

Smooth and sharp creation of a spherical shell for a (3 + 1)-dimensional quantum field

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We study the creation of a spherical, finite radius source for a quantized massless scalar field in 3 + 1 dimensions. The goal is to model the breakdown of correlations that has been proposed to occur at the horizon of an evaporating black hole. We do this by introducing at fixed radius $r = a$ a one parameter family of self-adjoint extensions of the three dimensional Laplacian operator that interpolate between the condition that the values and the derivatives on the two sides of $r = a$ coincide for $t \leq 0$ (no wall) and the two-sided Dirichlet boundary condition for $t \geq 1/\lambda$ (fully-developed wall). Creation of the shell produces null, spherical pulses of energy on either side of the shell, one ingoing and the other outgoing. The renormalized energy density $\langle T_{00} \rangle$ diverges to positive infinity in the outgoing energy pulse, just outside the light cone of the fully-formed wall at $t = 1/\lambda$. Unlike in the 3 + 1 point source creation, there is no persistent memory cloud of energy. As in the creation of a 1 + 1 dimensional wall, the response of an Unruh-DeWitt detector in the post-shell region is independent of the time scale for shell formation and is finite. The latter property casts doubt on the efficacy of this mechanism for firewall creation.

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I. INTRODUCTION

It has been more than thirty years since Hawking first suggested [1] that black hole formation might give rise to a fundamental breakdown of predictability. Many approaches have been taken in order to resolve this apparent dilemma [2]. Among these is the suggestion [3–6] that the horizon of a radiating black hole might be more singular than suggested by standard quantum field theory on a curved background [7,8]. Specifically, there might exist an “energetic curtain” [5] or “firewall” [6] whose purpose is to break correlations between objects falling into the horizon and those remaining on the outside. An important question then is whether there exists a local mechanism for creating such a firewall, one that does not rely on a detailed knowledge of the underlying theory of quantum gravity.

Recently Brown and Louko [9] explored a 1 + 1 dimensional model of firewall creation based on the imposition of time dependent boundary conditions at a fixed point in space. These boundary conditions were equivalent to the insertion of a wall that broke correlations between the quantum field on the left side of the wall and that on

the right. They found that in the rapid creation limit this scenario resulted in the creation of a divergent null pulse of energy emanating from the point at which the boundary conditions were imposed. It is this pulse of energy that plays the role of the firewall in this model because in the spacetime of an evaporating black hole, the firewall is supposed to be forming near the horizon, which is a null surface. Brown and Louko [9] found that the response of an Unruh-DeWitt detector crossing this pulse remained finite, irrespective of how rapidly the wall was created, suggesting that this mechanism could not produce sufficient energy to break all correlations as required.

A potentially more realistic 3 + 1 dimensional model was studied in [10], where time dependent boundary conditions interpolating between Neumann-type (ordinary Minkowski dynamics) and Dirichlet-type were imposed at the spatial origin. This corresponds to the smooth creation of a point source, as opposed to a wall separating two regions of space. As one might expect from standard quantum field theory, the field’s energy and the detector response are more divergent in 3 + 1 dimensions than in 1 + 1 dimensions. The 3 + 1 renormalized energy density $\langle T_{00} \rangle$ was shown to be well defined everywhere away from the source but unbounded both above (after) and below (before) the energetic pulse. Moreover, in this model a cloud of positive energy lingers near the source after the

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source is fully formed. The total energy of this cloud is positive infinity. At fixed radius, r , $\langle T_{00} \rangle$ is not static and diverges as $t \rightarrow \infty$. In the limit of rapid source creation $\langle T_{00} \rangle$ diverges everywhere in the timelike future of the creation event. The response of an Unruh-DeWitt detector traversing the shell is divergent as desired for the firewall mechanism, but the divergence appears to be primarily due to the energetic cloud that surrounds the source.

The purpose of the present work is to extend the study of this mechanism further by considering the creation of a spherical wall in $3 + 1$ dimensions. We do this by introducing at fixed radius $r = a$ a one parameter family of self-adjoint extensions of the three dimensional Laplacian operator that interpolate between the condition that the values and the derivatives on the two sides of $r = a$ coincide for $t \leq 0$ (no wall) and the two-sided Dirichlet boundary condition for $t \geq 1/\lambda$ (fully-developed wall). As in the previous studies, wall creation produces a null pulse of energy on either side of the shell, one ingoing and the other outgoing. We find that $\langle T_{00} \rangle$ diverges to positive infinity in the outgoing energy pulse, just outside the light cone of the fully-formed wall at $t = 1/\lambda$. Unlike in the $3 + 1$ point source creation, there is no persistent memory cloud of energy. As in the $1 + 1$ dimensional wall, the response of the detector in the post-shell region is independent of the time scale $1/\lambda$ and finite, once again casting doubt on the efficacy of this mechanism for breaking all entanglement at black hole horizons.

The paper is organized as follows. In Sec. II, we set up the massless Klein-Gordon equation with the boundary conditions at $r = a$ required for shell formation. We solve for the mode functions for time $t < a$ and quantize the field. We restrict to $t < a$ in order to ensure that the ingoing pulse does not have time to reach the origin and re-disperse, thereby simplifying the calculation significantly. In Sec. III, we discuss the quantized total energy with focus on the energy density in the regions to the “future” of the outgoing pulses. A discussion of the energy density in the intermediate regions can be found in Appendix D. Section IV investigates the response of an Unruh-DeWitt detector. Section V concludes with a summary of the results and conclusions. Technical details are given in five Appendices.

II. QUANTIZATION

We consider a real massless scalar field ϕ in $(3 + 1)$ -dimensional Minkowski spacetime with field equation

$$(\partial_t^2 - \nabla^2)\phi = 0. \quad (2.1)$$

In spherical coordinates, the Laplacian is

$$\nabla^2 = \frac{1}{r^2} \partial_r (r^2 \partial_r) + \frac{1}{r^2} \nabla_{S^2}^2 \quad (2.2)$$

where $\nabla_{S^2}^2$ is the Laplacian on the unit S^2 sphere. We consider only the spherically symmetric sector and therefore we define

$$\phi = \frac{f(t, r)}{\sqrt{4\pi r}}, \quad (2.3)$$

so that the field equation (2.1) becomes

$$(\partial_t^2 - \partial_r^2)f(t, r) = 0. \quad (2.4)$$

We want to consider the formation of a wall, or spherical shell, at position $r = a$ between times $t = 0$ and $t = 1/\lambda$. In order to do this, we replace the Laplacian with a one parameter family of self-adjoint extensions defined on $L_2(\mathbb{R}^3)$ with the sphere at $r = a$ removed. Some details are given in Appendix A; we summarize the important points below. The self-adjoint extensions of the Laplacian are parametrized by the function $\theta(t)$, and we assume $\theta \in [0, \pi/2]$ so that the spectrum of the self adjoint extensions of the Klein-Gordon equation has no tachyonic modes. The angle $\theta(t)$ can be written in terms of a function h of a dimensionless variable $T = \lambda t$ which is defined by the equation

$$\theta(t) = \cot^{-1}[L\lambda \cot(h(\lambda t))]. \quad (2.5)$$

L is a positive constant of dimension length which is introduced for convenience; its length is considered fixed. The real solutions of the Klein-Gordon operator satisfy the boundary conditions

$$f(t, 0) = 0 \quad (2.6)$$

$$f(t, a^-) = f(t, a^+), \quad (2.7)$$

$$\frac{f'(t, a^+)}{f(t, a^+)} - \frac{f'(t, a^-)}{f(t, a^-)} = 2\lambda \cot(h(\lambda t)). \quad (2.8)$$

