A fully coupled computational framework for fluid pressurised crack evolution in porous media

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Abstract

This paper presents a computational framework for modelling hydraulic fracture on the basis of combining continuum porous media and damage theories. By considering the continuum as two separate domains of damaged and intact porous domains, model components are isolated and considered separately. This simplifies the whole modelling approach. The mathematical model used consists of a set of coupled partial differential equations in continuum space that govern compressible flow in damaged and intact porous media, mechanical deformation of the domains and damage evolution. We particularly focus on the flow of fluid within the intact and damaged porous zones. The porous domain typically has a lower permeability than the fractured zone, therefore a more complicated flow of fluid is expected within the damage zone. To model the exchange of fluid in the interface of damage zone and intact porous domain, a double permeability concept has been utilized. The evolution of cracks is modelled using Francfort and Marigo's variational theory which approximates the fracture by a diffusive damage zone using a phase field variable. The governing model equations are discretised and solved using finite element method. The framework capabilities are verified using experimental data from a one-dimensional consolidation test and a plane stress pressured penny crack benchmark example. The framework performance highlights its capabilities in analysing hydraulic driven fracture process and the associated permeability variations.

Keywords: Hydraulic fracture, dual porous media, geoenergy, continuum damage modelling

Nomenclature

Symbol	Description	Unit
b_i	Strain	ND
$\mathbf{B}_i$	Shape function derivatives	ND
c	Compressibility of fractured region	Pa^{-1}
c_p	Compressibility of porous region	Pa^{-1}
c_f	Coefficient of compressibility for fluid	Pa^{-1}
C_v	Coefficient of consolidation	mS^{-1}
$\mathbf{C}$	Coupling matrix	Pa

$\mathbf{C}_{ijkl}$	Compliance matrix	Pa
dE	Bulk energy	J
E	Young's modulus	Nm^{-2}
f	Deformation	m
F	Deformation Gradient	ND
$\dot{\mathbf{F}}$	Mechanical force derivative	ms^{-1}
G	Fracture energy	J
$\mathbf{g}$	Gravity	ms^{-2}
H	Drainage height	m
$\mathbf{H}_i$	Flow matrix	Pa
$\mathcal{H}^{N-1}$	Hausdroff measure	ND
i	Region designation (i=1 for porous, i=2 for fractured)	ND
$\mathbf{Ic}$	Trace of left Cauchy strain tensor	ND
J	Volume change	ND
$\mathbf{k}_i$	Permeability	m^2
K	Fractured domain	ND
$\mathbf{K}$	Stiffness matrix	Pa
m_v	Modulus of volume change	Pa^{-1}
M_v	Numerical term	ND
$\mathbf{M}$	Mass matrix	Pa
$\mathbf{N}$	Shape function	ND
p_i	Fluid Pressure	Pa
q_c	Surcharge	Nm^{-2}
$\mathbf{Q}_i$	Boundary flow matrix	Pa
$\mathbf{R}$	Node reaction forces	Pa
s	Damage	ND
t	Time	s
t_c	Ramp loading time	s
T_v	Time factor before loading	ND
T_c	Time factor after loading	ND
$\mathbf{u}_i$	Deformation	m
$\mathbf{v}_i$	Absolute fluid velocity	ms^{-1}
$\mathbf{v}_i^r$	Relative fluid velocity to continuum	ms^{-1}
$\mathbf{v}_s$	Continuum velocity	ms^{-1}
V_{total}	Total volume	m^3
W	Total energy	J
z	Depth	m

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33 Nomenclature - Greek

Symbol	Description	Unit
α_1	Relative compressibility porous to fractured	ND
α_2	Relative compressibility fractured to porous	ND
β	Time interpolation factor	ND
γ	Leakage parameter	m^2
Γ	Boundary	ND

$d\tau$	Shear energy	J
ε	material length scale	ND
Θ	Domain	ND
κ_ε	regularization parameter	ND
λ	Láme's first constant	Pa
μ	Shear ,modulus	Pa
μ_f	Dynamic viscosity	Pas
ν	Poisson ratio	ND
ρ_f	Fluid density	kgm^{-3}
σ_i	Stress	Pa
ϕ_i	Porosity	ND
Ω	Whole continuum	ND

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35 1 Introduction

36 Interest in hydraulic driven fractures has grown in the past two decades with the increasing
37 popularity of fracking as a natural gas extraction method (Mayer, 2016). The increase is due
38 to both the decline in the prosperity of more traditional natural gas resources and the advent
39 of improved fracking techniques overcoming the difficulty of shale extraction from shale rock
40 (Christenson et al., 2017).

41 Shale rock typically has poor interconnection between its pores (Zhang et al., 2015) hence
42 pumping gas directly does not produce high shale gas yields. To solve the interconnectivity
43 issue fracking is utilised, hydrofracturing improves pore connectivity and permeability (Addis,
44 Hughes & Yassir, 2010). The application of high fluid pressure breaks apart the shale rock by
45 propagating fractures, however the direction and magnitude of these fractures are difficult
46 to predict. Hydraulic fracture models are needed for improved shale gas resource mining in
47 the shale gas extraction. Additionally in the extraction of shale gas, a key concern is the
48 leakage of additives in pumped fluid or shale gas into surrounding aquifers (e.g.,
49 (Montgomery and Smith, 2010, Osborn et al., 2011), aquifer pollution can effect fracking
50 reputation due to the requirement of clean up processes. Analysis of the resulting flow
51 characteristics of a fracking site post pressure events, again demands the use of complex
52 prediction tools that consider the evolution of flow characteristics with fracture
53 developments.

