A Current Sensorless Computationally Efficient Model Predictive Control for Matrix Converters

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Abstract—Model Predictive Control (MPC) is becoming more popular than ever as an alternative to conventional modulations such as Space Vector Modulation methods to control matrix converters (MCs). However, the implementation of MPC is computationally expensive, because control objectives are required to evaluate all admissible switching states of the converter. Additionally, a large number of sensors to measure the 3-phase load currents, source currents, source voltages, and input voltages of MCs increases the overall cost. To sort this out, an efficient MPC is proposed for MCs to enable fast computation and low cost. This approach eliminates the calculations of future load currents and source currents for all possible switching states, requiring only two predictions for the calculation of output voltage and input current references. Further, it removes all current sensors by employing a Luenberger observer. A simulation study has demonstrated that the proposed method can reduce the computation overhead and hardware cost dramatically, leading to high-frequency operation and good converter performance.

Index Terms—Matrix converter, model predictive control, Luenberger observer.

I. INTRODUCTION

Matrix converter (MC) is a future topology providing advantages of compact design, sinusoidal input/output currents with controllable input power factor and bidirectional power flow [1]–[4]. In order to attain good performance in the MCs, it is imperative to use appropriate modulation techniques and control methods [5]–[7]. Since Alesina and Venturini presented the first modulation strategy for MCs [8], [9], Literature reports several strategies such as scalar modulation, space vector modulation, carrier-based modulation, direct torque control and model predictive control [10].

Model predictive control is one of the emerging solutions that are becoming widely accepted [11]. MPC selects the switching state in MCs that generates the least error between the reference value and predicted value the following period from the 27 possible switching states. MPC methods have the advantage of achieving fast dynamic response without a modulator. A Finite Control Set Model Predictive Control (FCS-MPC) method is a type of MPC method which has been widely applied in recent years due to the intuitive nature of its concept and implementation. In FCS-MPC the discrete nature of the power converter is taken into account, and the multi-objective optimization problem is solved over a finite switching state of the converter. The control action in every control interval is to actuate only one switching state and requires no modulation. Consequently, the switching frequency in FCS-MPC is not constant, and a much shorter control time interval is needed to achieve the same quality of control as modulation-based control [12]. However, the duration required for FCS-MPC calculation makes it difficult to shorten control time intervals. The number of switching states of the converter increases the computation overhead. Adding more constraints or control objectives also affects the calculation time for FCS-MPC. In the case of a matrix converter, FCS-MPC would require 27 times predictions of load currents and 27 times predictions of input reactive power or source currents. Additionally, a cost function has to be calculated 27 times to find the optimal switching state. Moreover, FCS-MPC requires modelling of the load, MC, and input filter, thus making its implementation complex. FCS-MPC implementation for matrix converters requires powerful hardware, which limits their industrial applications [13].

The computational effort in MPC is reduced by a variety of methods in recent years [14]–[20]. The introduced techniques in [14], [15] present methods that reduce the computational
burden of MPC for long-term prediction horizons. In [16], the concept of DTC method has been used to decrease the number of candidate switching states for prediction. MPC can be streamlined in several other ways, such as using a graphical algorithm [17], a modified switching method [18], a double-vector approach [19], and considering a subset of adjacent vectors [20]. In [21], [22], a new approach has been developed to reduce the computational burden of MPCs. In these methods, rather than predicting control objectives for all possible voltage vectors for the power converter, one prediction is made to determine the voltage vectors required to achieve the desired control objectives. Similar technique has been proposed in [23] for matrix converters. By using this method, the number of candidate vectors decreases from 27 to 10, which results in higher ripple contents in comparison with the typical 27-vector FCS-MPC method. An interesting method is described in [24], in which delta-sigma modulation (DSM) and MPC are used to control MC. The technique aims to minimize the integrated error of a sinusoidal reference signal and a discrete PWM signal within a hysteresis band. However, a number of look-up tables are required in order to select the best vectors. In addition, MPC-based methods can outperform DSM due to the hysteresis band control used in DSM.

For the industrial applications of matrix converters, reducing manufacturer costs is one of the major issues, and current sensors directly contribute to that cost. As a result, the development of sensorless control methods are gaining more attention lately. The sliding mode observer, the Luenberger observer, and the Kalman filter are some examples of observers in this context [25], [26]. In order to perform the Kalman filter, many computations are required, including recursive optimization and matrix inversion. The sliding mode observer requires less time. Since it only requires a sign calculation and a gain multiplication. The Luenberger observer, however, only requires a gain multiplication, which requires less computing time than other observers [27].

