Title: **Ultrasonic Guided-Wave Based System Identification for Beams**

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ABSTRACT

Structural health monitoring (SHM) usually requires several stages of information, starting from damage detection, localisation, and identification. Ultrasonic guided waves can travel long distances with relatively low attenuation, which enables them to interact with any potential damage present in the structure. This paper focuses on the use of a novel ultrasonic guided wave propagation model in order to provide both damage localisation and identification. The wave propagation model used here is a state of the art method for transient simulation of ultrasonic guided waves in one dimensional structures both isotropic and anisotropic. This is embedded in a framework for generating excitation signals and capturing scattered signals from damage at any point in the structure. The methodology computes the complete transient response at a fraction of computational cost of full finite element (FE) method. Two kind of damages are modelled: (1) a transverse crack and (2) a delamination in composite beams. To address damage identification and quantification, a model based Bayesian inverse problem is formulated so that both damage scenarios are identifiable. The proposed methodology is exemplified in a beam using the case study of a simulated delamination between two layers. The results show that the proposed framework classify and localise the damage accurately.

INTRODUCTION

Ultrasonic guided waves have been demonstrated to be suitable for SHM in thin structures [1]. They can travel long distances, require minimal equipment and are sensitive to defects which makes them a cost-effective SHM solution [2, 3]. The guided wave propagation characteristics of a structural waveguide contains information about the health of the structure, the presence of potential defects and their locations. In an

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SHM framework, this can be used to evaluate the structure by comparing the signal with a database of possible responses [4]. The damage identification problem can be addressed by the model-based inverse approaches which can reconstruct the response through model updating. Conventional FE simulations are reliable but not practical due to large simulation times which cannot be used in model updating approaches.

Over the years, different methodologies have been developed to overcome this issue, such as the semi-analytical finite element method (SAFE) [5, 6], scaled boundary finite element method (SBFEM) [7], and direct solution of Rayleigh-Lamb wave equations [8]. One such methodology is the hybrid wave and finite element (WFE) method [9], which is used in this paper. It uses a combined analytical and numerical framework to reduce computational complexity. The fast and efficient approach can be integrated within a probabilistic SHM framework for damage identification and localisation. To this end, Bayesian approaches have been applied for damage identification and quantification of composite laminates using ultrasonic through the thickness technique [11], and for damage localisation in plate-like structures using ultrasonic guided waves [10]. In this paper, a framework based on the Bayesian inverse problem (BIP) and the aforementioned wave propagation model is proposed to rigorously identify and localise damage, while quantifying uncertainties stemming from several sources, such as the model parameters and the epistemic uncertainties.

The principal novelty introduced in this paper is the stochastic embedding of a WFE based guided wave simulation model in a hierarchical Bayesian framework for damage identification and localisation. The paper is organised as follows. The WFE model for guided wave simulation is presented in the next section followed by the section on Bayesian approach for damage identification. Then a numerical case study is presented followed by concluding remarks.

GUIDED WAVE SIMULATION MODEL

The transient ultrasonic guided wave simulation model is based on the WFE scheme. It is presented in detail in [12] and will be briefly reviewed here. It is a frequency domain method which uses the divide and conquer strategy. A typical structure with piezoelectric excitation on its surface is shown in Fig. 1. It can be divided into three sections: (i) a coupling section (CS) for arbitrary excitations, (ii) a scatterer containing arbitrary damage and (iii) the rest of the waveguide. The key to a fast and efficient method is to handle each of these sections separately and combining them together in a semi-analytical way using periodic structure theory. The WFE method is used to obtain wave propagation characteristics by modelling a periodic section of the waveguide in FE software. The FE model is used to extract the stiffness ($\mathbf{K}$), mass ($\mathbf{M}$) and damping ($\mathbf{C}$) matrices. These are used to set up the dynamic equilibrium as follows:

$$\mathbf{D}(\omega)\mathbf{q} = \mathbf{f}. \tag{1}$$

Here, $\mathbf{D}(\omega) = \mathbf{K} + \mathrm{i}\omega\mathbf{C} - \omega^2\mathbf{M}$ is the frequency dependent dynamic stiffness matrix, $\mathbf{q}$ is the vector of degrees of freedom and $\mathbf{f}$ is the internal forces vector. These vectors can be internally partitioned into left ($\mathbf{L}$) and right ($\mathbf{R}$) degrees of freedom with respect to direction of wave propagation. According to Bloch's theorem, the free wave propagation

