{t}_{ij} = \bar{t}^m_{ij}$, $\bar{t}^x_{ji} = 1$, and $\bar{s}_j = 1$. Additionally, to encompass the Melitz-Pareto-destination model, use ρ instead of ζ for the trade elasticity with respect to wages.

Expenditure shares (3) evaluated at pre-tariff import prices can be written as

$$\Lambda_{ij} = \frac{A^{\varepsilon}_i \bar{t}^{-(\zeta+1)}_{ij} \tau^{-\varepsilon}_{ij} w^{-\rho}_i L^{\alpha}_i}{n A^{\varepsilon}_0 \bar{t}^{-(\zeta+1)}_{0j} \widetilde{\tau}^{-\varepsilon}_{0j} w^{-\rho}_0 \widetilde{L}^{\alpha}_0 + \sum_{l=1}^{N} A^{\varepsilon}_l \bar{t}^{-(\zeta+1)}_{lj} \tau^{-\varepsilon}_{lj} w^{-\rho}_l L^{\alpha}_l}, \quad i, j = 1, \ldots, N; \tag{A.31}$$

$$\frac{\Lambda_{0j}}{n} = \frac{A^{\varepsilon}_0 \bar{t}^{-(\zeta+1)}_{0j} \widetilde{\tau}^{-\varepsilon}_{0j} w^{-\rho}_0 \widetilde{L}^{\alpha}_0}{n A^{\varepsilon}_0 \bar{t}^{-(\zeta+1)}_{0j} \widetilde{\tau}^{-\varepsilon}_{0j} w^{-\rho}_0 \widetilde{L}^{\alpha}_0 + \sum_{l=1}^{N} A^{\varepsilon}_l \bar{t}^{-(\zeta+1)}_{lj} \tau^{-\varepsilon}_{lj} w^{-\rho}_l L^{\alpha}_l}, \quad j = 1, \ldots, N; \tag{A.32}$$

$$\Lambda_{i0} = \frac{A_i^{\varepsilon} \bar{t}_{i0}^{-(\zeta+1)} \widetilde{\tau}_{i0}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{A_0^{\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} A_l^{\varepsilon} \bar{t}_{l0}^{-(\zeta+1)} \widetilde{\tau}_{l0}^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}, \quad i = 1, \ldots, N; \tag{A.33}$$

$$\Lambda_{00} = \frac{A_0^{\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha}}{A_0^{\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} A_l^{\varepsilon} \bar{t}_{l0}^{-(\zeta+1)} \widetilde{\tau}_{l0}^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}. \tag{A.34}$$

Trade balance conditions (2) can be written as

$$w_i L_i = \Lambda_{i0} w_0 n \widetilde{L}_0 + \sum_{j=1}^{N} \Lambda_{ij} w_j L_j, \quad i = 1, \ldots, N, \tag{A.35}$$

$$w_0 \widetilde{L}_0 = \Lambda_{00} w_0 \widetilde{L}_0 + \sum_{j=1}^{N} \left(\Lambda_{0j}/n\right) w_j L_j. \tag{A.36}$$

Expressions (A.35)–(A.36) correspond to expressions (8)–(9) in the main text.

Appendix B. SOE equilibrium system of equations

The equilibrium system of equations for the single-sector version of the SOE is obtained from expressions (A.32)–(A.34) and (A.36) by setting $n = 0$ or, more formally, by taking the limit as $n \to 0$. This system is detailed in Section 3.2.

In this appendix, we focus on the equilibrium system of equations for the SOE in the multi-sector case. We present a set of equilibrium conditions in a form that facilitates the analysis of equilibria and comparative statics. This set of equilibrium equations is the direct counterpart of the equations in the single sector case introduced in Section 3.2. In Appendix D, we provide an alternative (and equivalent) set of equilibrium conditions better suited for the optimal policy analysis discussed in Section 6.

The equilibrium conditions presented here are obtained from (A.2)–(A.4), (A.6), (A.8), and (A.10) provided in Appendix A.1 by taking the limit as $n \to 0$. To formulate the resulting system of equilibrium conditions, let

$$D_{j,k} \equiv \frac{\left[\bar{t}_{0j,k}^{m}\right]^{-\zeta_k - 1} \widetilde{\tau}_{0j,k}^{-\varepsilon_k}}{\sum_{l=1}^{N} A_{l,k}^{\varepsilon_k} \bar{t}_{lj,k}^{-\zeta_k} \tau_{lj,k}^{-\varepsilon_k} w_l^{-\zeta_k} L_{l,k}^{\alpha_k}} \beta_{j,k} \left(w_j L_j + T_j\right), \quad j = 1, \ldots, N, \; k = 1, \ldots, K,$$