54 Current fracking planning procedure is isolating where natural gas is stored in the highest
55 density within the rock (Prud'homme, 2014), drilling down to these dense regions and drilling

holes at regular intervals to ensure maximum extraction (Flekkøy et al., 2002). Drilling many pumping points decreases the efficiency of building pressure due to excess cracking in and pressure leakage to the surrounding areas. In the case of excess cracking, pore fluid that surrounds the cracks, effectively closing the crack, impeding the progress of future fractures (e.g., Peng et al., 2016, Bungler and Detournay, 2015). Therefore it is important to compare several drilling options when planning for the fracking process to optimise the efficiency of natural gas extraction. In order to model several fracking outcomes numerical prediction tools must have practical computational demand for several fracking scenarios to be tested and compared.

Hydrofracture increases the permeability of geomaterials by increasing the number of pore paths (Wu et al., 2004), void space and smoothing pore walls. Numerical modelling techniques are constantly under development in order to predict how permeability evolves with varying fracking approaches. Calculating the flow through fracture systems encapsulated within porous rocks and their interactions to the surrounding solid continuum proves challenging, (Tahir et al., 2014) due to the required coupling of behaviours. Current popular methods include discrete element model (DEM) with high accuracy (Tarokh and Fakhimi, 2014) but very demanding in terms of computational resources; extended finite element method (XFEM) (e.g., Khoei, 2014, Mohammadnejad and Khoei, 2013) is another approach which considers a continuum divided by shape functions that can simulate how a fluid filled fracture progresses without re-meshing; however, the related numerical algorithm requires large amounts of computational resources to converge to the solution. Finally continuum damage models (CDM) (Shojaei et al., 2014) are the most popular due to their simplicity and the fact that they are often pliable when developed with additional concepts.

The numerical algorithms on the basis of dual porous media concept has the ability to capture two pressure systems interacting with different pressure heads and swelling characteristics. This is useful as in practical application there are two flow systems involved, the surrounding porous domain and an evolving fracture system (Rohan et al., 2017), such as in shale rock. Porous media has a far lower permeability than the fractured system (Presho et al., 2011), therefore the fractured system is more efficient at fluid transport complicating fluid flow prediction. Pore network modelling can be used to replicate the same behaviour albeit at a high computational cost (Gao et al., 2016). Dual porosity/permeability models can solve the

same problem at reduced computational demand, this is because the dual porosity model solves the overlapping pressure fields as partial differential equations (Gerke and Van Genuchte, 1993, Jerbi et al., 2017). Achieving this efficiency requires a mesh representing the cracks to be discretised to a sufficient fidelity to accurately represent the fracture network. Within the dual porosity framework the interface between the fracture and porous system is dictated by a leak off coefficient (Guo et al., 2012), allowing the duality of pore/fracture flow to be simulated.

Using dual porous media concept for modelling flow in fractured porous media are becoming a more popular method in modelling the fracking processes (e.g., Thararoop et al., 2012, Chen et al., 2008, Mousavi Nezhad et al., 2011, Mousavi Nezhad et al., 2011). Hence an evolving flow and mechanical system which effects both permeability and strength of rock subjected to fracking is required (Jiang et al., 2010). Capturing the changes in flow and mechanical properties is important in the analysis of hydrofracture application (Wong et al., 1997), therefore in this approach a methodology based on the dual porous media concept has been combined with a continuum damage model to capture an evolving continuum.

A major cost in the fracking process is related to determining material properties of shale rocks. Naturally the more properties are required for designing the operation processes, the cost are higher, therefore in this work a model will be proposed which requires only five properties of the porous medium (e.g., shale rock), simplifying the modelling process (Rabbani et al., 2017).

An energy based fracture system is used, so that element damage condition can be employed to dictate if an element is porous or fractured (Mousavi Nezhad et al., 2018). This paper presents a new fully coupled practical computational framework developed by combining two simple and scientifically approved concepts. Variational formulation, proposed by Del Piero and co-workers' (Del Piero et al., 2007), is coupled with dual porous media concept in order to introduce an energy-based fluid driven crack propagation scheme that are solved numerically using the finite element method (Mousavi Nezhad et al., 2018, Gironacci et al., 2018). A key component of the proposed framework is the introduction of an energy-based fluid driven crack propagation for compressible fluid and porous structure. The theory used in the double permeability concept followed by the fracture framework used for the model

development are presented in sections 2 and 3. The paper then explains two verification tests, and the results are summarised in the conclusion.

2. Methodology

A fully coupled set of mathematical equations governing fluid flow, mechanical behaviour of deformable porous materials and damage evolution are used in this work.

2.1. Coupled flow formulation

This method relies on a dual porous media concept, in that any point within the considered continuum can vary between porous and fractured behaviour, or somewhere in between. Furthermore, only the fracture boundary allows fluid pressure exchanges. Porous medium is considered as intact domain with higher strength and lower permeability than damaged zone.

The basis for the flow formulation used in the coupled equations will be explained below. The mathematical equation for Darcy flow can be expressed as

$$\mathbf{v}_i^r = \frac{\mathbf{k}_i}{\mu_f} (\nabla \mathbf{p}_i + \rho_f \mathbf{g}) \quad (1)$$

where $\mathbf{v}_i^r$ is the relative velocity of fluid in the respective region, $\mathbf{k}_i$ is the permeability of the respective region, μ_f is the dynamic viscosity of the fluid considered, $\mathbf{p}_i$ is the pressure of the respective zone, ρ_f is the fluid density and $\mathbf{g}$ is the gravity vector, applied to all points.

