Journal of Mathematical Economics 101 (2022) 102658

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Information, Bertrand–Edgeworth competition and the law of one price

Robert A. Edwards [a,*,1], Robert R. Routledge [b,1]

[a] Business School, University of Nottingham, Nottingham, NG8 1BB, UK
[b] Management School, University of Liverpool, Chatham Street, Liverpool, L69 7ZH, UK

ARTICLE INFO

Article history:
Received 4 August 2021
Received in revised form 31 January 2022
Accepted 6 February 2022
Available online 14 February 2022
Manuscript handled by Editor Carmen Beviá

JEL classification:
C72
C62
D43
L11

Keywords:
Incomplete information
Bertrand–Edgeworth competition
Ambiguity aversion
Law of one price

ABSTRACT

Homogeneous goods often sell at different prices within the same market. This paper proposes a theoretical foundation for this phenomenon in the context of a capacity-constrained price game. Sellers have asymmetric information about the market demand, modelled by a partition of the state space, and evaluate uncertain profits in a way consistent with ambiguity aversion. We demonstrate that a pure strategy price equilibrium exists if the market demand is uniformly elastic in each state. Interestingly, the sellers may choose different prices, violating the law of one price. Moreover, market demand may be rationed between the sellers, resulting in consumers purchasing at different prices.

1. Introduction

A persistent challenge for Industrial Economists has been to credibly explain homogeneous goods selling at different prices within the same market.[2] The classical models of Bertrand and Bertrand–Edgeworth price competition struggle to explain pure strategy price dispersion, especially where consumers purchase at different prices, because in these models a pure strategy equilibrium would usually occur at the minimum price posted in the market. Therefore, price dispersion has tended to be explained as the outcome of sellers following mixed strategies, as a consequence of incomplete seller information or non-standard consumer decision-making procedures.[3]

The idea that firms randomise their price following a mixed strategy process has always remained contentious. As Friedman (1988, p.608) remarked "it is doubtful that the decision-makers in firms shoot dice as an aid to selecting output or price". In practice, prices also do not appear to oscillate as frequently as is implied by the ex post regret associated with the outcome of a mixed strategy for any firm; a further profitable deviation always exists, theoretically causing prices to perpetually cycle. Furthermore, the use of mixed strategies by individuals has been consistently refuted in experimental examinations of price competition and, in particular, Bertrand–Edgeworth competition where sellers face capacity constraints (Buchheit and Feltovich, 2011; Fonseca and Normann, 2013; Heymann et al., 2014; Kruse et al., 1994). This motivates our search for a new explanation for non-random price dispersion in markets with capacity-constrained sellers of homogeneous goods.

In this paper we present a novel and intuitively appealing explanation for pure strategy price dispersion arising from sellers

* Corresponding author.
E-mail addresses: Robert.Edwards@Nottingham.ac.uk (R.A. Edwards), R.R.Routledge@Liverpool.ac.uk (R.R. Routledge).

1 We are grateful to the Co-Editor, Carmen Bevia, and two anonymous referees for their very helpful comments and suggestions. The authors would like to thank the participants at EARIE 2019 (Barcelona), OLIGO Workshop 2019 (Nottingham), Economic Theory & Computation Workshop 2019 (Liverpool) and the Liverpool economic research seminar. We are grateful to Christian Bach, Dominique Demougin, Christian Ewerhart, Olga Gorelkina, Yiquan Gu, Erin Hengel, Sergei Izmalkov, Sanna Laksa, Eunyoung Moon, Alexei Parakhonyak, David Ronayne, Attila Tasnádi, Harvey Upton and Yunchou Wu for their comments and encouragement during the writing of this paper.

2 See Baye et al. (2004) for a large empirical study of this phenomenon examining internet prices.

3 Some classic and recent papers using mixed strategies to illustrate price dispersion include Shilony (1977), Varian (1980), Burdett and Judd (1983), Vives (1986), Baye and Morgan (2001) and Janssen and Rasmusen (2002). See Vives (1999) for a textbook treatment of Bertrand–Edgeworth games.

https://doi.org/10.1016/j.jmateco.2022.102658
0304-4068/

holding incomplete and asymmetric information regarding the future market demand.[4] We start with the classical Bertrand–Edgeworth duopoly, where capacity constrained sellers compete directly in prices, and we introduce asymmetric information of the type usually studied in the context of general equilibrium models.[5] The uncertainty that sellers face is modelled by an information partition. Sellers cannot distinguish between demand states within the same partition and prices must be measurable with respect to their private information. Intuitively, this requires that sellers set the same price for future demand states over which they are uncertain. The market demand is distributed in proportion to the sellers' capacities if prices are tied and efficient rationing occurs if different prices are posted in the market and the cheapest seller is unable to satisfy all of their forthcoming demand.

Beyond providing a new explanation for non-random price dispersion, our framework also contributes new results on the existence of pure strategy equilibria in Bertrand–Edgeworth competition under incomplete information. Even with complete information, it is well-known that a pure strategy equilibrium generally fails to exist because it can be profitable for a seller to deviate to a higher price than their rival and sell only to the residual demand that their competitor cannot meet due to their capacity constraint. This induces price cycles, referred to as the Edgeworth paradox (Dasgupta and Maskin, 1986; Dixon, 1992; Maskin, 1986). This problem of non-existence of pure strategy equilibrium is exacerbated by asymmetric information amongst sellers because we must specify how each seller evaluates ex ante uncertain profits. To address this, we consider ambiguity averse sellers with Maximin expected utilities (MEU), following Gilboa and Schmeidler (1989). Using this ex ante decision rule, sellers focus on the lowest possible ex post profits they know could be realised from each partition of the possible demand states. In this context, we provide conditions that guarantee the existence of a pure strategy equilibrium, which are straight-forward to understand, interpret and implement.