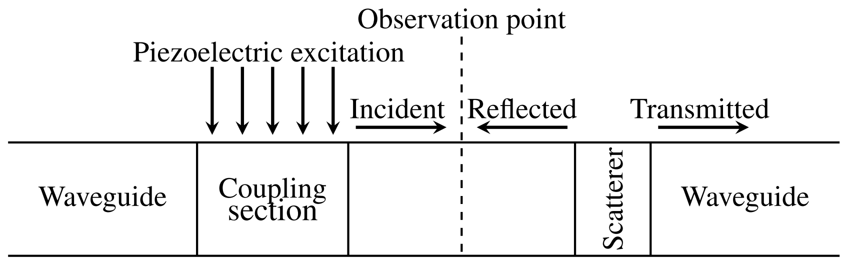


Figure 1: Wave propagation due to external excitation.

in a waveguide of length l_x has the propagation constant $\lambda = \mathrm{e}^{ikl_x}$ which gives $\mathbf{q}_R = \lambda \mathbf{q}_L$ and $\mathbf{f}_R = -\lambda \mathbf{f}_L$. An eigenvalue problem for λ is formulated by substituting this into Eq. (1), that is,

$$\lambda \left\{ \begin{array}{c} \mathbf{q}_L \\ \mathbf{f}_L \end{array} \right\} = \mathbf{T} \left\{ \begin{array}{c} \mathbf{q}_L \\ \mathbf{f}_L \end{array} \right\}, \tag{2}$$

where $\mathbf{T}$ is the transfer matrix and λ are the eigenvalues of $\mathbf{T}$. The propagation constants λ exist in pairs for positive $(+)$ and negative $(-)$ travelling waves. The eigenvectors $\boldsymbol{\phi}$ are the wavemodes as given below:

$$\boldsymbol{\phi} = \left\{ \begin{array}{c} \boldsymbol{\phi}_q \\ \boldsymbol{\phi}_f \end{array} \right\}, \quad \text{where } \boldsymbol{\phi}^+ = \left\{ \begin{array}{c} \boldsymbol{\phi}_q^+ \\ \boldsymbol{\phi}_f^+ \end{array} \right\}, \ \boldsymbol{\phi}^- = \left\{ \begin{array}{c} \boldsymbol{\phi}_q^- \\ \boldsymbol{\phi}_f^- \end{array} \right\}. \tag{3}$$

The wavemodes are used as basis functions to transform the problem into wave domain where the forces and displacements are represented by a linear combination of incoming $(\mathbf{a}^+)$ and outgoing amplitudes $(\mathbf{a}^-)$ as shown below:

$$\mathbf{q} = \boldsymbol{\phi}_q^+ \mathbf{a}^+ + \boldsymbol{\phi}_q^- \mathbf{a}^-, \quad \mathbf{f} = \boldsymbol{\phi}_f^+ \mathbf{a}^+ + \boldsymbol{\phi}_f^- \mathbf{a}^-. \tag{4}$$

The piezoelectric excitation generates outgoing amplitudes $(\mathbf{a}^-)$ into the structure. These amplitudes are obtained by modelling the CS in FE software and extracting matrices to set up the dynamic equilibrium similar to Eq. (1). The interface between the waveguide and the CS must be consistent in order to satisfy the continuity and equilibrium conditions Then the outgoing amplitudes can be obtained as follows:

$$\mathbf{a}^- = \left(\mathbf{D}_{\mathbf{cs}} \mathbf{R} \boldsymbol{\phi}_q^- - \mathbf{R} \boldsymbol{\phi}_f^- \right)^{-1} \mathbf{f}_{\mathbf{cs}}. \tag{5}$$