The relative velocity of the fluid to the continuum can take the form

$$\mathbf{v}_i^r = \phi_i (\mathbf{v}_i - \mathbf{v}_s) \quad (2)$$

where $\mathbf{v}_s$ is the velocity of the soil skeleton considered in the continuum, ϕ_i is the porosity and $\mathbf{v}_i$ is the absolute velocity of the fluid. The absolute velocities are defined as shown in the following

$$\mathbf{v}_i = \frac{d\mathbf{u}_i}{dt} \text{ and } \mathbf{v}_s = \frac{d\mathbf{u}}{dt} \quad (3)$$

where t is time, $\mathbf{u}_i$ and $\mathbf{u}$ are the velocities of the fluid and the soil skeleton, respectively. Taking the mass balance for fluid at a considered boundary between the porous medium and fractured regions leads to

$$\nabla (\phi_i \rho_f v_i) + \frac{\partial(\phi_i \rho_f)}{\partial t} + (-1)^i \Omega = 0 \quad (4)$$

where $\nabla (\phi_i \rho_f v_i)$ represents changes in porosity, density caused by the rate of continuum deformation and resulting mass change. $\frac{\partial(\phi_i \rho_f)}{\partial t}$ represents the changes in fluid density over time through changes in porosity with time and the corresponding change in mass. The last term $(-1)^i \Omega$ in the left hand side of the equation describes effects resulting from fluid leakage from one region to another. Substituting Eq. 3 into Eq. 4 produces.

$$\nabla (\phi_i \rho_f v_s) + \nabla(\phi_i v_i^r) = \frac{\partial(\phi_i \rho_f)}{\partial t} + (-1)^i \Omega. \quad (5)$$

Using the Lagrangian total derivative method concept for both a moving solid $\frac{d_s(^{\circ})}{dt} = \frac{\partial_s(^{\circ})}{dt} + \nabla(^{\circ})v_s$ and fluid $\frac{d_i(^{\circ})}{dt} = \frac{\partial_s(^{\circ})}{dt} + \nabla(^{\circ})v_i$ with an identity vector $\nabla [(^{\circ})v_i] = (^{\circ})\nabla(v_i) + (^{\circ})\nabla v_i$, Eq. 5 can be rewritten as

$$\rho_f \nabla(v_i^r) = \phi_i \frac{d_i \rho_f}{dt} + \rho_f \frac{d_s \phi_i}{dt} + \phi_i \rho_f \nabla(v_s) + (-1)^i \Omega = 0. \quad (6)$$

To consider the change in fluid density change in response to pressure with time in a more appropriate manner, fluid compressibility is considered with the change in fluid pressure in time as

$$\frac{d_i \rho_f}{dt} = \rho_f c_f \frac{d_s p_i}{dt} \quad (7)$$

and the change in continuum volume over time is considered in terms of continuum velocity as

$$\frac{1}{V_{total}} \frac{d_s V_{total}}{dt} = \nabla(v_s) \quad (8)$$

where V_{total} is the total element volume and c_f is fluid compressibility.

In order to remove the absolute continuum velocity term $\mathbf{v}_s$, Eqs. 7 and 8 are substituted into Eq. 6 and expanding terms yields

$$\begin{aligned} \rho_f \nabla \left(\frac{\mathbf{k}_i}{\mu_f} (\nabla \mathbf{p}_i + \rho_f \mathbf{g}) \right) \\ = \phi_1 \rho_f c_f \frac{d_s \mathbf{p}_1}{dt} + \phi_2 \rho_f c_f \frac{d_s \mathbf{p}_2}{dt} + \frac{1}{V_{total}} \frac{d_s V_{total}}{dt} + \phi_i \rho_f \nabla (\mathbf{v}_s) \\ + (-1)^i \Omega \end{aligned} \quad (9)$$

Eq. 9 describes flow in both regions of porous and fractured. This equation has been separated to model each flow separately but still coupled to formulate Eqs. 11 and 12. The α terms are used to improve readability of the equations and $(-1)^i \Omega$ has been replaced with the leakage term: $\gamma(\mathbf{p}_1 - \mathbf{p}_2)$ where γ is a leakage constant acting on the boundary between the porous and the fractured regions (Feng et al. 2016). Each point within the defined mesh as assigned a leak-off coefficient value depending on if the point is being used to define a fracture or porous section, the same is true for the alpha coupling terms used to determine the rate of swelling or dilation due to fracture pressure.

Eq. 10 is used to describe the pressure effects on the porous solid continuum, depending from which region the fluid pressure acts.

$$\nabla [\mathbf{C}_{ijkl} \nabla \dot{\mathbf{u}} + \alpha_1 \dot{\mathbf{p}}_1 \boldsymbol{\delta} + \alpha_2 \dot{\mathbf{p}}_2 \boldsymbol{\delta}] + \dot{\mathbf{F}} = 0 \quad (10)$$

$$\nabla \left[\frac{\mathbf{k}_1}{\mu_f} (\mathbf{p}_1 + \rho_f \mathbf{g}) \right] = -\alpha_1 \boldsymbol{\delta} \nabla \dot{\mathbf{u}} + \alpha_{11} \dot{\mathbf{p}}_1 - \alpha_{12} \dot{\mathbf{p}}_2 + \gamma(\mathbf{p}_1 - \mathbf{p}_2) \quad (11)$$

$$\nabla \left[\frac{\mathbf{k}_2}{\mu_f} (\mathbf{p}_2 + \rho_f \mathbf{g}) \right] = -\alpha_2 \boldsymbol{\delta} \nabla \dot{\mathbf{u}} + \alpha_{22} \dot{\mathbf{p}}_2 - \alpha_{12} \dot{\mathbf{p}}_1 + \gamma(\mathbf{p}_2 - \mathbf{p}_1) \quad (12)$$