denote the market access abroad, and let $p_{i,k}^m \equiv \left(\bar{s}_{i,k}/\bar{t}_{i0,k}^{x}\right) \left[\widetilde{\tau}_{i0,k}/\left(A_{i,k} L_{i,k}^{\phi_k}\right)\right]^{\varepsilon_k/\zeta_k} w_i$ denote a shifter of the SOE's price of imports from country i in sector k. The definition of $p_{i,k}^m$ is the same as expression (A.15) in Appendix A.1. Both $D_{j,k}$ and $p_{i,k}^m$ are taken by the SOE as given. Dropping the tildes and the country 0 subscript, the SOE's expenditure share on goods produced in sector k of country i can be written as

$$\lambda_{i,k} = \frac{\left[\bar{t}_{i0,k}^{m} p_{i,k}^m\right]^{-\zeta_k}}{A_k^{\varepsilon_k} \bar{s}_k^{-\zeta_k} w^{-\zeta_k} L_k^{\alpha_k} + \sum_{l=1}^{N} \left[\bar{t}_{l0,k}^{m} p_{l,k}^m\right]^{-\zeta_k}},$$

and the SOE's expenditure share on domestic goods in sector k is $\lambda_k = 1 - \sum_{i=1}^{N} \lambda_{i,k}$. The SOE's net tariff revenue is given by

$$T = \sum_{k=1}^{K} \sum_{i=1}^{N} \frac{\bar{t}_{i,k}^{m} - 1}{\bar{t}_{i,k}^{m}} \lambda_{i,k} \beta_k (wL + T) + \sum_{k=1}^{K} \sum_{j=1}^{N} \left(1 - \bar{t}_{j,k}^{x}\right) \left(\bar{s}_k/\bar{t}_{j,k}^{x}\right)^{-\zeta_k} A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} D_{j,k}$$
$$- \sum_{k=1}^{K} \left(1 - \bar{s}_k\right) w L_k.$$

With these expressions, the SOE's system of equilibrium conditions is given by the goods and labor market clearing conditions,

$$w L_k = \left(\lambda_k/\bar{s}_k\right) \beta_k (wL + T) + \sum_{j=1}^{N} \left(\bar{s}_k/\bar{t}_{j,k}^{x}\right)^{-\zeta_k - 1} A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} D_{j,k}, \quad k = 1, \ldots, K;$$

$$\sum_{k=1}^{K} L_k = L.$$

Also, using expressions (A.28), (A.29), and (A.30) for $\widetilde{P}_{0,k}$, $\widetilde{P}_0$, and $\widetilde{W}_0$, we can write the SOE's sector-k price index as

$$P_k = \delta_k \cdot \left(\beta_k X\right)^{1 - \frac{\zeta_k}{\varepsilon_k}} \left[\bar{s}_k^{-\zeta_k} A_k^{\varepsilon_k} w^{-\zeta_k} L_k^{\alpha_k} + \sum_{i=1}^{N} \left(\bar{t}_{i,k}^{m} p_{i,k}^m\right)^{-\zeta_k}\right]^{-\frac{1}{\varepsilon_k}},$$

the SOE's price index as $P = \prod_{k=1}^{K} \left(P_k/\beta_k\right)^{\beta_k}$, and the SOE's welfare as $W = (X/L)/P$, where $X \equiv wL + T$ is the total consumer expenditure in the SOE.

Appendix C. Optimal policy in the single-sector SOE

The SOE wage is given by the solution to (11), which can be written as

$$X(w) = (1 - \Lambda(w))\, wL, \tag{C.1}$$

with $X(w) \equiv DA^{\varepsilon} L^{\alpha} w^{-\rho}$ and

$$\Lambda(w) \equiv \frac{A^{\varepsilon} L^{\alpha} w^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{i=1}^{N} \bar{t}_i^{-(\zeta+1)} p_i^{-\rho}}.$$

Logarithmically differentiating (C.1), and denoting $\hat{x} \equiv d\ln x$, we obtain

$$-\rho\hat{w} = \left[-\frac{\Lambda}{1-\Lambda}\right]\hat{\Lambda} + \hat{w}. \tag{C.2}$$

Logarithmically differentiating the definition of $\Lambda(w)$, we obtain

$$\hat{\Lambda} = -\rho(1-\Lambda)\hat{w} + (\zeta+1)\sum_{i=1}^{N} \Lambda_i \hat{\bar{t}}_i, \tag{C.3}$$

where

$$\Lambda_i \equiv \frac{\bar{t}_i^{-(\zeta+1)} p_i^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{l=1}^{N} \bar{t}_l^{-(\zeta+1)} p_l^{-\rho}}$$

is the SOE's import expenditure share from i evaluated at pre-tariff import prices. Substituting (C.3) into (C.2), and solving for $\hat{w}$, we get

$$\hat{w} = \frac{\zeta+1}{\Delta} \cdot \frac{\Lambda}{1-\Lambda} \sum_{i=1}^{N} \Lambda_i \hat{\bar{t}}_i, \tag{C.4}$$

where $\Delta \equiv 1 + \rho(1+\Lambda) > 1$. This implies (13) in the main text.

