Any-degrees-of-freedom (anyDOF) registration for the characterization of freeform surfaces

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Abstract

This paper presents an any-degrees-of-freedom (anyDOF) registration method for the characterization of freeform surfaces. The method attempts to fill the research gap regarding traditional surface registration methods which are normally dedicated to solving the global optimization problem with all DOF but they lack flexibility. The proposed anyDOF method is capable of registering surfaces with any specified combination of DOF. This is particularly useful when some of the DOF are known to be unchanged according to the *a priori* knowledge. The anyDOF surface registration method is regarded as a typical optimization problem of finding the minimum distance from target surface to the reference surface, with constraints of the unwanted DOF. The problem is solved by the Levenberg-Marquardt method. Simulated experiments for a two-dimensional (2D) profile and a three-dimensional (3D) surface were undertaken, together with three measurement experiments including a fluid-jet polished surface, a bonnet polished surface and a diamond machined freeform surface. Experimental results show that the anyDOF registration method is highly flexible in the characterization of freeform surfaces.

Keywords: any-degrees-of-freedom (anyDOF); registration; characterization, freeform surfaces; ultra-precision machining; precision surface measurement

1. Introduction

Freeform surfaces [1] have attracted a lot of research attention in the last few decades. Technologies for manufacturing [2], measurement [3], and characterization [4] of freeform surfaces have been rapidly developed to meet the stringent requirements of freeform surfaces [5]. Characterization plays an important role in the development of precision freeform surfaces since one can never know whether the machined surfaces meet the design requirements without the characterization information. During characterization, the measured surface is compared with the design surface to determine the difference between them, which is a sum of machining errors, measurement errors and characterization errors. Measurement and characterization errors must be relatively small in order to accurately determine the machining errors and enable improvement to the accuracy of machined surfaces through error compensation.

Since the designed surface and the measured surface were obtained from different coordinate systems, i.e. the designed surface was within the coordinates of the design space and the measured surface was within the coordinates of the measurement instrument, alignment of the two surfaces was necessary for comparison. Alignment is typically achieved by a surface registration process that transforms the measured surface to the designed surface. Research on the registration of 3D surfaces has a long history and the topic has been intensively investigated. The most widely used is probably the iterative closest points (ICP) method developed by Besl and McKay [6] and Zhang [7], from which several variants have been developed [8]. Other frequently used methods include the least square method [9], the intrinsic feature-based method [10], and methods based on image registration [11]. Most of these methods perform surface transformation in all six degrees of freedom (DOF), which in most cases provides accurate alignment of the measured and designed surfaces. Take the ICP for example: the core algorithm is a singular value decomposition (SVD) method which determines the rotation matrix in all DOF and it is not easy to separate specified DOF during registration.

However, the complete freedom of transformation sometimes results in false alignment due to similarities in the surface form and measurement error. Such false alignment occurs more often in freeform surfaces that are asymmetric or contain periodic features. For example, when evaluating fluid jet polished [12] and bonnet polished [13] surfaces where large inhomogeneous errors are present, registration with full six DOF may result in unwanted tilting or shifting, even though the overall deviation from the designed surface is minimized. Such unwanted registration error can be avoided using *a priori* knowledge of the surface, i.e. utilizing pre-existing highly accurate reference features on the surface such as flat or spherical features [14] to assist the registration process. In such cases, the registration process is performed in two steps. The first step is registration of the reference features, which will constrain some of the DOF; and the second step is to register the entire surface using the remaining DOF. In this paper, a two-step any-degrees-of-freedom (anyDOF) registration method is proposed to enable the alignment of surfaces using less than six DOF.

In the first step, the target surface is pre-aligned using reference features, resulting in some of the DOF being constrained. In the second step, the remaining unconstrained DOF are used to minimize the overall difference between the target and reference surfaces. With the proposed method, the unwanted misalignment can be avoided and therefore more accurate characterization of the surface can be achieved. The rest of the article is structured as follows: Section 2 describes the alignment algorithm used in the anyDOF method; in Section 3, two simulated examples are used to demonstrate the limitations of the traditional ICP method and the potential improvement to be achieved by the proposed anyDOF method, the effectiveness of which is further verified using the measurement data of three real machined surfaces. The results demonstrated that the proposed method is highly robust and suitable for the characterization of freeform surfaces with inhomogeneous errors. Section 4 summarizes the findings and contributions of this work.

2. Any-degrees-of-freedom (anyDOF) registration method

A schematic diagram of the any-degrees-of-freedom (anyDOF) method is shown in Fig. 1. The target surface is first subjected to outlier removal to remove spurious points due to measurement noise, dirt on the surface and/or measurement artefacts often present in optical measurement instruments [15-17]. The surface is subsequently transformed using *a priori* knowledge, such as removing tilt or pre-alignment using reference features. Depending on the type of features used during pre-alignment, some of the DOF will be constrained. For example, pre-alignment using a planar reference feature will constrain two rotations and one translation, while pre-alignment using a spherical feature will constrain all three translations. The pre-aligned surface is compared to the reference surface and the root-mean-squared (RMS) distance from each point on the transformed surface to the reference surface is determined and used as the cost function. To find the solution of the specified DOF it is required to minimize the cost function, which is a typical nonlinear optimization problem. The nonlinear minimization problem is solved by the Levenberg-Marquardt algorithm (LMA) [18], which is iteratively executed until the minimum tolerance is found or the maximum number of iterations is reached. As a result, the unknown variables are determined numerically and the final transformation matrix for the anyDOF registration is obtained.

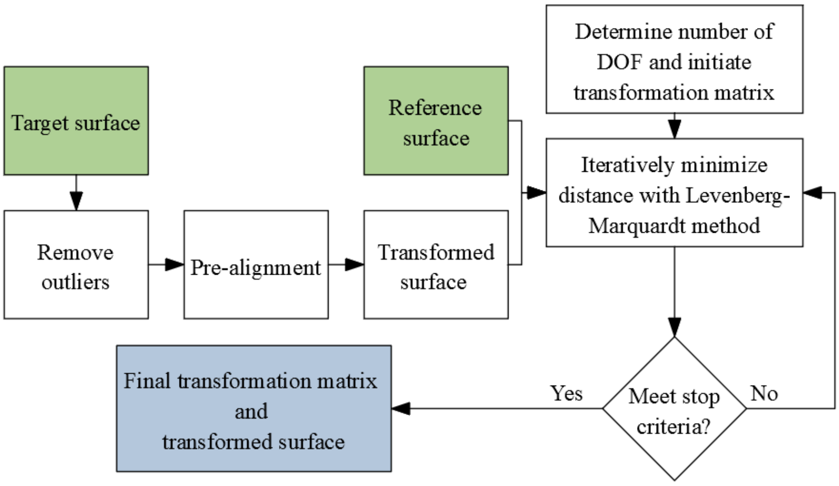


Figure 1. Diagram of the anyDOF registration method

In the case of 2D profiles, at most three DOF (i.e. two translations and one rotation) are available for optimization during registration. In the case of 3D surfaces, all or part of six DOF could be available for optimization. In this section, the discussion is focused on the registration of 3D surfaces, which is a superset of the case for 2D profiles.