where $\mathbf{C}_{ijkl}$ is the compliance matrix, $\dot{\mathbf{u}}$ is deformation rate, α_{ii} and α_i represent the relationships between porous and fractured domains demonstrated in Eqs (13)- (15), $\boldsymbol{\delta} = [1, 1, 0, 0]$, $\dot{\mathbf{F}}$ is the matrix of mechanical force derivative. Eq. (10) models the mechanical

components, the first term calculates internal deformations, the second and third terms show the impact that pore and fracture pressures have on the porous media skeleton. The $\vec{F}$ term accounts for external loads and self-weight. Eqs. (11) and (12) consider fluid flow throughout the porous and fracture systems, respectively. The first term of Eqs. (11) and (12) describes Darcy's law, the second describes deformations effect on fluid pressure in the respective media, the third and fourth account for the effect of the fluid's compressibility and the final term determines leakage between the two media, the boundary adapting to fracture evolution when necessary.

In the considered porous media, the pores are assumed to be equally distributed in size, shape and position through each particular element, all expanding or constricting equally in response to pressure variations.

$$\alpha_1 = c_p/c \quad (13)$$

where c_p is the compressibility of the porous system and c is the compressibility of the fractured media. Hence we can define

$$\alpha_{12} = \alpha_{21} = (\alpha_1\alpha_2 - \frac{\phi_1\phi_2}{\phi_1 + \phi_2})c \quad (14)$$

where α_2 is the reverse

$$\alpha_2 = 1 - \alpha_1 \quad (15)$$

where α_{12} and α_{21} relate both fluid pressures to each other by considering their relative compressibility and porosity on the boundary between fractured and porous media. Coefficients α_{ii} to include the compressibility of fluids inside both considered regions are defined as

$$\alpha_{ii} = \phi_i c_f + \alpha_{12}. \quad (16)$$

2.2. Fracture propagation formulation

The crack methodology developed by Del Piero et al. (Del Piero et al., 2007) was used for this framework. Energy minimization approach together with variational approach proposed by Francfort and Marigo (Francfort and Marigo, 1998) are employed for crack paths prediction. Deformation of the porous structure under load increases the bulk energy stored. When this value reaches to a critical value in a given zone, it is energetically favourable for the system to release its energy. Therefore, crack growth is deduced by successive minimization of total energy function which is defined as the sum of the bulk and surface energies as defined as

$$\mathcal{E}(f, K) = \int_{\Omega \setminus K} W(F(x)) dx + G \mathcal{H}^{N-1}(K) \quad (17)$$

where f is the body deformation, K is the fractured zone, W is the stored energy function, F is the deformation gradient, G is the fracture energy and $\mathcal{H}^{N-1}$ is the Hausdorff measure of K which provides the measure of the length of the crack for sufficiently regular fractured zone. The first and the second terms on the right hand side of the Eq. (17) represent bulk and surface energy of the body, respectively.

A standard finite deformation Ogden model (Ogden, 1997) is used for the constitutive model in this case to calculate mechanical stresses and strains as shown in equation (18) (Bonet and Wood, 1997).

$$\sigma_{ij} = \frac{\mu}{J} (\mathbf{b}_{ij} - \delta_{ij}) + \frac{\lambda}{J} \ln(J) \delta_{ij} \quad (18)$$

where λ is lame's constant, μ is shear strength, $\mathbf{b}_{ij}$ is strain, δ_{ij} is the Kronkecker delta and σ is stress.

A finite deformation constitutive model is utilised as experimental investigations have shown (Hwang et al., 2002) that model based on infinitesimal assumptions break down at the tips of fractures due to the large local strains involved (Martínez-Pañeda and Betegón, 2015).

The deformation gradient is then used to formulate the deformation gradient traced tensor ($\mathbf{I}_c$) and the Cauchy left strain tensor and change in volume (J). Eqs. (19) and (20) using shear modulus (μ) and Lamé's constant (λ) calculate energy accumulated due to strains within the continuum (Bonet and Wood, 1997).

$$dE = 1/2 \mu (\mathbf{I} \mathbf{c} - 3) \quad (19)$$

$$d\tau = \frac{1}{2} \lambda \log(J)^2 - \mu \log(J) \quad (20)$$

228

229 where dE is the shear fracture energy, $d\tau$ is the bulk energy accumulated (Bonet and Wood,
230 1997).

231 For minimization of the Eq. (17) with respect to any kinematically admissible displacement
232 and any set of crack paths, based on the regularized formulation of the energy function for
233 brittle fracture problems (Francfort and Marigo, 1998), an auxiliary variable s which is called
234 damage parameter, is introduced. s is a regularized representation of the damaged zone
235 defining the jump set in Eq. (17). Therefore, a matrix space X is considered which its elements
236 are pairs of (f, s) . The total energy formulation for the system can be represented as:

$$\mathcal{F}_\varepsilon(f, s) = \int_{\Omega_0} (s^2 + \kappa_\varepsilon) W(\nabla_0 f) d\Omega_0 + \frac{G}{2} \int_{\Omega'_0} \left(\varepsilon |\nabla_0 s|^2 + \frac{1}{\varepsilon} (1 - s)^2 \right) d\Omega_0 \quad (21)$$

237 where κ_ε is a positive regularization parameter and ε is related to the material length scale,
238 Ω_0 and Ω'_0 represent initial unfractured and stress-free configuration of the system in the
239 physical and logical domains, respectively. For a fixed s or f , the total energy of the system
240 can be minimised by solving a linear system of equations with the appropriate boundary
241 conditions.