By extending the ideas introduced in [23], this paper simplifies the implementation of FCS-MPC on a matrix converter in three ways: 1) instead of prediction of load currents and source currents for all switching states, predicting the desired output voltage reference and input current reference for MC, 2) presenting two methods to generate source current references which one of them does not require mathematical modelling of the input filter, and 3) using the Luenberger observer in order to remove all current sensor requirements involving in the traditional MPC control method of MCs. Based on simulation results, it appears that the proposed FCS-MPC algorithm drastically reduced the calculation effort without compromising control performance. Also, the observer can accurately estimate both of the source current and load current under various operating conditions.

II. MATRIX CONVERTER SYSTEM DESCRIPTION

Fig. 1. shows a three-phase to three phase matrix converter, a RL load, and an input filter that are studied in this paper.

| Table I
| POSSIBLE SWITCHING CONFIGURATION OF THE MC |
| State | A | B | C | \(v_o\) | \(i_t\) |
| -1 | a | b | b | 2/3\(v_{ab}\) | 2/\sqrt{3}\(v_{a}\) |
| -2 | c | b | b | -2/3\(v_{ab}\) | -2/\sqrt{3}\(v_{a}\) |
| 1 | c | a | a | 2/3\(v_{ac}\) | 2/\sqrt{3}\(v_{a}\) |
| -3 | b | a | a | -2/3\(v_{ab}\) | -2/\sqrt{3}\(v_{a}\) |
| 2 | b | c | c | 2/3\(v_{bc}\) | 2/\sqrt{3}\(v_{a}\) |
| -4 | b | a | a | -2/3\(v_{ab}\) | -2/\sqrt{3}\(v_{a}\) |
| 3 | a | a | c | 2/3\(v_{ac}\) | 2/\sqrt{3}\(v_{a}\) |
| -5 | a | b | a | -2/3\(v_{ab}\) | -2/\sqrt{3}\(v_{a}\) |
| 4 | a | a | a | 2/3\(v_{ac}\) | 2/\sqrt{3}\(v_{a}\) |
| -6 | c | c | c | -2/3\(v_{ac}\) | -2/\sqrt{3}\(v_{a}\) |
| 5 | b | b | b | 2/3\(v_{bc}\) | 2/\sqrt{3}\(v_{a}\) |
| -7 | a | a | b | -2/3\(v_{ab}\) | -2/\sqrt{3}\(v_{a}\) |
| 6 | c | c | b | 2/3\(v_{bc}\) | 2/\sqrt{3}\(v_{a}\) |
| -8 | b | b | c | -2/3\(v_{bc}\) | -2/\sqrt{3}\(v_{a}\) |
| 7 | a | a | b | 2/3\(v_{ab}\) | 2/\sqrt{3}\(v_{a}\) |
| -9 | c | c | a | -2/3\(v_{ac}\) | -2/\sqrt{3}\(v_{a}\) |
| 8 | b | b | b | 2/3\(v_{bc}\) | 2/\sqrt{3}\(v_{a}\) |
| -10 | a | a | a | 0 | 0 |
| 9 | b | b | b | 0 | 0 |
| -11 | c | c | c | 0 | 0 |
| 10 | a | a | a | 0 | 0 |
| -12 | c | b | b | 0 | 0 |
| 11 | a | b | b | 0 | 0 |

As shown in Fig. 1, the space vectors of the input current and output voltage of the matrix converter are defined by (1) and (2), respectively.

\[ i_t = \frac{2}{3} (i_{ta} + a \cdot i_{tb} + a^2 \cdot i_{tc}) = |i_t|e^{\beta t} \]  

\[ v_o = \frac{2}{3} (v_{oa} + a \cdot v_{ob} + a^2 \cdot v_{oc}) = |v_o|e^{\alpha t} \]  

where \( a = e^{j\frac{2\pi}{3}} \) and \( i_{ta} \), \( i_{tb} \) and \( i_{tc} \) are input phase currents of the MCs and \( v_{oa} \), \( v_{ob} \) and \( v_{oc} \) are output phase voltages of the MCs. \( \beta \) is the angle of input current vector and \( \alpha \) is the angle of output voltage vector. There is also an equation for the source voltage vector \( v_s \), the source current vector is \( i_s \) and the output current vector \( i_o \). Following is a relationship between MC’s output voltage and its input voltage:

\[ v_o = \begin{bmatrix} v_{oa} \\ v_{ob} \\ v_{oc} \end{bmatrix} = \begin{bmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ba} & S_{bb} & S_{bc} \\ S_{ca} & S_{cb} & S_{cc} \end{bmatrix} \begin{bmatrix} i_{ta} \\ i_{tb} \\ i_{tc} \end{bmatrix} = S \cdot i_s \]  