In order to model the creation of a shell at $r = a$ we choose a smooth function $h(T)$ that interpolates between $h(0) = \pi/2$ and $h(1) = 0$

$$h(T) = \pi/2 \quad \text{for } T \leq 0, \quad (2.9a)$$

$$0 < h(T) < \pi/2 \quad \text{for } 0 < T < 1, \quad (2.9b)$$

$$h(T) = 0 \quad \text{for } T \geq 1. \quad (2.9c)$$

From Eqs. (2.7), (2.8), (2.9) we see that the boundary condition at $r = a$ evolves from the condition at $t = T = 0$ that the derivatives on the two sides of $r = a$ coincide, to Dirichlet at $\lambda t = T = 1$, which can be thought of as the creation of a wall at $r = a$. In order to show that this wall has a physical interpretation which can be related to the

firewall scenario, we would have to show that it produces a pulse of energy, as explained in Sec. III.

We will quantize the scalar field by writing

$$\phi(t, r) = \int_0^\infty (a_k \phi_k(t, r) + a_k^\dagger \bar{\phi}_k(t, r)) dk, \quad (2.10)$$

or equivalently

$$f(t, r) = \int_0^\infty (a_k U_k(t, r) + a_k^\dagger \bar{U}_k(t, r)) dk, \quad (2.11)$$

where

$$U_k(t, r) = \frac{\phi_k(t, r)}{\sqrt{4\pi r}}, \quad (2.12)$$

and the annihilation and creation operators have the commutators $[a_k, a_{k'}^\dagger] = \delta(k - k')$. The mode functions $U_k(t, r)$ will be normalized so that the field and its time derivative have the correct equal-time commutator. The vacuum $|0\rangle$ is the state that is annihilated by all a_k . We work in the radial null coordinates $u := t - r$ and $v := t + r$, and write the field equation (2.1)

$$\partial_u \partial_v f = 0. \quad (2.13)$$

We construct an ansatz for the mode functions in two regions inside ($r < a$) and outside ($r > a$) the shell. We consider the case when $a > 1/\lambda$ which means that (in units where $c = 1$) the timescale for wall formation is less than the distance between the location of the wall and the origin. Physically this means that during the wall formation process only left-movers are modified inside the shell, and only right-movers are modified outside the shell. Our ansatz is

$$U_k(t, r) = \begin{cases} \frac{1}{\sqrt{4\pi k}} (e^{-ikv} + E_k(u)) & \text{for } r > a, \\ \frac{1}{\sqrt{4\pi k}} (G_k(v) - e^{-iku}) & \text{for } 0 < r < a. \end{cases} \quad (2.14)$$

This form of U_k satisfies the Klein-Gordon equation for any choice of the functions $G_k(u)$ and $E_k(v)$. Our goal is to find a solution for these functions so that the boundary conditions (2.6)–(2.8) are satisfied. Substituting (2.14) into (2.7), (2.8) we obtain

$$-e^{-ik(t-a)} + G_k(t+a) = e^{-ik(t+a)} + E_k(t-a) \quad (2.15a)$$

$$2\lambda \cot h(\lambda t) = \frac{-ike^{-ik(t+a)} - E'_k(t-a)}{e^{-ik(t+a)} + E_k(t-a)} - \frac{-ike^{-ik(t-a)} + G'_k(t+a)}{-e^{-ik(t-a)} + G_k(t+a)} \quad (2.15b)$$

where the prime indicates differentiation with respect to the argument. To solve (2.15), we use the dimensionless time variable $T = \lambda t$ introduced previously and define the auxiliary function $B(T)$

$$B(T) = \begin{cases} 1 & \text{for } T \leq 0, \\ \exp\left(\int_0^T \cot(h(z)) dz\right) & \text{for } 0 < T < 1. \end{cases} \quad (2.16)$$

From (2.9) it is easy to see that $B(0) = 1$ and that for $0 \leq T < 1$, $B(T)$ is smooth and satisfies

$$\frac{B'(T)}{B(T)} = \cot(h(T)). \quad (2.17)$$

In Appendix B we show that $1/B(T)$ and all of its derivatives approach zero as $T \rightarrow 1_-$, and therefore $1/B(T)$ is smooth at $T = 1$, but $B(T) \rightarrow \infty$ as $T \rightarrow 1_-$.

Rearranging (2.15) we obtain first order differential equations for the derivatives $\partial_t E_k(t, a)$ and $\partial_t G_k(t, a)$. Introducing the additional dimensionless variables $K = k/\lambda$ and $A = a\lambda$, $y = \lambda u$ and $w = \lambda v$, and defining the functions $R_K(y) = E_k(u)$ and $S_K(w) = G_k(v)$ we obtain

$$R_K(y) = \begin{cases} -e^{-iKy} & y < -A \\ -e^{-iKy} + \frac{2i \sin(AK)}{B(y+A)} \int_0^{y+A} e^{-iKz} B'(z) dz & y \in (-A, 1-A) \\ -e^{-iK(y+2A)} & y > 1-A \end{cases} \quad (2.18)$$

$$S_K(w) = \begin{cases} e^{-iKw} & w < A \\ e^{-iKw} + \frac{2i \sin(AK)}{B(w-A)} \int_0^{w-A} e^{-iKz} B'(z) dz & w \in (A, 1+A) \\ e^{-iK(w-2A)} & w > 1+A \end{cases} \quad (2.19)$$

Substituting these expressions into (2.14) we obtain expressions for the mode functions $U_k(t, r)$ in the regions inside and outside of the shell. A spacetime diagram is shown in Fig. 1.

In order to write equations in a more compact form we sometimes use a shorthand notation in which functions that depend only on $y + A$ are written without their arguments, for example $B := B(y + A)$. This notation is used throughout the Appendices.

The solutions in Eqs. (2.18), (2.19) have the following features:

- (1) The regions in Fig. 1 marked P_{out} and P_{in} correspond to the early time regions, relative to the

$$U_k(t, r) = \frac{-ie^{-ik(t+a)} \sin[k(r-a)]}{\sqrt{\pi k}},$$

$$U_k(t, r) = \frac{-ie^{-ik(t-a)} \sin[k(r-a)]}{\sqrt{\pi k}},$$

$y > 1 - A$ and $r > a$ [region F_{out}],

$w > 1 + A$ and $r < a$ [region F_{in}].

Comparing with the above expression we see that when $ka = 2n\pi$, which means that wall formation occurs at a node of the original mode, the mode function in the late time region is the same as the original Minkowski mode.

