

A Comparative Study on the Control of Quadcopter UAVs by using Singleton and Non-Singleton Fuzzy Logic Controllers

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Abstract—Fuzzy logic controllers (FLCs) have extensively been used for the autonomous control and guidance of unmanned aerial vehicles (UAVs) due to their capability of handling uncertainties and delivering adequate control without the need for a precise, mathematical system model which is often either unavailable or highly costly to develop. Despite the fact that non-singleton FLCs (NSFLCs) have shown more promising performance in several applications when compared to their singleton counterparts (SFLCs), most of UAV applications are still realized by using SFLCs. In this paper, we explore the potential of both standard and the recently introduced centroid based NSFLCs, i.e., Sta-NSFLC and Cen-NSFLC, for the control of a quadcopter UAV under various input noise conditions using different levels of fuzzifier, and a comparative study has been conducted using the three aforementioned FLCs. We present a series of simulation-based experiments, the simulation results show that the control performances of NSFLCs are better than those of SFLC, and the Cen-NSFLC outperforms the Sta-NSFLC especially under highly noisy conditions.

I. INTRODUCTION

Unmanned aerial vehicles (UAVs) have been widely utilized in a variety of civilian applications to save time, money and even lives, e.g., disaster rescue [1], target tracking [2], orchard monitoring [3], wildlife protection [4], infrastructure inspection [5] and 3D environment reconstruction [6]. Most of the time, to achieve fully autonomous flight in these UAV applications, classical controllers, e.g. proportional-integral-derivative (PID) [7], linear quadratic regulators (LQRs) [8] or sliding mode control (SMC) [9], have been utilized. There is no doubt that these well-known control algorithms are very successful when a precise mathematical model of the system is obtained and no significant internal or external uncertainties exist. On the other hand, modelling of complex dynamics systems is a tedious, costly and time consuming task [10]. Unfortunately, lack of modeling, uncertainty, inaccuracy in the sensors, approximation and incompleteness problems are inevitable in a typical UAV application. For instance, considerable uncertainties exist in the different types of

onboard UAV sensors, e.g., global positioning system (GPS), monocular/stereo cameras, laser range finders (LRFs) and inertial measurement units (IMUs).

In this study, we explore the potential of both standard and recently introduced centroid-based variants of non-singleton FLCs (NSFLCs) to address uncertainty affecting the inputs of quadcopter UAV control. Although there are a number of FLC applications for navigating a quadcopter UAV in literature, e.g., [11], most of these are SFLCs, which focus on high level navigation, rather than exploring the effect of various levels of input uncertainty on the control performance. On the other hand, it is reported in literature that NSFLCs give more promising results when compared to their singleton counterparts for non-linear servo system [12], chaotic time series prediction [13], etc. Both FLCs use the same fuzzy rule base, inference engine and defuzzifier. However, in the NSFLC, there is a different fuzzifier which treats the inputs as fuzzy sets (FSs). In this paper, we employ Gaussian input fuzzy sets with different standard deviations to capture different levels of uncertainty.

Another motivation of this investigation is to compare and contrast the performances of different types of NSFLCs, in addition to the standard NSFLC (Sta-NSFLC), centroid NSFLC (Cen-NSFLC) is also used. In spite of Sta-NSFLC is capable of handling uncertainties by capturing them from inputs, the adopted prefiltering approach in the Sta-NSFLC does not offer a fine-grained uncertainty information tracking, i.e., the prefilter is not highly sensitive to the shape of the input of FSs, leading to significant loss of information regarding the intersection of input and antecedent models. Therefore, an alternative type of the NSFLC initially developed in [14], [15], i.e., Cen-NSFLC, has been developed for controlling the quadcopter UAV. In [14], [15], the novel approach to NSFLCs showed promising results in the context of time-series prediction with different levels of injected uncertainty. The aim of the work discussed in this paper is to move beyond simulated levels of uncertainty to real-world uncertainty affecting real world sensors.

To the best of our knowledge, this is the first time in the literature that the performances of NSFLCs and SFLCs are compared for the control of quadcopter UAVs. The rest of this paper is organized as follows: Section II introduces the dynamic model of quadcopter UAV. Section III presents a brief background for the SFLC, Sta-NSFLC and Cen-NSFLC. Section IV evaluates and discusses the control performances with these three FLCs under different input noise levels, where a number of fuzzifiers are explored.

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Finally, some conclusions are drawn from this study and possible future works are also presented in Section V.

II. QUADCOPTER UAV DYNAMICS

In this section, we introduce the configuration, equations of motion as well low level velocity control of the quadcopter.

A. Configuration definition

Let the world fixed inertial reference frame be $\{\vec{x}_I, \vec{y}_I, \vec{z}_I\}$ and the body frame be $\{\vec{x}_B, \vec{y}_B, \vec{z}_B\}$. The origin of the body frame is located at the center of mass of the quadcopter. The axes $\vec{x}_B$ and $\vec{y}_B$ lie in the plane defined by the centres of the four rotors and respectively point toward motor 1 and motor 2, as illustrated in Fig. 1. The axis $\vec{z}_I$ points downward, as well as axis $\vec{z}_B$ which is opposite to the direction of the total thrust.