The input currents and output currents of MCs are related as follows:

\[ i = \begin{bmatrix} i_{ta} \\ i_{tb} \\ i_{tc} \end{bmatrix} = \begin{bmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ba} & S_{bb} & S_{bc} \\ S_{ca} & S_{cb} & S_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = S^T \cdot i_o \]
where $S^T$ is the transpose of matrix $S$ and $S_{xy}$ with $x \in \{a, b, c\}$ and $y \in \{a, b, c\}$, is the state of each switch as

$$S_{xy} = \begin{cases} 0 & \text{if switch off} \\ 1 & \text{if switch on} \end{cases}$$ (5)

There are two switching restrictions should be taken into consideration for the matrix converter in Fig 1. Since the input side of the matrix converter is connected to a voltage source, the input side of the matrix converter must not be short-circuited, and the output side cannot be open-circuited, because of the inductive nature of the load. The matrix converter is limited to 27 switching states based on these restrictions as shown in Table I.

III. CONVENTIONAL FCS-MPC FOR A MATRIX CONVERTER

According to the conventional FCS-MPC, a matrix converter feeding a RL load is controlled by predicting load currents and source currents for all possible switching states, comparing the predicted variables with their respective reference values, and determining the best switching state [28]. In order to predict load current behavior for a given voltage vector, a discrete-time model of the load is needed. In a stationary reference frame, the load current equation is as follows:

$$L \frac{di_o}{dt} = v_o - Ri_o$$ (6)

where $R$ is the load resistance and $L$ is the load inductance. Using forward-Euler approximation, (6) can be expressed as follows

$$i_{o,\text{ref}}(k+1) = (1 - \frac{RT_o}{L}) i_o(k) + \frac{T_o}{L} v_o(k)$$ (7)

where $i_o(k)$ and $i_{o,\text{ref}}(k+1)$ are the load currents at instants $k$ and $k+1$ respectively for $\text{sw} \in \{1, 2, 3, \ldots, 27\}$. A discrete-time model of the input filter can be used to predict the source current. According to Fig. 1, the input filter can be represented as a continuous-time space-state model:

$$\begin{bmatrix} v_i \\ i_i \end{bmatrix} = A \begin{bmatrix} v_i \\ i_i \end{bmatrix} + B \begin{bmatrix} v_s \\ i_i \end{bmatrix}$$ (8)

where

$$A = \begin{bmatrix} 0 & \frac{1}{L_f} \\ -\frac{1}{L_f} & -\frac{1}{L_f} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -\frac{1}{L_f} \\ \frac{1}{L_f} & 0 \end{bmatrix}$$ (9)

where $C_f$ and $L_f$ are the input filter capacitance and inductance, respectively, and $R_f$ is the leakage resistance of $L_f$. The discrete-time state-space form of (8) is as follows

$$\begin{bmatrix} v_i(k+1) \\ i_i(k+1) \end{bmatrix} = A \begin{bmatrix} v_i(k) \\ i_i(k) \end{bmatrix} + B \begin{bmatrix} v_s(k) \\ i_i(k) \end{bmatrix}$$ (10)

where

$$A_d = e^{AT_o}, \quad B_d = \int_0^{T_o} e^{A(T_o - \tau)} Bd\tau$$ (11)

The prediction of the source current is obtained by solving $i_{s,\text{ref}}(k+1)$ from (8) as

$$i_{s,\text{ref}}(k+1) = A_d(2, 1)v_i(k) + A_d(2, 2)i_i(k) + B_d(2, 1)v_s(k) + B_d(2, 2)i_{i,\text{ref}}(k)$$ (12)

According to equation (12), the source current at instant $k + 1$ is related to the matrix converter input current vector of $i_i$ that can be determined by that switching state $\text{sw}$ and the output current according to (4).

For all possible switching states of the matrix converter, assuming each switching state is applied from instant $k$ to instant $k + 1$, (7) and (12) are used to determine the load current and source current at instant $k + 1$. To determine the best switching state, these predicted variables are compared with their respective references. To accomplish this, a classical cost function is as follows:

$$CF_{sw} = |i_o^* - i_o(k+1)| + k_C |i_o^* - i_o(k)|$$ (13)

where $i_o^*$ is the source current reference value and $i_o$ is the load current reference value, respectively. $k_C$ is a weighting factor that is calibrated empirically to achieve the desired system performance. The conventional FCS-MPC is realized by evaluating the cost function (13) for all viable switching states of the matrix converter and then selecting the state that minimizes it. Accordingly, FCS-MPC for a matrix converter entails 27 predictions of load current, 27 predictions of source current, and 27 cost function evaluations. Due to the computational expenses, these calculations require powerful hardware and low sampling frequency. Due to this shortcoming, FCS-MPC cannot be used in matrix converters in industries.

