# An Analytical Approach to Evaluate Point Cloud Registration Error Utilizing targets

Ronghua Yanga;b;c , Xiaolin Mengd, Yibin Yaoe, Bi Yu Chenf , Yangsheng Youa;b, Zejun Xiangc

a Key Laboratory of New Technology for Construction of Cities in Mountain Area (Chongqing University), Ministry of Education, Chongqing, 400045, China
b School of Civil Engineering, Chongqing University, Chongqing 400045, China
c Chongqing Survey Institute, Chongqing, 401121, China
d Nottingham Geospatial Institute, the University of Nottingham, Nottingham, NG7 2TU, UK
e School of Geodesy and Geomatics, Wuhan University, Wuhan, 430079, China
f State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University, Wuhan, 430079, China

## Abstract

Point cloud registration is essential for processing terrestrial laser scanning (TLS) point cloud datasets. The registration precision directly in uences and determines the practical usefulness of TLS surveys. However, in terms of target based registration, analytical point cloud registration error models employed by scanner manufactures are only suitable to evaluate target registration error, rather than point cloud registration error. This paper proposes an new analytical approach called the registration error (RE) model to directly evaluate point cloud registration error. We verify the proposed model by comparing RE and root mean square error (RMSE) for all points in three point clouds that are approximately equivalent.

Keywords: Point cloud, Registration error, Target, Terrestrial laser scanning

## 1. Introduction

Terrestrial laser scanning (TLS) is used for a rapid collection of dense, three-dimensional (3D) spatial point cloud datasets of an entire object. Usually several scans are required with di erent stations to survey a relatively

Preprint submitted to ISPRS Journal of photogrammetry and remote sensingApril 10, 2018

large and complex object completely due to occluded surfaces and scanner  eld of view limitations [1]. To obtain the object's complete 3D model, the point cloud datasets must  rst be registered to a chosen coordinate system [2].

Previous registration studies mainly include: 1) Matric representation for rotation transformation, such as Euler angle [3, 4], unit quaternion [3{5], direction cosines [3, 5], dual quaternions [6], etc.; 2) Algorithms to compute 3-D rigid body transformation, such as singular value decomposition [7, 8], unit quaternion [7, 9, 10], dual quaternions [6, 7], orthonormal matric [7, 11], Lodrigues matric [12], etc.; 3) Iterative closest point method (ICP) (and variants), such as the feature correspondences [13{16], registration strategy [13, 17, 18], correspondence search [13, 19, 20], robustness [13, 19, 20], etc.; 4) Point cloud registration error models, such as error propagation for two scans [21], error propagation for multiple scans [2, 21, 22], directly geo-referenced TLS data precision [23, 24], the relationship between target precision and distribution relationships [1, 25{27], etc..

For target registration, point cloud registration error models and their statistics employed by scanner manufacturer software are based on how well the targets match. These approaches have been shown to be inadequate [24], since target registration error is not equal to the point cloud registration error. Although Fan et al. [24] recommended a model to evaluate registration error based on how well the point clouds matched, However, the model was derived from simulations, which are not always consistent with actual outcomes since practical situations are often very complicated. Therefore, this paper derives the target based point cloud registration error model analytically, and verifies the model by evaluating real-world point cloud registration precision.

## 2. Estimation of registration parameters

We first introduce the common registration model to provide true observation and transformation parameter values. We then consider true and approximate errors for these parameters, and derive the registration model error analytically using the estimation value and transformation parameter variances. Finally, we derive the analytical model to evaluate target based point cloud registration error.

### 2.1. Registration Model

Target based registration of two scans is the most common registration approach and is most often performed using 3D rigid body transformation algorithm [4, 7, 12]. The registration model can be expressed as point clouds in Scan i+1 are transformed into Scan i using the true values of three translation parameters $\tilde{t}_x$, $\tilde{t}_y$, $\tilde{t}_z$ and three rotation parameters $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ [4, 5],

$$\tilde{p}^i_j = \begin{bmatrix} \tilde{x}^i_j \\ \tilde{y}^i_j \\ \tilde{z}^i_j \end{bmatrix} = \tilde{R} \begin{bmatrix} \tilde{x}^{i+1}_j \\ \tilde{y}^{i+1}_j \\ \tilde{z}^{i+1}_j \end{bmatrix} + \tilde{T} = \tilde{R}\,\tilde{p}^{i+1}_j + \tilde{T}. \quad (1)$$