Next, expression (12) for the gains from trade is

$$\mathrm{GT} = \lambda^{-1/\varepsilon}\left(\lambda + \sum_{i=1}^{N} \lambda_i / \bar{t}_i\right)^{-\zeta/\varepsilon},$$

where

$$\lambda_i(w) = \frac{\bar{t}_i^{-\zeta} p_i^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{l=1}^{N} \bar{t}_l^{-\zeta} p_l^{-\rho}},$$
$$\lambda(w) = \frac{A^{\varepsilon} L^{\alpha} w^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{l=1}^{N} \bar{t}_l^{-\zeta} p_l^{-\rho}}$$

are the expenditure shares (at domestic prices) devoted by the SOE to imports from country i and to itself. Logarithmically differentiating these expressions, we obtain

$$\widehat{\mathrm{GT}} = -\frac{1}{\varepsilon}\hat{\lambda} - \frac{\zeta}{\varepsilon} \cdot \left[\Lambda\hat{\lambda} + \sum_{i=1}^{N}\left(\hat{\lambda}_i - \hat{\bar{t}}_i\right)\Lambda_i\right] \tag{C.5}$$

and

$$\hat{\lambda}_i = -\left(\zeta\hat{\bar{t}}_i - \lambda\rho\hat{w} - \sum_{l=1}^{N} \lambda_l \zeta \hat{\bar{t}}_l\right), \tag{C.6}$$

$$\hat{\lambda} = -\left(\rho\hat{w} - \lambda\rho\hat{w} - \sum_{l=1}^{N} \lambda_l \zeta \hat{\bar{t}}_l\right). \tag{C.7}$$

Substituting (C.6) and (C.7) into (C.5), and after doing some algebra, we get

$$\widehat{\mathrm{GT}} = \frac{\rho}{\varepsilon}\left[1 - \lambda + \zeta(\Lambda - \lambda)\right]\hat{w} - \frac{\zeta}{\varepsilon}(\zeta+1)\sum_{i=1}^{N}\left(\lambda_i - \Lambda_i\right)\hat{\bar{t}}_i. \tag{C.8}$$

Combining (C.4) and (C.8), we obtain (14) in the main text.

Let us now turn to the proof of Proposition 3 in the main text. The first-order condition for welfare maximization with respect to $\bar{t}_i$ is given by

$$\frac{\partial \ln \mathrm{GT}}{\partial \ln \bar{t}_i} = \frac{\rho}{\varepsilon}\left[1 - \lambda + \zeta(\Lambda - \lambda)\right]\frac{\zeta+1}{\Delta} \cdot \frac{\Lambda}{1-\Lambda}\Lambda_i - \frac{\zeta}{\varepsilon}(\zeta+1)\left(\lambda_i - \Lambda_i\right) = 0. \tag{C.9}$$

Given the definitions

$$\Lambda \equiv \frac{\lambda}{\lambda + \sum_{l=1}^{N} \lambda_l / \bar{t}_l} \quad \text{and} \quad \Lambda_i \equiv \frac{\lambda_i / \bar{t}_i}{\lambda + \sum_{l=1}^{N} \lambda_l / \bar{t}_l},$$

we have $\lambda + \sum_{l=1}^{N} \lambda_l / \bar{t}_l = \lambda / \Lambda$ and

$$\lambda_i - \Lambda_i = \Lambda_i \left[\left(\lambda + \sum_{l=1}^{N} \lambda_l / \bar{t}_l \right) \bar{t}_i - 1 \right] = \Lambda_i \left(\frac{\lambda}{\Lambda} \bar{t}_i - 1 \right).$$

Substituting the above expression for $\lambda_i - \Lambda_i$ into (C.9), and cancelling out Λ_i, we get

$$\frac{\rho}{\varepsilon} \left[1 - \lambda + \zeta (\Lambda - \lambda) \right] \frac{\zeta + 1}{\Delta} \cdot \frac{\Lambda}{1 - \Lambda} - \frac{\zeta}{\varepsilon} (\zeta + 1) \left(\frac{\lambda}{\Lambda} \bar{t}_i - 1 \right) = 0.$$

Since $\bar{t}_i$ is the only term in the above expression that depends on i, this implies that $\bar{t}_i$ is the same across i, and so we can use $\bar{t}_i = \bar{t}$ for all $i = 1, \dots, N$.