242 3. Numerical solution

243 3.1. Spatial discretisation

244 Galerkin's weighted residual approach is used to discretise the equations across the domain.
245 Applying shape functions $\mathbf{N}$ and their derivatives $\mathbf{B}$ to the continuum and considering
246 boundary effects transform Eqs. (10)-(12) into (22)-(24), the $\mathbf{B}$ terms used to determine the
247 coupling and permeability matrices as shown in Eqs.(27) - (28).

248

$$\begin{aligned}
& - \int_{\Theta} [\nabla' N]^T [C_{ijkl}] [\nabla' N] \dot{u} d\theta + \int_{\Gamma} [N]^T [C_{ijkl}] [\nabla' N] \dot{u} d\Gamma \\
& - \int_{\Theta} \alpha_1 [\nabla N]^T \delta[N] \dot{p}_1 d\Theta + \int_{\Gamma} \alpha_1 [N]^T \dot{p}_1 [N] d\Gamma \\
& - \int_{\Theta} \alpha_2 [\nabla N]^T \delta[N] \dot{p}_2 d\Theta \\
& - \int_{\Gamma} \alpha_2 [N]^T \delta[N] \dot{p}_2 d\Gamma + \int_{\Theta} [N]^T \dot{F} d\Theta = 0
\end{aligned} \tag{22}$$

$$\begin{aligned}
& \int_{\Theta} \alpha_1 [N]^T \delta^T [\nabla N]^T \dot{u} d\theta - \int_{\Theta} \alpha_{11} [N]^T [N] \dot{p}_1 d\Theta + \int_{\Theta} \alpha_{12} [N]^T [N] \dot{p}_2 d\Theta \\
& - \int_{\Theta} [\nabla N]^T \frac{[k_1]}{\mu} [\nabla N] p_1 d\Theta + \int_{\Gamma} [N]^T \frac{[k_1]}{\mu} [\nabla N] p_1 d\Gamma \\
& + \int_{\Theta} \gamma [N]^T [N] p_2 d\Theta - \int_{\Theta} \gamma [N]^T [N] p_1 d\Theta = 0
\end{aligned} \tag{23}$$

$$\begin{aligned}
& \int_{\Theta} \alpha_2 [N]^T \delta^T [\nabla N]^T \dot{u} d\theta - \int_{\Theta} \alpha_{22} [N]^T [N] \dot{p}_1 d\Theta + \int_{\Theta} \alpha_{21} [N]^T [N] \dot{p}_2 d\Theta \\
& - \int_{\Theta} [\nabla N]^T \frac{[k_2]}{\mu} [\nabla N] p_2 d\Theta + \int_{\Gamma} [N]^T \frac{[k_2]}{\mu} [\nabla N] p_2 d\Gamma \\
& - \int_{\Theta} \gamma [N]^T [N] p_2 d\Theta + \int_{\Theta} \gamma [N]^T [N] p_1 d\Theta = 0
\end{aligned} \tag{24}$$

where, Θ represents the whole continuum, the differential operator: ∇' represents the definition of strain and Γ represents the continuum boundaries.

For simplicity, boundary mechanical forces are ignored. After unit substitution for simplification, the equations can be put into matrix format to appear as shown in Eq. (25) in the framework, the two flows are related via relative compressibility terms and a leakage term. This means that the compressible nature of fractured media has to be known but this can be found from triaxial tests on damaged material samples. By referencing the strength of the two flow models, the framework is capable of determining how each region will deform according to internal pore and fracture pressures (Shojaei et al., 2014). The leakage term

determines how fluid transfers from one region to the other and helps to model fluid lag. The transfer itself causes indirect deformation; the higher the leakage parameter the less bearing the fracture network has on mechanical behaviour as all fluid pressure would leak into the porous region.

Once simplified and rearranged into a practical matrix format, the coupled equations can be presented as

$$\begin{bmatrix} [K] & \alpha_1[C] & \alpha_2[C] \\ \alpha_1[C]^T & -\alpha_{11}[M] - \beta\Delta t([H_1 + \gamma[M]]) & \alpha_{12}[M] + \beta\Delta t[M] \\ \alpha_2[C]^T & \alpha_{21}[M] + \beta\Delta t\gamma[M] & -\alpha_{22}[M] - \beta\Delta t([H_2 + \gamma[M]]) \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta p_1 \\ \Delta p_2 \end{bmatrix} \quad (25)$$

$$= \begin{bmatrix} \Delta R \\ 0 \\ 0 \end{bmatrix} + \Delta t \begin{bmatrix} 0 \\ (1-\beta)(Q_1)_t + \beta(Q_1)_{t+\Delta t} \\ (1-\beta)(Q_2)_t + \beta(Q_2)_{t+\Delta t} \end{bmatrix} + \Delta t \begin{bmatrix} 0 \\ (H_1 + \gamma M)(p_1)_t + \gamma M(p_2)_t \\ (H_2 + \gamma M)(p_2)_t + \gamma M(p_1)_t \end{bmatrix}$$

where K is the mechanical stiffness matrix M is the mass matrix

$$[M] = \int_{\Theta} [N]^T [N] d\Theta, \quad (26)$$

C is the coupling matrix

$$[C] = \int_{\Theta} [B_2]^T [N] d\Theta, \quad (27)$$

and H_i is the discretised permeability matrix which is defined as

$$[H_i] = \int_{\Theta} [B_3]^T \frac{[k_i]}{\mu} [B_3] d\Theta \quad (28)$$

Q_i is the boundary flow matrix

$$[Q_i] = \int_{\Gamma} [N]^T \frac{[k_i]}{\mu} [\nabla N] \mathbf{p}_i d\Gamma \quad (29)$$

In the above equations, the matrix terms B_2 and B_3 are the derivatives of respective shape functions, demonstrated in Eqs. (30) and (31) for plane strain problems. Furthermore ΔR is the loading matrix and β is the time finite difference interpolation factor.

$$\mathbf{B}_2 = \begin{bmatrix} \frac{dN_1}{dx_1} & \frac{dN_1}{dy_1} & \dots \end{bmatrix} \quad (30)$$

$$\mathbf{B}_3 = \begin{bmatrix} \frac{dN_1}{dx_1} & \dots \\ \frac{dN_1}{dy_1} & \dots \end{bmatrix} \quad (31)$$

3.2. Time discretization

The finite difference scheme is used to solve for pressure changes over time

$$\int_{\{t\}y(t)dt}^{\{t+\Delta t\}} = [(1 - \beta)y_t + \beta y_{t+\Delta t}] \quad (32)$$

where y_t is the solution at time t and $y_{t+\Delta t}$ is the solution at time $t + \Delta t$.