where $\tilde{p}^i_j$ and $\tilde{p}^{i+1}_j$ represent the coordinate true values of the same point in Scan i and Scan i+1, respectively, i.e., $(\tilde{x}^i_j; \tilde{y}^i_j; \tilde{z}^i_j)$ and $(\tilde{x}^{i+1}_j; \tilde{y}^{i+1}_j; \tilde{z}^{i+1}_j)$; $\tilde{T}$ is a $3 \times 1$ translation vector,

$$\tilde{T} = \begin{bmatrix} \tilde{t}_x \\ \tilde{t}_y \\ \tilde{t}_z \end{bmatrix}; \quad (2)$$

and $\tilde{R}$ is a 3 × 3 rotation matrix,

$$\tilde{R} = \frac{1}{1+\tilde{a}^2+\tilde{b}^2+\tilde{c}^2}\begin{bmatrix} 1+\tilde{a}^2-\tilde{b}^2-\tilde{c}^2 & 2(\tilde{c}+\tilde{a}\tilde{b}) & 2(\tilde{a}\tilde{c}-\tilde{b}) \\ 2(\tilde{a}\tilde{b}-\tilde{c}) & 1-\tilde{a}^2+\tilde{b}^2-\tilde{c}^2 & 2(\tilde{a}+\tilde{b}\tilde{c}) \\ 2(\tilde{b}+\tilde{a}\tilde{c}) & 2(\tilde{b}\tilde{c}-\tilde{a}) & 1-\tilde{a}^2-\tilde{b}^2+\tilde{c}^2 \end{bmatrix}; \quad (3)$$

$$\tilde{R}^T = \tilde{R}^{-1}; \ |\tilde{R}| = 1: \quad (4)$$

Let ~ = $[\tilde{a}; \tilde{b}; \tilde{c}; \tilde{t}_x; \tilde{t}_y; \tilde{t}_z]^T$ be the vector of transformation parameters. To uniquely determine ~ between Scan i and Scan i+1, we normally use three or more targets with known 3D coordinates [1, 27], placed in the overlaps between the two point clouds. This paper assumes the number of targets is k(≥ 3), hence

$$\begin{bmatrix} \tilde{p}^i_1 & \tilde{p}^i_2 & \cdots & \tilde{p}^i_k \end{bmatrix} = \begin{bmatrix} \tilde{R}\tilde{p}^{i+1}_1 & \tilde{R}\tilde{p}^{i+1}_2 & \cdots & \tilde{R}\tilde{p}^{i+1}_k \end{bmatrix} + \begin{bmatrix} \tilde{T} & \tilde{T} & \cdots & \tilde{T} \end{bmatrix}: \quad (5)$$

2.2. Error Equation of Target Based Registration Model
Errors inevitably occur in TLS measurements (including instrumental
errors, environmental errors, object related errors, target centroid errors,
saturation errors, blooming errors, etc. [1]). If the observation values of ~pi
j
and ~pi+1
j are pi
j and pi+1
j , respectively, and approximate values of ~R
, ~ T, ~ are
R0, T0, 0 ( 0 = [a0; b0; c0; tx0; ty0; tz0]T can be calculated by the method
in
Appendix C ), then true errors of pi
j , pi+1
j , R0, T0, and 0 are pi
j
, pi+1
j
, R,
T , and respectively, where
~pi
j = pi
j + pi
j
; ~pi+1
j = pi+1
j + pi+1
j
;
~R
= R0 + R0 ; ~ T = T0 + T0 ;
and
~ = 0 + 0 :
Hence, from eq. (5),
vj = R pi+1
j + T .. lj ; (6)
where lj = pi
j .. R0pi+1
j .. T0, j 2 f1; 2; ; kg, vj = ..(R0 pi+1
j
+ R pi+1
j
)
is residual error.
Using the linearization theorem [28],
8>>>><
>>>>:
R dR = @R
@a da + @R
@b db + @R
@c dc
T dT = [dtx; dty; dtz]T
d = [da; db; dc; dtx; dty; dtz]T
; (7)
where dR, dT, d are the approximate values for R, T , , respectively.
we can construct the error equations of the target based registration model
from eqs. (6) and (7),
V B d .. l; (8)
where V and l are 3k 1 matrices, B is a 3k 6 matrix,
V =
2
6664
v1
v2
...
vk
3

7775
;B =
2
6664
B1
B2
...
Bk
3
7775
; l =
2
6664
l1
l2
...
lk
3
7775
; vj dR pi+1
j +dT ..lj = Bj d ..lj ;
(9)
4