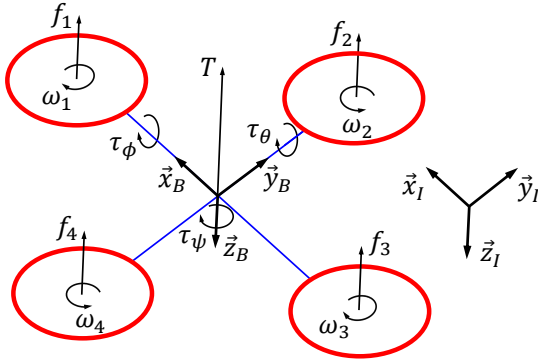


Fig. 1: Quadrotor model with reference frames.

The four propellers generate four forces (f_1, f_2, f_3 and f_4), directed along the axis of rotation $\vec{z}_B$ and with module proportional to the speed of rotation, and four torques (τ_1, τ_2, τ_3 and τ_4), around the axis of rotation $\vec{z}_B$ and with module proportional to the speed of rotation [16]. The forces and torques are proportional to the square of the angular speed of the propeller:

$$f_i = b\omega_i^2, \quad \tau_i = d\omega_i^2, \quad i = 1, \dots, 4, \quad (1)$$

where b is the propeller thrust coefficient, d is the propeller drag coefficient and ω_i is the rotational speed of the i^{th} propeller.

The quadrotor control vector $\mathbf{u}$ is considered as follows:

$$\mathbf{u} = [T \quad \tau_\phi \quad \tau_\theta \quad \tau_\psi]^T, \quad (2)$$

where T is the total thrust and acts along $\vec{z}_B$ axis, whereas τ_ϕ, τ_θ and τ_ψ are the moments acting around $\vec{x}_B, \vec{y}_B$ and $\vec{z}_B$ axes, respectively. Under these considerations, the relation between $\mathbf{u}$ and $\omega_i, i = 1, \dots, 4$, becomes [17]:

$$T = f_1 + f_2 + f_3 + f_4 \quad (3)$$

$$\tau_\phi = l(-f_2 + f_4) \quad (4)$$

$$\tau_\theta = l(f_1 - f_3) \quad (5)$$

$$\tau_\psi = -\tau_1 + \tau_2 - \tau_3 + \tau_4, \quad (6)$$

where l is the arm length.

The absolute position of a quadcopter (3 DOF) is described by the three Cartesian coordinates (x, y and z) of its center of mass in the world frame and its attitude by the three Euler's angles (ϕ, θ and ψ). These three angles are respectively called roll ($-\frac{\pi}{2} < \phi < \frac{\pi}{2}$), pitch ($-\frac{\pi}{2} < \theta < \frac{\pi}{2}$) and yaw ($0 \leq \psi < 2\pi$).

The time derivative of the position (x, y, z) is given by

$$\mathbf{V} = [\dot{x} \quad \dot{y} \quad \dot{z}]^T = [u \quad v \quad w]^T, \quad (7)$$

where $\mathbf{V}$ is the absolute velocity of the quadcopter's center of mass expressed with respect to the world fixed inertial reference frame. Let $\mathbf{V}_B \in \mathbb{R}^3$ be the absolute velocity of the quadcopter expressed in the body fixed reference frame. So, $\mathbf{V}$ and $\mathbf{V}_B$ are related by

$$\mathbf{V} = \mathbf{R}\mathbf{V}_B, \quad (8)$$

where $\mathbf{R} \in \text{SO}(3)$ is the rotation matrix from the body frame to the world frame:

$$\mathbf{R} = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - c_\phi s_\psi & s_\phi s_\psi + c_\phi c_\psi s_\theta \\ c_\theta s_\psi & c_\phi c_\psi + s_\phi s_\psi s_\theta & c_\phi s_\psi s_\theta - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\phi c_\theta \end{bmatrix}, \quad (9)$$

where c_θ and s_θ denote respectively $\cos \theta$ and $\sin \theta$.

Similarly, the time derivative of the angles (ϕ, θ, ψ) give the angular velocities:

$$\boldsymbol{\omega} = [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T, \quad (10)$$

and the angular velocities expressed in the body frame are

$$\boldsymbol{\omega}_B = [p \quad q \quad r]^T. \quad (11)$$

The relation between $\boldsymbol{\omega}$ and $\boldsymbol{\omega}_B$ is given by

$$\boldsymbol{\omega} = \mathbf{T}\boldsymbol{\omega}_B, \quad (12)$$

in which $\mathbf{T}$ is the transformation matrix given by

$$\mathbf{T} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{bmatrix}. \quad (13)$$

