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Abstract

Providing rail replacement bus services is a common strategy applied to handle track blockage situations in railway networks. Previously, a great deal of research has focused on modelling this strategy, particularly in the case of unplanned disruptions. However, little attention has been paid to planned disruptions where passengers know the situation in advance and the duration of the disruption is significantly longer. In this study, we propose a model that can be used to investigate the optimal solution of implementing a bus replacement service to minimise the impact of infrastructure possessions. The model is developed based on a discrete event simulation technique and uses a Genetic Algorithm to minimise passenger delays and the cost of operations. The interaction between trains and buses is taken into account. Thus, the passenger flow within the network can be simulated in microscopic detail. Finally, an application of the proposed model is presented using the Liverpool railway network in UK.
Notation

**RRBS** is the rail replacement bus service

**FEL** is the future event list

Clock is the variable representing the simulation time in the model (second unit)

**Is** is the mitigation model indicator

**d** is the disruption ID

**TD** is the departure time of a train at a station

**TA** is the arrival time of a train at a station

**Tt** is the turnaround of a train at a station

**TR** is the time that the train will be ready for the next service

**Tdw** is the dwell time of a train at a station

**Tad** is the allowable delay time of trains in the network

**Tos** is the original time of the first train service at the station

**Tst** is the time to start the first bus service

**To-d** is the travel time from the origin station to the destination station on the road

**Tbw** is the dwell time of a bus at a station

**f** is the bus service frequency

**NB** is the number of buses required for each service

**Ea** is the event when a train is arriving at a station

**Es** is the event when a train is stopping at a station

**Ed** is the event when a train is departing at a station
$E_{ds}$ is the event when replacement bus operations are deployed

$E_{st}$ is the event when replacement buses begin services at a terminal station

$E_{bs}$ is the event when a bus stops at a station along its route

$N_{Si}$ is the number of platforms available of station $i$

$S_{Si}$ is the number of sidings at station $i$

$TN_{Si}$ is the total number of platforms available at station $i$

$di$ is the bus service direction

$BR$ is the bus route ID

$n_{bpk}$ is the number of buses used in peak hours

$n_{b,opk}$ is the number of buses used in off-peak hours

$n_{b,nt}$ is the number of buses used in night hours

$du_{pk}$ is the rental duration (peak hours)

$du_{opk}$ is the rental duration (off-peak hours)

$du_{nt}$ is the rental duration (night hours)

$n_s$ is the number of bus services

$dt$ is the route distance

$u_{bc}$ is the bus rental cost

$u_{fc}$ is the bus fuel cost

$N_m$ is the maximum number of bus routes in each combination
Introduction

Railways are a vital part of the transport systems so essential to modern society. The railway assets thus require regular enhancement, maintenance and renewal to ensure long-term safety and reliability of train operations. However, conducting these engineering works will, at times, requires possession of the railway. This means some parts of the network might need to be closed, and the original timetable may be affected (Van Aken et al., 2017).

During possessions, train operators often provide a solution to reduce the impact on passengers. The solution is normally based on two strategies: short-turning train services on a disrupted route and providing rail replacement bus services at stations where trains cannot be reached. These strategies seem to be effective to enhance the connectivity of a railway network during disruption. However, they need to be planned carefully due to the nature of replacement operations such as higher number of interchanges and longer travel time of buses (Railfuture, 2016). In practice, the possible solutions during a possession are designed based on the experiences of senior traffic controllers (Ghaemi et al., 2017). Although this ad-hoc solution might be applicable to reduce the impact of a disruption, it is unlikely to be the optimal solution for a railway network (Gu et al., 2018).

This paper aims to develop a model for investigating the optimal solution to manage a railway network during possessions. Two strategies: short-turning and rail replacement bus services are considered, and the key performance indicators minimised are passenger delays and the cost of bus replacement operations. The outcomes of the model will be useful for train operators to operate both trains and buses during planned disruptions.
This paper is organised as follows: Section 2 presents a review of relevant literature. Section 3 describes the proposed modelling framework. Section 4 demonstrates the application of the proposed model on the case study. Finally, section 5 gives a conclusion of the paper.

Rail replacement bus service modelling

In recent years, there has been an increasing amount of literature on rail replacement bus service (RRBS) or ‘bus bridging’ modelling. Various studies have focused on investigating the optimum bus bridging solution to alleviate unplanned disruptions. For example, Kepaptsoglou and Karlaftis (2009) proposed a network flow-based method using a shortest path algorithm to design bus bridging routes during a metro disruption. The optimal option of bridging routes was found using a Genetic Algorithm, with the objective to minimise total unsatisfied demand and total travel time for all bridging routes. A RRBS model was also developed by De-Los-Santos et al. (2012) to evaluate the efficiency of bus bridging solutions. The passenger flow-based model was constructed, and the total travel time of all passengers was defined as the indicator.

Another study by Jin et al. (2014) formulated a Mixed Integer Program to design bus bridging routes in case of a metro service disruption. The study focused on the modification of the existing local bus services and attempted to integrate the new bus services to increase the resilience of a metro network. The indicator proposed was a fulfilled passenger demand after the implementation of different bus bridging strategies.

Jin et al. (2015) adapted the modelling framework by Kepaptsoglou and Karlaftis (2009). However, this study presented a column generation algorithm to generate candidate bus routes.
and developed an integrated optimisation algorithm to perform the route selection. The main indicator of this study was the average delays of all passengers.