The rigid-body transformation for a 3D surface has 6 DOF, i.e. translation along *x*, *y*, and *z* axes and rotation about *x*, *y*, and *z* axes, representing the yaw, pitch, and roll angles. The translation matrices can be determined by:

$$T_x(v_x) = \begin{bmatrix} 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$T_y(v_y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$T_z(v_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

where v_x, v_y and v_z are the translation variables along the *x*, *y*, and *z* axes. The rotation matrices can be determined by:

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$R_y(\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$R_z(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

where α, β, and γ are the rotation variables about the *x*, *y*, and *z* axes.

Assuming that the transformation is determined by first rotating about the *x*, *y*, and *z* axes and then translating along the *x*, *y*, and *z* axes, the final transformation matrix with all 6 DOF can be determined by:

$$M = T_z(v_z)T_y(v_y)T_x(v_x)R_z(\gamma)R_y(\beta)R_x(\alpha) \quad (7)$$

In the anyDOF method, any DOF previously used for pre-alignment becomes a unit matrix, and the final transformation matrix is the product of the remaining translation and/or rotation matrices. For the point set *P* of the target surface, the transformed point set *P'* can be determined by:

$$P' = MP \quad (8)$$

where *P* and *P'* are both $n \times 4$ matrix, *n* is the number of points, four elements are the coordinates of *x*, *y*, and *z* and a unit value padded for the matrix calculation. Assuming that point set *X* represents the reference surface, the cost function is determined by:

$$F = RMS(D) \quad (9)$$

where *D* is the vector of the distances from every point in *X* to *P'*.

In this study, the anyDOF method is implemented in Matlab. The LMA iterative procedure is started with the initial values of the unknown variables set to zero.

3. Experiments

In order to verify the effectiveness of the proposed anyDOF method, a series of experiments were conducted including a simulated 2D profile, a simulated 3D surface and three measurement experiments including a fluid-jet polished surface, a bonnet polished surface and a diamond machined freeform surface.

3.1 Simulations

The simulations included a 2D sinusoidal profile and a 3D sinusoidal surface. The target profile and surface were modified to create an inhomogeneous deviation. The modified profile and surface were then registered with the reference profile and surface to determine the errors. The widely used ICP method was applied to obtain full DOF registration, which was compared to the proposed anyDOF method. Simulation is an effective way to compare the two methods, as the determined registration error is not affected by other errors (e.g. measurement noise) that would have occurred in experiments.

(a) 2D profile

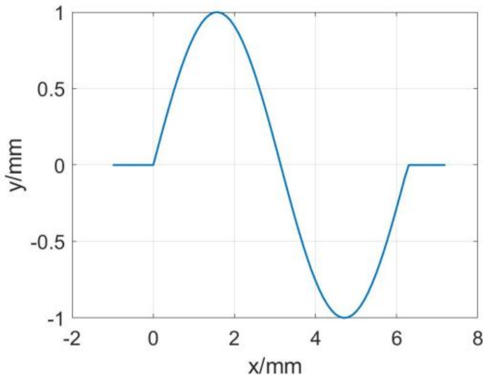
A sinusoidal profile was designed as shown in Fig. 2. The reference profile shown in Fig. 2(a) can be determined by:

$$y_{ref} = \begin{cases} \sin x, & \text{if } 0 < x < 2\pi \\ 0, & \text{others} \end{cases} \tag{10}$$

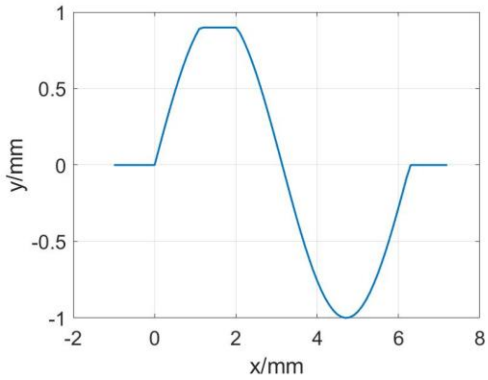
where $x \in [-1, 2\pi + 1]$ mm. The sampling distance was 0.1 mm. To simulate an inhomogeneous error in the target profile, thresholding by profile height was applied to the reference profile and replacing the missing points with new values. The modified profile, as shown in Fig. 2(b), can be determined by:

$$y_{tar} = \begin{cases} y_{ref}, & \text{if } y_{ref} < 0.9 \\ 0.9, & \text{others} \end{cases} \tag{11}$$

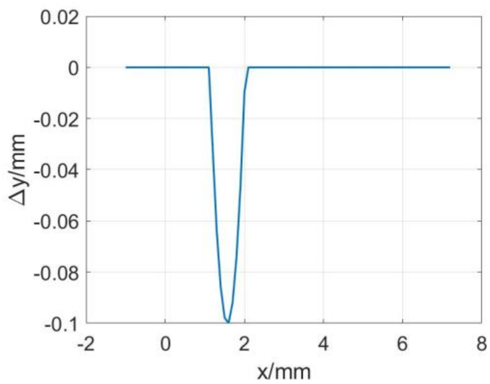
The error profile is determined by subtracting the reference profile from the target profile, as shown in Fig. 2(c). The RMS and peak-to-valley (PV) value of the error profile are 24.1 µm and 100 µm, respectively. The error profile in Fig. 2(c) is the ideal result to aim for.



(a) Reference profile



(b) Target profile



(c) Error profile

Figure 2. Simulated sinusoidal reference profile, target profile and error profile

The target profile was first added with some known transformations and it was then registered back to the reference profile using the ICP method and the anyDOF method. The performance of the two methods can then be compared using the known transformation information. Translations in both *x* and *y* directions with 1 mm were added and the result is shown in Fig. 3. As a result, the ideal transformation to register the target profile with the reference profile is translations in *x* direction for -1 mm and *y* direction for -1 mm, while the rotation should be zero.