B. Equations of motion of the quadrotor

Using the Newton-Euler equations about the center of mass, the dynamic equations for the quadcopter are [18]:

$$m\dot{\mathbf{V}} = \mathbf{F}_e \quad (14)$$

$$\mathbf{I}\dot{\boldsymbol{\omega}}_B = -\boldsymbol{\omega}_B \times \mathbf{I}\boldsymbol{\omega}_B + \boldsymbol{\tau}_e, \quad (15)$$

where m is the mass and $\mathbf{I}$ is the inertia matrix given by

$$\mathbf{I} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}, \quad (16)$$

and $\mathbf{F}_e$ is the vector of external forces and $\boldsymbol{\tau}_e$ is the vector of external torques. Some calculations yield the following form for these two vectors

$$\mathbf{F}_e = \begin{bmatrix} -(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) T \\ -(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) T \\ -\cos \phi \cos \theta T + mg \end{bmatrix} \quad (17)$$

$$\boldsymbol{\tau}_e = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}, \quad (18)$$

in which g is the gravity acceleration ($g = 9.81\text{m/s}^2$).

Using dynamic and kinematic differential equations (8), (12), (15) and (18), the quadcopter model is obtained [19]

$$\begin{cases} \dot{x} = u \\ \dot{y} = v \\ \dot{z} = w \\ \dot{\phi} = p + \sin \phi \tan \theta q + \cos \phi \tan \theta r \\ \dot{\theta} = \cos \phi q - \sin \phi r \\ \dot{\psi} = \frac{\sin \phi}{\cos \theta} q + \frac{\cos \phi}{\cos \theta} r \\ \dot{u} = -\frac{1}{m} (\cos \phi \cos \psi \sin \theta + \sin \phi \sin \psi) T \\ \dot{v} = -\frac{1}{m} (\cos \phi \sin \psi \sin \theta - \cos \psi \sin \phi) T \\ \dot{w} = -\frac{1}{m} \cos \phi \cos \theta T + g \\ \dot{p} = \frac{I_y - I_z}{I_x} qr + \frac{1}{I_x} \tau_\phi \\ \dot{q} = \frac{I_z - I_x}{I_y} pr + \frac{1}{I_y} \tau_\theta \\ \dot{r} = \frac{I_x - I_y}{I_z} pq + \frac{1}{I_z} \tau_\psi \end{cases} \quad (19)$$

Hence, the quadrotor's state $\mathbf{x}$ is

$$\mathbf{x} = [x \ y \ z \ \phi \ \theta \ \psi \ u \ v \ w \ p \ q \ r]^T. \quad (20)$$

C. Low level velocity control

After the quadcopter dynamic model is identified, the low level velocity controller is designed. The control law is based on a proportional-derivative (PD) algorithm. The inputs to the PD-based attitude controller are the desired linear velocity

$$\mathbf{V}^* = [u^* \ v^* \ w^*]^T \quad (21)$$

and the desired angular velocity $\dot{\psi}^*$ around the $\bar{z}_I$ axes, and the output is defined in (2).

III. BACKGROUND OF SFLC AND NSFLC

In this section, we present a brief background overview of the SFLC, Sta-NSFLC and Cen-NSFLC.

A. Structure of FLC

Figure 2 shows the general structure of a FLC. The fuzzy rule base (purple block), inference engine (grey block) and defuzzifier (green block) are the same in both NSFLC and SFLC. However, the difference between SFLC and NSFLC is the handling of the crisp inputs in the fuzzifier (red block). The NSFLC maps a given crisp input to a fuzzy input set,

rather than to a fuzzy singleton - as is the case in the SFLC, i.e. the fuzzifier of SFLC does not model any vagueness in the input.

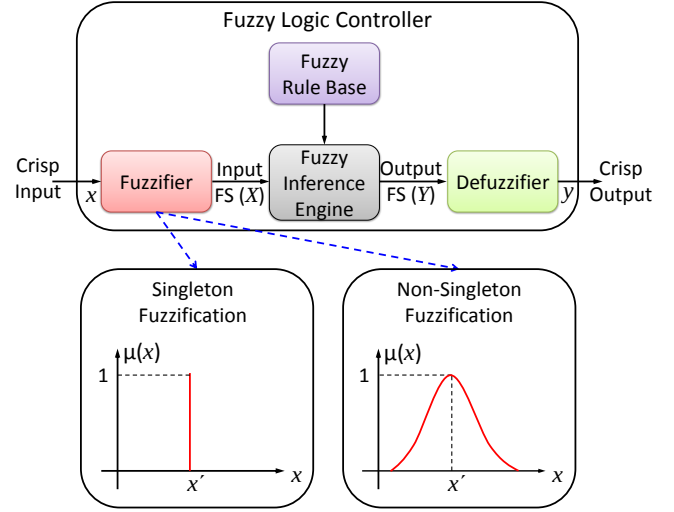


Fig. 2: Overview of FLC structure.