Recently, Gu et al. (2018) introduced a Integer Linear Programming model to simulate a new bus bridging pattern. Buses could flexibly serve different routes, namely local and express routes. The local route is the normal operation of buses to connect all disrupted stations, while the express routes provide direct connections between two disrupted stations. Buses on each route were operated on a loop operation without a given frequency. The indicators estimated were bus bridging time and total passenger delays.

In addition to the works on unplanned disruptions, Hurk et al. (2016) presented a Mixed Integer Programming model for designing a temporary bus services. The model was constructed using the path reduction concept and simplified by assuming that the demand and service frequencies are constant (i.e. not dependant on the times of the day). The indicators of the model were: travel cost, service frequency, waiting time of passengers and operating cost. Christoforou et al. (2016) applied a traffic assignment model, called Capacitated Transit Assignment (CapTA), to investigate different disruption management schemes for planned disruptions. This model is based on a mesoscopic approach. The passenger’s behaviour during disruptions and vehicle characteristics were considered as microscopic levels, while passenger flow and service operations are taken into account in macroscopic detail. The indicator proposed was the generalised cost of passengers. This was a function of passenger travel time, waiting time and comfort state (i.e. seated or standing).
It is obvious that the main aim of a RRBS model is to optimise temporary bus operations for connecting the affected parts of a railway network. Many studies have focused on unplanned disruptions. Literature on modelling this strategy for planned disruptions remains scarce. Moreover, most of the studies only attempt to design a temporary bus network by assuming the operation of a railway network during disruptions. As such, the interaction between the train and bus operations is not considered, and the impact of passengers using both systems during a disruption is not taken into the design of bus services. Therefore, there is a need to construct a model that takes train and bus operations, and passenger behaviour during a disruption into account. This will enable the indicators from the perspective of passengers to be predicted and used to compute the optimal solution for the whole system. The focus of this study will be mainly on addressing these research gaps. A new RRBS model is introduced as explained in the next section.

Modelling framework

Rail replacement bus services are normally deployed together with the short-turning operation of the trains on the disrupted routes. To find the optimal solution, this study attempts to construct a RRBS model based on a railway network simulation model (Meesit and Andrews, 2018). This model was developed using a stochastic-discrete event simulation technique. Its framework consists of three main modules: railway network modelling, passenger modelling and disruption scenario modelling. The first module simulates the operation of a railway network at a microscopic level. Its framework requires three sub-modules working together: infrastructure module, control system module and operational module. The infrastructure module creates the
railway network infrastructure assets, such as tracks, stations and junctions, as static entities. Then, the control system module and the operational module generate train movement events (dynamic entities) within the network based on the signalling rules and timetables. The results of the first module are the detailed schedule of train arrivals and departures at each station, in both normal and disruptive situations. These are then used as passenger information in the next module.

The second module imitates passengers using the train services in the network. Passengers are modelled as dynamic entities, and three significant activities are taken into account: arriving at a station, searching for routes and alighting/boarding a train. Passenger arrivals at a station are simulated by a Poisson process. Then, they are distributed based on an origin-destination matrix to each destination station. After that, the route selection process begins. This process searches for possible routes to the destinations (vector of interchange and destination stations) and selects the best option in terms of the shortest travel time for passengers. Once the first two processes are completed, the alighting and boarding functions are then used to transfer passenger entities between trains and stations. Passengers board a train if the first destination in their route vector matches the train calling stations. Meanwhile, passengers alight a train if the station stop is their interchange station or destination. Passengers complete their journeys when they arrive their final destinations, where statistic results (e.g. delay) are collected.

Finally, the last module is the disruption modelling. It is used to simulate disruption scenarios and imitate passenger behaviour during disruptions. The disruption scenarios are simulated by setting the occurrence time and the impact duration of a disruption and changing
the state of network components (e.g. track sections) to ‘unavailable’. Passenger behaviour during disruptions is related to three main tasks: using a disrupted timetable, reconsidering routes after disruption and cancelling rail journeys if no route to the destination is found or the expected travel time is longer than acceptable (i.e. assumed to follow a normal distribution).

Although the model by Meesit and Andrews (2018) can be applied to predict the system performance (e.g. passenger delays and passenger journey cancellations) of a railway network during disruptions, it does not take into account mitigation strategies implemented to reduce the impact of disruptions. Therefore, this paper extends the capability of the railway simulation model by introducing the mitigation model for short-turning operations and railway replacement bus services. This model is developed using a stochastic-discrete event simulation technique, which can be activated by changing the binary variable $I_s$ to 1 (Figure 1). Then, the key performance indicators (passenger delay and operating cost) can be predicted and used in a multi-objective Genetic Algorithm to investigate the optimal solutions of the mitigation strategies. The description of each part of the model is given in the following sections.

Short-turning operation modelling

Problem description

The short turning of train services is a mitigation option that can be applied by maintaining train services on a part or parts of the original route that is/are not impacted by the disruption. Trains can still run to stations close to the disruption location. Then, they are turned around to replace the service that is planned to operate in the opposite direction of their routes (Figures 2(a) and (b)). This strategy can be implemented to solve both unplanned and planned disruptions.
However, only planned disruptions, such as the possession of track on a Sunday, is the focus of this paper. Therefore, the transition from the original timetable to the disrupted timetable and vice versa, as in the case of unplanned disruptions, is not considered in the model. This is because it is assumed that all trains on the disrupted route will be operated based on the disrupted timetable from the beginning of the day until the end of a possession, which is normally the end of the operation.