IV. PROPOSED FCS-MPC FOR A MATRIX CONVERTER

A. Prediction simplification

The proposed method intends to simplify FCS-MPC. Instead of using 27 load current predictions and 27 source current predictions for instant $k + 1$, it uses the desired output voltage vector $v_o^*$ and the input current vector $i_i^*$ for prediction. In order to obtain the load reference current and input reference current at instant $k + 1$, the desired output voltage vector $v_o^*$
and input current vector \( i^*_n \) can be determined from (14) and (15), respectively, as follows:

\[
v^*_n(k) = \frac{L}{T_i} \left[ i^*_n(k+1) - i_n(k) \right] + R i_n(k) \tag{14}
\]

\[
i^*_n(k) = \left( i^*_n(k+1) - A_d(2,1) v_n(k) - A_d(2,2) i_n(k) - B_d(2,1) v_x(k) \right) \frac{1}{B_d(2,2)} \tag{15}
\]

Based on (14), the load current at instant \( k+1 \) will be the same as its reference current if the voltage vector exerted by the converter onto the load at instant \( k \) is the same as \( v^*_n(k) \). The matrix converter can produce only 27 voltage vectors, which may not be exactly the same as the targeted voltage vector. Nevertheless, the most appropriate method to reduce the error between the load current and its reference at the end of the next sampling time is to apply the voltage vector nearest to the desired voltage vector. The same procedure can be used to control source current by using (15). If the input current vector of the matrix converter at instant \( k \) is the same as \( i^*_n(k) \), then the source current at instant \( k+1 \) will be the same as that of the reference source current. It should be mentioned that \( i_n, v_n \) and \( v_x \) need to be measured by sensors. For generating the source current reference of \( i^*_n(k+1) \), two methods will be explained in subsequent sections.

In order to control the load current and the source current simultaneously, a cost function is required to decide the best switching state which can be defined as,

\[
C_F = |v^*_n(k) - v_{ref}| + k_r |i^*_n(k) - i_{ref}| \tag{16}
\]

All 27 switching states can be calculated for the cost function (16) and the one that minimizes the cost function is then applied to the matrix converter. Hence it is unnecessary to calculate 27 load current predictions and 27 source current predictions, since one prediction is required to calculate the output voltage vector, and another prediction to calculate the input current vector.

**B. Current reference calculation**

In order to calculate Eq. (15), the source current reference should be generated first.

1) **Method I:** Because of the voltage drop at the input filter, the voltage drop of a matrix converter \( v_i \) and the source voltage \( v_x \) are considered to be equal. Due to the absence of energy storage components in the matrix converter circuit, the power balance of \( (P_{in} = P_{out}) \) is always valid if the converter losses are neglected. Therefore, the source current reference can be derived as below,

\[
i^*_n = \left( R \times (\bar{I}^*_{an} + \bar{I}^*_{bn} + \bar{I}^*_{cn}) \right) \times \frac{v_{sn}}{v_{sa}^2 + v_{sb}^2 + v_{sc}^2} \tag{17}
\]

To determine the reference source current of the MC, this method requires the load parameters, the output current reference and the source voltage of the MC. As evident, mathematical modeling of input filter for source current generation is not required when Method I is employed.

**C. Source Current and load current Estimations**

To control matrix converter, both of source currents and load currents need to be measured or estimated. In this case, by applying Luenberger observer, the source currents and load currents are estimated and then used in the (15) and (14) in order to calculate input current reference and output voltage reference of matrix converter, respectively. In order to implementing the observer, the mathematical model of the input filter and RL load are required as,
TABLE III
THD OF THE SOURCE CURRENT AND LOAD CURRENT

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<tr>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Source Current THD (%)</td>
<td>10.48</td>
<td>4.23</td>
<td>3.89</td>
<td>5.59</td>
<td>2.1</td>
</tr>
<tr>
<td>Load current THD (%)</td>
<td>0.24</td>
<td>0.38</td>
<td>0.45</td>
<td>0.77</td>
<td>0.35</td>
</tr>
</tbody>
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\[
\begin{align*}
\frac{d\hat{i}_s}{dt} &= \frac{1}{L_f}(v_s(k) - v_i(k) - R_f i_s(k)) \\
\frac{d\hat{i}_i}{dt} &= \frac{1}{L_f}(i_s(k) - i_i(k)) \\
\frac{d\hat{i}_o}{dt} &= \frac{1}{L}(v_o(k) - R_i o(k))
\end{align*}
\]