Therefore

$$\Delta y = y_{t+\Delta t} - y \quad (33)$$

The value of β determines the type of approximation. When $\beta = 0$ forward interpolation is used, $\beta = 1$ is for backwards and $\beta = 0.5$ is for central interpolation, which the latter is the most stable.

3.3. Numerical strategy for crack modelling

As cracks propagate, newly damaged parts of the considered domain transfer from the porous type domain to the fractured one, with the corresponding changes in material strength, porosity and permeability (Bunger et al., 2013, Huang et al., 2017).

The newly damaged domain will take the high pressure fluid from the preceding crack at a rate determined by the change in permeability. The increase in permeability occurred during the time step and hence the fluid should realistically be accommodated with some fluid flow which is unknown, for this reason the step is repeated with the new permeability to calculate how the flow develops, and how that increase in flow dictates energy distribution around the hydrofracture. The case is repeated until the minimal total energy is found for the mechanical continuum; the full algorithm can be seen in Figure 1.

Algorithm

```
1 Specify run time  $t_{full}$ 
2 Create model geometry
3 Interpret geometry to mesh using Delaunay triangulation algorithm
4 Initialize material parameters;  $\mu, \nu, \lambda, R_0, K_1, K_2, \alpha_1, \alpha_2, \alpha_{12}, \alpha_{21}, \alpha_{11}, \alpha_{22}, \gamma$ 
5 Initialize damage parameter for starting crack
6 Initialize pore and fracture pressure on continuum
7 while  $t < t_{full}$  do
8     Calculate stiffness matrix  $C_{ijkl} = \frac{\lambda}{J} \delta_{ij} \delta_{kl} + \frac{2\mu - \lambda \ln(J)}{J} \delta_{ik} \delta_{jl}$ 
9     Calculate mass matrix  $[M] = \int_{\theta} [N]^T [N] d\theta$ 
10    Calculate internal forces  $[R] = \int_{\theta} [B_3]^T \sigma [B_3] d\theta$ 
11    Calculate Coupling matrices  $[C] = \int_{\theta} [B_2]^T [N] d\theta$ 
12    Calculate Permeability matrices  $[H_i] = \int_{\theta} [B_3]^T [k_i] \mu f [B_3] d\theta$ 
13    Form Global matrices
14    Solve system of Global equations for  $\Delta u$  and  $\Delta p_i$ 
15    Calculate stored elastic energy  $W = \frac{1}{2} \mu (Ic - 3) + \frac{1}{2} \lambda \log(J)^2 - \mu \log(J)$ 
16    Check for fracture  $W_e > K_{frat}$ 
17     $\Delta D_i = \min(\frac{W}{R_0}, 1)$ 
18    Assign damage variable accordingly
19    Redistribute permeability and leak off coefficient
20    Mesh refinement at newly damaged elements
21    Calculate total energy
22    if  $i == 0$  then
23         $E_i^{tot} = \text{total energy}$   $i = i + 1$  reset deformations and fluid pressures goto line 7
24    else
25        if  $E_i^{tot} \geq E_i^{tot} - 1$  then
26             $t = t + 1$ 
27             $i = 1$ 
28            Update deformations  $U_t = U_t - 1 + \Delta u_i - 1$ 
29            Update pressures  $P_t = P_t + \Delta p$ 
30            Update damage  $D_t = D_t - 1 - \Delta D_i - 1$ 
31            goto line 7
32        else reset deformations and fluid pressures  $E_i^{tot} = \text{total energy}$   $i = i + 1$ 
33            goto line 7
34    end
35 End
```

Figure 1 Hydrofracture algorithm

305 In the algorithm shown in Figure 1, step 20 mentions mesh refinement, this is in aid of
306 increasing accuracy in fracture propagation by improving the leakage terms' boundary of
307 definition. The Leakage term is determined computationally with a gradient operation on the

damage distribution, where the fields' gradient is highest is where the leakage term is attributed.

The highest rate of energy exchange is at the fracture tip, as fracture progresses the model will realise that if some sections of the crack heal then the loads incurred can be carried more efficiently with overall lower energy. In order to replicate realistic crack behaviour an irreversibility condition is employed (Bourdin et al., 2000), preventing cracks from healing, i.e. the damage parameter can only decrease, providing irreversible changes to the continuum model.

The interaction between fracture flow and pore flow is considered important as the fracture flow leaking can greatly impact both the fracture pressure itself and the fracture propagation. Thus the line between the two set domains is given the leak off coefficient, represented by γ allowing pressure to transfer between the two domains within the model. The leak off coefficient is derived from the ratio of connecting domains permeabilities.

4. Numerical examples

In this section, two numerical examples are considered to verify the developed framework and assess the robustness of the method.