Model description

To simulate short-turning operations, the first step is to obtain the new input data related to short-turning stations and the nearest stations to the disruption. Short-turning stations are the intermediate stations on the disrupted route used for short-turning, while the nearest stations to the disruption are the stations closed to the disruption, which can be either different or the same from/to the short-turning stations, depending on the decision of train operators. These data are collected in terms of station IDs ($i$) based on disruption IDs ($d$) in two separated 2D-vectors: short-turning stations vector ($STT$) and the nearest stations to the disruption vector ($NSD$), as shown in Equations 1 and 2.

$$STT = \{(i_0,i_1, i_1, i_0, i_1, i_1)\}$$

$$NSD = \{(i_0,i_1, i_1, i_0, i_1, i_1)\}$$

The second step is to propose two functions to the railway network simulation model, which are: starting train services and turning trains at short-turning stations. The explanation of each function is given as follows.
Starting train services

In the case of a possession, the short-turning operation might take place from the beginning of the day. Starting train services at the same location as in the case of normal operation may thus not be effective because it could lead to the situation where there is an insufficient number of trains to circulate in short-turned routes. This study attempts to propose the algorithm to reallocate trains to each short-turned route. This algorithm firstly allocates trains to their starting stations of the disrupted routes as the normal situation. However, instead of assigning the departure time to each train, the algorithm checks whether it is feasible to begin the train services at the current stations. This checking process is done by iterating through the train calling station vector and comparing the position of the first short-turning station ($P_{stn}$) with the first-nearest stations to the disruption ($P_{nsd}$) found. If $P_{nsd} \geq P_{stn}$, the trains can be started as planned. Otherwise, there is no short-turning station on the section related to the current station. Thus, the trains are moved to a spare train vector waiting for a new assignment.

The new assignment process then compares the departure times ($T_D$) to the first arrival time ($T_A$) of trains at short-turning stations. For example (Figure 3), considering that Station B is the short-turning station, if $T_{D0} < T_{A0} + \text{Turnaround time} (T_t)$, the service $T_{D0}$ cannot be run using the arrival train $T_{A0}$. Thus, a train from the spare train vector (e.g. Train 1) is called to take the service $T_{D0}$ at Station B. After this step, the next departure time ($T_{D1}$) is considered. In this case, since $T_{D1} \geq T_{A0} + T_t$, the process is thus terminated because Service $T_{D1}$ can be operated using the arrival train $T_{A0}$. Otherwise, another spare train will be called to take the service $T_{D1}$, and the process is repeated by considering the next departure time.
Turning trains at intermediate stations

Turning trains at intermediate stations is modelled using three events occurring at stations: arriving, stopping and departing (Figure 4). The arriving event \( (E_a) \) is an event when a train arrives at a station. A train can only stop at or run through a station if a platform is available. Otherwise, it must wait until a platform is free. The number of platforms available \( (N_{S_i}) \) is thus used to define the state of a station. However, it is assumed that operators can also use available sidings \( (S_{S_i}) \) to hold and reorder trains at turning stations. Thus, the total number of platforms available \( (TN_{S_i}) \) at a turning station is \( N_{S_i} + S_{S_i} \).

The next event represents an event when a train enters the turning station and stops at a platform. This event causes \( TN_{S_i} \) to be reduced by one and the passengers alighting function to be called. The passengers alighting function simulates passengers alighting a train and estimates the dwell time \( (T_{dw}) \) at the station. Then, the time that the train will be ready for the next service \( (T_R) \) is calculated by \( T_A + T_{dw} + T_i \) and compared with the departure time of the next service \( (T_D) \). If \( T_R \leq T_D \), the train will be scheduled to depart the turning station at \( T_D \). If \( T_R > T_D \) but \( T_R \leq T_D + \text{allowable delay} (T_{ad}) \), the train will be authorised to depart as soon as it is ready (at \( T_R \)). However, if \( T_R > T_D + T_{ad} \), the service at departure time \( T_D \) is cancelled (deleted). Then, the next \( T_D \) is considered, and the process is repeated until a suitable \( T_D \) is found. It is noted that all scheduled events in the simulation are stored in the future event list (FEL) where the events are sorted in chronological order and wait for execution.

Finally, the departure event refers to an event when the train is going to leave the station. The calling stations of the train are set, and the passenger boarding function is then called to
transfer passenger entities from the station vector to the train vector based on the calling stations.

The number of passengers boarding the train is limited by the available capacity of the train.

After this step, the next event (the train enters the next block section) is created and put into the FEL as the normal procedure of the simulation model.

**Rail replacement bus service modelling**

**Problem description**

In the case of planned disruptions, the train operators normally provide temporary bus services in parallel to the closed sections of a network. Buses are operated based on a given frequency, and they are scheduled to stop at every intermediate station to reconnect stations impacted by the closure (Figure 5(a)). Even though this standard replacement route seems to be useful to enhance the connectivity of the railway network, it may not be the optimal solution for all situations, especially when:

i) Some stations within the disrupted area have a large number of trip productions/attractions, and the major destinations/origins of these trips are the stations outside the disrupted area;

ii) A travel demand between the non-disrupted areas is high; thus, most passengers want to travel passing through the disrupted area as fast as possible.