89
R0 =
1
1 + a20
+ b20
+ c20
2
4
1 + a20
.. b20
.. c20
2(c0 + a0b0) 2(a0c0 .. b0)
2(a0b0 .. c0) 1 .. a20
+ b20
.. c20
2(a0 + b0c0)
2(b0 + a0c0) 2(b0c0 .. a0) 1 .. a20
.. b20
+ c20
3
5;
(10)
90
T0 =
2
4
tx0
ty0
tz0
3
5; pi+1
j =
2
4
xi+1
j
yi+1
j
zi+1
j
3
5; (11)
91
Bj =

@R
@a pi+1
j
@R
@b pi+1
j
@R
@c pi+1
j E3 3

;E3 3 =
2
4
1 0 0
0 1 0
0 0 1
3
5; (12)
92 8>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>><
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>:
@R
@a =
2
6666664
4a0(b20
+c20
)
(1+a20
+b20
+c20
)2
2b0(1..a20
+b20
+c20
)..4a0c0
(1+a20
+b20
+c20
)2
2c0(1..a20
+b20
+c20
)+4a0b0
(1+a20
+b20
+c20
)2
2b0(1..a20
+b20
+c20
)+4a0c0
(1+a20
+b20
+c20
)2
..4a0(1+b20
)
(1+a20
+b20
+c20
)2
2(1..a20
+b20
+c20)..4a0b0c0
(1+a20+b20
+c20
)2
2c0(1..a20

+b20
+c20
)..4a0b0
(1+a20
+b20
+c20
)2
..2(1..a20
+b20
+c20
)..4a0b0c0
(1+a20
+b20
+c20
)2
..4a0(1+c20
)
(1+a20
+b20
+c20
)2
3
7777775
@R
@b =
2
6666664
..4b0(1+a20
)
(1+a20
+b20
+c20
)2
2a0(1+a20
..b20
+c20
)..4b0c0
(1+a20
+b20
+c20
)2
..2(1+a20
..b20
+c20
)..4a0b0c0
(1+a20
+b20
+c20
)2
2a0(1+a20
..b20
+c20
)+4b0c0
(1+a20
+b20
+c20
)2
4b0(a20
+c20)
(1+a20
+b20+c20
)2
2c0(1+a20
..b20
+c20
)..4a0b0
(1+a20
+b20

+c20
)2
2(1+a20
..b20
+c20
)..4a0b0c0
(1+a20
+b20
+c20
)2
2c0(1+a20
..b20
+c20
)+4a0b0
(1+a20
+b20
+c20
)2
..4b0(1+c20
)
(1+a20
+b20
+c20
)2
3
7777775
@R
@c =
2
6666664
..4c0(1+a20
)
(1+a20
+b20
+c20
)2
2(1+a20
+b20
..c20
)..4a0b0c0
(1+a20
+b20
+c20
)2
2a0(1+a20
+b20
..c20
)+4b0c0
(1+a20
+b20
+c20
)2
..2(1+a20
+b20
..c20
)..4a0b0c0
(1+a20
+b20+c20
)2
..4c0(1+b20
)
(1+a20
+b20
+c20
)2
2b0(1+a20
+b20
..c20

)..4a0c0
(1+a20
+b20
+c20
)2
2a0(1+a20
+b20
..c20
)..4b0c0
(1+a20
+b20
+c20
)2
2b0(1+a20
+b20
..c20
)+4a0c0
(1+a20
+b20
+c20
)2
4c0(a20
+b20
)
(1+a20
+b20
+c20
)2
3
7777775
:
(13)

Assuming the weight matrix of l is P, by using the principle of indirect adjustment [28] and V TPV = min, we can obtain estimated ^ , ^R
, T^ for
5

transformation parameters ~ , ~R
, T~ as
~ ^ =

^a ^b
^c ^ tx ^ ty ^ tz
T
= 0 + d ; (14)
d = (BTPB)..1BTPl; (15)
and
~R
^R
=
1
1 + ^a2 +^b
2 + ^c2
2
4
1 + ^a2 ..^b
2 .. ^c2 2(^a^b
+ ^c) 2(^a^c ..^b
)
2(^a^b
.. ^c) 1 .. ^a2 +^b
2 .. ^c2 2(^b
^c + ^a)
2(^a^c +^b
) 2(^b
^c .. ^a) 1 .. ^a2 ..^b

2 + ^c2
3
5;
(16)
~ T ^ T =
2
4
^ tx
^ ty
^ tz
3
5: (17)

If 0 is the unit weight variance (usually determined in initial processing before registration), then from error propagation [28] and eq. (15), the variance and covariance of ^ can be expressed as

D^ ^ = 2
0Q^ ^ = 2
0Qd d = 2
0N..1
BB = 2
0(BTPB)..1; (18)

where D ^ ^ is a 6 6 matrix.