Getting back to (C.9), after we sum it across $i = 1, \dots, N$, and use $\sum_{i=1}^{N} \Lambda_i = 1 - \Lambda$ and $\sum_{i=1}^{N} \lambda_i = 1 - \lambda$, we get

$$\frac{\rho}{\varepsilon} \left[1 - \lambda + \zeta (\Lambda - \lambda) \right] \frac{\zeta + 1}{\Delta} \cdot \Lambda - \frac{\zeta}{\varepsilon} (\zeta + 1) (\Lambda - \lambda) = 0.$$

Using $\Delta \equiv 1 + \rho (1 + \Lambda)$, and after doing some algebra, this can further be simplified to

$$(1 - \lambda) \rho \Lambda - (1 + \rho) \zeta (\Lambda - \lambda) = 0.$$

Finally, substituting

$$\Lambda = \frac{\lambda}{\lambda + \sum_{l=1}^{N} \lambda_l / \bar{t}_l} = \frac{\lambda}{\lambda + (1 - \lambda) / \bar{t}}$$

into the above, and after doing some algebra, we get

$$\bar{t} - 1 = \frac{\rho}{(1 + \rho) \zeta - \rho}.$$

Appendix D. Optimal policy through micro-to-macro representation

D.1. An alternative system of equilibrium conditions in the multi-sector case

For the SOE described in Section 5.1, the price indices P_k^d, $P_{i,k}^m$, and $P_{j,k}^x$ are defined in terms of adjusted price indices $\widetilde{P}_{00,k}$, $\widetilde{P}_{i0,k}$, and $\widetilde{P}_{0j,k}$ as $P_k^d \equiv \lim_{n \to 0} \widetilde{P}_{00,k}$, $P_{i,k}^m \equiv \lim_{n \to 0} \widetilde{P}_{i0,k} / \bar{t}_{i0,k}^m$, and $P_{j,k}^x \equiv \lim_{n \to 0} \widetilde{P}_{0j,k} / \bar{t}_{0j,k}^m$, with the expressions for $\widetilde{P}_{00,k}$, $\widetilde{P}_{i0,k}$, and $\widetilde{P}_{0j,k}$ given by (A.19)–(A.21). This yields

$$P_k^d = \delta_k^{\frac{\varepsilon_k}{\zeta_k}} \cdot \left[Q_k^d \right]^{\frac{\varepsilon_k}{\zeta_k} - 1} \bar{s}_k A_k^{-\frac{\varepsilon_k}{\zeta_k}} L_k^{-\frac{\alpha_k}{\zeta_k}} w, \tag{D.1}$$

$$P_{i,k}^m = \left[Q_{i,k}^m \right]^{\frac{\varepsilon_k}{\zeta_k} - 1} \overline{S}_{i,k}^m, \tag{D.2}$$

$$P_{j,k}^x = \left[Q_{j,k}^x \right]^{\frac{\varepsilon_k}{\zeta_k} - 1} \left(\bar{s}_k / \bar{t}_{j,k}^x \right) A_k^{-\frac{\varepsilon_k}{\zeta_k}} L_k^{-\frac{\alpha_k}{\zeta_k}} w p_{j,k}^x, \tag{D.3}$$

where $\overline{S}_{i,k} \equiv \delta_k^{\frac{\varepsilon_k}{\zeta_k}} \cdot p_{i,k}^m$ is a foreign supply shifter exogenous to the SOE. Quantities Q_k^d, $Q_{i,k}^m$, and $Q_{j,k}^x$ are defined as $Q_k^d \equiv \lim_{n \to 0} \widetilde{Q}_{00,k}$, $Q_{i,k}^m \equiv \lim_{n \to 0} \widetilde{Q}_{i0,k}$, and $Q_{j,k}^x \equiv \lim_{n \to 0} \widetilde{Q}_{0j,k}$, with the expressions for $\widetilde{Q}_{00,k}$, $\widetilde{Q}_{i0,k}$, and $\widetilde{Q}_{0j,k}$ given by (A.16)–(A.18).

The SOE's price index in sector k can be written as

$$P_k = \left(\left[P_k^d \right]^{\frac{1}{1 - (1 + \zeta_k)/\varepsilon_k}} + \sum_{i=1}^{N} \left[\bar{t}_{i,k}^m P_{i,k}^m \right]^{\frac{1}{1 - (1 + \zeta_k)/\varepsilon_k}} \right)^{1 - (1 + \zeta_k)/\varepsilon_k}, \tag{D.4}$$

and the aggregate price index is given by $P = \prod_{k=1}^{K} \left(P_k / \beta_k \right)^{\beta_k}$.