Our approach for capturing ambiguity aversion is motivated by experimental and empirical evidence, which has spurred the adoption of Maximin utilities throughout the theoretical literature (Cerreia-Vioglio et al., 2013; Correia-da-Silva and Hervés-Beloso, 2009, 2012; De Castro et al., 2017; He and Yannelis, 2015a, 2016, 2017; Pulford and Colman, 2007). One appealing property of Maximin preferences stems from the ability to explain classic examples of behaviour that are incompatible with Subjective Expected Utility (SEU) theory, including the Ellsberg and Allais Paradoxes (Ellsberg, 1961; Halpern and Leung, 2016). Recent empirical evidence includes Giordani et al. (2010), who analyse responses to the European Values Survey to understand how individuals approach uncertain possibilities. Maximin utilities play a substantive role and underpin the behaviour of 23% of individuals surveyed, with significant geographical variation. In Italy, 46% of individuals exhibit Maximin preferences, whilst 25% act as Bayesians. This motivates our inclusion of Maximin utilities as a legitimate approach to decision-making under uncertainty for at least some market players and contexts.

The main mechanics of our results operate as follows: In the complete information benchmark, the only candidate for a pure strategy equilibrium is the competitive price (see Dixon, 1992; Shubik, 1959) but the incentive to charge a higher price and sell only to the residual demand that their competitor cannot meet generally destabilises this equilibrium. Therefore, following Tasnádi (1999), demand must be sufficiently elastic to shut down such upward price deviations and sustain a pure strategy equilibrium at the competitive level. We now develop this line of argument to a general incomplete information environment.

When sellers possess incomplete but symmetric information, we show that sellers choose the minimum competitive price for each of their information partitions. If a seller deviated above the lowest competitive price and the lowest demand state prevailed, the firm would have earned higher profit at the competitive price, violating the Maximin utilities of the sellers. This constitutes the incomplete information analogue of Shubik's (1959) well-known results in the complete information game. Interestingly, excess demand also arises in equilibrium whenever the realised demand is not the lowest possible demand from one of the sellers' (symmetric) information partitions.

When we introduce asymmetric information, sellers can charge different prices in a pure strategy equilibrium, violating the law of one price. Interestingly, demand can also be rationed across sellers charging different prices, which is consistent with empirical findings but rarely identified in the theoretical literature. Therefore, we are able to explain pure strategy price dispersion, consumers purchasing at different prices and equilibrium excess demand as direct consequences of intuitively plausible and empirically observed seller uncertainty regarding market demand.[6] These results are also salient for competition authorities as the Bertrand–Edgeworth framework continues to act as a benchmark for competition policy analyses.[7]

Equilibria in Bertrand–Edgeworth games can also be difficult to find and/or characterise. Our model has an additional advantage that the equilibrium is easy to construct and analyse. One simply has to find the competitive equilibrium for each state of the market demand and construct the sellers' price strategies based upon their information partitions (see the example in Section 2.4).

Following the literature review, Section 2 outlines the Bertrand–Edgeworth game. Section 2.1 explores the ex post payoffs for sellers and Section 2.2 analyses their ex ante payoffs under Maximin expected utilities. In Section 2.3 we present our main results on the existence of pure strategy equilibrium and we provide precise conditions under which the law of one price is violated. Section 2.4 provides a simple example that illustrates the intuitive and analytically tractable nature of our results. We conclude with a discussion of our findings in Section 3.

1.1. Related literature

The literature on capacity-constrained price competition has primarily focused on a complete information environment, where several remedies to the non-existence of pure strategy equilibria have been proposed. Tasnádi (1999) provides conditions on the elasticity of the demand function that restore pure strategy equilibrium by ensuring that upward price deviations, which generally destabilise the equilibrium, decrease revenue. In Section 2.3 we show that our model nests Tasnádi's (1999) restrictions as a special case when firms hold complete information. However, we go further by permitting incomplete information and we show

[4] Demand uncertainty can also stem from inter-temporal variations in demand as identified in domestic electricity markets (see Green and Newbery, 1992; Lemus and Moreno, 2017).

[5] Glycopantis and Yannelis (2005) contain many papers which analyse asymmetric information of the type which we introduce in the Bertrand–Edgeworth game.

[6] For example, *The Wall Street Journal* note CEO Tim Cook's statement in reference to iPhone sales: *"It's very hard to gauge demand, as you know, when you're selling everything you're making"* (Mims, 2017). Moreover, this statement is consistent with the excess demand that we observe in equilibrium.

[7] For example, see the European Commission's merger appraisals of Holcim/Cemex West (COMP/M.7009) and, in particular, Outokumpu/INOXUM (COMP/M.6471) where the Bertrand–Edgeworth framework "provides the best approximation to important industry features" (p. 166).

that the resulting pure strategy equilibrium can involve price dispersion, rationing of demand and excess demand.

Bade (2005) resolves the non-existence of pure strategy equilibrium by introducing incomplete preferences amongst sellers with multiple objectives, such as sales and profit maximisation. In contrast, incomplete information in our setting creates further challenges, rather than resolving the non-existence of pure strategy equilibrium. Iskakov et al. (2018) consider cautious sellers, where any profitable deviation must not induce a counter-deviation by another player that leaves the initial deviator worse off than their original position. Their solution of equilibrium in stable strategies is similar to the Von-Neumann Morgenstern stable strategies used in cooperative game theory.

Alternative methods of delivering pure strategy equilibria include modifying the timing of the game (Deneckere and Kovenock, 1992; Deneckere and Peck, 2012; Dudey, 1992), allowing sellers to choose list prices and subsequent discount prices (Garcìa Díaz et al., 2009; Myatt and Ronayne, 2019), requiring integer pricing (Chowdhury, 2008), imposing a cost on firms that turn customers away (Dixon, 1990) and introducing a public social-surplus maximising seller (Rácz and Tasnádi, 2016).

More recently, Bos et al. (2021) remedy the non-existence of pure strategy Nash equilibria by considering Myopic sellers who seek improvements on their current position, rather than more stringent Nash best responses. Using the Myopic Stable Set (MSS) solution concept, due to Demuynck et al. (2019), they show that MSS prices are equivalent to pure strategy Nash equilibria when the latter exist. When no pure strategy Nash equilibria exist, MSS offers a solution in the form of a (pure strategy) price range that contains the mixed strategy interval and can involve sellers pricing below the competitive level, leading to rationing. In contrast, our approach focuses on pure strategy Nash equilibria under incomplete information with ambiguity averse sellers, where rationing can arise as a consequence of demand uncertainty.