### 2.3. Target based Point Cloud Registration Error Evaluation

We can obtain the actual registration value p^i for any point pi+1 from eqs. (16) and (17),

^pi = ^R
pi+1 + ^ T; (19)

where the registration error of p^i is in
uenced by both ^ and pi+1 precision.
Therefore, partial di erentiation of eq. (19) shows that

d^pi = d^R
pi+1 + d ^ T + ^R
dpi+1; (20)

where

pi+1 =
2
4
xi+1
yi+1
zi+1
3
5;Bpi+1 =
h
@ ^R
@a pi+1 @ ^R
@b pi+1 @ ^R
@c pi+1 E3 3
i
; (21)
6

and

8>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>><
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>:
@ ^R
@a =
2
6666664
4^a(^b
2+^c2)
(1+^a2+^b
2+^c2)2
2^ b(1..^a2+^b
2+^c2)..4^a^c
(1+^a2+^b
2+^c2)2

2^c(1..^a2+^b
2+^c2)+4^a^b
(1+^a2+^b
2+^c2)2
2^ b(1..^a2+^b
2+^c2)+4^a^c
(1+^a2+^b
2+^c2)2
..4^a(1+^b
2)
(1+^a2+^b
2+^c2)2
2(1..^a2+^b
2+^c2)..4^a^b
^c
(1+^a2+^b
2+^c2)2
2^c(1..^a2+^b
2+^c2)..4^a^b
(1+^a2+^b
2+^c2)2
..2(1..^a2+^b
2+^c2)..4^a^b
^c
(1+^a2+^b
2+^c2)2
..4^a(1+^c2)
(1+^a2+^b
2+^c2)2
3
7777775
@ ^R
@b =
2
6666664
..4^ b(1+^a2)
(1+^a2+^b
2+^c2)2
2^a(1+^a2..^b
2+^c2)..4^b
^c
(1+^a2+^b
2+^c2)2
..2(1+^a2..^b
2+^c2)..4^a^b
^c
(1+^a2+^b
2+^c2)2
2^a(1+^a2..^b
2+c^c2)+4^b
^c
(1+^a2+^b
2+^c2)2
4^ b(^a2+^c2)
(1+^a2+^b
2+^c2)2
2^c(1+^a2..^b
2+^c2)..4^a^b
(1+^a2+^b
2+^c2)2
2(1+^a2..^b
2+^c2)..4^a^b
^c
(1+^a2+^b
2+^c2)2
2^c(1+^a2..^b
2+^c2)+4^a^b
(1+^a2+^b

2+^c2)2
..4^b
(1+^c2)
(1+^a2+^b
2+^c2)2
3
7777775
@ ^R
@c =
2
6666664
..4^c(1+^a2)
(1+^a2+^b
2+^c2)2
2(1+^a2+^b
2..^c2)..4^a^b
^c
(1+^a2+^b
2+^c2)2
2^a(1+^a2+^b
2..^c2)+4^b
^c
(1+^a2+^b
2+^c2)2
..2(1+^a2+^b
2..^c2)..4^a^b
^c
(1+^a2+^b
2+^c2)2
..4^c(1+^b
2)
(1+^a2+^b
2+^c2)2
2^b
(1+^a2+^b
2..^c2)..4^a^c
(1+^a2+^b
2+^c2)2
2^a(1+^a2+^b
2..^c2)..4^b
^c
(1+^a2+^b
2+^c2)2
2^ b(1+^a2+^b
2..^c2)+4^a^c
(1+^a2+^b
2+^c2)2
4^c(^a2+^b
2)
(1+^a2+^b
2+^c2)2
3
7777775
:
(22)

Assuming coordinate measurements for any point pi+1 have independent and identical distributions, and the variance of coordinate error of pi+1 is

Dpi+1pi+1 = 2
pi+1 E3 3, then from eq. (20),
D^pi ^pi = DPRE(pi+1) + DORE(pi+1); (23)

DPRE(pi+1) = Bpi+1D^ ^ BT
pi+1; (24)
and
DORE(pi+1) = ^R
Dpi+1pi+1 ^R
T = Dpi+1pi+1; (25)

where Dp^ip^i is the registration error (RE) of pi+1, DPRE(pi+1) is the propagated registration error (PRE) of pi+1, and DORE(pi+1) is the observation registration error (ORE) of pi+1.