For the SFLC, its fuzzifier is defined as below:

$$\mu_X(x_i) = \begin{cases} 1, & x_i = x'_i \\ 0, & x_i \neq x'_i. \end{cases} \quad (22)$$

For the NSFLC, the Gaussian distribution is selected for its fuzzifier in our case, i.e.:

$$\mu_X(x_i) = \exp \left[\frac{-(x_i - x'_i)^2}{2\sigma_F^2} \right], \quad (23)$$

where x'_i is the crisp value of the input and the mean value of the fuzzy set, and σ_F is the spread of this set. Larger values of the spread imply that more noise is anticipated to exist in the input data.

In the literature, standard NSFLCs have been defined for both type-1 and type-2 FLCs. However, we limit ourselves to type-1 non-singleton type-1 FLC in this work, i.e. type-1 input membership functions (MFs) are adopted for type-1 FLCs. The general mapping between the inputs and outputs of the FLCs, as the input set X and output set Y are shown in Fig. 2, is comprehensively introduced in [20]. This paper will not repeat the details of how the comprehensive formula for the mapping is derived. Since the prefiltering module of the NSFLC is the key to handling the input uncertainties, as shown in Fig. 3, we introduce the details of this prefiltering module below.

Specifically, the inference engine of a NSFLC can be considered as a prefilter module embedded to a SFLC inference engine. The prefilter module converts the crisp input in combination with the uncertain input set to a representative numerical value x_{max} . The handling of the input uncertainty of NSFLCs takes place in this prefiltering module. The other components are the same as the ones in a SFLC. In the following subsections, we briefly review the

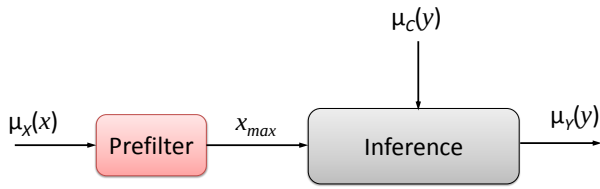


Fig. 3: Prefiltering of the input FS to a NSFLC.

nature of the prefiltering stage for both standard and centroid-based [14], [15] NSFLCs.

B. Prefiltering module in Sta-NSFLC

Figure 4 provides an example of a single-input, single-rule and single-output discrete standard NSFLC has been considered, and the Mamdani implication is utilized.

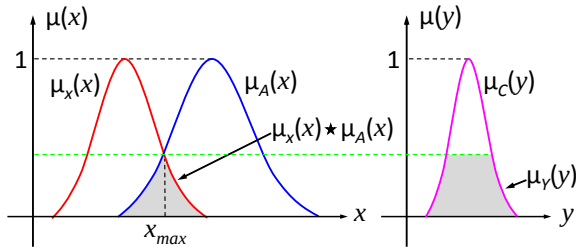


Fig. 4: Example of the calculation from input (X), antecedent (A) and consequent (C) fuzzy sets to output (Y). (Adapted from [14], [15])

Let x and y be the members of the input FS (X) and output FS (Y), and A and C are two FSs representing an antecedent and a consequent. The only rule is defined as:

$$\text{IF } x \text{ is } A \text{ THEN } y \text{ is } C. \quad (24)$$

The $\mu_X(x)$, $\mu_A(x)$, $\mu_C(y)$ and $\mu_Y(y)$ are the MFs of X , A , C and Y , respectively. Note that we only consider the Gaussian MFs in this work, then the input-output mapping is:

$$\mu_Y(y) = \mu_C(y) \star \max_{x \in X} [\mu_X(x) \star \mu_A(x)] \quad (25)$$

or

$$\mu_Y(y) = \mu_C(y) \star \mu_X(x_{max}) \star \mu_A(x_{max}), \quad (26)$$

where $\star$ is the t-norm operator and x_{max} is the value of x at which $\mu_X(x) \star \mu_A(x)$ takes its maximum.

Taking the minimum-operator as the t-norm, as shown in Fig. 4, $\mu_X(x) \star \mu_A(x)$ is the intersection of X and A , and $\mu_X(x_{max})$ and $\mu_A(x_{max})$ are equal according to the definition of x_{max} , (25) can be written as below:

$$\mu_Y(y) = \min[\mu_C(y), \mu_A(x_{max})] \quad (27)$$

Equation 27 represents the input-output mapping in the considered NSFLC. In addition, (27) shows that the firing level of an antecedent is the maximum of its intersection with the input set.

C. Prefiltering module in Cen-NSFLC

Figure 5 shows two different input FSs, i.e. X_1 and X_2 , which are intersected with an antecedent A . Although the actual input FSs are different, the firing levels calculated by the standard approach are the same in both cases, as shown in circle 1 in the Fig. 5. Thus, two different inputs or more specifically, inputs with a different associated uncertainty distribution result in the same firing level and thus FLC output. Hence, a method with a more detailed capture of input certainty and its intersection with the respective antecedent FS is desirable, i.e. the new method should have a higher sensitivity to the shape of the intersection.