Hunold and Muthers (2019) show that spatial differentiation can drive price dispersion in a capacity-constrained price game. Chao et al. (2018, 2019) identify price dispersion when a capacity constrained seller competes against an unconstrained seller with sequential pricing and scope for 'all-unit-discounts' (AUD).[8] Our model features symmetric cost structures and captures both symmetric and asymmetric capacities, but price dispersion is driven only by asymmetric information.

A related literature analyses Bertrand–Edgeworth competition with demand uncertainty. Dana (1999) considers identical sellers who know the probability of each demand state. The pure strategy equilibrium involves intra-firm price dispersion, where sellers specify the output available at each price. Price schemes are identical across sellers and the price of each specific unit is given by the marginal cost divided by the probability that it will be sold. In contrast, we do not require that sellers can attach probabilities to demand states that they are unable to distinguish between and we consider asymmetric sellers in terms of information and capacities. This generates our novel pure strategy equilibrium with inter-firm price dispersion, where consumers are rationed across firms charging different prices. The two frameworks provide complementary but distinct explanations for non-random price dispersion.

Other papers consider demand uncertainty when sellers choose their capacities before price competition. In that literature, however, demand is usually realised before price competition (De Frutos and Fabra, 2011; Lepore, 2012; Reynolds and Wilson, 2000) and there generally exists no pure strategy price equilibrium (Hviid, 1991). Our model abstracts from preceding capacity investment decisions to zoom in on the existence and nature of pure strategy price dispersion in Bertrand–Edgeworth markets. Recent research has also explored the consequences of asymmetric information across sellers on other dimensions, such as the number of firms a consumer considers (Bergemann et al., 2020).

2. The Bertrand–Edgeworth game

The model consists of a finite set of sellers $N = \{1, 2\}$, who are producing a single perfectly homogeneous good. The uncertainty will be modelled by a finite set $\Omega = \{\omega_1, \dots, \omega_m\}$, which is the set of possible **states of the world**. There is a probability distribution, μ, over the set Ω which describes the probability of each state occurring. It shall be assumed that $\mu(\omega) > 0$ for every $\omega \in \Omega$ so no state of the world is redundant. Each seller is endowed with a fixed quantity $q_i > 0$ of the good.[9] The total quantity of the good which can be traded in the market is $q_1 + q_2$. There is a state-contingent **market demand function** for the homogeneous good given by $D : \Re_{++} \times \Omega \to \Re_+$. The following conditions are imposed upon the demand function, where x denotes price.

Assumption 1. For every $\omega \in \Omega$ and every $x \in (0, \infty)$, $D(x, \omega) > 0$. The function $D(\cdot, \omega)$ is C^1 and $D'(x, \omega) < 0$ for every $\omega \in \Omega$ and every $x \in (0, \infty)$.

The **private information** of seller i is modelled by a partition, P_i, of the set Ω. Whenever two states of the world are in the same element of the partition P_i, it means that seller i is unable to distinguish between those two states. The information partitions are fixed as a primitive of the game and they are common knowledge amongst the players. This means that a player can know whether their rival has more information than they hold. If the partitions are symmetric, the sellers have identical information regarding the future market demand. If the partitions are asymmetric, the sellers possess asymmetric information regarding the future market demand. The probability distribution (μ) over each of the possible realised states of the market demand, however, is not common knowledge.

A function $f : \Omega \to \Re_+$ will be called P_i-measurable if, whenever $\omega_p \in E$ and $\omega_q \in E$ for some $E \in P_i$, then $f(\omega_p) = f(\omega_q)$. Facing these information restrictions, the strategy set of seller i in the game is:

$$L_i = \{f : \Omega \to \Re_{++} : f \text{ is } P_i - \text{measurable}\}.$$

Let $L = L_1 \times L_2$ be the joint strategy set. The primitives of a **Bertrand–Edgeworth game with asymmetric information** can be summarised as $G = \{N, \Omega, (P_i, q_i)_{i\in N}, D, \mu\}$. The price elasticity of the market demand in state $\omega \in \Omega$ is:

$$\epsilon(x, \omega) = D'(x, \omega)\frac{x}{D(x, \omega)}.$$

The market demand will be called **uniformly elastic** if $\epsilon(x, \omega) \le -1$ for every $x \in (0, \infty)$ and every $\omega \in \Omega$. Let $R(x, \omega) = xD(x, \omega)$ so $R(x, \omega)$ is the total revenue available in the market at price x in state $\omega \in \Omega$. The uniform elasticity condition on the market demand curve requires that a proportional increase in price results in a more than proportional decrease in the quantity demanded. Therefore, increases in price will reduce revenue.

8 All-unit-discounts involve a reduction in the price for all purchased units once total quantity crosses a threshold (Chao et al., 2018).

9 We are assuming that each seller has zero marginal cost to supply the good. However, one could easily add a constant marginal cost of production and this would make no difference to the results.

2.1. The Ex post payoffs

After fixing a set of strategies $f \in L$, to specify the payoffs which a seller receives ex post, a rationing rule is required because a seller may not set the lowest price, but the other seller may not be able to serve all the market demand.[10] We consider the most widely used rationing rule in the literature: efficient, or "surplus-maximising", rationing which is consistent with those buyers with the highest valuation of the good being served first.[11] Under this rule, the demand which the higher-priced seller faces is a horizontal displacement of the market demand. If the sellers tie at the same price, we shall make the standard assumption that they split the market demand in proportion to the quantities of the good they are endowed with.