- (2) The middle lines of (2.18) and (2.19) give, respectively, the expressions for the mode functions on the inward and outward light cones of the events where the boundary condition changes [at $r = a$ and $t \in (0, \lambda^{-1})$].
- (3) The middle lines of (2.18) and (2.19) give, respectively, the expressions for the mode functions on the inward and outward light cones of the events where the boundary condition changes [at $r = a$ and $t \in (0, \lambda^{-1})$].
- (4) For $t < a$ we have

$$\lim_{r \rightarrow a_-} (G_k(v) - e^{-iku}) = \lim_{r \rightarrow a_+} (e^{-ikv} + E_k(u))$$

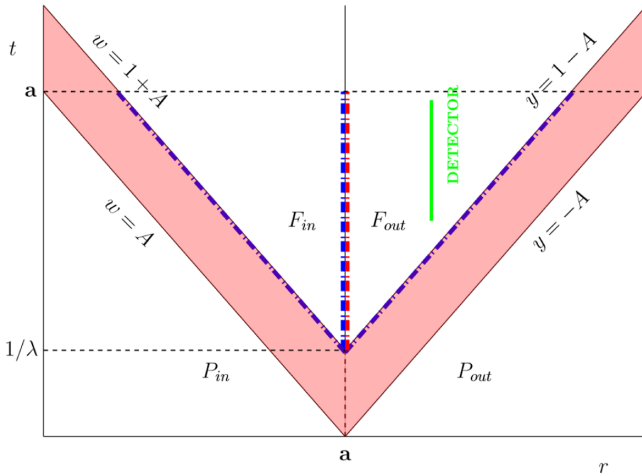


FIG. 1. Spacetime diagram of the evolving boundary conditions. The interpolation between $\theta = \pi/2$ and $\theta = 0$ at $r = a$ occurs over $0 < t < \lambda^{-1}$, and the null cones of the events where the boundary condition changes fill the regions $(-A < y < 1 - A, r > a)$ and $(A < w < 1 + A, r < a)$ in the spacetime. The triangular regions marked P_{out} , P_{in} , F_{out} and F_{in} indicate what we will refer to as the early time (P) and late time regions (F), outside and inside of the shell. The figure shows the case $a > \lambda^{-1}$. The blue dash-dotted line indicates that the energy density is positively divergent, while red dashed line shows where the energy density is negatively divergent.

wall creation. The top line of either (2.18) or (2.19) gives the mode functions in these regions, which are just those of the Minkowski vacuum: $U_k(t, r) = \frac{1}{\sqrt{4\pi k}} (e^{-ikv} - e^{-iku}) = -\frac{ie^{-ikt} \sin(kr)}{\sqrt{\pi k}}$.

- (2) The regions in Fig. 1 marked F_{out} and F_{in} correspond to the late time regions, relative to the wall creation. The mode functions in these two regions are obtained from the bottom lines of (2.18), (2.19) which give

and therefore the mode functions $U_k(t, r)$ are continuous at the location of the shell $r = a$.

- (5) It is easy to see that $R_K(y)$ is smooth for $y < 1 - A$ and in Appendix B we show that it is also smooth at $y = 1 - A$. Similarly $S_K(w)$ is smooth for $w \leq 1 + A$.
- (6) The mode functions are normalized so that

$$(\phi_k, \phi_{k'}) = i \int_0^\infty dr r^2 \int_{S^2} d\Omega (\overline{\phi_k} \partial_t \phi_{k'} - (\partial_t \overline{\phi_k}) \phi_{k'}) = \delta(k - k')$$

for any constant time hypersurface with $t < \lambda^{-1}$.

- (7) The late time region inside the shell ($w > 1 + A$ and $r < a$ which is denoted region F_{in} in Fig. 1) is not part of our calculation. One problem is that this part of the spacetime diagram would be influenced by waves reflected at the origin, so that we do not expect the ansatz (2.14) to be satisfied in this region. Physically, once the shell is fully formed, an infinite potential separates the regions inside and outside, which means there is no flow of probability between them.

Useful alternate expressions for the functions $R_K(y)$ and $S_K(w)$ are obtained by integrating the middle lines in (2.18) and (2.19) by parts:

$$R_K(y) = -e^{-iK(y+2A)} - \frac{2i \sin(AK)}{B(y+A)} - \frac{2K \sin(AK)}{B(y+A)} \int_0^{y+A} e^{-iKz} B(z) dz, \quad (2.20)$$

$$S_K(w) = e^{-iK(w-2A)} - \frac{2i \sin(AK)}{B(w-A)} - \frac{2K \sin(AK)}{B(w-A)} \int_0^{w-A} e^{-iKz} B(z) dz. \quad (2.21)$$

We note that the mode functions in the future regions F_{in} and F_{out} , from the third lines of (2.18) and (2.19), are λ independent. As we will see in the Sec. IV, this means that the detector response in the future regions does not depend on λ , which is the parameter that controls how fast the shell evolves at $r = a$.

III. TOTAL ENERGY

The expression for the energy density outside of the shell in terms of the mode functions $R_K(y)$ has exactly the same form as in Ref. [10]. We summarize the calculation and give the result below. The renormalized energy density of the quantized field in the state $|0\rangle$ is obtained by point-splitting the field operators, taking the expectation value in $|0\rangle$, subtracting the corresponding expectation value in the Minkowski vacuum $|0\rangle_M$, and taking the coincidence limit. This gives

$$\begin{aligned} \langle T_{00} \rangle &= \langle 0|T_{00}|0 \rangle_{\text{ren}} \\ &= \lim_{\substack{u_1, u_2 \rightarrow u \\ v_1, v_2 \rightarrow v}} (\partial_{u_1} \partial_{u_2} + \partial_{v_1} \partial_{v_2}) \\ &\quad \times [\langle 0|\phi(1)\phi(2)|0 \rangle - \langle 0_M|\phi(1)\phi(2)|0_M \rangle] \\ &= \frac{1}{4\pi} \left(\frac{\langle (\partial_u f)^2 \rangle}{r^2} + \frac{\langle f(\partial_u f - \partial_v f) \rangle + c.c.}{2r^3} + \frac{\langle f^2 \rangle}{2r^4} \right). \end{aligned} \quad (3.1)$$

Using Eq. (2.11) and the first line of (2.14) we obtain in the outside region ($r > a$)

$$\begin{aligned} \langle T_{00} \rangle &= \frac{\lambda^2}{16\pi^2 r^2} \int_0^\infty \frac{dK}{K} [|R'_K(y)|^2 - K^2] \\ &\quad - \frac{1}{32\pi^2 r^2} \frac{\partial}{\partial r} \left(\frac{\mathcal{G}_{\text{out}}(t, r)}{r} \right) \end{aligned} \quad (3.2)$$

where \mathcal{G}_{out} is defined as

$$\mathcal{G}_{\text{out}} = \int_0^\infty \frac{dK}{K} [|e^{-iKw} + R_K(y)|^2 - |e^{-iKw} - e^{-iKy}|^2]. \quad (3.3)$$