Here, $\mathbf{D}_{\mathbf{cs}}$ is the dynamic stiffness matrix of the coupling section, $\mathbf{R}$ is the rotation matrix to transform from local degrees of freedom to global and $\mathbf{f}_{\mathbf{cs}}$ is the frequency domain external excitation from the piezoelectric transducer. Then the scattering coefficients for a damage are computed to reconstruct the transient response at the observation point. The procedure follows the same steps of modelling the damaged section in FE software and extracting the matrices. Then applying the continuity and equilibrium at the interface of scatterer and the waveguide gives us the scattering matrix as follows:

$$\mathbf{S} = - \left[\mathbf{R} \boldsymbol{\phi}_f^- - \mathbf{D}_J \mathbf{R} \boldsymbol{\phi}_q^- \right]^{-1} \left[\mathbf{R} \boldsymbol{\phi}_f^+ - \mathbf{D}_J \mathbf{R} \boldsymbol{\phi}_q^+ \right]. \tag{6}$$

Here, $\mathbf{D}_J$ is the dynamic stiffness matrix of the scatterer and $\mathbf{S}$ is the scattering matrix. The scattered amplitudes $(\mathbf{a}_\mathrm{s})$ can be obtained from incident amplitudes $(\mathbf{a}_\mathrm{i})$

by $\mathbf{a}_\mathrm{s} = \mathbf{S}\mathbf{a}_\mathrm{i}$. Then the transient response is obtained by summing up all amplitudes reaching the selected observation point in the desired time window and expanding them over the individual degrees of freedom using Eq. (4) and performing an inverse discrete fourier transform.

BAYESIAN APPROACH FOR DAMAGE IDENTIFICATION

Bayesian model class assessment [13] is used for rigorous and robust damage identification and localisation. To this end, a set of different hypotheses or models $\mathbf{M} = \{\mathcal{M}_1, \mathcal{M}_2\}$ are classified, i.e., considering either the crack ($\mathcal{M}_1$) or the delamination ($\mathcal{M}_2$) as damage scenarios within the structural element. Let us also define the ultrasonic guided wave data $\mathbf{q}_\mathcal{D}$, obtained experimentally, and the output of each model $\mathbf{q}_\mathcal{M}$. A probabilistic version of the ultrasonic models may be obtained by adding an error term e to the modelled output, as follows:

$$\mathbf{q}_\mathcal{D} = \mathbf{q}_\mathcal{M} + e \tag{7}$$

Using the principle of Maximum Information Entropy [13, 14], a zero mean Gaussian distribution with covariance σ_e as $\mathscr{N}(0, \sigma_e)$ is adopted to model the error term in order to produce the largest uncertainty. Thus, the stochastic version of the model is given by a Gaussian distribution, as:

$$p\left(\mathbf{q}_\mathcal{D} | \mathbf{q}_M, \boldsymbol{\theta}, \mathcal{M}_j\right) = \left(2\pi\sigma_e^2\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2\sigma_e^2}\mathcal{J}(\boldsymbol{\theta}, \mathcal{D})\right) \tag{8}$$

where $\mathcal{J}(\boldsymbol{\theta}, \mathcal{D})$ is the goodness-of-fit function which is selected to be the L_2 norm of the measured and modelled data. As part of each model class $\mathcal{M}$, the prior distribution of the model parameters $p(\boldsymbol{\theta})$ can be defined. Next, the posterior distribution of the model parameters $p\left(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M}_j\right)$, given the data and a specific model class can be defined by applying the Bayes' Theorem, as follows:

$$p\left(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M}_j\right) = \frac{p\left(\mathcal{D}|\boldsymbol{\theta}, \mathcal{M}_j\right) p(\boldsymbol{\theta})}{p(\mathcal{D}|\mathcal{M}_j)} \tag{9}$$

where $p\left(\mathcal{D}|\boldsymbol{\theta}, \mathcal{M}_j\right)$ denotes the likelihood function explicitly expressed in Equation (8) and $p(\mathcal{D}|\mathcal{M}_j)$ is the evidence of the model class $\mathcal{M}_j$ in representing the data $\mathcal{D}$. Note that the computation of Equation (9) requires addressing multidimensional integrals, which usually do not have an analytical expression. Thus, the Metropolis-Hastings algorithm [15, 16] is adopted here as Markov chain Monte Carlo (MCMC) method to obtain samples from the posterior distribution, $p\left(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M}_j\right)$. The evidence term $p(\mathcal{D}|\mathcal{M}_j)$ is calculated in this paper by using samples from the posterior distribution, as provided in [17]. Finally, the model classes are ranked by using their posterior plausibilities obtained though Bayes' Theorem, as follows:

$$P\left(\mathcal{M}_j|\mathcal{D}, \mathbf{M}\right) = \frac{p\left(\mathcal{D}|\mathcal{M}_j\right) P\left(\mathcal{M}_j|\mathbf{M}\right)}{\Sigma_{l=1}^{N_m} p\left(D|\mathcal{M}_l\right) P\left(\mathcal{M}_l|\mathbf{M}\right)}. \tag{10}$$

Note that the posterior plausibilities are dependent on the evidence terms calculated for each model class. Therefore, both damage identification and model parameters inference are addressed by this hierarchical Bayesian approach.

NUMERICAL EXAMPLE

In this section, a numerical case study is presented of a composite beam with a delamination type damage present in it. The beam has a height of 3mm and width of 2mm. The delamination has a length of 5mm and located between the second and third layer. The reference signal is generated by performing FE simulations for a delamination type damage in Abaqus. The layup under consideration is a 6 layered carbon fibre beam with $[0_2/90_2/0_2]$ stacking sequence. A single carbon fibre layer has a density of 1560 kg/m^3 and the stiffness matrix as shown below:

$$C = \begin{bmatrix} 143.8 & 6.2 & 6.2 & 0 & 0 & 0 \\ 6.2 & 13.3 & 6.5 & 0 & 0 & 0 \\ 6.2 & 6.5 & 13.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.7 \end{bmatrix} GPa. \tag{11}$$

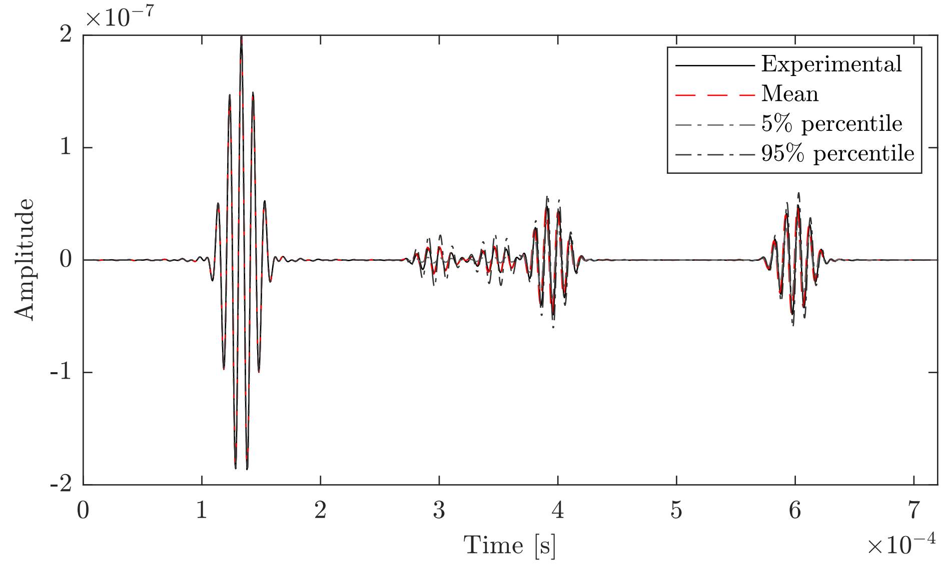
The uncertain parameters are the reflection coefficients and damage location. The antisymmetric (A_0) mode is selected for damage identification as it can detect both crack and delamination [18]. A Hanning windowed sinusoid at 100 kHz is applied as the excitation signal at one end of the beam. The in-plane response is observed at 0.2m from the point of excitation for a time window of $720\mu s$. The beam is assumed to be long enough such that the reflections from the far end of the beam do not reach the observation point in the selected time window. The delamination is modelled by node duplication in both the FE simulation and the WFE model.