To this end, train operators could introduce other bus service routes to reduce the impact of a disruption (Figure 5(b)). The new routes should be easy to implement, and their operating costs must be at a minimum. A goal of this study is to provide a tool to enable train operators to investigate the optimal sets of temporary bus service routes. This section presents a model that
can be applied to simulate the operation of bus services using multiple routes. The operating
details of each route are considered, and the service patterns in the different periods of the day
are also taken into account as described below.

Model description

Simulating bus replacement operations

Similar to the short-turning operation model, two steps are conducted to simulate the bus
replacement operations. The first step acquires input data to give the bus replacement strategy,
road distance and bus travel time between stations in the network. The bus replacement
strategy is the information used to operate the bus services. This information consists of bus
routes (a list of station stops) and their operating details: elapsed time to start the first service
\(T_{st}\) (based on the original departure time of the first train at the particular station, \(T_{os}\)) and
frequencies \(f\) and the number of buses required for each service \(NB\) during peak \(op\),
off-peak \(opk\) and night period \(nt\). The second set of data provides the shortest distance route
and the travel time of buses between stations. This data is collected in the form of an
Origin-Destination matrix and used to simulate the buses running along these routes. The
second step is to introduce three new events to the railway network simulation model:
deploying bus operations, starting bus services on each route and bus stopping a station. These
events are explained below.

The first event \(E_{ds}\) happens after the occurrence of a disruption. It enables all bus
replacement routes planned for a disruption to be deployed in the simulation. As depicted in
Figure 6, the bus replacement routes are deployed one by one. In each route, the information,
such as station stops and interchange stations, is collected for passengers, and a new event called “starting bus services (\(E_{st}\))” is generated to trigger the first bus service at each terminal station of the route. The occurrence time of \(E_{st}\) is set to \(T_{or} + T_{st}\). Once all bus routes are deployed, the route searching function is then called, this accounts for the new available routes and updates the passenger routes in the system.

The second event (\(E_{st}\)) starts a bus service at the terminal station. Once it happens, the calling stations of the bus are set based on direction of the service (\(di\)). If \(di = 0\), the calling stations will follow the route vector (e.g. A-B-C). Otherwise, it will be the return route (e.g. C-B-A). It is noted that the value of \(di\) can be obtained by checking the position of the current station in the route vector (\(P_{cs}\)). If \(P_{cs} = 0\), \(di = 0\). However, if \(P_{cs} = \) the last position in the route vector, \(di = 1\). At the next step, the frequency of the route and the number of buses required to operate for the service are obtained from the input data based on the time of the day. The frequency data is used to start the next service of the route at the current terminal station. This is done by generating the same event (\(E_{st}\)) but changing the occurrence time to Clock (i.e. current time in the model) + \(f\) and placing in the FEL. Meanwhile, the number of buses required is applied in the bus assignment process (Figure 7). This process checks the number of buses available in the queue at the station based on the disruption ID (\(d\)), route ID (\(BR\)) and direction (\(di\)). If the queue is empty, the new buses (dynamic objects like trains) are generated based on the number of buses required. However, if the queue is not empty, the buses in the queue will be used for the service, and the number of buses in the queue is reduced by one. After assigning buses, the total number of buses for the route is updated based on the time of the day. Then, the
passenger boarding function is called, and the new event “E₁₈” is created by setting its occurrence time to Clock + the travel time from the current station to the next station (t₀₋d). This t₀₋d can be obtained from the travel time O-D matrix. During off-peak and night periods, it is assumed that buses can run according to the plan. Thus, the t₀₋d from the O-D matrix can be used directly. However, during peak periods, the uncertainty of the bus travel time is taken into account. Thus, the increment of the travel time between stations during this period can be random based on a Uniform distribution (assumed to increase up to 50%).

The last event (E₁₉) represents an event when a bus stops at a station. The function of this event (Figure 8) is to firstly investigate whether the bus is at an intermediate station or a terminal station. The position of the current station in the route vector (P₁₈) is checked. If P₁₈ = 0 or the last position in the route vector, the bus is at one of the route terminal stations. The bus will then be pushed back into the queue, and only the passenger alighting function is called in order to transfer passenger entities from the bus vector to the station vector. However, if the above condition is false, the bus is at an intermediate station. Thus, the calling stations of the bus need to be updated based on its direction. At this step, both the passenger alighting and boarding functions are called, and the dwell time of the bus at the station (t₁₈) is estimated. Then, the next event (E₁₉), the bus stops at the next station, will be generated. The occurrence time of this event is equal to Clock + t₁₈ + t₀₋d.

Imitating passenger behaviour during planned disruptions

For planned disruptions, the disrupted timetable can be announced to passengers several days before the disruption occurrence. Thus, passengers can plan their journeys in advance. Some
passengers might change to other transport modes such as local buses or taxis, but most of the passengers (60-80%) still choose the modified rail services in the case of planned disruptions (Shires et al., 2018). This demand pattern is considered, and the model still simulates passenger arrivals to the network as normal. However, passengers involved with bus replacement services can choose whether to stay with the railways or change to other transport modes based on the probability given in each choice. The probability of passengers changing to other modes of transport is set to 0.3. Thus, if the passengers decide to travel by other transport modes, the travel time of passengers will be random based on the expected travel time of their journeys which is assumed to follow a Uniform distribution on the interval of -50% and +50%.