Given a set of strategies $f \in L$, let $D_j = \min\{D(f_j(\omega), \omega), q_j\}$. The demand which seller i faces under efficient rationing is:

$$D_i^E(f, \omega) = \begin{cases} \max\{0, D(f_i(\omega), \omega) - D_j\}, & \text{if } f_i(\omega) > f_j(\omega); \\ \frac{q_i}{q_1+q_2} D(f_i(\omega), \omega), & \text{if } f_i(\omega) = f_j(\omega); \\ D(f_i(\omega), \omega) & \text{if } f_i(\omega) < f_j(\omega). \end{cases}$$

If seller i has the highest price, then seller i receives only the residual demand that seller j cannot meet due to their capacity constraint, or zero if seller j satisfies the market demand. If both sellers have the same price, they share the forthcoming demand in proportion to their capacities. If seller i has the lowest price, the demand they face is the entire market demand forthcoming at that price.

As seller i is only endowed with q_i units of the good, the demand which seller i actually meets in state $\omega \in \Omega$ is given by $D_i^A(f, \omega) = \min\{q_i, D_i^E(f, \omega)\}$. Therefore, the ex post payoff of seller i in state $\omega \in \Omega$ is:

$$u_i(f, \omega) = f_i(\omega) D_i^A(f, \omega).$$

2.2. The Ex ante payoffs

Before each seller has received the information regarding which element in P_i the state of the world is in, how should the sellers evaluate their expected payoff? Given we are assuming that sellers cannot distinguish between different states of the world contained in the same element in P_i, it is not unreasonable to assume that sellers cannot assign probabilities to those states. In this context, it is not possible for sellers to calculate standard Bayesian expected utilities because they do not know the probabilities of each demand state being realised.

We consider a well-known alternative to Bayesian expected utilities: Maximin expected utilities (MEU). If a seller knows that the state of the world is contained in $E \in P_i$, we consider the case where the seller is pessimistic and assigns all the probability associated with event E, which is $\mu(E)$, to the minimum ex post payoff in E. Formally, $\mu(E) = \sum_{\omega \in E} \mu(\omega)$. Let H be the set of probability distributions over Ω:

$$H = \{h \in \Re^{\Omega} : h(\omega) \geq 0 \text{ for every } \omega \in \Omega \text{ and } \sum_{\omega \in \Omega} h(\omega) = 1\}$$

Let M_i be the set of probability distributions which **agree with seller i's private information**:

$$M_i = \{h \in H : h(E) = \mu(E) \text{ for every } E \in P_i\}.$$

Therefore, $h(E) = \mu(E)$. Given a set of strategies $f \in L$, the **ex ante payoff** of seller i is:

$$U_i(f) = \min_{h \in M_i} [\sum_{\omega \in \Omega} h(\omega) u_i(f, \omega)].$$

An alternative, but equivalent expression, is:

$$U_i(f) = \sum_{E \in P_i} \mu(E) [\min_{\omega \in E} u_i(f, \omega)].$$

Remark 1. The most prominent early application of Maximin expected utilities was in Gilboa and Schmeidler (1989) who characterised this type of decision rule and noted that it can explain the Ellsberg (1961) violations of subjective expected utility theory. Recently, Maximin expected utilities have been used in a wide range of papers, including Correia-da-Silva and Hervés-Beloso (2009), He and Yannelis (2015a) and De Castro and Yannelis (2018).

Remark 2. This model of a Bertrand–Edgeworth game with asymmetric information contains, as a special case, the standard complete information game. If one specifies the information partitions of the sellers to be $P_i = \{\{\omega_1\}, \{\omega_2\}, \ldots, \{\omega_m\}\}$ for every $i \in N$ then each seller can distinguish every state of the world and the model is a complete information game. Moreover, the calculation of Maximin expected utilities then coincides with standard Bayesian utilities.

2.3. Existence of pure strategy equilibrium and the law of one price

Now that the ex ante and ex post payoffs have been defined, we can introduce the equilibrium concept. A set of strategies $f \in L$ is a **pure strategy price equilibrium** if, for every $i \in N$;

$$U_i(f) \geq U_i(f_i', f_{-i}) \text{ for every } f_i' \in L_i.$$

We shall say that a pure strategy price equilibrium, $f \in L$, **violates the law of one price** if $f_1(\omega) \neq f_2(\omega)$ for some $\omega \in \Omega$. That is to say, a pure strategy price equilibrium violates the law of one price if there is at least one state of the world when the sellers post different prices in the market. The first result gives some useful properties of the market demand function, where $R(x, \omega) = xD(x, \omega)$ is the total revenue available in the market at price x in state $\omega \in \Omega$.

Proposition 1. *Fix a Bertrand–Edgeworth game with asymmetric information $G = \{N, \Omega, (P_i, q_i)_{i \in N}, D, \mu\}$. If $D(x, \omega)$ is uniformly elastic then the following are true:*

(i) $R'(x, \omega) \leq 0$ for every $x \in (0, \infty)$.

(ii) $\lim_{x \to 0} D(x, \omega) = \infty$ and $\lim_{x \to \infty} D(x, \omega) = 0$.

(iii) For each $\omega \in \Omega$ there exist unique prices $p^c(\omega)$ such that $D(p^c(\omega), \omega) = q_1 + q_2$.

Proof.

(i) From the definition $R(x, \omega) = xD(x, \omega)$, therefore:

$$R'(x, \omega) = D(x, \omega) + xD'(x, \omega) = D(x, \omega)(1 + \epsilon(x, \omega)).$$

As $\epsilon(x, \omega) \leq -1$ for every $x \in (0, \infty)$, $R'(x, \omega) \leq 0$ for every $x \in (0, \infty)$.

(ii) Suppose a contradiction: that $\lim_{x \to 0} D(x, \omega) = y > 0$. Then $\lim_{x \to 0} xD(x, \omega) = 0$. As $R'(x, \omega) \leq 0$, this implies $R(x, \omega) \leq 0$ for every $x \in (0, \infty)$ and contradicts $xD(x, \omega) > 0$ for every $x \in (0, \infty)$. Hence, $\lim_{x \to 0} D(x, \omega) = \infty$. Suppose a contradiction: $\lim_{x \to \infty} D(x, \omega) = y > 0$. Then $\lim_{x \to \infty} xD(x, \omega) = \infty$ and contradicts $R'(x, \omega) \leq 0$ for every $x \in (0, \infty)$. Hence, $\lim_{x \to \infty} D(x, \omega) = 0$.