(19)

It should be mentioned that \(i_s(k)\) and \(v_o(k)\) can be calculated by using \(i_s(k) = M^T \times i_o(k)\) and \(v_o(k) = M \times v_i(k)\), respectively. \(M\) and \(M^T\) are switching function of matrix converter and its transpose which can be found by the final switching information. The equation of estimator for the studied system is introduced as:

\[
\frac{d}{dt} \begin{bmatrix} \hat{i}_s \\ \hat{v}_i \\ \hat{i}_o \end{bmatrix} = \begin{bmatrix} \frac{1}{L_f}(v_s(k) - v_i(k) - R_f i_s(k)) \\ \frac{1}{L_f}(i_s(k) - i_i(k)) \\ \frac{1}{L}(v_o(k) - R_i o(k)) \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}(v_i - \hat{v}_i)
\]

(20)

where \(\hat{i}_s\), \(\hat{v}_i\) and \(\hat{i}_o\) represent the estimated source current, input voltage and load current of matrix converter, and \(L_1\), \(L_2\) and \(L_3\) are constant coefficients for the Luenberger estimator. The following values, when adjusted in the simulation, lead to the highest precision: \(L_1 = 0.0005\), \(L_2 = 1\) and \(L_3 = 0.0005\).

V. SIMULATION RESULTS

The effectiveness of the proposed method has been verified through simulation studies using MATLAB/Simulink. The system parameters used in [24] are listed in Table II. Fig. 3 depicts the steady state performance of source voltages, source currents, estimated source current by observer, output voltages, output currents and estimated output current by observer when the proposed method is applied for a load current reference of 10 A at 50 Hz. As this control method is implemented in one frame, it is easy to implement the switching between one method and another at any specific time by enabling another method in the next control cycle. Source current generation by the method I is enabled at the beginning of the control cycle, then method II is used at \(t = 0.04\) s, and satisfactory performance including sinusoidal load and source current and near unity input power factor of the proposed method have been obtained. Table III represents the THD values of conventional method [29], predictive DSM [24] and both of the sensor-based and sensorless-based proposed method in steady state when a load current reference of 10 A at 50 Hz is demanded. In order to establish a fair comparison, same load parameters, sampling time and input filter components are considered. When considering the two methods of source current generation and removing all current sensors incorporated in the control algorithm, as can be seen, both load and source currents exhibit acceptable harmonic performance which is completely compliant with IEEE standards. Taking into consideration the lower calculation effort of the proposed scheme and further reducing the sample time to 25\(\mu\)s, this strategy outperforms previous documented modulation schemes of MC.

The performance of the system is further demonstrated during amplitude and frequency dynamics so as to demonstrate the effectiveness of the algorithm. Fig. 4 shows the system response when amplitude and frequency are simultaneously changed. As shown in Fig. 4(a) and (b), which correspond to the use of a sensor and observer, respectively, the frequency and amplitude of the load current are changed from 25 Hz to 50 Hz and their values from 10 A to 5 A, respectively, and satisfactory results are obtained. According to Fig. 5, the effect of varying the load current reference manifest itself on the source side while ensuring near unity input power factor performance. As compared to the modulation technique proposed in [24], [30], [31], the overall performance of proposed method in this paper shows considerable improvement in terms of waveforms quality and Total Harmonic Distortion (THD). Also, neither switching tables nor look-up tables are needed in switching scheme in comparison with previously documented methods.

VI. CONCLUSIONS

An efficient current sensorless model predictive control with less computation is discussed for matrix converters in this
paper. Only two predictions are needed to make the matrix converter fully controllable leading to a drastic reduction of calculation overhead. Furthermore, two methods are presented for generating the source currents that provided a different view than another well-known method. In comparison with method II, method I has a tangible advantage of not requiring a mathematical modelling of input filter, which reduced the sensitivity of control strategy to parameter variations. Another advantage of the proposed method is the use of a Luennberger observer for estimating currents, which enables precise tracking of reference currents on both the load and source sides and completely removes the necessity of using current sensors in the system. However, mathematical modelling of input filter will be required when sensorless control is needed. Simulation results show that the algorithm is capable of working effectively with a variety of supply voltage frequencies and output frequencies, improves load/source current quality, and maintains the input power factor at unity.

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