**Optimising bus replacement operations using a multi-objective Genetic Algorithm**

A Genetic Algorithm is applied to investigate a Pareto set of optimal bus replacement operations. This metaheuristic is based on the concept of natural evaluation (Holland, 1975). Its process begins with creating an initial set of solutions (population). Then, the fitness value of each solution is evaluated based on the objective functions. After that the selection process chooses some solutions according to their fitness values to survive in the next generation. Additional solutions are then generated by mating some of the fittest solutions (crossover), and some variables of the new solutions obtained are randomly changed to ensure a diversity of solutions (mutation). At this step, the new population is ready to be evaluated, and the process is repeated until the solutions converge (e.g. the fitness of each solution is similar or has not changed for several generations). This study will adapt this process to analyse a trade-off between two objective functions. The procedure of this model is described below.
Objective functions

Two objective functions are used to minimise the total delay to passengers and the cost of the bus replacement operations. The total passenger delay is calculated based on the summation of the difference between the actual arrival time \( at_p \) and the expected arrival time \( et_p \) of each passenger \( p \) at their destination station (Equation 3). The bus replacement operating cost is estimated based on the summation of the operating costs for each service route. The operating cost for each route is a function of bus hire cost and fuel cost as shown in Equation 4.

\[
\text{min} \sum_p (at_p - et_p) \quad (3)
\]

\[
\text{min} \sum_p \left[ \left( (n_{b,pk})(du_{pk}) + (n_{b,opk})(du_{opk}) + (n_{b,m})(du_m) \right) \times u_{bc} + (n_r)(dr)(u_r) \right] \quad (4)
\]

Finding the sets of candidate bus replacement routes

As described in section 0, implementing other bus routes together with the standard route might be beneficial to improve the performance of a railway network during a disruption. Ideally, all potential bus routes to all stations within a network could be considered. However, the size of the problem will then be massive, and it will be very time consuming to calculate all options, some of which would be unnecessary routes. Therefore, a method is used to generate candidate bus routes that can provide a potential good connection between stations on the closed section and the important stations outside the disrupted area. The method requires three items of input data: all disrupted stations, intermediate-disrupted stations with high passenger numbers and the nearest important stations (e.g. interchanges or attractive stations), from both
short-turning stations, that are not on the disrupted route. These data are used to create candidate bus routes as follows (see Figure 4(b)):

i) The standard route connecting all stations in the disrupted part of the network (e.g. A-B-C-D-E);

ii) The direct route between short-turning stations (e.g. route R2 (A-E)) or a short-turning station and the terminal station (A-C, in Figure 4(a));

iii) The direct routes between short-turning stations and the intermediate-disrupted station that has a high number of passenger users. For example, it is assumed that station C is the station that has the highest number of users. Then, the routes provided will be from C to short-turning stations, like C-A and C-E;

iv) The route connecting short-turning stations to all high usage stations (e.g. A-C-E);

v) The routes connecting short-turning stations, all high usage stations and the nearest important stations. For example, if there is an interchange station (F) near station A, the bus route will be F-A-C-E.

After obtaining candidate routes, the maximum number of routes in each combination ($N_m$) are set, and the process of route combination is started. This process attempts to find all possible combinations of all candidate routes under the conditions that all disrupted stations must be accessible from other stations, and the number of routes in each combination must be less than or equal to $N_m$. Then, the combinations found for each disruption, $BP_d$, (i.e. vector of bus route IDs ($BR_r$) based on the combination IDs ($C$)) are collected in 3D vector ($BS$) and used as the strategy input to the optimisation model (Equations 5 and 6).
Coding and initialising the model variables

The set of bus replacement routes and their operating variables for each disruption are optimised simultaneously. As presented in Figure 9, a 2D-vector is created to contain a sub-strategy for each disruption. This sub-strategy is a set of candidate bus routes indicated by the combination ID described in section 0. The combination ID is recorded at the first position of the sub-strategy vectors. Then, the candidate route objects are created and stored at the following positions of the vector to carry the operating variables of each route in this combination set, which are: \( BR, T_p \) and \( f \) and \( NB \) during peak, off-peak and night period. It is noted that the operating variables of all strategies are randomly initialised for the first population based on the range of the input data. They will be used in the rail replacement bus service model to simulate the bus replacement operations.