[10] See Bos and Vermeulen (2021) for a study of price-quantity competition when not all excess demand for the lowest priced seller spills over to another seller.

[11] An alternative interpretation of the efficient rationing rule is that buyers are served randomly at first, but are then able to retrade the good amongst themselves, which then results in the same allocation of the good. See, amongst others, Vives (1999), pp. 124–5).

(iii) It follows from (ii) that the range of $D(\cdot, \omega)$ is $(0, \infty)$. Therefore, for each $\omega \in \Omega$ there exists a $p^c(\omega)$ such that $D(p^c(\omega), \omega) = q_1 + q_2$. The uniqueness of such prices follows from $D(\cdot, \omega)$ being decreasing on $(0, \infty)$. ■

The $p^c(\omega)$ prices correspond to the competitive price for each state of the market demand. Using the $p^c(\omega)$ prices defined in part (iii) of the previous result, define the strategies of the sellers to be as follows. For each $E \in P_i$ let:

$$f_i^*(E) = \min_{\omega \in E} p^c(\omega).$$

By construction these strategies are P_i-measurable, so $f^* \in L$. The following result gives some of the properties of these strategies.

Proposition 2. *Fix a Bertrand–Edgeworth game with asymmetric information $G = \{N, \Omega, (P_i, q_i)_{i \in N}, D, \mu\}$. Suppose the demand $D(x, \omega)$ is uniformly elastic and the sellers play the strategies $f^* \in L$. Then:*

(i) $D_i^A(f^*, \omega) = q_i$ for every $\omega \in \Omega$.

(ii) $U_i(f^*) = \sum_{E \in P_i} \mu(E) f_i^*(E) q_i$.

Proof.

(i) If the sellers use strategies $f^* \in L$ then $f_i^*(\omega) \leq p^c(\omega)$ for every $\omega \in \Omega$. Therefore $D(f_i^*(\omega), \omega) \geq q_1 + q_2$, $D_i^E(f^*, \omega) \geq q_i$, and consequently, $D_i^A(f^*, \omega) = q_i$ for every $\omega \in \Omega$.

(ii) Follows from (i) and the definition of the ex ante utilities. ■

The next result demonstrates that the strategies $f^* \in L$ are a pure strategy price equilibrium of the Bertrand–Edgeworth game.

Proposition 3. *Fix a Bertrand–Edgeworth game with asymmetric information $G = \{N, \Omega, (P_i, q_i)_{i \in N}, D, \mu\}$. If $D(x, \omega)$ is uniformly elastic, then the strategies $f^* \in L$ are a pure strategy price equilibrium.*

Proof. Suppose the sellers play the strategies $f^* \in L$. It follows from part (i) of Proposition 2 that using these strategies each seller is able to sell all their quantity of the good they are endowed with. If for some $E \in P_i$ seller i were to deviate and play $f_i(E) < f_i^*(E)$, then $u_i((f_i, f_j^*), \omega) = f_i(E) q_i < f_i^*(E) q_i = u_i(f^*, \omega)$ for every $\omega \in E$. This is not a profitable deviation.

Suppose for some $E \in P_i$ seller i were to deviate and play $f_i(E) > f_i^*(E)$. From part (i) of Proposition 2 we know that using strategies $f^* \in L$ seller i obtains the same payoff $f^*(E) q_i$ across all states in E. To show that deviating to $f_i(E) > f_i^*(E)$ is not a profitable deviation, given the Maximin ex ante utilities, we only have to find one state in E where the payoff does not increase above $f^*(E) q_i$. Consider the state $\omega_E = \{\omega \in E : p^c(\omega) \leq p^c(\omega') \quad \forall \quad \omega' \in E\}$. In state ω_E, using strategies f^*, seller i either ties at price $p^c(\omega_E)$ with seller j, or seller j posts a strictly lower price. Hence:

$$u_i(f^*, \omega_E) = p^c(\omega_E) q_i = p^c(\omega_E)(D(p^c(\omega_E), \omega_E) - q_j)$$

where the second equality follows from $D(p^c(\omega_E), \omega_E) = q_1 + q_2$. To show that deviating to $f_i(E) > f_i^*(E)$ is not a profitable deviation, we need to demonstrate that the function $g(x) = x(D(x, \omega_E) - q_j)$ is decreasing in x. The derivative is:

$$\begin{aligned} g'(x) &= D(x, \omega_E) - q_j + x D'(x, \omega_E) \\ &= D(x, \omega_E)(1 - q_j / D(x, \omega_E) + \epsilon(x, \omega_E)). \end{aligned}$$

As $\epsilon(x, \omega_E) \leq -1$ for every $x \in (0, \infty)$ it follows that $g'(x) \leq 0$. Therefore, deviating to $f_i(E) > f_i^*(E)$ is not a profitable deviation in state ω_E. ■

At this point, it is helpful to clarify that we are applying the standard Nash equilibrium solution concept. The key difference from Bayesian Nash equilibrium is that we are considering an alternative expected utility of the players in the form of Maximin expected utility due to the ambiguity that players face (He and Yannelis, 2015b, 2016).

In equilibrium, a seller will never earn a lower payoff than their calculated minimum profit associated with the lowest realisation of the market demand. Furthermore, neither seller has an incentive to be yet more pessimistic and set a price lower than the minimum competitive price (i.e. the competitive price corresponding to the lowest possible realisation of the market demand for each information partition) because the seller is already guaranteed to sell their entire capacity at the minimum competitive price. Therefore, choosing a lower price will generate no additional sales (as their capacity constraint is binding), reduce profit and only stimulate excess demand. Moreover, if one firm (firm 2) were to behave in this way following an out-of-equilibrium price, the competitor (firm 1) has no incentive to reduce their price further as they already exhaust their capacity at their minimum competitive price.