From eqs. (23)-(25), RE for any point pi+1 is related to its coordinate value in Scan i+1 (in
uencing Bpi+1), transformation parameter precision
7

(in
uencing D ^ ^), and observation precision (in
uencing Dpi+1pi+1). ORE for pi+1 is unchanged by the transformation.

3. Veri cation

We  rst introduce the experiment method (including constraint conditions), analyze RE model in
uencing factors, and propose a method to verifying RE model accuracy. We then design the experiment to verify that rotation parameters do not in
uence PRE. Finally, based on these outcomes, we design the experiment to verifying the proposed RE model accuracy, and analyze the experimental results.

3.1. Experiment Method

To verify RE model accuracy (eq. (23)), we design several processing schemes with realistic point clouds drawn from previous studies [5] using Riegl VZ-400 laser scanner, as shown in Figs. 1 and 2. The speci c experimental processes are as follows:

Step1: Point cloud extraction.

We included three practical point cloud types. case A: completely within (Fig. 2, red zone), case B: partially within and partially outside (Fig. 2, pink zone), and case C: completely outside (Fig. 2, yellow zone) the targets convex polyhedron. We extracted these three point cloud types from realistic point clouds.

Step2: Constraint conditions.

Similar to [24], we make the following assumptions:

(1) Unit weight variance  0 = 5mm, since Riegl VZ-400 laser scanner acquisition error = 5 mm@50 m [5].

(2) Target coordinate measurement error for Scan i+1 is isotropic, targets are independent and have equal standard deviation. Hence P, target measurement weight matrix, is diagonal matrix with equal diagonal elements.

Step3: Rotation parameter in
uences.

Since ORE is unchanged after transformation (eq. (25)), RE only depends on PRE magnitude (eq. (24)), PRE is related to Bpi+1 and D ^ ^, and Bpi+1 is only related to pi+1 coordinates and ^a, ^b and c^ (eqs. (21) and (22)). Therefore, we need only investigate whether di erent rotation parameter values in
uence PRE (eq. (24)).
8

Appendix A shows that the rotation parameters can be calculated from the rotation angle and axis, hence we can analyze PRE variation by  xing each of these independently.

Step4: Verify RE model accuracy .

We adopt the root mean square error (RMSE) to evaluate true errors magnitude [24]. For any point pi+1 in Scan i+1, we can calculate true registration errors, RMSE, from eqs. (1) and (19) as

RMSE =
vuut
1
m
Xm
s=1

2s
; (26)
where
s = (~R
.. ^R
)pi+1
s + ( ~ T .. ^ T); (27)
and pi+1
s is the s-th sampling value of point pi+1; s = 1; ;m; m is the total
number of random samples.
Thus, we compare RE from eqs. (23)-(25) with RMSE from eqs. (26)-(27).

Figure 1: Experimental target geometry.

9

Figure 2: Measured point cloud. (case A = red, case B = pink, and case C = yellow)

3.2. Rotation parameter in
uences

We randomly generate 1000 rotation axes for a xed rotation angle (eqs. (A.1) and (A.2)) and calculate D ^ ^ from eqs. (12), (13), and (18) using target observations. We then calculate target PRE, targets barycenter PRE, and point cloud barycenter PRE for case A, case B, case C using eqs. (21), (22), and (24), respectively. Similarly, we randomly generate 1000 rotation angles for a xed rotation axis, and calculate D ^ ^, target PRE, targets barycenter PRE, and point cloud barycenter PRE.

Figure 3 and Table 1 show that rotation parameters have no PRE in-
uence for any point, and PRE is inversely proportional to distance to the targets barycenter. Thus, target registration errors are not equal to point cloud registration errors.

10

Table 1 The relationship between the position and PRE.

| Position | | Distance (to Barycenter of targets) | Ratio (PRE to 0) |
|---|---|---|---|
| target01 | | 45.393m | 1.248 |
| target02 | | 36.263m | 1.161 |
| target03 | | 32.980m | 1.104 |
| target04 | | 14.745m | 0.840 |
| target05 | | 34.768m | 1.083 |
| point cloud barycenter | case A | 8.151m | 0.797 |
| | case B | 56.018m | 1.439 |
| | case C | 104.285 | 2.552 |
| targets barycenter | | 0 | 0.775 |

Figure 3: Rotation parameter in
uence. (Each x axis value represents a di erent rotation matrix case, i.e. di erent rotatin angle and axis; Each y axis represents a ratio of PRE to 0)

3.3. RE model accuracy

We calculate D ^ ^ from eqs. (12), (13), and (18) using target observations, and randomly generate 1000 di erent approximate errors, d , for the transformation parameters using D ^ ^. Since RE is independent of the rotation

11

parameters (Section 3.2), we can assume
~ = 0 =

0 0 0 100 100 100
T
: (28)
We then calculate 1000 di erent ^, and the RMSE for all points in case

A, B, C point clouds from eqs. (26), (27), (2), (3), (16), and (17). Finally, we set ^ = 0, and calculate RE for all points in case A, B, C point clouds from eqs. (21)..(25).