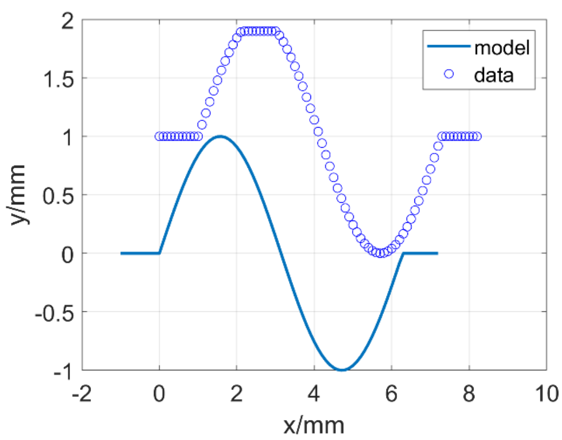
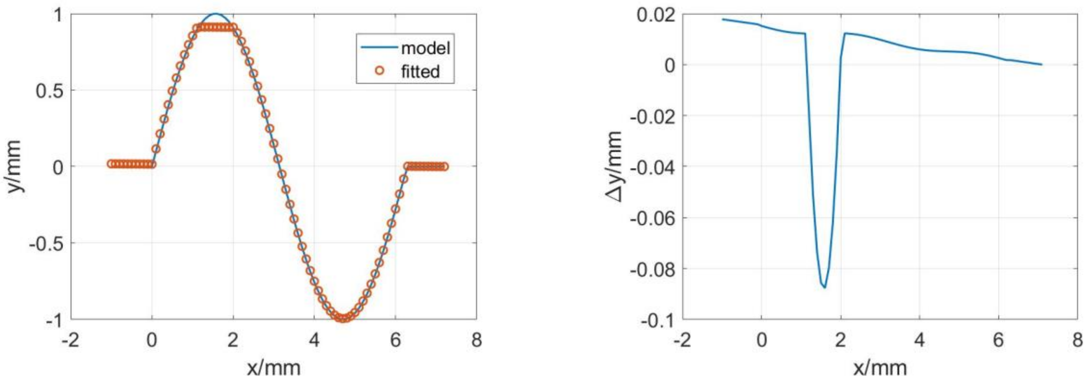


Figure 3. Reference profile and target profile with added transformations

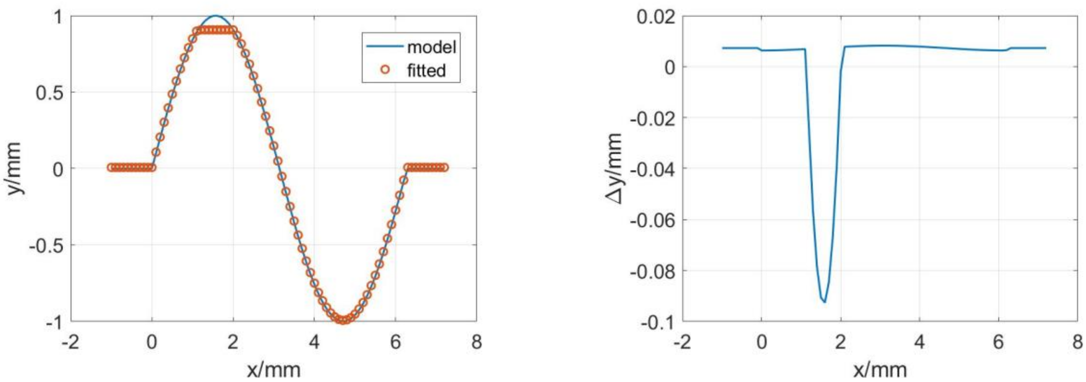
The registration result using the ICP method is shown in Fig. 4. The error profile shown in Fig. 4(b) indicates that a tilting error of approximately 20 μm along the 8 mm long profile was present as a result of the simulated inhomogeneous error. The tilting error was introduced in the registration process, where the ICP method aimed to find the minimum RMS distance for the entire profile. With the ICP method, the registration was performed in three DOF including two translations in the *x* and *y* directions and one rotation in the *x*-*y* plane. The transformation information determined by ICP method is translation in *x* direction for -1.0017 mm, translation in *y* direction for -0.9822 mm and rotation for -0.1259º. The RMS value of the error profile was 22.7 µm and the PV value of the error profile was 105.5 µm.



(a) Registered profile (b) Error profile

Figure 4. Registration results with the ICP method

With the anyDOF method, *a priori* knowledge was utilized to pre-align the target profile using the flat portion in the outer area, which constrained the rotation in the *x*-*y* plane. Hence, only translations in the *x* and *y* directions were part of the optimization problem. Fig. 5 shows the registration result using the anyDOF method. As a result of restricting the rotation of the profile, the registration error shown in Fig. 5(b) was much closer to the ideal registration. The translation distances in the *x* and *y* directions were -1.0009 mm and -0.9927 mm, respectively, and the rotation is 0 since it is a fixed DOF. The results are closer to the ideal case than the ICP method. The RMS value of the error profile was 23.0 µm and the PV value of the error profile was 100.8 µm, which were also closer to the ideal registration errors than those obtained from the ICP method. The results for the ICP method and anyDOF method are summarized in Table 1.



(a) Registered profile (b) Error profile

Figure 5. Registration results with the anyDOF method

Table 1. Summarized results for the ICP method and anyDOF method

	ICP	anyDOF	Ideal
DOF	Full DOF: T_x, T_y , R_z	2 DOF: T_x, T_y	2 DOF: T_x, T_y
Translation in the *x* direction (mm)	-1.0017	-1.0009	-1
Translation in the *y* direction (mm)	-0.9822	-0.9927	-1
Rotation (°)	-0.1259	0	0

The anyDOF method enables more accurate description of the true form error through the capability of pre-alignment using a priori information. In this case, if the error profile is used to correct the G-code, ICP would result in a tilt error in the surface, which is undesirable for various applications in precision engineering. The RMS error obtained using the anyDOF method (23.0 µm) was slightly larger than that obtained using the ICP method (22.7 µm), which was expected as the ICP method is a global optimization method to find the minimum distance from the target surface to the reference surface, while the anyDOF method was limited to one DOF, i.e. the DOF of rotation. Nevertheless, the anyDOF method has been shown to produce a registration result closer to what is deemed by the authors to be ideal.

(b) 3D surface

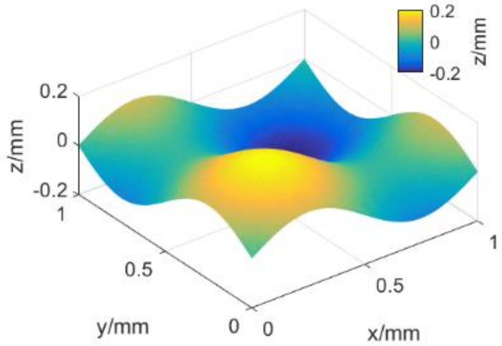
A simulated sinusoidal surface is shown in Fig. 6(a), and can be determined by:

$$z_{ref} = 0.1\left[\sin\left(2\pi x\right) + \sin\left(2\pi y\right)\right] \tag{12}$$

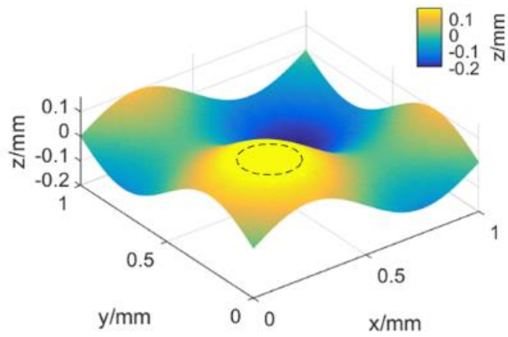
where $x, y \in [0,1]$ mm. The sampling distance was 0.01 mm. Thresholding by surface height was applied to the reference surface to create the target surface, which is shown in Fig. 6(b) and can be determined by:

$$z_{tar} = \begin{cases} z_{ref}, & \text{if } z_{ref} < 0.16 \\ 0.16, & \text{others} \end{cases} \tag{13}$$

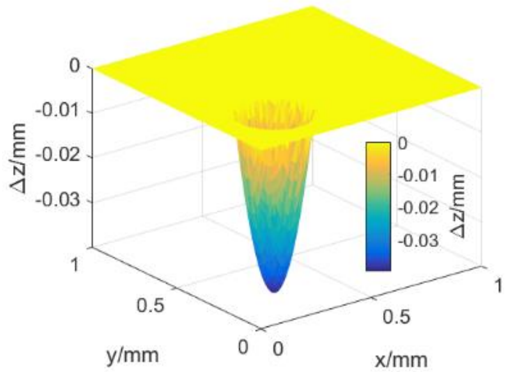
Hence, the error map of the target surface compared to the reference surface could be determined and it is shown in Fig. 6(c). The RMS and PV values of the error map were 5.8 µm and 40.0 µm, respectively.