4.1. One-dimensional consolidation example

For correct prediction of hydraulic fracture, porous flow must also be predicted correctly. In order to verify this component of the model, a one dimensional test has been chosen from the literature (Samini and Pak, 2012) where a generalised finite element method was used for the simulation. For direct comparison an analytical solution is also used, the solution is shown in Eqs. (34), (37)-(41).

For $T_v \leq T_c$:

$$p = \sum \frac{2 q_c}{M_v^3 T_c} \sin \frac{M_v z}{H} [1 - \exp(-M_v^2 T_v)] \quad (34)$$

For $T_v \geq T_c$:

$$p = \sum \frac{2 q_c}{M_v^3 T_c} \sin \frac{M_v z}{H} \exp(-M_v^2 T_v) [\exp(-M_v^2 T_v) - 1] \quad (35)$$

where q_c is surcharge, H is drainage height, z is height of interest and M_v is the modulus of volume change. T_c and T_v are dimensionless time factors, the equations to calculate these variables are found in Eqs. (37)-(41)

$$m_v = \frac{(1 + v)(1 - 2v)}{E(1 - v)} \quad (36)$$

$$m_v = \frac{(1 + v)(1 - 2v)}{E(1 - v)} \quad (37)$$

$$M_v = \frac{(2m_v + 1)\pi}{2} \quad (38)$$

$$C_v = \frac{k}{m_v \rho_f} \quad (39)$$

$$T_v = \frac{C_v t}{H^2} \quad (40)$$

$$T_c = \frac{C_v t_c}{H^2} \quad (41)$$

where m_v is the coefficient of volume change, E is the Young's modulus, v is the Poisson's ratio, M_v is a numerical term, C_v is consolidation coefficient and t_c is the ramp loading time.

In this test, a 10m by 1m column of porous media is considered under a rapidly applied pressure of 1kPa (see Figure 2 and Figure 3).

Figure 2 one dimensional consolidation schematic

At quarter height intervals, pore pressure is recorded over time, recording the increase in pressure from loading and the consequent slow dissipation due to drainage. The test was to replicate the results as mentioned in Samini and Pak, (2012) with similar model conditions, reported in Table 1.

Table 1. Material properties used for one-dimensional test

Young's modulus	E	10 Mpa
Poisson's ratio	ν	0.2
Permeability	k	10^{-8} m^2
Porosity	ϕ	0.25
Dynamic viscosity	μ_f	10^{-15} Pa.s

The one-dimensional consolidation test starts off initially with zero pore pressure and load. The 1kPa load compresses the medium as shown in Figure 3 and the excess pore pressures increase dramatically as expected.

Figure 3- Load-Time graph

As time progresses the rate of deformation subsides, the pore pressures returns to zero and the model becomes normally consolidated again. The pore pressure response being monitored and shown in Figure 4. In this figure each line represents the average pore pressure in the sample at 50% and 100% height intervals. The pressure change corresponds to the 1kPa load and dissipates to become consolidated as time progresses. The time plotted in Figure 4 varies from Figure 6, setting $t = 0$ at the beginning of the consolidation to ease result interpretation.

Figure 4 Excess pore pressures in 1D consolidation test at regular height intervals.

Figure 4 also compares the model generated solution against the analytical solution. The proposed coupled solution slightly over estimates the pore pressure peak when compared to the analytical solution. However, the coupled hydromechanics aspects behave as expected of dual permeability methodology (Jing et al. 2001), the highest pressure drops occurring quickly after the peak pressure. Pressure changes reduce in the expected asymptotic manner, slowly approaching full equilibrium as time progresses, a snapshot of the consolidation process is shown in Figure 5, the pressure distribution remaining constant throughout the simulation.

Figure 5 Pore pressure distribution and flow paths

Pressure distributions have been quantitatively evaluated over time and found to have good agreement between the theoretical results and the framework's result.

Figure 6 Pressure distributions at various times

Figure 6 represents the fluid pressure at the intervals at each quarter showing a different point of comparison of the consolidation process. The pressure is taken as an average of the finite points, and hence why the 4H/4 is not zero.

4.2. Pressurised fracture evolution example

This example concerns modelling of fluid pressurised fracture evolution in a rectangular domain with a penny shaped crack in the middle. This example has been simulated previously by Gunther and Leonhart (2015). In their work they proposed an enrichment function to account for a displacement jump with the associated pressure variation aiming to replicate the behaviour of a fluid driven fracture. The fluid flow was described using the Darcy flow model and the crack propagation was modelled based on an upper bound stress criterion. Schematic description of the problem is presented in Figure 7.

The penny crack is defined by an infinitesimal line with the length of 97.5mm in the centre of a rectangular domain 3875 mm by 975 mm. The material properties were replicated from the literature (Gunther and Leonhart 2015) and are reported in Table 2. The internal crack was pressurised instantly to 20MPa and gravity effects were neglected.

Figure 7 Schematic of pressurised fracture test.

Table 2. Material properties used for pressurised fracture evolution test

Young's modulus	E	2000 Mpa
Poisson's ratio	ν	0.3
Fracture strength	K_{frat}	1K J
Permeability	k	$10^{-15} m^2$
Porosity	ϕ	0.25
Dynamic viscosity	μ_f	$8 \cdot 10^{-4} Pa \cdot s$

The fracture strength is determined by how much elastic energy the material can withstand before failure. In their paper, Gunther and Leonhart, (2015) mentioned a stiffness of 2000Mpa and a tensile strength of 2Mpa, resulting in a failure strain of 0.001% and total energy of 1kJ, here described as the fracture energy.