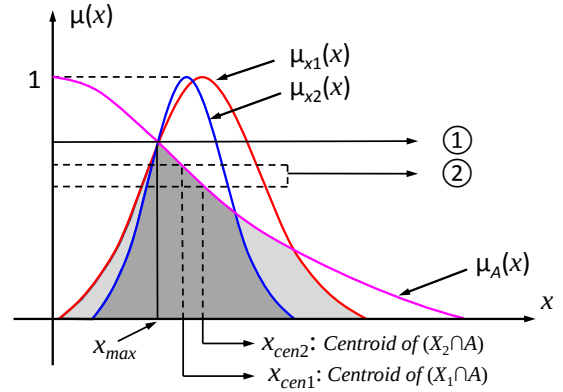


Fig. 5: The difference between the Sta-NSFLC and Cen-NSFLC. (Adapted from [14], [15])

Figure 5 shows how in Sta-NSFLCs the maximum of the intersection between the antecedent and different input FSs result in the same x_{max} and thus the same firing level. However, the firing strengths in the Cen-NSFLC, i.e. x_{cen1} and x_{cen2} , are different when the centroid of each intersection has been applied instead of their maximum.

Given a discrete FS X with a MF $x_X(x_i)$, the Centroid is defined as below:

$$x_{cen}(X) = \frac{\sum_{i=1}^n x_i \mu_X(x_i)}{\sum_{i=1}^n \mu_X(x_i)}, \quad (28)$$

where n is the number of discretization levels ($n = 100$ in our case) utilized in a discrete system.

As detailed in [14], [15], the centroid of the intersection of an input X and an antecedent A , i.e. centroid of $X \cap A$, the new input/output mapping is defined as:

$$\mu_Y(y) = \mu_{X \cap A}(x_{cen}(X \cap A)) \star \mu_C(y). \quad (29)$$

Alternatively, for minimum t-norm:

$$\mu_Y(y) = \min[\mu_{X \cap A}(x_{cen}(X \cap A)), \mu_C(y)]. \quad (30)$$

The above formulas represent that the firing level of an antecedent is the membership degree of its intersection with the input set at the centroid point of intersection.

IV. SIMULATION STUDIES

A series of quadcopter UAV simulation experiments are discussed using the SFLC, Sta-NSFLC and Cen-NSFLC introduced in the previous sections. Figure 6 shows the block diagram of the experimental setup where the output from each FLC, i.e., desired velocity (21), is sent as input to the UAV's velocity controller. A number of tests are made considering different levels of noise and fuzzifier for the FLCs.

A. Intrinsic parameters of the Quadcopter and its Initial State

Table I shows the quadcopter's intrinsic parameters, which are used in (1) and (19). These parameters are chosen to be close to the ones of a real quadcopter.

TABLE I: The quadcopter's intrinsic parameters.

Parameter	Value	Unit
b	10^{-5}	$[\text{N} \cdot \text{s}^2]$
d	10^{-7}	$[\text{N} \cdot \text{m} \cdot \text{s}^2]$
I_x	1	$[\text{kg} \cdot \text{m}^2]$
I_y	1	$[\text{kg} \cdot \text{m}^2]$
I_z	1	$[\text{kg} \cdot \text{m}^2]$
l	0.2	$[\text{m}]$
m	1	$[\text{kg}]$

The quadcopter is first hovering, therefore, the quadcopter initial state of (20) is

$$\mathbf{x}_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T.$$

B. Uncertainty Sources in Measurements

In our paper, the goal is to control the position and yaw angle of the quadcopter (x, y, z, ψ) without considering external disturbances, e.g., wind. Therefore, we need to measure the actual position and yaw angle of the quadcopter (x, y, z, ψ) . In order to obtain these measurements we simulate a key set of standard onboard quadcopter sensors with different levels of noise. In our case the noise level is

characterised by its standard deviation σ_N . Hence, we can convert σ_N to SNR, which is also commonly used to describe the noise, using the following relation:

$$\text{SNR} = 10 \log_{10} \left(\frac{1}{\sigma_N^2} \right). \quad (31)$$

In order to have the measurement of the quadcopter's position $(\bar{x}, \bar{y}$ and $\bar{z})$ we simulate the GPS. To produce the noisy position measurements we add white Gaussian noise to the true position:

$$\bar{x} = \mathcal{N}(x, \sigma_N^2) \quad (32)$$

$$\bar{y} = \mathcal{N}(y, \sigma_N^2) \quad (33)$$

$$\bar{z} = \mathcal{N}(z, \sigma_N^2), \quad (34)$$

where $\mathcal{N}(\mu, \sigma^2)$ is the Gaussian distribution with mean μ and variance σ^2 . Our simulated GPS computes a new position every 100ms, as the real one.