Remark 3. One might reasonably ask whether the uniform elasticity of the market demand can be relaxed and still guarantee the existence of a pure strategy price equilibrium. As is well-known, Nash equilibria in Bertrand–Edgeworth games often only exist in mixed strategies. However, in the current model, we can be more precise. Suppose there is a market demand $D^*(x, \omega)$ and $\epsilon(x, \omega) \in (-1, 0)$ for every $x \in (0, \infty)$ so demand is always inelastic in one state of the world. Then, it is possible to find a Bertrand–Edgeworth game with asymmetric information $G = \{N, \Omega, (P_i, q_i)_{i \in N}, D, \mu\}$ with $D = D^*$, such that a pure strategy price equilibrium fails to exist. This result follows directly from Remark 2, that the current model includes the complete information game as a special case, and Proposition 2.3 of Tasnádi (1999). Therefore, it does not seem possible to significantly weaken the condition of uniform elasticity and still guarantee the existence of a pure strategy equilibrium.

We are now able to present our main result which gives precise conditions under which a Bertrand–Edgeworth game with asymmetric information possesses a pure strategy price equilibrium that violates the law of one price.

Proposition 4. *Fix a Bertrand–Edgeworth game with asymmetric information $G = \{N, \Omega, (P_i, q_i)_{i \in N}, D, \mu\}$. Suppose the following three conditions are satisfied:*

(i) The demand $D(x, \omega)$ is uniformly elastic.

(ii) $p^c(\omega) \neq p^c(\omega')$ whenever $\omega \neq \omega'$.

(iii) $P_1 \neq P_2$.

Then the game possesses a pure strategy price equilibrium which violates the law of one price.

Proof. Let the sellers play the strategies $f^* = (f_1^*, f_2^*) \in L$. It follows from Proposition 3 that these are a pure strategy price equilibrium of the game. Suppose a contradiction: $f_1^*(\omega) = f_2^*(\omega)$ for every $\omega \in \Omega$. As $f_1^* \in L_1$ and $f_2^* \in L_2$, $f_1^*(\omega) = f_2^*(\omega)$ for every $\omega \in \Omega$, together with $p^c(\omega) \neq p^c(\omega')$ whenever $\omega \neq \omega'$ imply $P_1 = P_2$. This contradicts $P_1 \neq P_2$. Hence, there must be at least one $\omega \in \Omega$ such that $f_1^*(\omega) \neq f_2^*(\omega)$. ■

The conditions (i)–(iii) in Proposition 4 are tight in the following sense. If one were to dispense with (i), but retain (ii) and (iii), it follows from Remark 3 that a game can be found which fails to possess a pure strategy price equilibrium. More specifically, when the demand is not uniformly elastic, it can be profitable for a seller to choose a price above their competitor and sell only

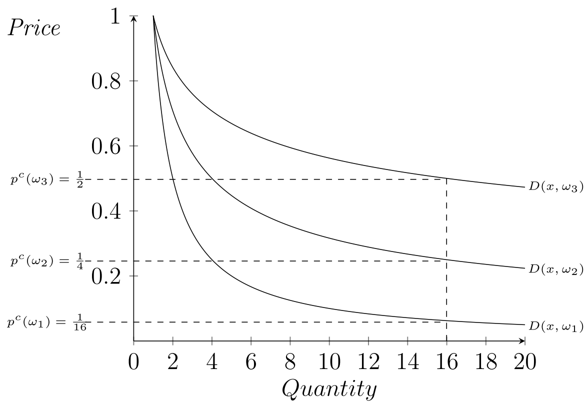

Fig. 1. State-contingent competitive prices.

to the residual demand that the rival cannot meet. This generates the classic non-existence of pure strategy price equilibrium in the Bertrand–Edgeworth framework.

If one retains condition (i), and dispenses with either (ii) or (iii), then a game can be found in which the pure strategy price equilibrium defined by the $p^c(\omega)$ prices does not violate the law of one price. More specifically, when (ii) is violated, the competitive equilibrium prices for multiple demand states coincide. Therefore, even if players have asymmetric information, the law of one price need not be violated. For example, consider an extreme case where the competitive price in every possible demand state is identical. This leads each seller to choose the same price, independently of their information partition or the realised state of the market demand, and the law of one price would not be violated.

If condition (iii) is violated, the players have the same information. Therefore, even if the competitive prices vary between each demand state and the demand is uniformly elastic, a pure strategy equilibrium would exist but the law of one price would not be violated. Therefore, all three conditions in Proposition 4 are required to ensure that a pure strategy price equilibrium exists, defined by the $p^c(\omega)$ prices, which violates the law of one price.

2.4. An illustrative example

Consider a market in which there are three states of the world, $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and the prior is $\mu(\omega_1) = \mu(\omega_2) = \mu(\omega_3) = 1/3$. The market demands in the three states are $D(x, \omega_1) = x^{-1}$, $D(x, \omega_2) = x^{-2}$ and $D(x, \omega_3) = x^{-4}$. The quantities of the good the sellers are endowed with are $q_1 = 4$ and $q_2 = 12$. The information partitions of the two sellers are $P_1 = \{\{\omega_1, \omega_2\}, \{\omega_3\}\}$ and $P_2 = \{\{\omega_1\}, \{\omega_2, \omega_3\}\}$. Given these market primitives, the p^c prices are $p^c(\omega_1) = \frac{1}{16}$, $p^c(\omega_2) = \frac{1}{4}$ and $p^c(\omega_3) = \frac{1}{2}$, as illustrated in Fig. 1.

It follows from Proposition 3 that the strategies:

$$f_1^*(\{\omega_1, \omega_2\}) = \frac{1}{16} \quad \text{and} \quad f_1^*(\omega_3) = \frac{1}{2}$$

$$f_2^*(\omega_1) = \frac{1}{16} \quad \text{and} \quad f_2^*(\{\omega_2, \omega_3\}) = \frac{1}{4}.$$

are a pure strategy price equilibrium. The ex ante expected utilities of the sellers at the equilibrium are $U_1(f^*) = \frac{5}{6}$ and $U_2(f^*) = 2\frac{1}{4}$. This example violates the law of one price because $f_1^*(\omega_2) \neq f_2^*(\omega_2)$ and $f_1^*(\omega_3) \neq f_2^*(\omega_3)$. As $\mu(\{\omega_2, \omega_3\}) = 2/3$, the sellers post different prices in the market with ex ante probability 2/3.