The SOE's version of expression (A.24) for $\widetilde{L}_{0,k}$ is

$$L_k = \left(\delta_k^{\frac{\varepsilon_k}{\zeta_k}} A_k^{-\frac{\varepsilon_k}{\zeta_k}} \left[Q_k^d \right]^{\frac{\varepsilon_k}{\zeta_k}} + \sum_{j=1}^{N} A_k^{-\frac{\varepsilon_k}{\zeta_k}} p_{j,k}^x \left[Q_{j,k}^x \right]^{\frac{\varepsilon_k}{\zeta_k}} \right)^{\frac{1}{1 + \alpha_k / \zeta_k}}, \tag{D.5}$$

which is simply another way to represent the goods market clearing condition,

$$\bar{s}_k w L_k = P_k^d Q_k^d + \sum_{j=1}^{N} \bar{t}_{j,k}^x P_{j,k}^x Q_{j,k}^x.$$

The net tax revenue of the SOE is given by

$$T = \sum_{k=1}^{K} \left\{ \sum_{i=1}^{N} \left(\bar{t}^m_{i,k} - 1 \right) P^m_{i,k} Q^m_{i,k} + \left(1 - \bar{t}^x_{j,k} \right) P^x_{j,k} Q^x_{j,k} - \left(1 - \bar{s}_k \right) w L_k \right\}. \tag{D.6}$$

The SOE's expenditure share on goods produced in sector k of country i is given by

$$\lambda_{i,k} = \frac{\left[\bar{t}^m_{i,k} P^m_{i,k} \right]^{\frac{1}{1-(1+\zeta_k)/\epsilon_k}}}{\left[P^d_k \right]^{\frac{1}{1-(1+\zeta_k)/\epsilon_k}} + \sum_{i=1}^{N} \left[\bar{t}^m_{i,k} P^m_{i,k} \right]^{\frac{1}{1-(1+\zeta_k)/\epsilon_k}}}, \tag{D.7}$$

while the SOE's expenditure share on domestically produced goods in industry k is given by $\lambda_k = 1 - \sum_i \lambda_{i,k}$. The supply-equals-demand conditions that determine quantities Q^d_k, $Q^m_{i,k}$, and $Q^x_{j,k}$ are

$$P^d_k Q^d_k = \beta_k \lambda_k (wL + T), \tag{D.8}$$

$$\bar{t}^m_{i,k} P^m_{i,k} Q^m_{i,k} = \beta_k \lambda_{i,k} (wL + T), \tag{D.9}$$

$$P^x_{j,k} = \left[Q^x_{j,k} \right]^{-\frac{1+\zeta_k-\epsilon_k}{1+\zeta_k}} \overline{D}^x_{j,k}, \tag{D.10}$$

where the last condition is the limit of (A.27) as $n \to 0$, with $\overline{D}^x_{j,k} \equiv \lim_{n\to 0} \left(D^x_{j,k} / \bar{t}^m_{0j,k} \right)$ and $D^x_{j,k}$ given by (A.26). $\overline{D}^x_{j,k}$ is a foreign demand shifter exogenous to the SOE.

A SOE equilibrium is given by prices P^d_k, $P^x_{j,k}$, and $P^m_{i,k}$, quantities Q^d_k, $Q^x_{j,k}$, and $Q^m_{i,k}$, labor allocations L_k, and wage w such that: prices satisfy (D.1)–(D.3); labor allocations satisfy (D.5); quantities satisfy (D.8)–(D.10); and the labor market clears, $\sum_{k=1}^{K} L_k = L$.

D.2. Welfare maximization by the social planner

Here we use the conditions introduced in Appendix D.1. Welfare is given by

$$W = \frac{I}{P} = \frac{w + \Pi/L + T/L}{P},$$

where I is the income per capita,

$$\Pi = \sum_{k=1}^{K} \left\{ P^d_k Q^d_k + \sum_{j=1}^{N} \bar{t}^x_{j,k} P^x_{j,k} Q^x_{j,k} - \bar{s}_k w L_k \right\} \tag{D.11}$$

are aggregate profits, T are net tax revenues given by (D.6), L_k is given by (D.5), and $P = \prod_{k=1}^{K} \left(P_k / \beta_k \right)^{\beta_k}$ is the aggregate price index, with the price index in sector k given by (D.4). The social planner chooses policy instruments $\bar{s}_k$, $\bar{t}^m_{i,k}$, and $\bar{t}^x_{j,k}$, as well as quantities Q^d_k, $Q^m_{i,k}$, and $Q^x_{j,k}$ so as to maximize welfare. This is done by taking the SOE's demand for domestic and foreign goods as given by (D.8) and (D.9), the foreign demand for the SOE's goods as given by (D.10), and the supply of foreign goods to the SOE as given by (D.2).