To measure the yaw angle of the quadcopter $\bar{\psi}$, we simulate the IMU. The IMU provides us the measurements of orientation $(\bar{\phi}, \bar{\theta}$ and $\bar{\psi})$ and angular velocities in body frame $(\bar{p}, \bar{q}$ and $\bar{r})$. In order to produce the noisy measurements for the yaw angle we also add white Gaussian noise to the true yaw angle:

$$\bar{\psi} = \mathcal{N}(\psi, \sigma_N^2 (1 + 3r)^2). \quad (35)$$

C. Fuzzifiers for NSFLC

As the fuzzifier discussed in Section III, the Gaussian distribution is applied for the inputs of the NSFLCs. Figure 7 shows the different levels of fuzzifier utilized in our experiments, representing different levels of expected uncertainty or noise.

D. Membership Function and Rule Base of FLC

In our work, Gaussian distribution is employed for the input and output membership functions (MFs) of FLC. Each input variable, i.e., error or time derivative of the error, has

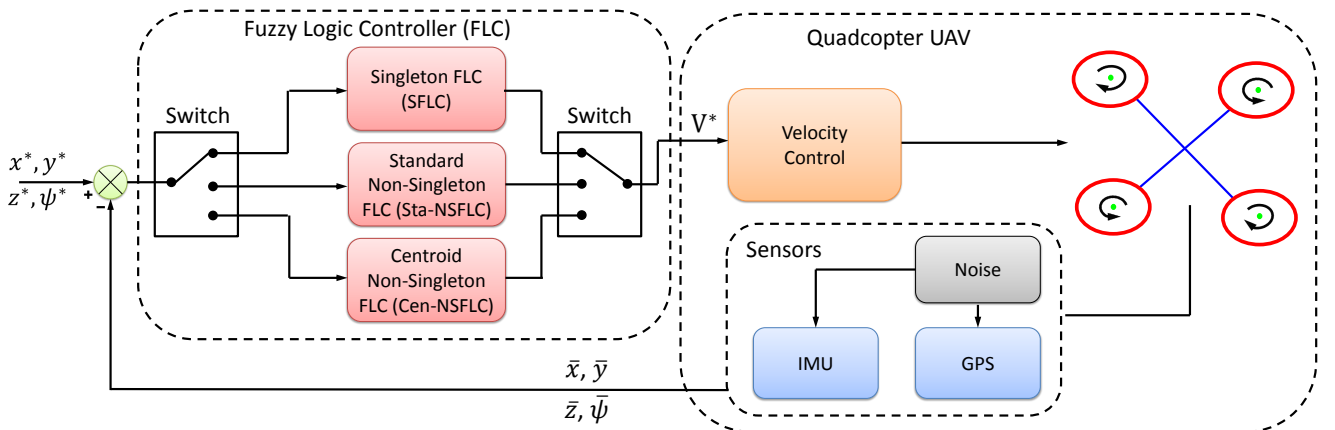


Fig. 6: Experimental setup for quadcopter UAV control performance evaluations using the SFLC, Sta-NSFLC and Cen-NSFLC.

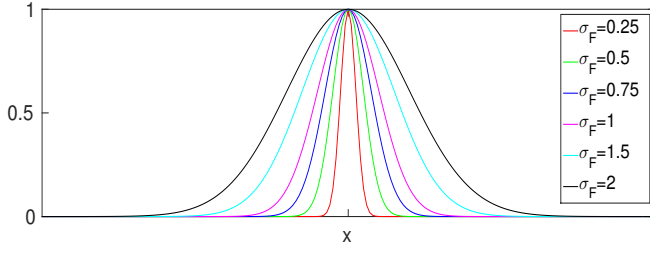


Fig. 7: Different levels of fuzzifier for the NSFLC.

five MFs, and the output variable has seven MFs. Table II shows the rule base of FLC, as utilized in [21], where each abbreviation N, Z, P, B, M or S represents negative, zero, positive, big, medium or small, respectively.

TABLE II: Rule Base of FLC

Dot error/error	BP	SP	Z	SN	BN
BP	BN	BN	MN	SP	SP
SP	BN	BN	SN	SP	SP
Z	MN	MN	Z	MP	MP
SN	SN	SN	SP	BP	BP
BN	SN	SN	MP	BP	BP

E. Performance Evaluation

The control performance evaluation is carried out in terms of the mean squared error (MSE) of the 3D position, e_{pos} :

$$e_{pos} = \frac{1}{n} \sum_{k=1}^n \sqrt{(x(k) - x^*)^2 + (y(k) - y^*)^2 + (z(k) - z^*)^2}, \quad (36)$$

where $x(k)$, $y(k)$ and $z(k)$ are the translations along x -, y - and z -axis, and x^* , y^* and z^* represent the desired 3D position. And the mean absolute error (MAE) of yaw angle, e_{yaw} :

$$e_{yaw} = \frac{1}{n} \sum_{k=1}^n |\psi(k) - \psi^*| \quad (37)$$

where $\psi(k)$ is the yaw angle and ψ^* represents the desired yaw angle.