Using the second line of (2.14) gives for the inside region ($r < a$)

$$\begin{aligned} \langle T_{00} \rangle &= \frac{\lambda^2}{16\pi^2 r^2} \int_0^\infty \frac{dK}{K} [|S'_K(w)|^2 - K^2] \\ &\quad - \frac{1}{32\pi^2 r^2} \frac{\partial}{\partial r} \left(\frac{\mathcal{G}_{\text{in}}(t, r)}{r} \right) \end{aligned} \quad (3.4)$$

with the definition

$$\mathcal{G}_{\text{in}} = \int_0^\infty \frac{dK}{K} [|S_K(w) - e^{-iKy}|^2 - |e^{-iKw} - e^{-iKy}|^2]. \quad (3.5)$$

In the early time regions, which are denoted P_{out} and P_{in} in Fig. 1, $\langle T_{00} \rangle$ vanishes by construction. In the late time regions F_{out} and F_{in} , the first terms in (3.2) and (3.4) vanish, as can be seen from (2.18), (2.19). The functions \mathcal{G}_{out} and \mathcal{G}_{in} are easily calculated. We obtain

$$\mathcal{G}_{\text{out future}} = \mathcal{G}_{\text{in future}} = \ln[(r-a)^2] - \ln(r^2) \quad (3.6)$$

$$\langle T_{00} \rangle_{\text{future}} = \frac{1}{16\pi^2 r^4} (\ln|r-a| - \ln(r)) - \frac{1}{16\pi^2 r^3} \left(\frac{1}{r-a} - \frac{1}{r} \right). \quad (3.7)$$

This result shows that the energy density goes discontinuously from positive infinity just inside the shell to negative infinity energy just outside the shell. However, the spatial integral of the energy density along a constant time hypersurface which crosses $r = a$ produces a finite energy, if interpreted at $r = a$ in the principal value sense. In Appendix D we consider the intermediate outside region $r > a$ and $-A < y < 1 - A$ and consider the continuity of the energy density as $y + A \rightarrow 1_-$. We show that the energy density is finite everywhere except to the immediate past of the light cone of the point where boundary condition finishes changing, where it is positively divergent.

IV. DETECTOR RESPONSE

In this section we consider an inertial Unruh-DeWitt (UDW) detector [11,12] that is linearly coupled to the quantum field and at a fixed spatial location. Using first-order perturbation theory, the probability that the detector undergoes a transition from a state with energy 0 to a state with energy ω is proportional to the response function [7,8,11,12]

$$\mathcal{F}(\omega) = \int_{-\infty}^\infty dt_1 \int_{-\infty}^\infty dt_2 e^{-i\omega(t_1-t_2)} \chi(t_1) \chi(t_2) \mathcal{W}(t_1, t_2), \quad (4.1)$$

where the smooth real-valued switching function χ specifies how the detector's interaction with the field is turned on and off, and \mathcal{W} is the pull-back of the field's Wightman function to the detector's worldline. We consider a detector located outside the spherical shell ($r > a$) and operating only in the future region $y > 1 - A$. The response function is

$$\Delta\mathcal{F}(\omega) = \int_{-\infty}^\infty dt_1 \int_{-\infty}^\infty dt_2 e^{-i\omega(t_1-t_2)} \chi(t_1) \chi(t_2) \Delta\mathcal{W}(t_1, t_2), \quad (4.2)$$

where

$$\Delta\mathcal{W}(t_1, t_2) = \frac{1}{4\pi r^2} \int_0^\infty (U_k(t_1 - r, t_1 + r) \overline{U_k(t_2 - r, t_2 + r)} - U_k^M(t_1 - r, t_1 + r) \overline{U_k^M(t_2 - r, t_2 + r)}) dk. \quad (4.3)$$

The functions U_k and $\overline{U_k}$ are obtained from the first line in (2.14) with $E_k(u) = R_K(y)$ and $R_K(y)$ given by the last line of (2.18); U_k^M and $\overline{U_k^M}$ are obtained in the same way except using the first line of (2.18). Substituting these expressions and collecting terms we find

$$16\pi^2 r^2 \Delta\mathcal{W}(t_1, t_2) = \int_0^\infty \frac{dk}{k} (e^{ik(2r-t_1+t_2)} + e^{-ik(2r+t_1-t_2)} - e^{ik(2a-2r-t_1+t_2)} - e^{-ik(2a-2r+t_1-t_2)}). \quad (4.4)$$

Doing the k integral we get

$$16\pi^2 r^2 \Delta\mathcal{W}(t_1, t_2) = \ln \left[\frac{|4(a-r)^2 - (t_1 - t_2)^2|}{|4r^2 - (t_1 - t_2)^2|} \right] + i\pi \operatorname{sgn}(t_1 - t_2) \theta(r - |t_1 - t_2|/2) \times \theta(a - r + |t_1 - t_2|/2). \quad (4.5)$$

The change in the Wightman function is λ independent, and therefore one does not obtain a divergent response in the limit $\lambda \rightarrow 0$, which would correspond to instantaneous wall creation. There is a divergence when $|t_1 - t_2| = 2r$, but this divergence is only logarithmic.

V. CONCLUSION

We have extended previous work on firewall creation via time dependent boundary conditions by considering the smooth and sharp creation of a fixed radius shell in (3 + 1)-dimensional Minkowski space. This was implemented by introducing at $r = a$ a one parameter family of self-adjoint extensions of the three dimensional Laplacian operator that interpolate between the condition that the values and derivatives on the two sides of $r = a$ coincide for $t \leq 0$ (no wall) and the two-sided Dirichlet boundary condition for $t \geq 1/\lambda$ (fully-developed wall). Wall creation produces null pulses of energy on either side of the shell, one ingoing and the other outgoing. As in the previous two calculations, [9,10], the boundary condition is being changed by an external agent whose dynamics is not included in the field action. The total energy of the field hence need not be conserved on its own since the external agent may inject energy into the field. Since we are primarily interested in the detector response across the energy pulse, we have not calculated all the contributions to the field energy on hypersurfaces of constant Minkowski time, in particular the contributions from inside the shell, and therefore have

not established whether the total energy of the field can be unambiguously defined.

We found that the process of shell creation is significantly more divergent than in (1 + 1)-dimensions [9], but less divergent than for point-like source creation in (3 + 1)-dimensions [10]. In the present case, the energy density $\langle T_{00} \rangle$ diverges to positive infinity in the outgoing energy pulse, just outside the light cone of the fully-formed wall at $t = 1/\lambda$. Unlike in the 3 + 1 point source creation case, there is no persistent memory cloud of energy. As in the 1 + 1 dimensional wall case, the response of the detector in the post-shell region is independent of the time scale $1/\lambda$ and finite, casting doubt once again on the viability of wall creation as a possible mechanism for breaking entanglement at the event horizon of an evaporating black hole.

Finally, we note that in order to determine how well breaking correlations by changing boundary conditions models the breaking of correlations in the proposed quantum gravitational firewall scenario, one would need to address the gravitational dynamics. This remains a question for future work.