Ranking and selecting strategies for the next generation

The non-dominated sorting process by Deb et al. (2002) is applied to rank each strategy in the population. The process starts by checking each strategy to establish if it is dominated by the other strategies. A strategy is dominated by another strategy if all of its fitness values represent a worst solution for all corresponding objective functions. For example, Figure 10(a), if the objective functions are to minimise both \( F_1 \) and \( F_2 \), then \( S_6 \) is dominated by \( S_3 \) because both conditions: \( F_1(S6) > F_1(S3) \) and \( F_2(S6) > F_2(S3) \) are true. However, if we compare \( S_3 \) to \( S_5 \),
only one condition: \( F_2(S5) > F_2(S3) \) is true. Thus, S3 and S5 are not strictly dominated by each other. In this example, the first set of non-dominated strategies is \{S1, S2, S3, S4 and S5\}, and the boundary that they form is called “the optimal-Pareto frontier or 1st rank”. After the Pareto rank is obtained, the strategies in the rank are removed from the list, and the process is repeated to find the other Pareto ranks from the rest of the strategies. Once all Pareto ranks are found, the next step is to sort the strategies in each Pareto rank. This is done by comparing their crowding-distances (\( CD \)). This crowding-distance is defined as the average distance from a strategy \( (i) \) to its neighbouring strategies (Figure 10(b) and Equation 7). The strategy with larger crowding-distances are preferred over other strategies in the same Pareto rank in order to maintain the diversity in the solutions. Finally, the last step is to select the best strategies that will survive to the next generation. In this study, the selection rate is set to 0.5. Therefore, half of the strategies in the population will be chosen according to their Pareto ranks and the crowding-distances’ ordered within the Pareto rank.

\[
CD = \frac{F_1(i-1) - F_1(i+1)}{F_{1(max)} - F_{1(min)}} + \frac{F_2(i+1) - F_2(i-1)}{F_{2(max)} - F_{2(min)}} \tag{7}
\]

Creating the new population

The best strategies selected from the previous iteration are applied to create the new population. The pair of these strategies are randomly made as parents. Then, the crossover process is conducted using a uniform crossover method to obtain two new strategies as the children of the parents. The uniform crossover method produces a random number (0 or 1) for each operating variable in the route object. If the random number is equal to 1, the first parent gives its variable
to the second child. Similarly, the second parent also gives its variable to the first child. Otherwise, the parents will give the variables to each child based on their IDs. It is noted that if the number of bus routes is not the same in each parent, the parent with the lower number of bus routes will be considered as a basis for crossover as depicted in Figure 11. After obtaining the new population, the mutation process is started to prevent the solution becoming stuck in a local minimum. The variables in the child strategy vectors are randomly changed based on their input data, and the number of changes is dependent on the mutation rate given in the model. Once the mutation process is completed, the fitness of each strategy in the new population will be evaluated, then the overall process described is repeated until the results have converged, where most of the strategies in the population are the Pareto optimal solutions.

**Case study and results**

*The Liverpool railway network*

The Liverpool railway network is used to demonstrate the application of the proposed model. This network serves more than 100,000 passengers on an average weekday. The structure of the network comprises 67 stations and 72 links (double track), and the total length of this network is about 120 km (Figure 12). For train operations, seven service routes are operated daily from 6:00 to 24:00. These service routes include three common routes: Southport to Hunts Cross (R0), Ormskirk to Liverpool Central (R1) and Kirkby to Liverpool Central (R2), and four loop routes from four terminal stations: Ellesmere Port (R3), Chester (R4), West Kirkby (R5) and New Brighton (R6), via the Liverpool Central station. The trains on each
route are the British rail class 507/508 (3 coaches), and they are scheduled to provide services at every intermediate station along their routes.

**Input data and parameters setting**

The data required to simulate the Liverpool railway network, such as operating data (e.g. timetables and train characteristics) and passenger data (e.g. passenger arrival rate at each station and passenger O-D matrix), was obtained from Meesit et al. (2019). Meanwhile, the data and parameters used for optimising bus replacement operations were set as follows. The shortest distance and the travel time between stations were acquired from the car-driving option in Google map (2018). Then, the bus operating variables were given as the range of values based on the experience of train operators. These variables were: the elapsed time to start the first bus service \{0, 1, 2…10 minutes\}, the number of buses per service \{1, 2 buses\} and frequency \{5, 6, 7… 30 minutes\}. Moreover, the bus hire cost and fuel consumption rate (80 seats-buses) were set according to the bus hire quote suggested by the bus hiring company. These two variables were £80 per hour and £0.412 per km (estimated from 3.4 km/litre and £1.40 per litre), respectively. Finally, the full list of GA parameters used in the simulation was inputted into the model as depicted in Table 1.

**Experimental scenarios**

Two scenarios were considered: single and multiple possessions. These scenarios were assumed to be taken place on a Sunday, and their durations were set to 24 hours, affecting the
network from the beginning of the day until the end of the operation. The detail of each scenario and its optimal solutions are illustrated in the following sections.

In this study, the model was constructed in C++ 11 environment with Microsoft Visual Studio 2015. The computational experiments were performed using a computer with a dual core Intel i3 processor CPU 3.50 GHz and 8 GB of RAM running on Window 7, 64-bit. With regard to the stochastic behaviour of the model, the average results from each objective function were calculated from the results of 500 simulations, where the statistics sufficiently converged and used in the optimisation model.

It is noted that, due to the limited availability of data logged during an actual bus replacement service operation, the validation of the model was accomplished through a systematic process of examining the rules which govern the treatment of each event in the simulation. Then, the model structure was checked with the industrialists running the operation that each event has been treated correctly.

Scenario 1: Single possession

The first scenario was related to a possession on Route 0, between three stations: Cressington (ID21), Liverpool South Parkway (ID42) and Hunts Cross station (ID34). To mitigate this situation, Liverpool Central station (ID39) was used as the short-turning station, and the result from the simulation is presented in Figure 13. After the short-turning strategy was applied, the train services between Southport (ID58) and Liverpool Central station can be operated based on the original timetable. However, to obtain this result, the model suggested that one of the trains planned to start the service at Hunts Cross station needs to be reallocated to take the first
service at Liverpool Central toward Southport station. Thus, the adequate number of trains can be balanced against the number of services on the short-turned route.