3. Discussion and conclusion

This paper provides a theoretical foundation for the commonly observed phenomenon of perfectly homogeneous goods selling at different prices within the same market, without resorting to the usual, but contentious, device of sellers using mixed strategies. Our main result demonstrates that if the market demand is uniformly elastic, the competitive equilibrium prices differ in each state of the world, and if the sellers have different information partitions, then a pure strategy price equilibrium exists which violates the law of one price. Furthermore, given the rationing rule, even the seller posting a higher price may make positive sales in equilibrium.

If firms have the same information, the law of one price is restored but excess demand exists whenever the lowest possible demand from one of the sellers' information partitions is not realised. The intuition is that with Maximin utilities, a seller cannot increase their price ex ante because they would earn a lower payoff than at the minimum competitive price if the lowest demand state occurred.

The advantage of the models of Bertrand and Bertrand–Edgeworth competition is that they provide a direct foundation for prices in the marketplace without resorting to the fiction of the Walrasian auctioneer. The model we have analysed assumed constant zero marginal costs. Most of the literature indicates that with more general convex costs an equilibrium only exists in mixed strategies, and is often difficult to characterise. However, Dixon (1992) noted that if sellers are permitted to specify both price and quantity pairs, and provided all but one seller could supply the whole market demand subject to a no bankruptcy constraint, then the competitive equilibrium of the market could be sustained as a pure strategy equilibrium. More recently, Bos and Vermeulen (2021) demonstrate that the existence of a pure strategy equilibrium is driven primarily by demand and costs, rather than whether firms also choose quantities simultaneous to prices. It would therefore be interesting to explore whether the framework of sellers posting both price and quantity pairs in the market could be extended to permit asymmetries of information of the type studied in this paper, and what the implications are for the law of one price.

It would also be of interest to explore a deeper degree of uncertainty, where the capacity of each seller is state dependent. In this case, each seller's endowment would be a function $q_i : \Omega \to \Re_{++}$. One could impose that q_i is measurable with respect to each seller's private information so that no seller could infer more about the state of the market by observing their endowment. In this richer model, it is an open question whether the strategies defined by the $p^c(\omega)$ prices still constitute a pure strategy price equilibrium, and whether conditions, such as those in Proposition 4, which determine when the law of one price is violated could be found.

Whilst Maximin utilities are a standard approach for modelling utilities under ambiguity, it would also be interesting to explore the extent to which our results continue to pass through in models of ambiguity that adopt alternative preferences. One alternative approach could be to consider the Hurwicz criterion (Eichberger and Kelsey, 2014), where decision-makers adopt a weighted utility of the best and worst possible outcomes. In this context, Maximin preferences correspond to the case where all weight is assigned to the worst possible outcome.

The extent to which our results extend to an oligopoly setting with $n > 2$ sellers also remains an interesting direction for further research. We suspect that the main insights will continue to survive in an oligopoly setting. However, in our environment of asymmetric information, (possibly) asymmetric capacities and many possible realisations of the market demand, the number of case distinctions makes a formal proof more challenging.

References

Bade, S., 2005. Nash equilibrium in games with incomplete preferences. Econom. Theory 26, 309–332.

Baye, M.R., Morgan, J., 2001. Information gatekeepers on the internet and the competitiveness of homogeneous product markets. Amer. Econ. Rev. 91, 454–474.

Baye, M.R., Morgan, J., Scholten, P., 2004. Price dispersion in the small and in the large: Evidence from an internet price comparison site. J. Ind. Econ. 52, 463–496.

Bergemann, B., Brooks, B., Morris, S., 2020. Search, Information and Prices. Cowles Foundation Discussion Paper, 2224.

Bos, I., Marini, M.A., Saulle, R., 2021. Myopic Oligopoly Pricing. FEEM Working Paper No. 9. 2021, Available at ssrn.com/abstract=3817311.

Bos, I., Vermeulen, D., 2021. On pure-strategy Nash equilibria in price-quantity games. J. Math. Econom. 96, 102501.

Buchheit, S., Feltovich, N., 2011. Experimental evidence of a sunk-cost paradox: A study of pricing behaviour in Bertrand–Edgeworth duopoly. Internat. Econom. Rev. 52, 317–347.

Burdett, K., Judd, K.L., 1983. Equilibrium price dispersion. Econometrica 51, 955–969.

Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M., Montrucchio, L., 2013. Ambiguity and robust statistics. J. Econom. Theory 148, 974–1049.

Chao, Y., Tan, G., Wong, A.C.L., 2018. All-units discounts as a partial foreclosure device. Rand J. Econ. 49, 155–180.

Chao, Y., Tan, G., Wong, A.C.L., 2019. Asymmetry in capacity and the adoption of all-unit discounts. Int. J. Ind. Organ. 65, 152–172.

Chowdhury, P.R., 2008. Bertrand–Edgeworth equilibrium with a large number of firms. Int. J. Ind. Organ. 26, 746–761.

Correia-da-Silva, J., Hervés-Beloso, C., 2009. Prudent expectations equilibrium in economies with uncertain delivery. Econom. Theory 39, 67–92.

Correia-da-Silva, J., Hervés-Beloso, C., 2012. General equilibrium with uncertain delivery. Econom. Theory 51, 729–755.

Dana, J.D., 1999. Equilibrium price dispersion under uncertainty: the roles of costly capacity and market structure. Rand J. Econ. 30, 632–660.

Dasgupta, P., Maskin, E., 1986. The existence of equilibrium in discontinuous games, I: Theory. Rev. Econom. Stud. 53, 1–26.

De Castro, L., Liu, Z., Yannelis, N.C., 2017. Implementation under ambiguity. Games Econom. Behav. 101, 20–33.