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APPENDIX A: SCALAR LAPLACIAN ON \mathbb{R}^3 WITH A SPHERICAL SHELL

Defining the short hand notation $\int_0^\infty dr = \int_0^{a^-} + \int_{a^+}^\infty$ and scaling $g = f/r$, we can map the 3 dimensional Laplacian and the L_2 inner product on a positive half line with a spherical shell at $r = a$ to

$$\nabla^2 \Rightarrow \nabla^2 = \partial_r^2 + \frac{1}{r^2} \nabla_{S^2}^2, \quad (g_1, g_2) \Rightarrow (f_1, f_2) = \int_{S^2} d\Omega \int_0^\infty dr \bar{f}_1 f_2. \quad (A1)$$

We consider only the spherically symmetric case [see Eq. (2.3)], and Hermiticity of ∇^2 requires

$$(f_1, \nabla^2 f_2) - (\nabla^2 f_1, f_2) = \int_0^\infty dr \partial_r [\bar{f}_1 (\partial_r f_2) - (\partial_r \bar{f}_1) f_2] = 0. \quad (A2)$$

Taking $f_1 = f_2 \equiv f$, the square bracket $[\bar{f}_1 (\partial_r f_2) - (\partial_r \bar{f}_1) f_2]$ can be written

$$S(t, x) = \frac{1}{(2iL)} (|L\partial_r f - if|^2 - |L\partial_r f + if|^2). \quad (\text{A3})$$

Following the method in [13], we find that the boundary conditions are given by

$$\begin{pmatrix} Lf'_+ - if_+ \\ Lf'_- + if_- \\ Lf'_0 - if_0 \end{pmatrix} = U \begin{pmatrix} Lf'_+ + if_+ \\ Lf'_- - if_- \\ Lf'_0 + if_0 \end{pmatrix} \quad (\text{A4})$$

where we have used the shorthand notation $f(t, 0) = f_0$, $f(t, a^-) = f_-$ and $f(t, a^+) = f_+$, and the choice of a unitary 3×3 matrix U specifies the boundary condition. Generally, the matrix U would be decomposed by Gell-Mann matrices. We choose an expression of U that ensures no flow of probability through the origin and depends on one free parameter which is a time dependent function chosen to model the formation of the wall at $r = a$. We use

$$U = \begin{pmatrix} e^{-i\theta(t)} \begin{pmatrix} \cos \theta(t) & i \sin \theta(t) \\ i \sin \theta(t) & \cos \theta(t) \end{pmatrix} & \\ & 1 \end{pmatrix} \quad (\text{A5})$$

and Eq. (A4) gives the conditions

$$f(0) = 0 \quad (\text{A6})$$

$$f_- = f_+ \quad (\text{A7})$$

$$\frac{2}{L} \cot \theta(t) = \frac{f'_+}{f_+} - \frac{f'_-}{f_-} \quad (\text{A8})$$

We note that the continuity of the wave function at $r = a$ is a result of our choice of $U(\theta)$, and does not have to be imposed as an extra condition.

APPENDIX B: MODE FUNCTION REGULARITY

In this Appendix we show that the mode function $R_K(y)$ is C^{26} at $y = 1 - A$. We follow closely Appendix B of [10].

1. Smoothness of $1/B(y)$

First we show that $1/B(y)$ and all its derivatives go to zero as $y \rightarrow 1_-$. We write $g(y) := \tan(h(y))$ and from Eq. (2.9) we see that $g(y) > 0$, and $g(y)$ and all its derivatives approach 0 as $y \rightarrow 1_-$.

Using (2.16) we obtain

$$B(y) = \exp \left(\int_0^y \frac{dz}{g(z)} \right) \quad (\text{B1})$$

$$B'(y) = B(y)/g(y). \quad (\text{B2})$$

Defining $B_{\text{in}}(y) = 1/B(y)$, we have $B'_{\text{in}}(y) = -B_{\text{in}}(y)/g(y)$. For $n \in \mathbb{N}$, induction gives

$$B_{\text{in}}^{(n)}(y) = (-1)^n P_n(y) f_n(y), \quad (\text{B3a})$$

$$f_n(y) = \frac{B_{\text{in}}(y)}{(g(y))^n}, \quad (\text{B3b})$$

where each P_n is a polynomial in g and its derivatives and $P_n(y) \rightarrow 1$ as $y \rightarrow 1_-$. We show below that $f_n(y) \rightarrow 0_+$ as $y \rightarrow 1_-$, and therefore $B_{\text{in}}^{(n)}(y) \rightarrow 0$. From (B3b) we have

$$\ln(f_n(y)) = - \left(\int_0^y \frac{dz}{g(z)} \right) \left(1 + \frac{n \ln(g(y))}{\int_0^y \frac{dz}{g(z)}} \right). \quad (\text{B4})$$

As $y \rightarrow 1_-$, the first parentheses in (B4) tend to ∞ , while the second parentheses tend to 1 by l'Hôpital. Hence $\ln(f_n(y)) \rightarrow -\infty$ as $y \rightarrow 1_-$ and therefore $f_n(y) \rightarrow 0_+$ as $y \rightarrow 1_-$.

2. Differentiability of $R_K(y)$

We make the definitions

$$J_K(y) = \int_0^y B(z) e^{-iKz} dz \quad (\text{B5})$$

$$\mathcal{F}_K(y) = J_K(y)/B(y) \quad (\text{B6})$$

$$\mathcal{H}_K(y) = J_K(y)/(g(y)B(y)), \quad (\text{B7})$$

and rewrite the mode function (2.20) as

$$R_K(y) = -e^{-iK(2A+y)} - \frac{2i \sin(AK)}{B} - 2K \sin(AK) \mathcal{F}_K(y+A). \quad (\text{B8})$$

We will show that $R_K(y)$ is C^{26} at $y+A \rightarrow 1_-$ by showing that $\mathcal{F}_K(y)$ is C^{26} as $y \rightarrow 1_-$. We outline the steps below.

- (1) Calculate derivatives $\mathcal{F}_K^{(n)}(y)$ using (B2) to remove factors of $B'(y)$, so that the result does not contain derivatives of $B(y)$.
- (2) Define $\text{den}(y) = B(y)(g(y))^n$ and $\text{num}(y) = \mathcal{F}_K^{(n)}(y)\text{den}(y)$. In order to use l'Hôpital n times, calculate $\text{num}^{(n)}(y)$ and $\text{den}^{(n)}(y)$, again using (B2) after taking each derivative.
- (3) Define $\mathbf{num}^{(n)}(y) = \text{num}^{(n)}(y)/B(y)$ and $\mathbf{den}^{(n)}(y) = \text{den}^{(n)}(y)/B(y)$ and replace all remaining factors of $J_K(y)$ by $\mathcal{F}_K(y)B(y)$. The limit of the n th derivative of $\mathcal{F}(y)$ as $y \rightarrow 1_-$ now has the form

$$\lim_{y \rightarrow 1_-} \mathcal{F}^{(n)}(y) \xrightarrow{\text{l'Hôpital}} \lim_{y \rightarrow 1_-} \frac{\mathbf{num}^{(n)}(y)}{\mathbf{den}^{(n)}(y)}. \quad (\text{B9})$$

Using computer algebra we have verified up to order $n = 26$ that the quantities $\mathbf{num}^{(n)}(y)$ and $\mathbf{den}^{(n)}(y)$ have the form

$$\mathbf{num}^{(n)}(y) = P(y), \quad \mathbf{den}^{(n)}(y) = 1 + Q(y)$$

where $P(y)$ and $Q(y)$ are polynomials of $g(y)$ and its derivatives, $\mathcal{F}(y)$, and factors e^{-iKy} , and each term contains at least one power of $\mathcal{F}(y)$ or $g^{(j)}(y)$ $j \in (0, n)$. This means that both $P(y)$ and $Q(y)$ go to 0 as $y \rightarrow 1_-$. We therefore have from (B9) that $\mathcal{F}(y)$ is C^{26} at $y = 1$. We stopped the calculation at $n = 26$ because of limitations of computing time and memory. If the proof extends to $n \in \mathbb{N}$, we would have that $R_K(y)$ is smooth at $y = 1$.