Log differentiation of welfare yields

$$d \ln W = \frac{dw + d\Pi/L + dT/L}{I} - \sum_{k=1}^{K} \beta_k d \ln P_k. \tag{D.12}$$

Expression (D.4) for P_k implies

$$d \ln P_k = \lambda_k d \ln P^d_k + \sum_{i=1}^{N} \lambda_{i,k} d \ln \left(\bar{t}^m_{i,k} P^m_{i,k} \right), \tag{D.13}$$

where $\lambda_{i,k}$ is given by (D.7) and $\lambda_k = 1 - \sum_{i=1}^{N} \lambda_{i,k}$. Substituting expressions (D.11), (D.6), and (D.13) for Π, T, and $d \ln P_k$ into (D.12), and multiplying both sides on IL, we get

$$\begin{aligned} IL \cdot d \ln W = {} & L dw + \sum_{k=1}^{K} d \left\{ P^d_k Q^d_k \right\} + \sum_{k=1}^{K} \sum_{j=1}^{N} d \left\{ \bar{t}^x_{j,k} P^x_{j,k} Q^x_{j,k} \right\} - \sum_{k=1}^{K} d \left\{ \bar{s}_k w L_k \right\} \\ & + \sum_{k=1}^{K} \sum_{i=1}^{N} d \left\{ \left(\bar{t}^m_{i,k} - 1 \right) P^m_{i,k} Q^m_{i,k} \right\} + \sum_{k=1}^{K} \sum_{j=1}^{N} d \left\{ \left(1 - \bar{t}^x_{j,k} \right) P^x_{j,k} Q^x_{j,k} \right\} \\ & - \sum_{k=1}^{K} d \left\{ \left(1 - \bar{s}_k \right) w L_k \right\} \\ & - \sum_{k=1}^{K} \beta_k \lambda_k IL d \ln P^d_k - \sum_{k=1}^{K} \sum_{i=1}^{N} \beta_k \lambda_{i,k} IL d \ln \left(\bar{t}^m_{i,k} P^m_{i,k} \right). \end{aligned}$$

Using

$$d\left\{P_k^d Q_k^d\right\} = P_k^d dQ_k^d + P_k^d Q_k^d d\ln P_k^d,$$
$$d\left\{\bar{t}_{j,k}^x P_{j,k}^x Q_{j,k}^x\right\} + d\left\{\left(1-\bar{t}_{j,k}^x\right) P_{j,k}^x Q_{j,k}^x\right\} = P_{j,k}^x dQ_{j,k}^x + P_{j,k}^x Q_{j,k}^x d\ln P_{j,k}^x$$
$$Ldw - \sum_{k=1}^K d\left\{\bar{s}_k w L_k\right\} - \sum_{k=1}^K d\left\{\left(1-\bar{s}_k\right) w L_k\right\} = -w\sum_{k=1}^K dL_k,$$

we get

$$\begin{aligned} IL\cdot d\ln W = {} & \sum_{k=1}^K P_k^d dQ_k^d + \sum_{k=1}^K P_k^d Q_k^d d\ln P_k^d + \sum_{k=1}^K\sum_{j=1}^N P_{j,k}^x dQ_{j,k}^x + \sum_{k=1}^K\sum_{j=1}^N P_{j,k}^x Q_{j,k}^x d\ln P_{j,k}^x \\ & - w\sum_{k=1}^K dL_k + \sum_{k=1}^K\sum_{i=1}^N d\left\{\left(\bar{t}_{i,k}^m - 1\right) P_{i,k}^m Q_{i,k}^m\right\} \\ & - \sum_{k=1}^K \beta_k\lambda_k IL d\ln P_k^d - \sum_{k=1}^K\sum_{i=1}^N \beta_k\lambda_{i,k} IL d\ln\left(\bar{t}_{i,k}^m P_{i,k}^m\right). \end{aligned}$$

Also, $P_k^d Q_k^d = \beta_k\lambda_k IL$ and $\bar{t}_{i,k}^m P_{i,k}^m Q_{i,k}^m = \beta_k\lambda_{i,k} IL$, and thus,

$$P_k^d Q_k^d d\ln P_k^d - \beta_k\lambda_k IL d\ln P_k^d = 0,$$
$$d\left\{\left(\bar{t}_{i,k}^m - 1\right) P_{i,k}^m Q_{i,k}^m\right\} - \beta_k\lambda_{i,k} IL d\ln\left(\bar{t}_{i,k}^m P_{i,k}^m\right) = \left(\bar{t}_{i,k}^m - 1\right) P_{i,k}^m dQ_{i,k}^m - P_{i,k}^m Q_{i,k}^m d\ln P_{i,k}^m,$$