(a) Reference surface



(b) Target surface



(c) Error map

Figure 6. Simulated sinusoidal reference surface, target surface and error map

The target surface was first added with some known transformations and it was then registered back to the reference surface by the ICP method and the anyDOF method. The performance of the two methods can then be compared using the known transformation information. Translations in *x*, *y* and *z* directions with 0.1 mm, 0.1 mm and 0.5 mm, respectively, were added and the result is shown in Fig. 7. As a result, the ideal transformation value to register the target surface with the reference surface is translations in *x* direction for -0.1 mm, *y* direction for -0.1 mm and *z* direction for -0.5 mm, while the rotations in all directions should be 0.

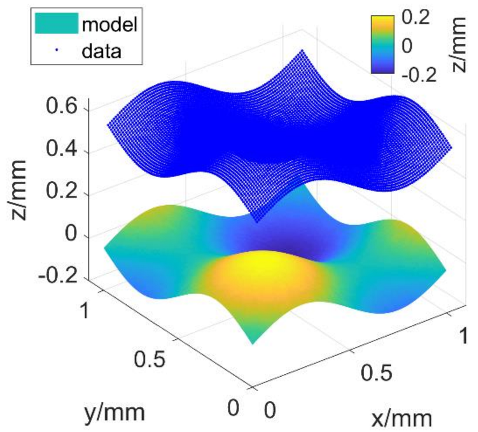
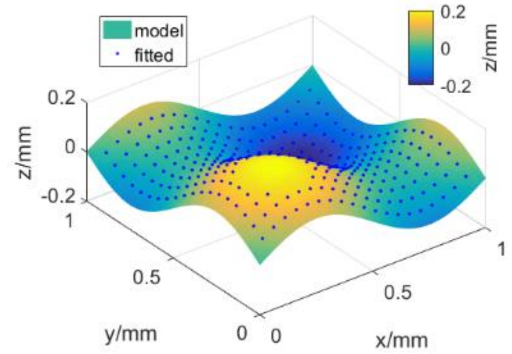
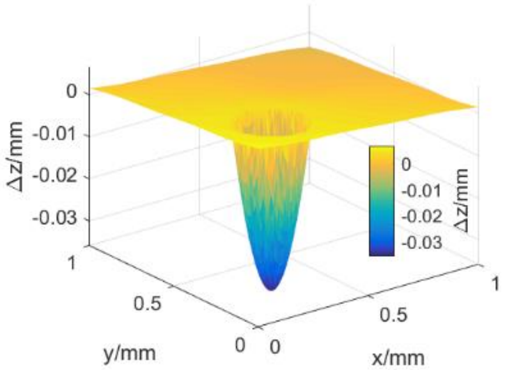


Figure 7. Reference surface and target surface with added transformations

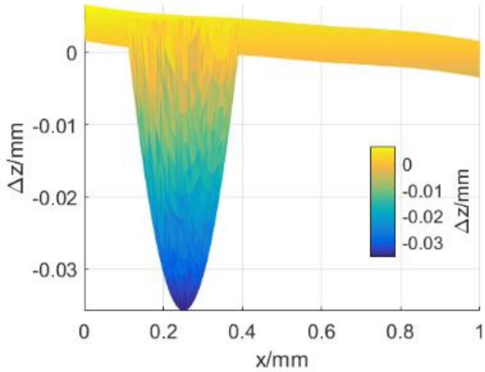
The registration result obtained using the ICP method is shown in Fig. 8. The registered surface was downsampled for better visualization. The error map is shown in Fig. 8(b). Fig. 8(c) shows a viewing angle from which the tilting error is better visualized. Translations in the *x*, *y* and *z* directions were -0.1024 mm, -0.1024 mm, and -0.4923 mm, respectively. Rotations about the *x*, *y*, and *z* axes were -0.2908°, 0.2907°, and -9.2849×10^{-04} °, respectively.



(a) Registration result



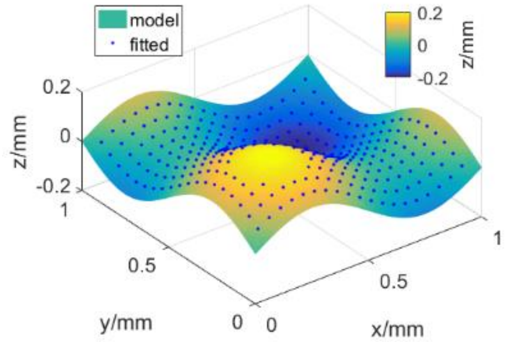
(b) Error map



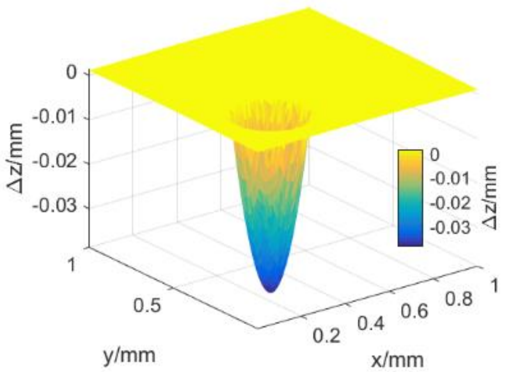
(c) Error map showing tilting

Figure 8. Registration results using the ICP method

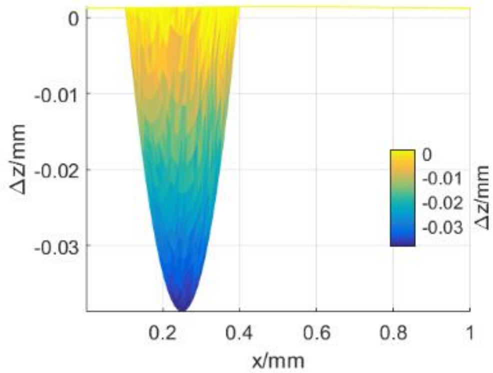
The target surface was then registered to the reference surface using the anyDOF algorithm and the result is shown in Fig. 9. The DOF used in the anyDOF were three translations in *x*, *y* and *z* directions, according to the *a priori* knowledge. Fig. 9(a) shows the registered surface with the reference surface. The registered surface was down sampled for better visualization. The error map is shown in Fig. 9(b) and in Fig. 9(c) is viewed along the *y* axis to demonstrate the lack of tilting in the error map, which was expected since the DOF of rotations were excluded. Translations in the *x*, *y* and *z* directions were -0.0998 mm, -0.0999 mm, and -0.4987 mm, respectively, while rotations in the *x*, *y* and *z* directions were all zeros since the rotations were fixed DOF, which are closer to the ideal case than the ICP method. The results for the ICP method and anyDOF method are summarized in Table 2.