APPENDIX C: LEMMA AND CONDITIONS ON g

Lemma C.1.

- (i) For a complex-valued function $f(y, z)$ that is bounded for $y \in [0, 1]$ and $z \in [0, 1 + A]$, we have

$$\lim_{y+A \rightarrow 1_-} \frac{1}{B(y+A)} \int_0^{y+A} B(z) f(y, z) dz = 0. \quad (\text{C1})$$

- (ii) If in addition $\partial_y f(y, z)$ is bounded for $y \in [0, 1]$ and $z \in [0, 1 + A]$, we have

$$\lim_{y+A \rightarrow 1_-} \partial_y \left(\frac{1}{B(y+A)} \int_0^{y+A} B(z) f(y, z) dz \right) = 0. \quad (\text{C2})$$

Proof.

- (i) By boundedness of f , there is a positive constant C such that $|f(y, z)| \leq C$. For $0 < y + A < 1$, using the triangle inequality and the positivity of B , we have

$$\begin{aligned} & \left| \frac{1}{B(y+A)} \int_0^{y+A} B(z) f(y, z) dz \right| \\ & \leq \frac{C}{B(y+A)} \int_0^{y+A} B(z) dz. \end{aligned} \quad (\text{C3})$$

When $y + A \rightarrow 1$, the rightmost expression in (C3) goes to zero, using l'Hôpital and (B2).

- (ii) For $0 < y + A < 1$, we expand out the derivative in (C2) using (B2) and make some cancellation. After repeatedly applying l'Hôpital as $y + A \rightarrow 1$ and using lemma (C1), the proof is done. ■

Conditions C.2. We introduce the technical assumption that $g'''(y) < 0$ for $0 < y < 1$. It follows that $g''(y) > 0$ and $g'(y) < 0$ for $0 < y < 1$. For $0 < z \leq y + A < 1$, this implies

$$g'(z) \leq \frac{g(y+A) - g(z)}{y+A-z} \leq g'(y+A), \quad (\text{C4})$$

$$g''(y+A) \leq \frac{g'(y+A) - g'(z)}{y+A-z} \leq g''(z), \quad (\text{C5})$$

where the quotients are understood at $z = y + A$ in the limiting sense. (C4) can be verified by writing the numerator as the integral of g' and using the monotonicity of g' , and (C5) can be verified similarly by writing the numerator as the integral of g'' . We will derive three consequences which are used in Secs. D 2 and D 3.

- (i) *First consequence:* $\lim_{y+A \rightarrow 1_-} \frac{g'(y+A)}{g(y+A)} = -\infty$.

For $0 < y + A < 1$, using the monotonicity of g' , we have

$$\begin{aligned} g(y+A) &= - \int_{y+A}^1 g'(z) dz \leq -g'(y+A) \\ &\times \int_{y+A}^1 dz = -(1-y-A)g'(y+A). \end{aligned} \quad (\text{C6})$$

Hence $g'(y+A)/g(y+A) \leq -1/(1-y-A)$, which implies $\lim_{y+A \rightarrow 1_-} \frac{g'(y+A)}{g(y+A)} = -\infty$.

- (ii) *Second consequence:* $\lim_{y+A \rightarrow 1_-} \mathcal{C}(y) = 0$

For $0 < y + A < 1$, we define

$$\begin{aligned} \mathcal{C}(y) &= \frac{1}{B(y+A)} \int_0^{y+A} dz \cos(y+A-z) B'(z) \\ &\times \frac{g(y+A) - g(z)}{y+A-z}, \end{aligned} \quad (\text{C7})$$

$$\mathcal{C}_-(y) = \frac{1}{B(y+A)} \int_0^\kappa dz \left[\frac{g(y+A) B'(z)}{y+A-z} - \frac{B(z)}{y+A-z} \right], \quad (\text{C8})$$

$$\mathcal{C}_+(y) = \frac{1}{B(y+A)} \int_\kappa^{y+A} dz B'(z) \frac{g(y+A) - g(z)}{y+A-z}, \quad (\text{C9})$$

$$\tilde{\mathcal{C}}_-(y) = - \frac{1}{B(y+A)} \int_0^\kappa dz \frac{B(z)}{y+A-z}, \quad (\text{C10})$$

$$\tilde{\mathcal{C}}_+(y) = \frac{1}{B(y+A)} \int_\kappa^{y+A} dz B'(z) g'(z), \quad (\text{C11})$$

where $0 < \cos(y+A-z) \leq 1$, $B'(z) > 0$ and $g'(z) < 0$ for $0 < z < y+A$, and we have introduced a constant κ with $0 < \kappa < y+A < 1$. We have $\tilde{\mathcal{C}}_-(y) < \mathcal{C}_-(y)$ and $\tilde{\mathcal{C}}_+(y) \leq \mathcal{C}_+(y)$ using (C4). Therefore

$$\tilde{\mathcal{C}}_+(y) + \tilde{\mathcal{C}}_-(y) < \mathcal{C}_+(y) + \mathcal{C}_-(y) < \mathcal{C}(y) < 0 \quad (\text{C12})$$

As $y + A \rightarrow 1_-$, both integral $\tilde{\mathcal{C}}_-(y)$ and integral $\tilde{\mathcal{C}}_+(y)$ tend to 0. From (C12) it then follows that $\lim_{y+A \rightarrow 1_-} \mathcal{C}(y) = 0$.