Figures 4 and 5 compare the RE and RMSE outcomes for the various cases. Maximum RE and RMSE di erences are less than -0.022 0, -0.035 0, and 0.03 0 for in case A, B, C, respectively. These di erences are su ciently small that we can consider RE RMSE, i.e., the proposed RE model is correct.

Commercial software can only calculate target registration errors of targets, and for these experimental data, target registration error calculated by Leica cyclone are 1:163 0, 1:070 0, 0:998 0, 0:746 0, and 0:962 0, for targets 01, 02, 03, 04, and 05, respectively. Each point in the point cloud has di erent accuracy, which cannot be evaluated by several numerical values (such as target registration errors). Hence, the proposed RE model is superior to current commercial software to evaluate point cloud registration error.

12

Figure 4: The di erence between RE and RMSE. (Each x axis value represents a di erent point in the point cloud of case A, B, C; Each y axis represents a ratio of RE-RMSE to 0)

13

Figure 5: Point cloud registration error from the proposed method (RE) for case A, B, C point cloud. (Each x axis value represents a di erent point; Each y axis represents a ratio of RE to 0)

14

## 4. Conclusion

This paper investigate point cloud registration error (RE) magnitude analytically, and derive a new competent evaluation model of point cloud RE model. We verify the registration error from the proposed RE model and the true error statistics RMSE are signi cantly smaller (<0.035 0). Thus, the proposed RE model can directly evaluate point cloud registration error.

Several relevant conclusions are evident: (1) Registration error (RE) for any point in space included propagated registration error (PRE) and observation registration error (ORE); (2) ORE for any point in a point cloud is only related to its observation precision, and is unchanged after registration, provided coordinate measurements for any point have independent and identical distribution; (3) PRE for any point in a point cloud is related to its position and registration parameter precisions, but is independent of rotation parameters; (4) PRE is related to the distance from the targets barycenter, i.e., increased PRE with increasing distance, thus the commercial evaluation models of point cloud registration error are only suitable to evaluate target registration errors, and are unsuitable to evaluate point cloud registration errors.

However, it should be noted that "before we use the proposed model, the coordinates information of targets need to be extracted using feature extraction algorithms", "our model is only suitable to evaluate the feature based registration error, including sphere target, plane target, natural features, building corner, etc.", "the relationship between the PRE and the rotation-parameter requires further analytical investigation" and "we do not consider the e ects of linearization errors or coe cient matrix errors".

Appendix A. Rotation Parameters from Rotation Axis and Angle

Following [5, 12], if the rotation angle is and rotation axis is ~n, then

we
can express the quaternions, q, of rotation-matrix ~R
as
q =

cos
2
sin
2 ~n

: (A.1)
and hence the rotation parameters are
2
4
~a
~b
~c
3
5 = tan

2
~n: (A.2)
15

Appendix B. Lodrigues Matrix
The Lodrigues Matrix [12] is a rotation matrix composed of real skew symmetric matrix, and we can express the Lodrigues Matrix of the rotationmatrix
~R
as
~R
= (E3 3 + ~ S)..1(E3 3 .. ~ S) = (E3 3 .. ~ S)(E3 3 + ~ S)..1; (B.1)
where S~ is a real skew symmetric matrix, and
~ S =
2
4
0 ..~c ~b
~c 0 ..~a
..~b
~a 0
3
5: (B.2)

Thus, from eq. (3) and eq. (B.2), we can get

(E3 3 + ~ S)~R
= E3 3 .. ~ S: (B.3)

Assuming T~ = 0, from eq. (1) and eq. (B.3), we can get
(E3 3 + ~ S)
2
4
~xi
~yi
~zi
3
5 = (E3 3 .. ~ S)
2
4
~xi+1
~yi+1
~zi+1
3
5: (B.4)
and hence,
2
4
0 ..(~zi + ~zi+1) ~yi + ~yi+1