De Castro, L., Yannelis, N.C., 2018. Uncertainty, efficiency and incentive compatibility: Ambiguity solves the conflict between efficiency and incentive compatibility. J. Econom. Theory 177, 678–707.

De Frutos, M.A., Fabra, N., 2011. Endogenous capacities and price competition: The role of demand uncertainty. Int. J. Ind. Organ. 29, 399–411.

Demuynck, T., Herings, P.J-J., Saulle, C., Seel, R.D., 2019. The myopic stable set for social environments. Econometrica 86, 111–138.

Deneckere, R., Kovenock, D., 1992. Price leadership. Rev. Econom. Stud. 59, 143–162.

Deneckere, R., Peck, J., 2012. Dynamic competition with random pricing and costless search: A theory of price posting. Econometrica 80, 1185–1247.

Dixon, H., 1990. Bertrand–Edgeworth equilibria when firms avoid turning customers away. J. Ind. Econ. 39, 131–146.

Dixon, H., 1992. The competitive outcome as the equilibrium in an Edgeworthian price-quantity model. Econ. J. 102, 301–309.

Dudey, M., 1992. Dynamic Edgeworth-Bertrand competition. Q. J. Econ. 107, 1461–1477.

Eichberger, J., Kelsey, D., 2014. Optimism and pessimism in games. Internat. Econom. Rev. 55, 483–505.

Ellsberg, D., 1961. Risk, ambiguity, and the savage axioms. Q. J. Econ. 75, 643–669.

Fonseca, M.A., Normann, H.T., 2013. Excess capacity and pricing in Bertrand–Edgeworth markets: Experimental evidence. J. Inst. Theor. Econ. 169, 199–228.

Friedman, J.W., 1988. On the strategic importance of prices versus quantities. Rand J. Econ. 19, 607–622.

Garcìa Díaz, A., Hernán González, R., Kujal, P., 2009. List pricing and discounting in a Bertrand–Edgeworth duopoly. Int. J. Ind. Organ. 27, 719–727.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with non unique prior. J. Math. Econom. 18, 141–153.

Giordani, P.E., Schlag, K.H., Zwart, S., 2010. Decision makers facing uncertainty: Theory versus evidence. J. Econ. Psychol. 31, 659–675.

Glycopantis, D., Yannelis, N.C., 2005. Differential Information Economies. Springer-Verlag.

Green, R.J., Newbery, D.M., 1992. Competition in the british electricity spot market. J. Polit. Econ. 100, 929–953.

Halpern, J.Y., Leung, S., 2016. Maxmin weighted expected utility: A simpler characterization. Theory and Decision 80, 581–610.

He, W., Yannelis, N.C., 2015a. Equilibrium theory under ambiguity. J. Math. Econom. 61, 86–95.

He, W., Yannelis, N.C., 2015b. Discontinuous games with asymmetric information: An extension of Reny's existence theorem. Games Econom. Behav. 90, 26–35.

He, W., Yannelis, N.C., 2016. A remark on discontinuous games with asymmetric information and ambiguity. Econ Theory Bull. 5, 119–126.

He, W., Yannelis, N.C., 2017. Equilibria with discontinuous preferences: New fixed point theorems. J. Math. Anal. Appl. 450, 1421–1433.

Heymann, D., Kawamura, E., Perazzo, R., Zimmerman, M.G., 2014. Behavioral heuristics and market patterns in a Bertrand–Edgeworth game. J. Econ. Behav. Organ. 105, 124–139.

Hunold, M., Muthers, J., 2019. Spatial competition and price discrimination with capacity constraints. Int. J. Ind. Organ. 67, 1–20.

Hviid, M., 1991. Capacity constrained duopolies, uncertain demand and non-existence of pure strategy equilibria. Eur. J. Political Econ. 7, 183–190.

Iskakov, M., Iskakov, A., d'Aspremont, C., 2018. Games for cautious players: The equilibrium in secure strategies. Games Econom. Behav. 110, 58–70.

Janssen, M., Rasmusen, E., 2002. Bertrand competition under uncertainty. J. Ind. Econ. 50, 11–21.

Kruse, J.B., Rassenti, S., Reynolds, S.S., Smith, V.L., 1994. Bertrand–Edgeworth competition in experimental markets. Econometrica 62, 343–371.

Lemus, A.B., Moreno, D., 2017. Price caps with capacity precommitment. Int. J. Ind. Organ. 50, 131–158.

Lepore, J.J., 2012. Cournot outcomes under Bertrand–Edgeworth competition with demand uncertainty. J. Math. Econom. 48, 177–186.

Maskin, E., 1986. The existence of equilibrium with price-setting firms. Amer. Econ. Rev. 76, 382–386.

Mims, C., 2017. Why there are never enough new iphones. Wall Street J. September 17th.

Myatt, D.P., Ronayne, D., 2019. Stable Price Dispersion. Oxford Working Paper Series, October, 873.

Pulford, B.D., Colman, A.M., 2007. Ambiguous games: Evidence for strategic ambiguity aversion. Q. J. Exp. Psychol. 60, 1083–1100.

Rácz, Z., Tasnádi, A., 2016. A Bertrand–Edgeworth oligopoly with a public firm. J. Econ. 119, 253–266.

Reynolds, S.S., Wilson, B.J., 2000. Bertrand–Edgeworth competition, demand uncertainty and asymmetric outcomes. J. Econom. Theory 92, 122–141.

Shilony, Y., 1977. Mixed pricing in oligopoly. J. Econom. Theory 14, 373–388.

Shubik, M., 1959. Strategy and Market Structure. Wiley, New York.

Tasnádi, A., 1999. Existence of pure strategy Nash equilibrium in Bertrand–Edgeworth oligopolies. Econom. Lett. 63, 201–206.

Varian, H., 1980. A model of sales. Amer. Econ. Rev. 70, 651–659.

Vives, X., 1986. Rationing rules and Bertrand–Edgeworth equilibria in large markets. Econom. Lett. 21, 113–116.

Vives, X., 1999. Oligopoly Pricing: Old Ideas and New Tools. MIT Press, Cambridge.