For the optimal bus replacement operations, the results of the simulation are presented in Figure 14. The Pareto optimal solutions were obtained at generation 50. The overall Pareto front shows the trade-off between the operating cost and the total passenger delays in the network (Figure 14(b)). Increasing the operating cost led to a reduction in the passenger delays. However, there was only a little reduction in the passenger delays once the operating cost is more than £40,000. This implies that providing more bus services beyond this point will not be effective because it causes the supply greater than the demand in the network. Thus, the solutions located in this part of the Pareto front are not recommend.

Consequently, any solutions in the other part of the Pareto front can be selected to implement during this possession. However, to select the suitable solution, the acceptable budget and available resources in the system might need to be considered. For example (Figure 14(b) and Table 2), if the acceptable budget for this possession is £40,000, it is possible to implement solution 1 to reduce the passenger impact to a minimum. However, if there are only 20 buses available on the possession day, Solution 1 is not the suitable option anymore because it requires 32 buses to operate during peak hours. Thus, Solution 2 might be the most suitable option in this example. Even though this solution leads to a reduction of 13% performance compared to the first solution, it needs only 19 buses to run on two routes: BR1 and BR2 during peak hours (Figure 15).
Scenario 2: Multiple possessions

The second scenario is more complicated than the first scenario. It was assumed that two possessions occur at the same time in the network. The first possession takes place between Hall Road (ID28) and Seaforth & Litherland station (ID57). Meanwhile, the second possession is between Leasowe (ID37) and Meols station (ID45). Two service routes: Route 0 and 5, were disrupted due to these possessions. Therefore, to mitigate this scenario, three stations were selected as the short-turning stations: Hall Road (ID28), Bootle New Strand (ID13) and Bidston station (ID7). The first two stations were used for Route 0, and the last station was applied for Route 5. The time-distance graph for the short-turning operations on these two routes are shown in Figure 16 and 17. It is apparent that the trains on the non-disrupted parts can run according to the original timetable. Route 0 was split into two short-turned routes. One provided services between Hall Road and Southport station (R0.1), and another one connected all stations between Bootle New Strand and Hunts Cross station (R0.2). R0.1 required four trains to start services at Southport station, while R0.2 needs two trains at Bootle New Strand and other three trains at Hunts Cross station to deliver services as in the original timetable. For Route 5, the track section between West Kirkby (ID66) and Bidston station (ID7) was disrupted. Thus, three trains that used to be at West Kirkby (ID66) were reallocated to begin the services at Bidston station and continued running as the loop operation via Liverpool Central station (ID39).

For the optimal bus replacement operations, Figure 18(b) presents the Pareto optimal solutions of this scenario at generation 50. The total passenger delays still decreased as the cost
of the operations increased as explained in the first scenario. Thus, train operators can select any optimal solutions to implement in this scenario based on the budgets and resources that they have. For example, if the operator would like to select solution “A”, the operators might need to have approximately £35,000 and 32 buses in order to provide services on four bus routes during peak hours (Figure 19 and Table 3).

**Conclusion**

During possessions, it is essential to maintain services within a railway network and keep the impact on passenger to a minimum. Unlike most of the studies in literature that only focused on the bus replacement services, this study proposes a new mitigation model that can be applied to simulate the short-turning operations of rail traffic and investigate the optimal solutions for bus replacement services simultaneously. The model is developed using a stochastic-discrete event simulation technique. The interaction between trains and buses is considered, and the passenger flow within the network is imitated in microscopic detail. In this way, the impact on passengers traveling on both modes (railways and buses) can be predicted and used to find the optimal solutions for the whole system. The model then applies a multi-objective Genetic Algorithm to optimise the results. Two main objectives considered are to minimise total passengers delays and the cost of bus replacement operations. The outcomes of the model can thus be used to support a decision-making process of infrastructure managers and train operators.

For the application of the proposed model, the study selected the Liverpool railway network as a case study. Two scenarios: single and multiple possessions were tested, and the
results from each scenario illustrated that the proposed model is capable of providing the significant information for operating trains and rail replacement buses to mitigate the impact of possessions. Even though the computational time of the case study seem to be quite long (8 hrs), however, it is sufficient to apply for planning a possession in advance.

In the future, this model will be further developed to include the modelling of other transport systems such as road networks to cover the overall logistic problem. Then, the capabilities for multi-short-turning stations and unplanned disruptions will be considered. The transition from the original timetable to the disrupted timetable and vice versa will be taken into account, and the computational performance of the model will be improved to apply for real-time disruption management.