(a) Registration result



(b) Error map



(c) Error map in another angle

Figure 9. Registration results with the anyDOF method

Table 2. Summarized results for the ICP method and anyDOF method

	ICP	anyDOF	Ideal
DOF	Full DOF: T_x, T_y, T_z, R_x, R_y, R_z	3 DOF: T_x, T_y, T_z	3 DOF: T_x, T_y, T_z
Translation in the *x* direction (mm)	-0.1024	-0.0998	-0.1
Translation in the *y* direction (mm)	-0.1024	-0.0999	-0.1
Translation in the *z* direction (mm)	-0.4923	-0.4987	-0.5
Rotation about the *x* axis (°)	-0.2908	0	0
Rotation about the *y* axis (°)	0.2907	0	0
Rotation about the *z* axis (°)	-9.2849×10^{-04}	0	0

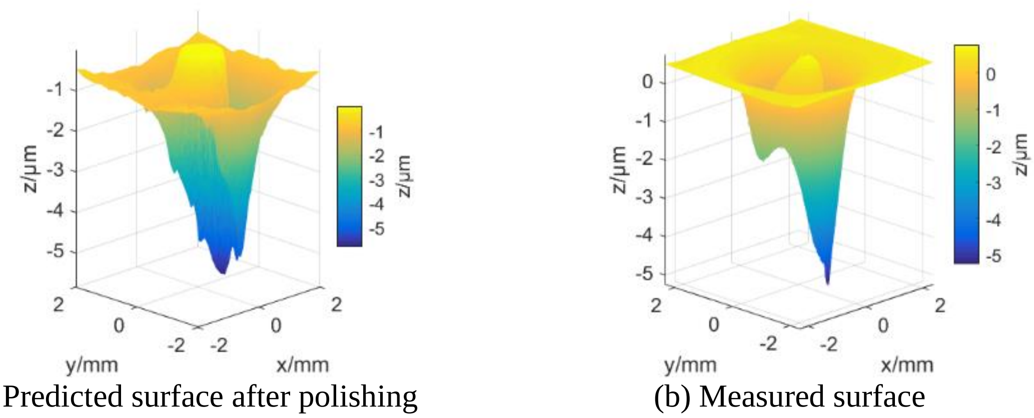
The results show that the anyDOF method has more accurate description of the true form error through the capability of pre-alignment using a priori information. The RMS error obtained using the anyDOF method (5.7 µm) was slightly larger than that obtained using the ICP method (5.5 µm). However, the error map obtained using the anyDOF can better represent the error simulated in the ideal registration. Limiting the DOF according to the *a priori* knowledge using the anyDOF method provided high flexibility and uniqueness compared to the ICP method, which can specify any DOF for the registration process.

3.2 Measurement experiments

Three measurement experiments were conducted to verify the effectiveness of the proposed anyDOF method: a fluid jet polished sample, a bonnet polished sample and a diamond machined sample. Due to the nature of the polishing processes [13, 19], the polished workpieces had significant inhomogeneous deviation to the predicted models. Characterization of the polished workpieces using traditional full-DOF registration methods such as the ICP method resulted in unwanted rotational error [12], while using the proposed anyDOF method could avoid this problem. The diamond machined sample was measured by an optical sensor and there were outliers which affect the registration result. The proposed anyDOF method also demonstrates the improvement over ICP method.

(a) Fluid jet polished sample

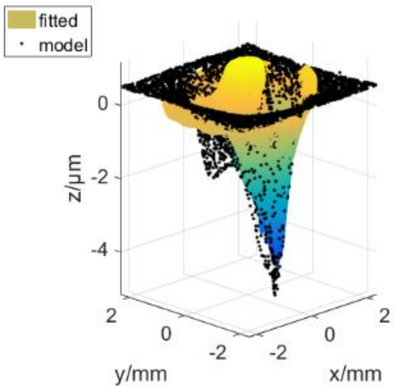
Fluid jet polishing is one of the most promising polishing processes, especially for freeform surface finishing, depending on its unique advantages, such as high adaptability to the freeform surface, no temperature increase of the workpiece, etc. [12]. Modelling of the tool influence function is critical for the modelling of the surface generation during the fluid jet polishing process, to predict the polished surface form. In this experiment, one footprint of fluid jet polishing on a BK7 optical glass surface was conducted using 5 wt.% silicon carbide polishing slurry. The diameter of the nozzle was 1.4 mm. The impinging angle was 75° and the dwell time was 3 minutes. The surface after polishing predicted using a process model [12] is shown in Fig. 10(a), and the actual polished surface, measured using a coherence scanning interferometer (CSI) Zygo Nexview, is shown in Fig. 10(b). It should be noted that the surface was levelled in advance using the unpolished flat surface of the sample.



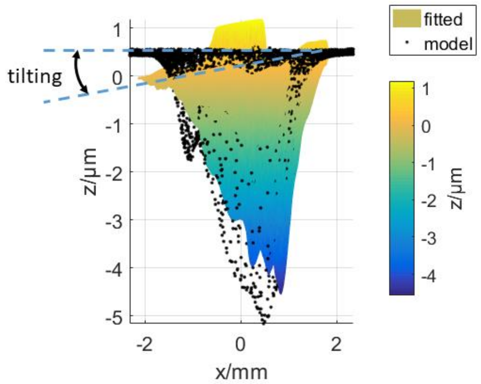
(a) Predicted surface after polishing (b) Measured surface

Figure 10. A fluid jet polishing experiment

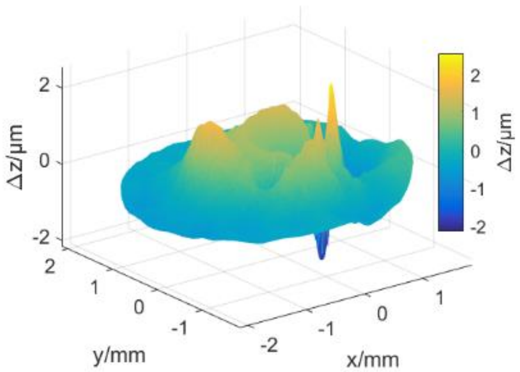
Registration was first performed using the ICP method and the result is shown in Fig. 11. A tilting angle between the target surface and the reference surface was observed in Fig. 11(b). The RMS and PV values of the error map were 0.5550 µm and 4.7013 µm, respectively. Translations in the *x*, *y* and *z* directions were -153.5782 µm, 137.0135 µm, and 1.1019 µm, respectively. Rotations about the *x*, *y*, and *z* axes were 0.0062°, -0.0079°, and -31.3300°, respectively. Tilting errors about the *x* and *y* axes were small but they existed visually, although the surface had already been levelled in advance. The large rotation angle about the *z* axis was due to the fact that there was no pre-alignment process in regard to this axis.