which gives

$$\begin{aligned} IL\cdot d\ln W = {} & \sum_{k=1}^K P_k^d dQ_k^d + \sum_{k=1}^K\sum_{j=1}^N P_{j,k}^x dQ_{j,k}^x + \sum_{k=1}^K\sum_{j=1}^N P_{j,k}^x Q_{j,k}^x d\ln P_{j,k}^x \\ & - w\sum_{k=1}^K dL_k + \sum_{k=1}^K\sum_{i=1}^N \left(\bar{t}_{i,k}^m - 1\right) P_{i,k}^m dQ_{i,k}^m - \sum_{k=1}^K\sum_{i=1}^N P_{i,k}^m Q_{i,k}^m d\ln P_{i,k}^m. \end{aligned}$$

Given the expression (D.10) for foreign demand for the SOE's goods, we have

$$P_{j,k}^x Q_{j,k}^x d\ln P_{j,k}^x = P_{j,k}^x Q_{j,k}^x \frac{\partial\ln P_{j,k}^x}{\partial\ln Q_{j,k}^x} d\ln Q_{j,k}^x = -\epsilon_{j,k}^x P_{j,k}^x dQ_{j,k}^x,$$

where $\epsilon_{j,k}^x = \frac{1+\zeta_k-\epsilon_k}{1+\zeta_k}$. Similarly, given the expression (D.2) for foreign supply of goods to the SOE, we have

$$P_{i,k}^m Q_{i,k}^m d\ln P_{i,k}^m = P_{i,k}^m \frac{d\ln P_{i,k}^m}{d\ln Q_{i,k}^m} dQ_{i,k}^m = \epsilon_{i,k}^m P_{i,k}^m dQ_{i,k}^m,$$

where $\epsilon_{i,k}^m = \frac{\epsilon_k}{\zeta_k} - 1$. The above two expressions then give

$$\begin{aligned} IL\cdot d\ln W = {} & \sum_{k=1}^K P_k^d dQ_k^d + \sum_{k=1}^K\sum_{j=1}^N \left(1-\epsilon_{j,k}^x\right) P_{j,k}^x dQ_{j,k}^x \\ & - w\sum_{k=1}^K dL_k + \sum_{k=1}^K\sum_{i=1}^N \left(\bar{t}_{i,k}^m - 1 - \epsilon_{i,k}^m\right) P_{i,k}^m dQ_{i,k}^m. \end{aligned}$$

Finally, expression (D.5) implies that $L_k \equiv L_k\left(Q_k^d, Q_{1,k}^x, \ldots, Q_{N,k}^x\right)$, and thus, we have

$$dL_k = \frac{\partial L_k}{\partial Q_k^d} dQ_k^d + \sum_{j=1}^N \frac{\partial L_k}{\partial Q_{j,k}^x} dQ_{j,k}^x.$$

Therefore,

$$\begin{aligned} IL\cdot d\ln W = {} & \sum_{k=1}^K \left(P_k^d - w\frac{\partial L_k}{\partial Q_k^d}\right) dQ_k^d + \sum_{k=1}^K\sum_{j=1}^N \left[\left(1-\epsilon_{j,k}^x\right) P_{j,k}^x - w\frac{\partial L_k}{\partial Q_{j,k}^x}\right] dQ_{j,k}^x \\ & + \sum_{k=1}^K\sum_{i=1}^N \left(\bar{t}_{i,k}^m - 1 - \epsilon_{i,k}^m\right) P_{i,k}^m dQ_{i,k}^m. \end{aligned}$$

D.3. Derivation of (26)

We consider three different types of variations that determine optimal subsidies, export taxes, and import tariffs.

First, consider a variation in Q_k^d while adjusting Q_1^d to maintain $\sum_k L_k = L$. This entails

$$\frac{\partial L_k}{\partial Q_k^d} dQ_k^d + \frac{\partial L_1}{\partial Q_1^d} dQ_1^d = 0,$$

hence $d \ln W = 0$ implies

$$\frac{P_k^d}{P_1^d} = \frac{\partial L_k/\partial Q_k^d}{\partial L_1/\partial Q_1^d}.$$

Using

$$\bar{s} = P_1^d \left(w \frac{\partial L_1}{\partial Q_1^d} \right)^{-1}$$

we then have

$$P_k^d = \bar{s} w \partial L_k/\partial Q_k^d.$$

Using (27) we then get $\bar{s} \equiv \left(\frac{\alpha_1+\zeta_1}{\varepsilon_1} \right) \bar{s}_1$.