F. Results

To illustrate the performance comparison of the different FLCs, the obtained results are presented in Table III and Table IV. Figure 8 shows four visualised results of the squared errors in 3D position and absolute errors in yaw angle under different input noise levels (σ_N). In general, the control performances of NSFLCs are better than the ones of SFLC. When we increase the spread parameter (σ_F , as the parameters shown in the brackets in Fig. 8) for the fuzzifier in the NSFLC, the performance of the NSFLC gets better, and the Cen-NSFLC outperforms the Sta-NSFLC, especially with larger input noise levels. Finally, we can observe that all the FLCs provide better performances when noise level decreases. In addition, we can also clearly find the performance differences among the SFLC, Sta-NSFLC and Cen-NSFLC in Fig. 9 below.

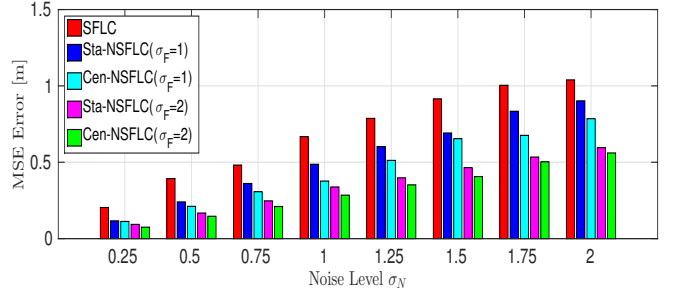


Fig. 9: Performance comparison of SFLC and NSFLCs.

V. CONCLUSION AND FUTURE WORK

In this paper, a comprehensive evaluation of quadcopter UAV control performance has been conducted with three different types of FLCs, i.e., a SFLC and two novel NSFLCs (Sta-NSFLC and Cen-NSFLC), under different levels of input noise (uncertainty). The objective of this work was not only to evaluate the performances of the Sta-NSFLC and the SFLC, but also to compare the control performances between the Sta-NSFLC and the Cen-NSFLC. The extensive simulated experiments show that the Sta-NSFLC outperforms the SFLC in the most of cases, and the Cen-NSFLC can obtain better control performances compared to the Sta-NSFLC, especially at the higher input noise levels. Additionally, the various levels of fuzzifier show the different capabilities to capture a specified amount of uncertainty, i.e. the higher level fuzzifier has more capability to manage higher level input noise. These results support the results in [14], [15].

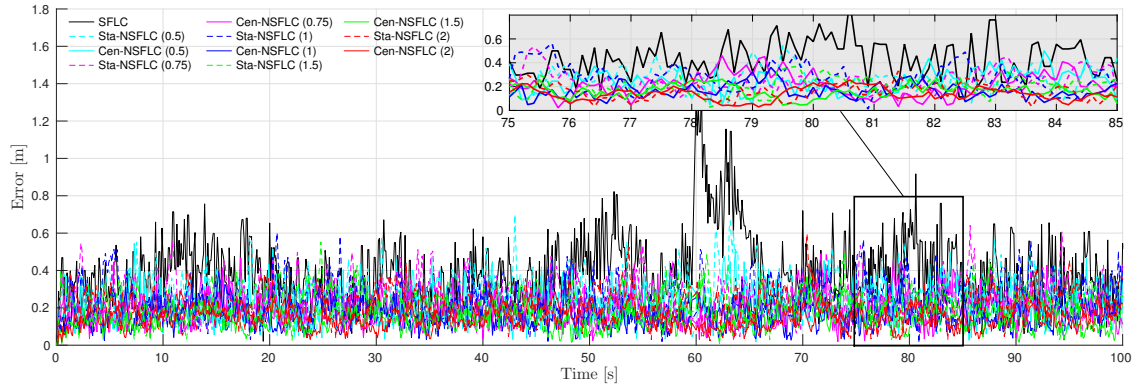
For future work, we will focus on conducting experiments on real-world quadcopter UAVs while also exploring different Type-2 FLCs for navigating quadcopter UAV, and compare their performances in a variety of input noise levels.