(a) Registration result



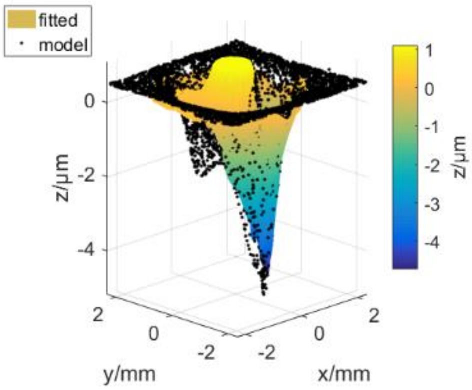
(b) Registration result showing tilting



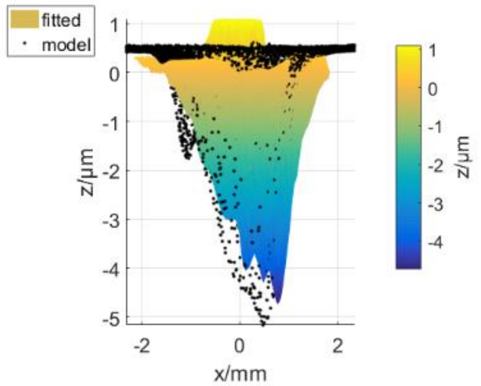
(c) Error map

Figure 11. Registration result using the ICP method

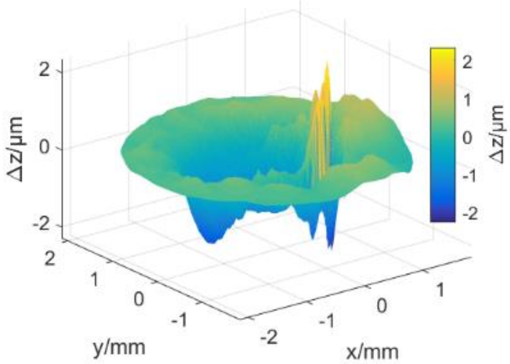
The target surface was also registered using the anyDOF method and the result is shown in Fig. 12. Three translations (in the *x*, *y* and *z* directions) and one rotation about the *z* axis were enabled, as the other two DOF (rotations about the *x* and *y* axes) were removed in advance during pre-alignment, using the unpolished flat surface. The tilting error introduced during ICP registration was successfully avoided, as shown in Fig. 12(b). The error map is shown in Fig. 12(c). Translations in the *x, y* and *z* directions were -160.9646 µm, 139.6536 µm, and 1.1020 µm, respectively. Rotation about the *z* axis was -37.7597°. The results for the ICP method and anyDOF method are summarised in Table 3.



(a) Registration result



(b) Registration result showing no tilting



(c) Error map

Figure 12. Registration result using anyDOF method

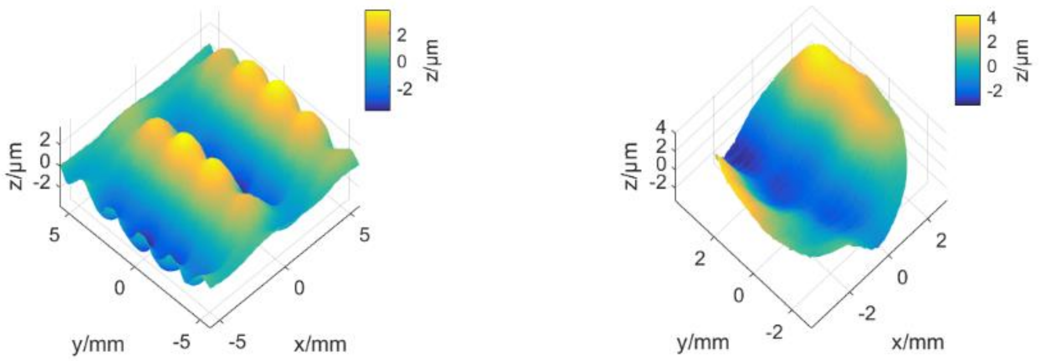
Table 3. Summarized results for the ICP method and anyDOF method

	ICP	anyDOF
DOF	Full DOF: T_x, T_y, T_z, R_x, R_y, R_z	4 DOF: T_x, T_y, T_z, R_z
Translation in the *x* direction (μm)	-153.5782	-160.9646
Translation in the *y* direction (μm)	137.0135	139.6536
Translation in the *z* direction (μm)	1.1019	1.1020
Rotation about the *x* axis (°)	0.0062	-
Rotation about the *y* axis (°)	-0.0079	-
Rotation about the *z* axis (°)	-31.3300	-37.7597

The results obtained with both the ICP method and the anyDOF method showed similar RMS error, PV error, translation distances in all the *x*, *y* and *z* directions and even a rotation angle about the *z* axis, which demonstrated the effectiveness of the proposed anyDOF method. While the ICP method produced slightly lower RMS error, it was achieved at the expense of unwanted tilting error, which cannot correctly represent the characterization result. The result demonstrates the advantage of the proposed anyDOF method.

(b) Bonnet polishing sample

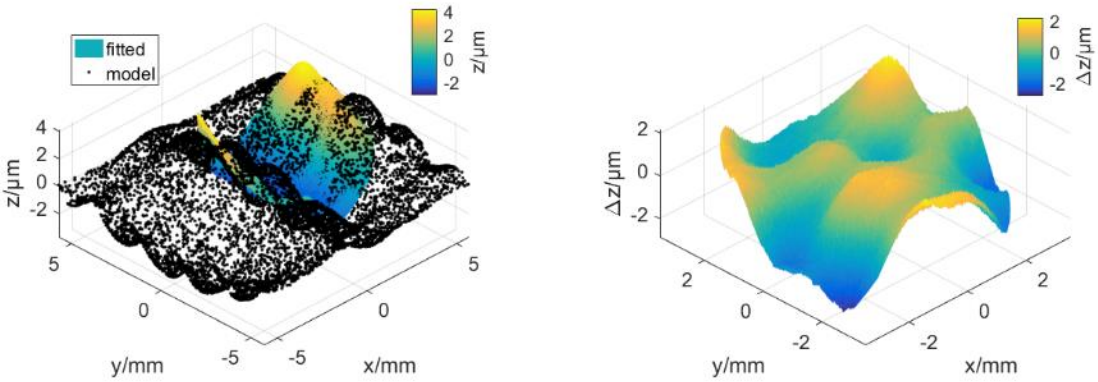
Bonnet polishing is another promising method to achieve ultra-fine surface finishing. The mechanics of the polishing process were studied based on the contact mechanics, kinematics theory, abrasive wear mechanism, as well as the relative and cumulative removal process of surface generation [13]. The polished pattern was compared to the simulated model through registration. In this section, a polishing experiment with the bonnet polishing method was conducted to evaluate the effectiveness of the proposed anyDOF method. The evaluated surface was a bonnet polished surface with the following machining parameters: the tool pressure was 1.2 bar, the spindle speed was 1,500 rpm, the precess angle was 15°, the tool offset was 0.28 mm, the feed rate was 50 mm/min, the tool spacing was 0.6 mm, and the vertical swing speed was 250 degrees per minute. The surface after polishing was predicted using the process model [13] and it is shown in Fig. 13(a). The measured surface using CSI is shown in Fig. 13(b). It is noted that the tilting was removed from the measurement result in advance, using the unpolished flat surface. In this experiment, the predicted surface was regarded as the reference surface.