The predicted values of the crack width over time are presented in Figure 8 that shows a fairly good agreement with those predicted by Gunther and Leonhart, (2015). The crack width was calculated as the sum of deformations in the top and bottom of the middle points of the crack. The increase in crack pressure leads to further opening of the existing crack, and eventually building stress locally at the tips which causes the crack to propagate. The results show that crack width decreases as the crack starts to propagate. This is because of the mutual fluid interactions between fluid and solid phases during the exchange of fluid between the two zones. Both internal crack pressure and the surrounding pore pressures cause their respective media to expand. The crack pressure pushes the crack boundaries and intends to open the crack, while the porous pressure acts inversely and tends to close it. As a result, the crack width increases in a linear fashion until the crack begins to propagate. During the fracture opening stages fluid leaks into the surrounding media, equalising the pore pressure around the crack to the internal crack pressure. As the crack length increases, the rate of the exchange of the fluid between fracture and the surrounding porous domain also increases, and consequently, the pressure of the whole domain reaches to an equilibrium condition with a higher rate. Therefore, the enhanced pore pressure around the crack reduces the crack width as fracture propagates, this two-dimensional consolidation like behaviour is the primary influence on the crack width over time in both the proposed solution and the Gunther and Leonhart's solution. The earlier onset of pressure decreasing at time 20s is due to the $\alpha_{11}\dot{p}_1$ and $\alpha_{12}\dot{p}_2$ terms opening the crack quicker, however still failing at the same crack width.

Figure 8 Crack opening with time (s).

The predicted pressure at which the crack propagates, is higher than the one predicted by Gunther and Leonhart, (2015). This is because we consider the compressibility of the fluid. As the porous media above and below the crack are compressed, fluid is compressed and then its pressure increases. Therefore, a higher pressure is required to satisfy the fracture criterion.

Figure 9 shows that the crack evolves horizontally and it is in agreement with what has been reported in the literature about the hydraulic fracturing of the porous samples with penny shaped initial fracture (e.g., Samini and Pak, 2012, Wangen, 2017, Wilson and Landis, 2016). The fluid stream lines in Figure 9 show that the fluid flows from the fracture to the surrounding porous region, as the fracture propagates.

Figure 9- Damage and pressure distribution development with fracture.

Similar to Gunther and Leonhart's, (2015) work, to verify the developed computational framework in predicting the flow behaviour, fluid pressure along four lines A-A, B-B, C-C and D-D have been studied at intervals 1.75m, 1.544m and 1.2725m, respectively. Figure 10 shows the distribution of pressure along these cross sections. A non-linear behaviour for pressure distribution is predicted with our model for each cross section, and it is seen that pressure along the section C-C significantly increases after 80s when crack reaches this section. These findings are in agreement with discussions about the pressure behaviour in compressible porous media presented by Boutin and Venegas, (2016). However, a linear pressure distribution was predicted by Gunther and Leonhart, (2015). The possible reason of the difference between the results of our model and those presented by Gunther and Leonhart is that we considered the compressibility of both fluid and porous media. This is proved by results of our simulations repeated with the assumption of the incompressible fluid and porous domain (i.e., ignoring third and fourth terms of equation (11) which are presented in Figure 11 and is in good agreement with results of Gunther and Leonhart, (2015). The deformation profiles along A-A, B-B and C-C at time 50s is presented in Figure 10, and shows that porous media below and above the crack compressed during crack propagation. As pores compress during crack expansion, pore pressure increases due to the consequences of the third $\alpha_{11}\dot{p}_1$ and fourth term, $\alpha_{12}\dot{p}_2$ of the equation (11), giving cause to the bulge seen in Figure 10b.

(a) (b)

Figure 10-pressure profiles through sample

Where n is the solution from the proposed numerical framework. With the terms $\alpha_{11}\dot{p}_1$ and $\alpha_{12}\dot{p}_2$ removed a more accurate result is achieved as seen in figure 11b.

Figure 11 - pressure profile through sample not assuming compressibility of porous media and the fluid

Figure 12 demonstrates the pressure profile along D-D at time 50s to compare against data given by Gunther and Leonhart, (2015). In both D-D snapshots, pressure sharply decreases to about 5MPa up to a distance of 1.8m from the left boundary and then stays constant throughout the rest of the sample due to the zero flux condition applied to the external boundary of the domain.

Figure 12 Damage and Pressure distribution along D-D.

5. Conclusion

In this paper a double porosity concept and a fracture propagation methodology have been combined together to create a new practical modelling framework for simulation of fracture in deformable porous media. The framework solves fully coupled nonlinear partial differential equations for pressures, deformation and damage evolutions. We have shown that it is possible to combine Simo and Pister's (Simo and Pister, 1984) fracture methodology with Ma et al.'s (Ma et al., 2016). Ma et al.'s (Ma et al., 2016) double permeability model to produce a simple yet effective hydraulic fracture prediction tool, neither methodology interfering with the other coupled such that the model can replicate mutual interaction between fluid and porous domains and all associated behaviours within an elastic scenario.

It has been shown that the proposed model can capture one-dimensional soil consolidation behaviour to a very good degree of accuracy in the pressure increase and asymptotic decrease. The model is highly flexible in what it can model with permeability and strengths evolving with fracture growth. Also, this paper investigates fluid injection wells, the pressure hike and drop of both pores and pressure fluids along with the pressurised crack propagation.

The developed methodology has great capability in capturing interaction between fluid and deformable porous media as it is based on the dual porosity fracture framework, it is able to interpret two coupled flows and their interactions with each other. It considers compressibility of both porous structure and fluids and the mechanical to fluid pressure coupling. The fracture methodology relies on the basis of maximum energy release rate criterion and is expandable to more complex constitutive models. The model presented involved non-linear elasticity, however work is undergoing on the inclusion of an elastic-plastic constitutive model in the framework to better represent rock like material behaviour.

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