We will identify the damage between a crack and a delamination, hence the set of model classes M consists of a model class for crack and another one for delamination, with equal prior probability. The posterior PDF is obtained through Metropolis Hasting (M-H) algorithm with $T_s = 400000$ and a Gaussian proposal distribution. The standard deviation of M-H random walk is selected such that the acceptance rate r lies in the interval $[0.2, 0.4]$ [19,20]. The evidence of each model class is then computed and used for model class assessment. The resulting posterior probabilities from Eq. (10) determine the most plausible model class as shown in Table I. The delamination model class has the higher probability, which should be the case considering the synthetic data is from the beam with delamination damage.

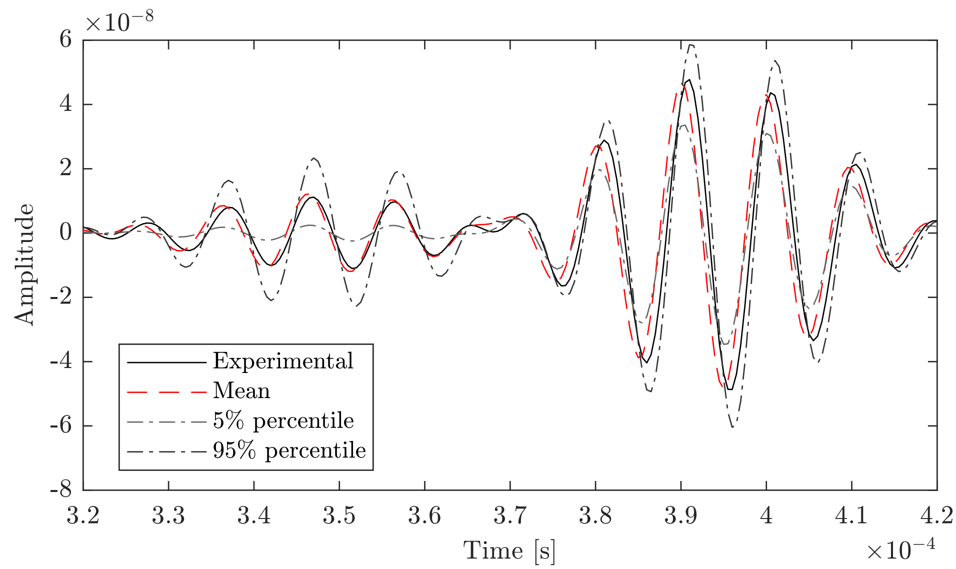
The model parameters $\boldsymbol{\theta}$ estimation is simultaneously performed. The prior information for all model parameters is defined as a uniform distribution with the scattering coefficients going from 0-no damage to 1-maximum damage. The location of damage can be from 0.3m to 0.7m. The posterior PDF for $\boldsymbol{\theta}$ is obtained from the MCMC algorithm with a proposed Gaussian distribution. The resulting reconstructed signal is shown in Fig. 2 along with the synthetic data. The approach successfully reconstructs the data

TABLE I: BAYESIAN MODEL CLASS SELECTION RESULT.

Model class	Log-likelih.	Expected Inf. Gain	Log-evidence	Probability [%]
Crack	-1.5669	5.5424	-7.1093	15.98
Delamination	-0.7357	4.7135	-5.4493	**84.02**



(a) Complete signal.



(b) Zoom of the signal in (a).

Figure 2: Comparison between the experimental signal and the reconstruction using the mean of the model parameters and the 5 and 95% percentiles.

as well as provides the confidence intervals. The mean value of the damage position parameter is correctly inferred as 0.4494m which is remarkably similar to the location of damage 0.45m in measured data, i.e., only a 0.13% of deviation.

CONCLUDING REMARKS

A comprehensive damage identification and localisation approach is presented in this paper. A WFE method based guided wave simulation model is used which is capable of handling complex damage scenarios in both isotropic and anisotropic structures. The model is incorporated in a hierarchical Bayesian framework designed for damage classification and model parameters inference. A case study using a multi-layered composite beam has also been presented which demonstrates the ability of the approach to identify damage type with sufficient degree of confidence and also obtain the location with a very high degree of accuracy.

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