(iii) *Third consequence:* $\lim_{y+A \rightarrow 1_-} \mathcal{D}(y) = 0$

We define

$$\mathcal{D}(y) = \frac{1}{B} \int_{\kappa}^{y+A} dz B'(z) \frac{\cos(\alpha)}{\alpha} \times \left(\frac{g(A+y) - g(z)}{\alpha} - g'(A+y) \right), \quad (\text{C13})$$

$$\mathcal{D}_2(y) = \frac{1}{B} \int_{\kappa}^{y+A} dz B'(z) \frac{1}{\alpha} \times \left(\frac{g(A+y) - g(z)}{\alpha} - g'(A+y) \right), \quad (\text{C14})$$

$$\mathcal{D}_3(y) = \frac{1}{B} \int_{\kappa}^{y+A} dz B'(z) \frac{1}{\alpha} (g'(z) - g'(A+y)), \quad (\text{C15})$$

$$\mathcal{D}_4(y) = -\frac{1}{B} \int_{\kappa}^{y+A} dz B'(z) g''(z), \quad (\text{C16})$$

where $\alpha := y + A - z$, $0 < \cos(y + A - z) \leq 1$ and $B'(z) > 0$ for $0 < z < y + A$. Using Eqs. (C4) and (D.2), there is

$$\mathcal{D}_4(y) \leq \mathcal{D}_3(y) \leq \mathcal{D}_2(y) < \mathcal{D}(y) < 0. \quad (\text{C17})$$

Further use of l'Hôpital on $\mathcal{D}_4(y)$ we find $\lim_{y+A \rightarrow 1_-} \mathcal{D}_4(y) = 0$, which means $\lim_{y+A \rightarrow 1_-} \mathcal{D}(y) = 0$.

APPENDIX D: BEHAVIOR OF $\langle T_{00} \rangle$ OUTSIDE THE SHELL IN THE INTERMEDIATE REGION

In this section we study the behavior of the energy density outside the shell ($r > a$) in the intermediate region ($-A \leq y \leq 1 - A$), which is given in Eq. (3.2) with the function \mathcal{G}_{out} defined in (3.3), and $R_K(y)$ given in the middle line of (2.18). We rewrite these expressions as

$$\langle T_{00} \rangle = \frac{\lambda^2}{16\pi^2 r^2} [F(y) + \tilde{F}(y)] \quad (\text{D1})$$

$$F(y) = \int_0^\infty \frac{dK}{K} [|R'_K(y)|^2 - K^2] \quad (\text{D2})$$

$$\tilde{F}(y) = -\frac{1}{w-y} \left[\partial_w - \partial_y - \frac{2}{w-y} \right] \mathcal{G}_{\text{out}}, \quad (\text{D3})$$

where the notation suppresses the dependence of $F(y)$ and $\tilde{F}(y)$ on w . We comment that the denominators in (D3) are produced by the factor $1/r$ in Eq. (3.2), and the action of the derivative ∂_r on this factor.

1. Preliminaries

From (2.20) we have for fixed $y \in (-A, 1 - A)$ the small K estimates

$$R_K(y) = -1 + O(K), \quad (\text{D4})$$

$$|R'_K(y)|^2 = O(K^2), \quad (\text{D5})$$

and the large K estimates

$$|R_K(y)|^2 = 1 + \frac{2B' \sin(2AK)}{B K} + O(K^{-2}), \quad (\text{D6})$$

$$|R'_K(y)|^2 = K^2 + \frac{2B'}{B} K \sin(2AK) + O(K^{-1}). \quad (\text{D7})$$

Using these results it is straightforward to show that the integrals in Eqs. (D2) and (D3) are well defined for $y \in (-A, 1 - A)$.

At $y = -A$ both integrands vanish. At $y = 1 - A$ the integrand in (D2) vanishes, and a straightforward calculation gives $\mathcal{G}_{\text{out}}|_{y=1-A} = 2[\ln(r-a) - \ln r]$ in agreement with (3.6). In the remainder of this Appendix we study the continuity of $F(y)$ and $\tilde{F}(y)$ in Eqs. (D2) and (D3) as $y + A \rightarrow 1_-$.

2. Term $F(y)$

We divide the integral in (D2) into two pieces defined as

$$F_-(y) = \int_0^1 \frac{dK}{K} [|R'_K|^2 - K^2], \quad (\text{D8})$$

$$F_+(y) = \int_1^\infty \frac{dK}{K} [|R'_K|^2 - K^2]. \quad (\text{D9})$$

To show that $F_-(y)$ is finite, we use the form of $R_K(y)$ in Eq. (2.20). Differentiating and substituting into (D8), we get an expression with \mathcal{H}_K as defined in Eq. (B7) and $f_i := f_i(y + A)$ as defined in Eq. (B3b). It is then straightforward to show with l'Hôpital that $\lim_{y+A \rightarrow 1_-} F_-(y)$ exists and is finite.

Now we study the second term $F_+(y)$. Our strategy is as follows:

- (1) Integrate by parts in z , taking the antiderivative of the factor $e^{\pm iKz}$, until we have enough powers of $1/K$ so that the K -integral is convergent.
- (2) Switch the order of the z and K integrals and do the K integral.
- (3) Integrate by parts again so that derivatives are removed from factors $B(z)$.
- (4) Analyse the behavior of the remaining integrals.

Defining

$$V_K = \frac{1}{B} \int_0^{y+A} dz B''(z) e^{-iKz}, \quad (\text{D10})$$

the result of step (1) is

$$F_+(y) = \int_1^\infty dK \left[\frac{4V_K \overline{V}_K \sin^2(AK)}{K^3 g^2} - \frac{4\sin^2(AK)(V_K e^{iK(A+y)} + \overline{V}_K e^{-iK(A+y)})}{K^3 g^3} + \frac{4i\sin^2(AK)(V_K e^{iK(A+y)} - \overline{V}_K e^{-iK(A+y)})}{K^2 g^2} \right. \\ \left. + \frac{2i\sin(AK)(V_K e^{iKy} - \overline{V}_K e^{-iKy})}{Kg} + \frac{4\sin^2(AK)}{K^3 g^4} + \frac{2\sin(2AK)}{g} \right]. \quad (\text{D11})$$

The integral over large K has terms of the form $\int_1^\infty dK \sin(2AK)$ which is regularized with a factor $e^{-\delta K}$ to obtain a finite result in the limit $\delta \rightarrow 0$. The other K integrals can be done using

$$\int_1^\infty \frac{dK}{K} (e^{iK\alpha} + e^{-iK\alpha}) = -2\text{Ci}(\alpha) \quad (\text{D12})$$

$$\int_1^\infty \frac{dK}{K^2} (e^{iK\alpha} + e^{-iK\alpha}) = 2\cos(\alpha) + 2\alpha\text{Si}(\alpha) - \pi\alpha \quad (\text{D13})$$

$$\int_1^\infty \frac{dK}{K^3} (e^{iK\alpha} + e^{-iK\alpha}) = \cos(\alpha) - \alpha\sin(\alpha) + \alpha^2\text{Ci}(\alpha) \quad (\text{D14})$$

where $\alpha > 0$ and Ci and Si are the cosine and sine integrals in the notation of [14].

First, for terms that include V_K as defined in (D10), we interchange the integration by the absolute convergence of multiple integral, and we evaluate the integration over K to reach an expression containing elementary functions and expressions like (D12). Many terms would contain factor B' or B'' , but we simply integrate by part to reduce them to combinations involving (C1) and (C2).