~zi + ~zi+1 0 ..(~xi + ~xi+1)
..(~yi + ~yi+1) ~xi + ~xi+1 0
3
5
2
4
~a
~b
~c
3
5 =
2
4
~xi+1 .. ~xi
~yi+1 .. ~yi
~zi+1 .. ~zi
3
5:
(B.5)

## Appendix C. Approximate Target Transformation Parameters

We can compute the approximation 0 = [a0; b0; c0; tx0; ty0; tz0]T of ~ from eq. (B.5) [12] using the following steps

Step1: Compute targts barycenter coordinates,
pi
c =
2
664
Pk
j=1 xi
j
P k k
j=1 yij
P k k
j=1 zij
k
3
775
; pi+1
c =
2
664
Pk
j=1 xi+1
j
P k k
j=1 yi+1
j
P k k
j=1 zi+1
j
k
3
775
: (C.1)
16

Step2: Centralize the target coordinates,
pi
jc = pi
j .. pi
c; pi+1
c = pi+1
j .. pi+1
c : (C.2)

Step3: Calculate the coe cient matrices from the centralized target coordinates,
AC =

$$A_c = \begin{bmatrix} A_{1c} \\ \vdots \\ A_{kc} \end{bmatrix}; \quad \text{(C.3)}$$

$$l_c = \begin{bmatrix} l_{1c} \\ \vdots \\ l_{kc} \end{bmatrix}; \quad \text{(C.4)}$$

where $A_c$ is a $3k \times 3$ matrix, $l_c$ is a $3k \times 1$ matrix, $j = 1, \cdots, k$, and

$$A_{jc} = \begin{bmatrix} 0 & -(z_{ijc} + z_{i+1jc}) & y_{ijc} + y_{i+1jc} \\ z_{ijc} + z_{i+1jc} & 0 & -(x_{ijc} + x_{i+1jc}) \\ -(y_{ijc} + y_{i+1jc}) & x_{ijc} + x_{i+1jc} & 0 \end{bmatrix}; \quad \text{(C.5)}$$

$$l_{jc} = \begin{bmatrix} x_{i+1jc} - x_{ijc} \\ y_{i+1jc} - y_{ijc} \\ z_{i+1jc} - z_{ijc} \end{bmatrix}; \quad \text{(C.6)}$$

Step4: Compute approximate rotation parameters

$$\begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix} = (A_c^T P A_c)^{-1} A_c^T P l_c; \quad \text{(C.7)}$$

where $P$ is the target weight matrix.

Step5: Compute the approximate rotation matrix, $R_0$, from eq. (10).

Step6: Compute the approximate translation parameters

4
tx0
ty0
tz0
3
5 = pi+1
c .. R0pi
c; (C.8)



## Acknowledgment

This work is supported by the National Natural Science Foundation of China (grant 41304001), Chongqing Natural Science Foundation and Frontier Research Planning Project (grant cstc2014jcyjA00011), Chongqing Postdoctoral Research Project (grant xm2017097), Chongqing Natural Science Foundation and Technology Innovation Special Project of Social Undertaking and Peoples Livelihood Guarantee (grant cstc2016shmszx0299).

## References

[1] Y. Reshetyuk, 2009. Self-calibration and direct georeferencing in terrestrial laser scanning. Ph.D. dissertation, Royal Institute of Technology, Swedish.

[2] Y. Zhang, 2012. Research on error propagation of point cloud registration. In: Proceedings of IEEE International conference on Computer Science and Automation Engineering (CSAE), vol. 2, Zhangjiajie, China, pp. 18-21, 25-27 May, 2012.

[3] D. Lichti, J. Skaloud, 2010. Registration and calibration (Chapter 3). In: G. Vosselman, H.G. Maas (Eds.). Airborne and terrestrial laser scanning, Whittles Publishing, ISBN: 190444587X. 2010.

[4] Y. Zhang, 2008. Research on point cloud processing of terrestrial laser scanning. Ph.D. dissertation, Wuhan University, Wuhan China.

[5] R.H. Yang, 2011. Research on point cloud angular resolution and processing model of terrestrial laser scanning. Ph.D. dissertation, Wuhan University, Wuhan China.

[6] M.W.Walker, L.J. Shao, 1991. Estimating 3-D location parameters using dual number quaternions. CVGIP: Image Understanding, vol. 54, no. 3, pp. 358-367.

[7] D.W. Eggert, A. Lorusso, R.B. Fisher, 1997. Estimating 3-D rigid body transformations: a comparison of four major algorithms. Machine vision and applications, vol. 9, no. 5-6, pp. 272-290.