(a) Predicted surface after polishing (b) Measured surface

Figure 13. A bonnet polishing experiment

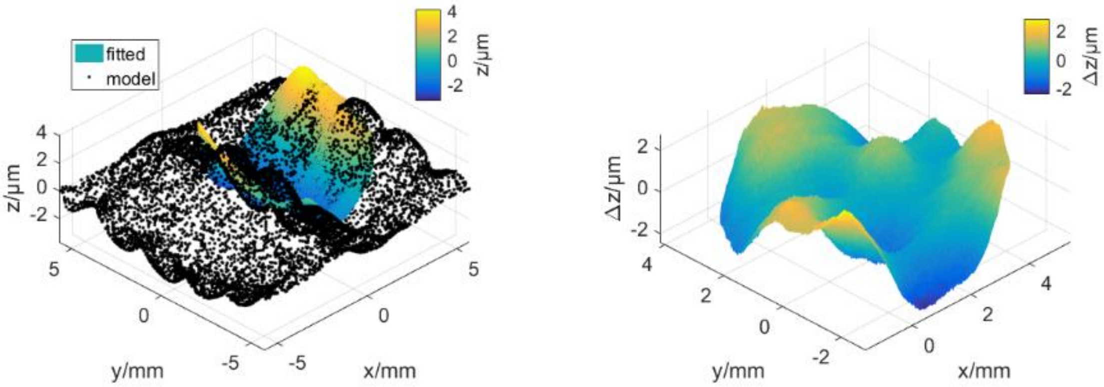
The registration result using the ICP method is shown in Fig. 14, which showed significant deviation from the reference surface due to the complexity of the polishing process. The error map is shown in Fig. 14(b). Translations in the *x*, *y* and *z* directions were 2.2763×10^{3} µm, 83.9525 µm, and -0.08158 µm, respectively. Rotations about the *x*, *y*, and *z* axes were 0.0044°, 0.0029°, and -8.3323°, respectively. The result shows that there were still tilting errors in the rotation about the *x* and *y* axes, although the tilting was removed in advance using the unpolished flat surface of the workpiece.



(a) Registration result (b) Error map

Figure 14. Registration result using the ICP method

The target surface was also registered to the reference using the anyDOF method. Since the tilting of the target surface was removed in advance, the DOF considered in the registration were four DOF including three translations in the *x*, *y* and *z* directions and one rotation about the z axis. The result is shown in Fig. 15. Fig. 15(b) shows the error map. Translations in the *x*, *y* and *z* directions were 2.3089×10^3 μm, 98.4012 μm, and -0.01754 μm, respectively. Rotation about the *z* axis was -7.8028°. The results for the ICP method and anyDOF method are summarized in Table 4.



(a) Registration result (b) Error map

Figure 15. Registration result using the anyDOF method

Table 4. Summarized results for the ICP method and anyDOF method

	ICP	anyDOF
DOF	Full DOF: T_x, T_y, T_z, R_x, R_y, R_z	4 DOF: T_x, T_y, T_z, R_z
Translation in the *x* direction (μm)	2.2763×10^3	2.3089×10^3
Translation in the *y* direction (μm)	83.9525	98.4012

Translation in the *z* direction (μm)	-0.08158	-0.01754
Rotation about the *x* axis (°)	0.0044	-
Rotation about the *y* axis (°)	0.0029	-
Rotation about the *z* axis (°)	-8.3323	-7.8028

The results showed a similarity to those obtained from the fluid jet polishing experiment. The RMS error for the anyDOF method was larger than that for the ICP method. However, the result from the anyDOF provided more confidence in the characterization of the polished surface, according to the *a priori* knowledge of the registration for the specified DOF, i.e., removing tilting using an unpolished flat surface of the workpiece.

(c) Diamond machined freeform surface

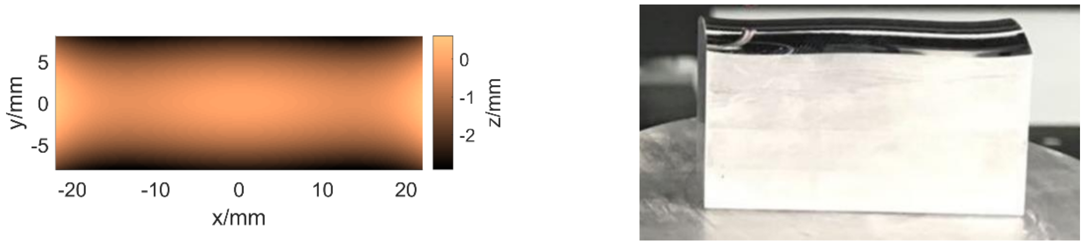
To further demonstrate the merit of the proposed anyDOF method and compare it with the full DOF method such as ICP method quantitatively, an experiment was designed for a diamond machined freeform surface. The measured surface was incorporated with known translations and rotations before conducting registration. The experiment is designed as follows:

1) Perform measurement and obtain original measurement result, which includes some outliers which affect the registration result.
2) Remove the outliers to eliminate their influence.
3) The data without outliers are used to register with the design surface, which can achieve better alignment result without the influence of the outliers. After the transformation information is determined, the data with outlier is transformed to the same position.
4) Introduce some translations and rotations to the data after previous alignment.
5) Register the transformed data back to the design surface using both ICP method and anyDOF method. Hence, the associated translations and rotations are determined.
6) Compare the results between the ICP method and the anyDOF method with the *a priori* knowledge, i.e., the introduced translations and rotations in step 4.

A diamond machined freeform surface was used in this experiment. The surface is an f-theta surface and it can be determined by Eq. (14):

$$z = ax^2 + bx^4 + cy^2 \quad (14)$$

where $a = -1/250$, $b = 1/92000$ and $c = -1/25$, $x \in [-22, 22]$ mm and $y \in [-8, 8]$ mm. Figure 16 shows the design surface and the machined workpiece using a precision diamond fly cutting machine (Precitech Freeform 705G).