Acknowledgement

Ratthaphong Meesit obtains a PhD scholarship from the Royal Thai Government to conduct this research at the University of Nottingham. Professor John Andrews is the Royal Academy of Engineering and Network Rail Chair in Infrastructure Asset Management. Both authors would like to appreciatively acknowledge these organisations.
References


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Table 1. Genetic Algorithm parameters set in the model

<table>
<thead>
<tr>
<th>GA parameters</th>
<th>value</th>
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<tr>
<td>Maximum number of route combinations for each disruption</td>
<td>3</td>
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<tr>
<td>Population size</td>
<td>40</td>
</tr>
<tr>
<td>The number of parents (selection rate)</td>
<td>20 (0.5)</td>
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<tr>
<td>Crossover probability</td>
<td>0.5</td>
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<tr>
<td>Mutation probability</td>
<td>0.01</td>
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<tr>
<td>Number of generations</td>
<td>50</td>
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Table 2. Bus operating detail of the chosen solutions (Scenario 1)

<table>
<thead>
<tr>
<th>No.</th>
<th>BR</th>
<th>$T_w$ (mins)</th>
<th>$f_{pk}$ (mins)</th>
<th>$f_{opk}$ (mins)</th>
<th>$f_{nt}$ (mins)</th>
<th>NBpk</th>
<th>NBopk</th>
<th>NBnt</th>
<th>npk</th>
<th>nopk</th>
<th>nnt</th>
<th>Delays $(\text{mins} \times 10^5)$</th>
<th>Cost $(\text{£} \times 10^5)$</th>
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<tbody>
<tr>
<td>1</td>
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<td>8</td>
<td>17</td>
<td>28</td>
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<td>1</td>
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<td>24</td>
<td>8</td>
<td>6</td>
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<td>3.67</td>
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<td>30</td>
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<td>1</td>
<td>1</td>
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<td>30</td>
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<td>1</td>
<td>1</td>
<td>9</td>
<td>6</td>
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<td>2.63</td>
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Table 3. Bus operating detail of the chosen solution (Scenario 2)

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<th>( T_M ) (mins)</th>
<th>( f_{pk} ) (mins)</th>
<th>( f_{opk} ) (mins)</th>
<th>( f_{nt} ) (mins)</th>
<th>( N_{Bpk} )</th>
<th>( N_{Bopk} )</th>
<th>( N_{nt} )</th>
<th>Delays (mins ( \times 10^5 ))</th>
<th>Cost (£( \times 10^4 ))</th>
</tr>
</thead>
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<td>6</td>
<td>10</td>
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<td>6</td>
</tr>
<tr>
<td></td>
<td>BR2</td>
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<td>13</td>
<td>21</td>
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<td>1</td>
<td>6</td>
<td>4</td>
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<td>24</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>12</td>
<td>5</td>
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<tr>
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<td>26</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
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Figure 1. Modelling framework
Figure 2. Short-turning services on a part (a) or both parts (b) of the original route
Figure 3. Reallocating trains to the short-turning station
Figure 4. Short-turning trains at stations algorithm
**Figure 5.** Bus replacement services, connecting to a short-turning station (a), both short-turning stations (b)
Figure 6. Event (E_{ds}): Deploying bus service route algorithm
Figure 7. Event (Eₐ): Starting bus services at route terminal station algorithm

- Check the direction (dᵢ) to operate (toward right (0) or left (1) hand side of route vector);
- Get the calling stations of the service based on the direction obtained.

- Get the operational variables, the number of buses per service (NB) and frequency (f), based on the time of the day;
- The duration of peak, off-peak and night can be set in the model;

- Create the start bus service event Eₛₐ based on the route frequency;
- The occurrence time = Clock + f;
- Push the event to FEL.

- Create a counter variable (b);
- And, set b = 0.

If b < NB?

Yes

- If the bus queue[d][BR][dᵢ] is empty?
- Generate a new bus for operation;
- Update the number of buses in the system (based on the time of operation).

No

- Use the bus in the queue to run the service;
- Delete one bus in the queue;

No

End
Figure 8. Event ($E_{bs}$): A bus stops at a station algorithm

**Event: A bus stops at a station ($E_{bs}$)**

- Get the information of bus running event
  - Get the bus ID
  - Get bus route ID ($BR_r$)
  - Get the disruption ID ($d$)
  - Get the current station ID
  - Get the previous station ID

---

If the bus is at the terminal station?

- No
  - Get the calling stations of the bus;
  - Passenger alighting function is called;
  - Passenger boarding function is called;
  - Estimate the dwell time of the bus ($t_{bw}$).

- Yes
  - Push the bus back to the queue at the terminal station;
  - Passenger alighting function is called in order to transfer passenger entities from the bus vector to the station vector.

---

- Create the next event $E_{bs}$;
- Its occurrence time = Clock + $t_{bw}$ + travel time from the current station to the next station ($t_{v-q}$);
- Push the event to FEL
Figure 9. Example of the population of bus replacement strategies based on the disruption IDs.
Figure 10. Example of Pareto ranking (a) and crowding distance calculation (b)
Figure 11. Example of the crossover process
Figure 12. Liverpool railway network
Figure 13. Time-distance graph for the short-turning operation on Route 0 (Scenario 1)
Figure 14. Optimal solutions of the bus replacement operations, Generation 1(a) and Generation 50 (b) (Scenario 1)
Figure 15. Optimal bus routes of the chosen solutions (Scenario 1)
Figure 16. Time-distance graph for the short-turning operation on Route 0 (Scenario 2)
Figure 17. Time-distance graph for the short-turning operation on Route 5 (Scenario 2)
Figure 18. Optimal solutions of the bus replacement operations, Generation 1(a) and Generation 50 (b) (Scenario 2)
Figure 19. Optimal bus routes of the chosen solution on Route 0 (a) and Route 5 (b) (Scenario 2)