Next we introduce the parameters $\alpha := A + y - z$, $\beta := A - y + z$, $\gamma := 3A + y - z$, which will be used in 2nd, 3rd and 4th terms of (D11) for which we have $z \in (0, y + A)$; and $\rho := 2A - x + z$, $\sigma := 2A + x - z$ and $\tau := x - z$ for the 1st term of (D11) where $x \in (0, y + A)$ and $z \in (0, x)$. Using our original assumption $A > 1$ these parameters are all positive over the full range of the corresponding integrals. We therefore find out that single integrals not having factors Ci(α) and double integrals not having Ci(τ) can be easily proven to be bounded when $y + A \rightarrow 1_-$.

Third, we consider the remaining contributions to (D9) which contain factors Ci(α) and Ci(τ). These are the difficult terms and we label them I_{hard} . The terms containing Ci(τ) are double integrals of variable x and z , we handle them by first integrating by part in z then x and then integrating by part in x then z . This two different ways of integration by parts generate two equivalent expressions of the double integral. A lot of terms cancel out when we represent the double integral using its two equivalent expressions, further use of Lemma C.1, $\lim_{y+A \rightarrow 1_-} \mathcal{C}^+(y) = 0$ and $\lim_{y+A \rightarrow 1_-} \mathcal{D}^+(y) = 0$ in Appendix C.2 eventually reduce the result to

$$\lim_{y+A \rightarrow 1_-} I_{\text{hard}} = - \lim_{y+A \rightarrow 1_-} \frac{g}{g^2}. \quad (\text{D15})$$

Therefore $F(y)$ is divergent as $y + A \rightarrow 1_-$.

3. The term $\tilde{F}(y)$

The integral in (D3) can be divided into two pieces

$$\mathcal{G}_{\text{out}}^- = \int_0^1 \frac{dK}{K} [|e^{-iKw} + R_K(y)|^2 - |e^{-iKw} - e^{-iKy}|^2], \quad (\text{D16})$$

$$\mathcal{G}_{\text{out}}^+ = \int_1^\infty \frac{dK}{K} [|e^{-iKw} + R_K(y)|^2 - |e^{-iKw} - e^{-iKy}|^2]. \quad (\text{D17})$$

We use two different forms for $R_K(y)$: in $\mathcal{G}_{\text{out}}^-$ we use Eq. (2.20), and in $\mathcal{G}_{\text{out}}^+$ we use Eq. (2.18).

First we look at the easy piece (D16). We define the factors

$$l_1(z) = \frac{\cos(z) - 1}{z}, \quad l_2(z) = \frac{\cos(z) - 1}{z^2} + \frac{\sin(z)}{z}, \quad (\text{D18})$$

and in addition to the definitions $\alpha, \beta, \gamma, \rho, \sigma, \tau$, we use $\mu := 2A - z$, $\nu := 2A + z$, $\xi := A + w - z$ and $\chi := w - z - A$. It is straightforward to show that the parameters μ, ν, ξ , and χ are all non-negative for $A > 1$, $z \in (0, y + A)$ and $y \in (-A, 1 - A)$. We also note that $l_1(z)$ and $l_2(z)$ and their derivatives are bounded on $z \in (0, y + A)$. Using this notation the result after doing the K integral can be expressed in compact form, where the integrands can be expressed using factors $l_1(z), l_1(\alpha), l_2(\rho) \dots$ etc. Further use of results from Appendix B 1, l'Hôpital's rule and Lemma C.1 renders

$$\lim_{y+A \rightarrow 1_-} \mathcal{G}_{\text{out}}^- = \lim_{y+A \rightarrow 1_-} \left[2 \ln \left(\frac{w - y - 2A}{w - y} \right) - 2\text{Ci}(w - y - 2A) + 2\text{Ci}(w - y) \right]. \quad (\text{D19})$$

Next we consider $\mathcal{G}_{\text{out}}^+$. Interchanging the order of integration and performing the K integrals we obtain

$$\begin{aligned}
\mathcal{G}_{\text{out}}^+ &= \frac{2}{B} \int_0^{y+A} dz B'(z) \text{Ci}(\alpha) \\
&+ \frac{2}{B} \int_0^{y+A} dz B'(z) (\text{Ci}(\chi) - \text{Ci}(\beta) - \text{Ci}(\xi)) \\
&- \frac{4}{B^2} \int_0^{y+A} dx B'(x) \int_0^x dz B'(z) \text{Ci}(\tau) \\
&+ \frac{2}{B^2} \int_0^{y+A} dx B'(x) \int_0^x dz B'(z) (\text{Ci}(\rho) + \text{Ci}(\sigma)).
\end{aligned} \tag{D20}$$

We pass the lengthy procedures of carefully manipulating the double integrals using Lemma C.1 and l'Hôpital and point out that the nonvanishing terms in $\mathcal{G}_{\text{out}}^+$ cancel out the cosine integrals in (D19). Final result is

$$\lim_{y+A \rightarrow 1_-} \mathcal{G}_{\text{out}} = \lim_{y+A \rightarrow 1_-} \left[2 \ln \left(\frac{w-y-2A}{w-y} \right) \right] \tag{D21}$$

which is in agreement with (3.6).

APPENDIX E: INSIDE REGION

In this section we look at the energy density in the region $r < a$. The idea is to see if there is a symmetry between the inside and outside regions that would allow us to extract the final result for the inside region from the results we have already calculated which are valid outside the shell. We consider each of the four pieces: F_- , F_+ , \mathcal{G}^- , and \mathcal{G}^+ .

We have already outlined how to calculate F_-^{out} in the outside region, doing the same calculation in the inside region we find out

$$F_-^{\text{in}}(y, w)|_{A \rightarrow -A} - F_-^{\text{out}}(y, w) = 0. \tag{E1}$$

Here, $F_-^{\text{in}}(y, w)|_{A \rightarrow -A}$ is the inside expression transformed by making sign change on $A \rightarrow -A$ and switches $(y, w) \rightarrow (w, y)$. A similar expression holds for $\mathcal{G}_{\text{in}}^-(w, y)$ through transformation $(y, w, A) \rightarrow (w, y, -A)$

$$\mathcal{G}_{\text{in}}^-(w, y)|_{A \rightarrow -A} - \mathcal{G}_{\text{out}}^-(y, w) = 0. \tag{E2}$$

Defining positive definite variables throughout the inside region

$$\begin{aligned}
\hat{\alpha} &= -A + w - z, & \hat{\gamma} &= A + w - z, & \hat{\beta} &= 3A - w + z, \\
\hat{\delta} &= 2A + x - z, & \hat{\rho} &= 2A - x + z,
\end{aligned}$$

we transform the inside result again and find

$$F_+^{\text{in}}(w, y)|_{A \rightarrow -A} - F_+^{\text{out}}(y, w) = \text{imaginary}. \tag{E3}$$

With additional definitions $\hat{\chi} = A + y - z$, $\hat{\eta} = A - y + z$, we obtain

$$\mathcal{G}_{\text{in}}^+(w, y)|_{A \rightarrow -A} - \mathcal{G}_{\text{out}}^+(y, w) = \text{imaginary}. \tag{E4}$$

From Eqs. (E1)–(E4) we see that the energy density inside the shell can be obtained from the outside results by performing the transformation $(y, w, A) \rightarrow (w, y, -A)$ and dropping any imaginary parts that are produced.

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