(a) Design surface (b) Machined workpiece

Figure 16. Design surface and the machined sample

The workpiece was measured by a multi-sensor CMM machine (Werth VideoCheck UA) using a laser auto-focus probe. The measurement result is shown in Fig. 17. It is found that there are outliers in the measurement result which may be caused by the instrument noise and this influences the registration result, e.g., introducing unwanted rotational errors as the registration process tends to compromise orientation in order to minimize the RMS error.

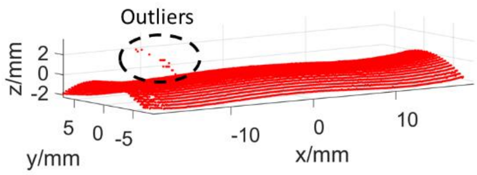
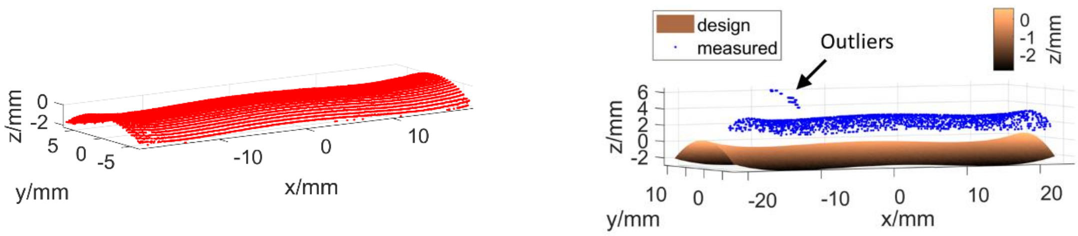


Figure 17. Original measurement result

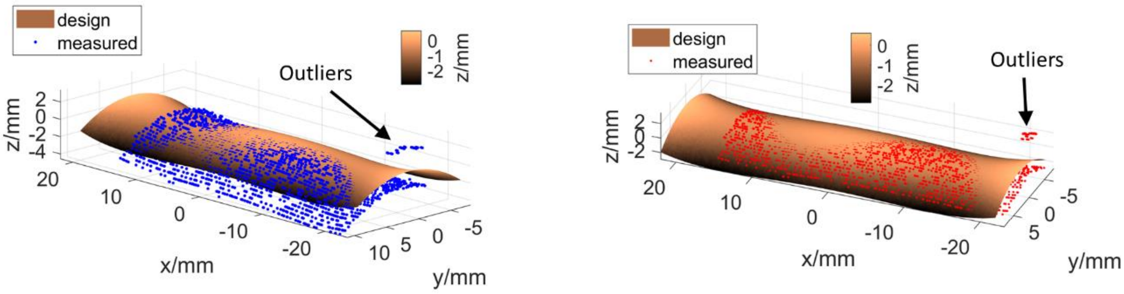
Figure 18(a) shows the result after removing outliers by a statistical method [20]. The result shows that most of the outliers have been removed and the influence of outliers can thus be greatly reduced. The data were then used to register with the design surface and the registration process can be done with full DOF method such as ICP method or anyDOF with all DOF enabled. The process aims to find the initial position for the design surface and the measured data. The original data with outliers was then transformed by the transformation information determined and this is the reference position for the latter performance comparison of the ICP method and the anyDOF method. Translations in *x, y, z* directions are 5 mm, 4 mm, 3 mm, respectively and the rotations around *x, y, z* directions are 0º, 0º, 0º (i.e., no rotations in the experiment), respectively were added to the pre-aligned data in the previous step. The result of the intentionally transformed data is shown in Fig. 18(b). The rotations around *x, y* and *z* directions are designed as 0º to demonstrate DOF constrained by reference features. The known translation and rotation can be used as the target values to evaluate the performance of the registration process achieved.



(a) Removed outliers (b) Added transformations

Figure 18. Measurement result after removing outliers and original data after pre-alignment and added transformations

The data werethen registered by ICP method and anyDOF method. The DOF used in the anyDOF method are translations in *x, y, z* directions. The registration results for ICP and anyDOF are shown in Fig. 19. The differences between them are obvious visually and the results are also summarised in Table 5. The result shows that the outliers introduce large rotation error about the *x*, *y* and *z* axes for -3.7152º, -1.1299º and -15.0191º, respectively, which should be zeros according to the *a priori* knowledge and they can be controlled using the anyDOF method. The translations in *x, y* and *z* axes for anyDOF method are also better than those from ICP method, as compared to the ideal transformation determined from the *a priori* knowledge.



(a) Result of ICP

(b) Result of anyDOF, where rotations around *x* and *y* axes are fixed

Figure 19. Registration results for ICP and anyDOF

Table 5. Summarized results for the ICP method and anyDOF method compared with the purposely added transformation values

	ICP	anyDOF	Ideal
DOF	Full DOF: T_x, T_y, T_z, R_x, R_y, R_z	3 DOF: T_x, T_y, T_z	3 DOF: T_x, T_y, T_z
Translation in the *x* direction (μm)	-9.9379	-9.3901	-5
Translation in the *y* direction (μm)	-0.1225	-3.4151	-4
Translation in the *z* direction (μm)	-2.5974	-3.0569	-3
Rotation about the *x* axis (°)	-3.7152	0	0
Rotation about the *y* axis (°)	-1.1299	0	0
Rotation about the *z* axis (°)	-15.0191	0	0

All experiments including simulated 2D profile, 3D surface and actual measurements demonstrated that the proposed anyDOF method can successfully register a target surface with the reference surface and hence further characterize the target profile/surfaces by calculating the error map. The advantage of the anyDOF method is that it can specify any combination of all or part of six DOF. For a 2D profile, it can be any of the three DOF including two translations and one rotation. For a 3D surface, it can be any of the six DOF. The anyDOF method is particularly useful when the target profile/surface has inhomogeneous deviations which introduce unwanted tilting if registration is performed using all DOF. With the flexibility of the anyDOF method and *a priori* knowledge of the surface, pre-alignment of reference features such as planes or spheres can be performed in advance, and characterization of complex freeform surfaces can be more accurate. Furthermore, the anyDOF method can also utilize all DOF in the registration process when it is deemed needed. Hence, it is a more generalized method which is expected to be able to have wide application in the field of characterization of 2D profiles and 3D surfaces.

4. Conclusion

In this paper, an any-degrees-of-freedom (anyDOF) registration method is presented to provide a more flexible solution for the characterization of freeform surfaces. Unlike the traditional full DOF methods such as the ICP method, the enabled and disabled DOF can be specified in the anyDOF method with any combination of all available DOF. Solving the anyDOF problem is achieved using the Levenberg-Marquardt method, which is a classical optimization procedure. A number of experiments including simulations and actual measurement were conducted and the results demonstrated that the method is effective in providing accurate characterization results when limiting some unwanted DOF. Experimental results also show that, the anyDOF method can performance better than the ICP method according to the a priori knowledge with given known added transformation to the datasets. This method can be used as a generic method and it is particularly useful when *a priori* knowledge of the surface is utilized, e.g. the surface is pre-aligned with reference features such as reference planes or spheres, or the surface is pre-processed such as by removing tilting.

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