[8] K.S. Arun, T.S. Huang, S.D. Blostein, 1987. Least-squares  tting of two 3D point sets. IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 9, no. 5, pp. 698-700.

[9] O.D. Faugeras, M. Hebert, 1986. The representation, recognition, and locating of 3-D objects. The International Journal of Robotics Research, vol. 5, no. 3, pp. 27-52.

[10] B.K.P. Horn, 1987. Closed-form solution of absolute orientation using unit quaternions. Journal of the Optical Society of America A, vol. 4, no. 4, pp. 629-642.

[11] B.K.P. Horn, H.M. Hilden, S. Negahdaripour, 1988. Closed-form solution of absolute orientation using orthonormal matrices. Journal of the Optical Society of America A, vol. 5, no. 7, pp. 1127-1135.

[12] Jili Yao, Yufei Xu, Wei Xiao, 2007. Application of Lodrigues Matrix in 3D coordinate transformation. Geomatics and information science of Wuhan University, vol. 10, no. 3, pp. 173-176.

[13] J. Salvi, C. Matabosch, D. Fo , J. Forest, 2007. A review of recent range image registration methods with accuracy evaluation. Image and vision computing, vol. 25, no. 5, pp. 578-596.

[14] P.J. Besl, H.D. Mckay, 1992. A method for registration of 3-D shapes. IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 14, no. 2, pp. 239-256.

[15] J.P. Tarel, H. Civi, D.B. Cooper, 1998. Pose estimation of free-form 3D objects without point matching using algebraic surface models. In: Proceedings of IEEE Workshop on Model-based 3D, pp. 13-21.

[16] I. Stamos, M. Leordeanu, 2003. Automated feature-based range registra-

tion of urban scenes of large scale. In: Proceedings of IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 2003, pp. 555-561.

[17] D.H. Chuang, I.D. Yun, S.U. Lee, 1998. Registration of multiple-range views using the reverse-calibration technique. Pattern Recognition, vol. 31, no. 4, pp. 457-464.



[18] O. Carmichael, D.F. Huber, M. Hebert, 1999. Large data sets and confusing scenes in 3-D surface matching and recognition. In: Proceedings of 2nd International Conference on 3D Digital Imaging and Modeling, 1999, pp. 358-367.

[19] C.K. Chow, H.T. Tsui, T. Lee, 2004. Surface registration using a dynamic genetic algorithm. Pattern Recognition, vol. 37, no. 1, pp. 105-117.

[20] T. Zinsser, J. Schmidt, H. Niemann, 2003. A re ned ICP algorithm for robust 3-D correspondence estimation. In: Proceedings of IEEE International Conference on Image Processing, 2003, pp. 695-698.

[21] G.L. Wang, 2006. Research on registration of ground lidar range images. Masters dissertation. Beijing University of Civil Engineering and Architecture, Beijing China.

[22] G.C. Sharp, S.W. Lee, D.K. Wehe, 2004. Multiview registraion of 3D scenes by minimizing error between coordinate frames. IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 26, no. 8, pp. 1037-1050.

[23] D.D. Lichti, S.J. Gordon, T. Tipdecho, 2005. Error models and propagation in directly georeferenced terrestrial laser scanner networks. Journal of Surveying Engineering, vol. 131, no. 4, pp. 135-142.

[24] L. Fan, J.A. Smethurst, P.M. Atkinson, W. Powrie, 2015. Error in target-based georeferencing and registration in terrestrial laser scanning. Computers and Geosciences, vol. 83, pp. 54-64.

[25] S.J. Gordon, D.D. Lichti, 2004. Terrestrial laser scanners with a narrow eld of view: the e ect on 3D resection solutions. Survey review, vol. 37, no. 292, pp. 448-468.

[26] B.R. Harvey, 2004. Registration and transformation of multiple site terrestrial laser scanning. Geomatics Research Australasia, no.80, pp. 33-50.

[27] L. Bornaz, A. Lingus, F. Rinaudo, 2003. Multiple scan registration in LiDAR close- range applications. In: Proceedings of the International Archives of the Photogrammery: Remote Sensing and Spatial Information Sciences, vol. XXXIV, 2003, Part 5/W12, pp. 72-77.



[28] W.N. Qiu, B.Z. Tao, Y.B. Yao, Y. Wu, H.L. Huang, 2008. The theory and method of surveying data processing. Wuhan University press, 1st ed., Wuhan China, pp. 1-6.