Next, consider a variation in $Q_{j,k}^x$ while adjusting $Q_{j,k}^d$ to maintain $\sum_k L_k = L$, and $Q_{1,1}^m$ to maintain trade balance $\sum_k \sum_j P_{j,k}^x Q_{j,k}^x = \sum_k \sum_i P_{i,k}^m Q_{i,k}^m$. This entails

$$\frac{\partial L_k}{\partial Q_k^d} dQ_k^d + \frac{\partial L_k}{\partial Q_{j,k}^x} dQ_{j,k}^x = 0$$

and

$$\left(1 - \epsilon_{j,k}^x\right) P_{j,k}^x dQ_{j,k}^x + \left(-1 - \epsilon_{1,1}^m\right) P_{1,1}^m dQ_{1,1}^m = 0$$

Combining these equations with $d \ln W = 0$ yields

$$P_k^d \frac{\partial L_k/\partial Q_{j,k}^x}{\partial L_k/\partial Q_k^d} = \bar{t} \left(1 - \epsilon_{j,k}^x\right) P_{j,k}^x.$$

where $\bar{t} \equiv \frac{\bar{t}_{1,1}^m}{1+\epsilon_{1,1}^m}$. This can be rewritten as

$$\bar{t} \left(1 - \epsilon_{j,k}^x\right) P_{j,k}^x = \bar{s} w \partial L_k/\partial Q_{j,k}^x,$$

as in the text.

Finally, consider a variation in $Q_{i,k}^m$ while adjusting $Q_{1,1}^m$ to maintain trade balance. This entails

$$P_{i,k}^m \left(-1 - \epsilon_{i,k}^m\right) dQ_{i,k}^m + P_{1,1}^m \left(-1 - \epsilon_{1,1}^m\right) dQ_{1,1}^m = 0.$$

Combined with $d \ln W$ we then have

$$P_{1,1}^m \bar{t}_{1,1}^m dQ_{1,1}^m + P_{i,k}^m \bar{t}_{i,k}^m dQ_{i,k}^m = 0.$$

From these equations, we then get

$$\bar{t}_{i,k}^m = \bar{t} \left(1 + \epsilon_{i,k}^m\right).$$

D.4. Macro pricing formula

From (25), $\frac{\partial L_k}{\partial Q_k^d}$ and $\frac{\partial L_k}{\partial Q_{j,k}^x}$ are calculated as

$$\begin{aligned}\frac{\partial L_k}{\partial Q_k^d} &= \frac{\zeta_k}{\zeta_k + \alpha_k} \left(L_k^{\frac{\zeta_k+\alpha_k}{\zeta_k}} \right)^{\frac{\zeta_k}{\zeta_k+\alpha_k}-1} \frac{\varepsilon_k}{\zeta_k} \left(\frac{\delta_k Q_k^d}{A_k} \right)^{\frac{\varepsilon_k}{\zeta_k}-1} \frac{\delta_k}{A_k} \\ &= \frac{\varepsilon_k}{\zeta_k + \alpha_k} L_k^{-\frac{\alpha_k}{\zeta_k}} \left(\delta_k/A_k\right)^{\frac{\varepsilon_k}{\zeta_k}} \left[Q_k^d\right]^{\frac{\varepsilon_k}{\zeta_k}-1}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial L_k}{\partial Q_{j,k}^x} &= \frac{\zeta_k}{\zeta_k + \alpha_k} \left(L_k^{\frac{\zeta_k+\alpha_k}{\zeta_k}} \right)^{\frac{\zeta_k}{\zeta_k+\alpha_k}-1} \frac{\varepsilon_k}{\zeta_k} \left(\frac{\tau_{j,k}^x \delta_{j,k} Q_{j,k}^x}{A_k} \right)^{\frac{\varepsilon_k}{\zeta_k}-1} \frac{\delta_{j,k} \tau_{j,k}^x}{A_k} \\ &= \frac{\varepsilon_k}{\zeta_k + \alpha_k} L_k^{-\frac{\alpha_k}{\zeta_k}} \left(\tau_{j,k}^x \delta_{j,k}/A_k\right)^{\frac{\varepsilon_k}{\zeta_k}} \left[Q_{j,k}^x\right]^{\frac{\varepsilon_k}{\zeta_k}-1}.\end{aligned}$$

Combining these with (D.1) and (D.3), and noting that $p_{j,k}^x \equiv \delta_{j,k}^{\varepsilon_k/\zeta_k} \tilde{\tau}_{0j,k}^{\varepsilon_k/\zeta_k} = \delta_{j,k}^{\varepsilon_k/\zeta_k} \left[\tau_{j,k}^x\right]^{\varepsilon_k/\zeta_k}$, we obtain (27).

Appendix E. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jinteco.2024.103997.

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