ACKNOWLEDGMENT

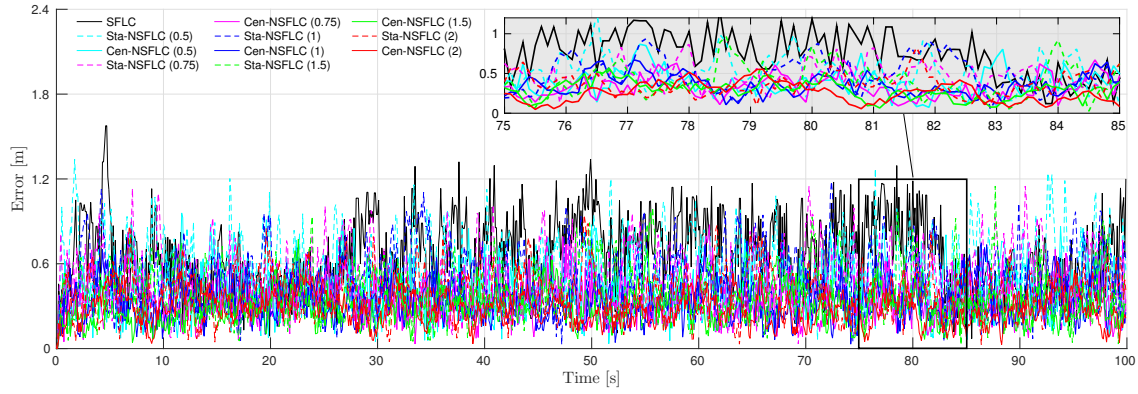
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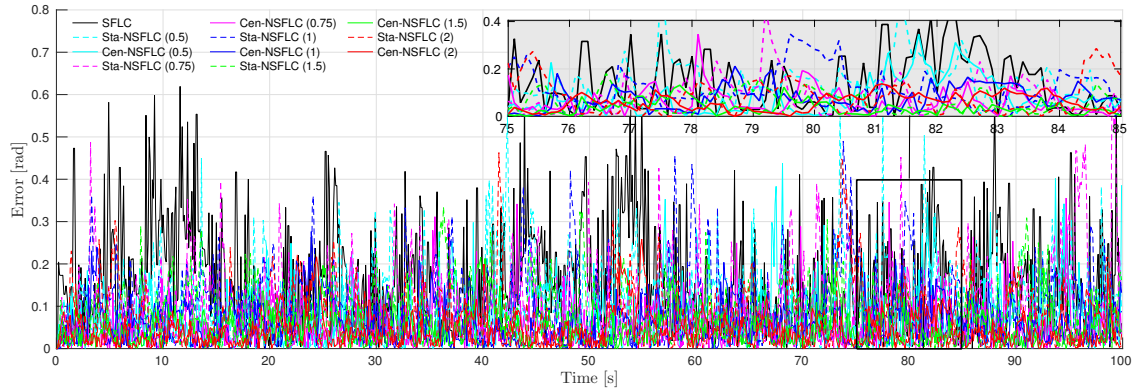
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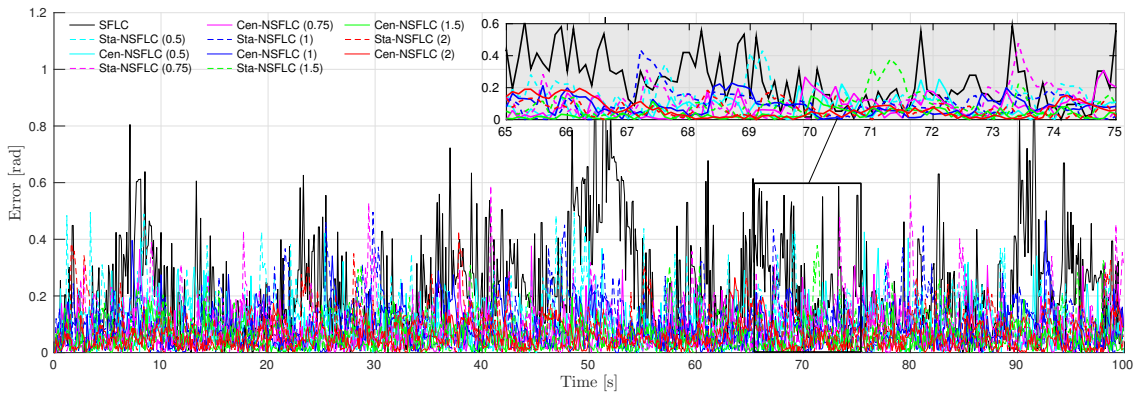
(a) Squared Error of 3D position with noise level $\sigma_N = 0.5$



(b) Squared Error of 3D position with noise level $\sigma_N = 1$



(c) Absolute Error of yaw with noise level $\sigma_N = 1.75$



(d) Absolute Error of yaw with noise level $\sigma_N = 2$

Fig. 8: Squared errors of 3D position and absolute errors of yaw angle. The parameter in the bracket is the standard deviation σ_F for the fuzzifier in the NSFLC.

TABLE III: Average MSE of 3D Position (Unit: Meter)

FLC / Noise Level (σ_N)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Singleton FLC	1.4447×10^{-15}	0.2035	0.3930	0.4813	0.6678	0.7871	0.9152	1.0040	1.0393
Sta-NSFLC ($\sigma_F=0.25$)	1.2694×10^{-12}	0.1501	0.2900	0.4366	0.5830	0.7708	0.9088	1.0823	1.2533
Cen-NSFLC ($\sigma_F=0.25$)	1.8813×10^{-12}	0.1663	0.3057	0.4272	0.5422	0.7080	0.8303	0.9975	1.1680
Sta-NSFLC ($\sigma_F=0.5$)	1.9668×10^{-12}	0.1506	0.2555	0.4113	0.5477	0.6580	0.8138	0.9816	1.0387
Cen-NSFLC ($\sigma_F=0.5$)	1.6627×10^{-12}	0.1125	0.2158	0.3308	0.4059	0.5500	0.6164	0.7262	0.8465
Sta-NSFLC ($\sigma_F=0.75$)	1.1863×10^{-12}	0.1365	0.2486	0.3840	0.4853	0.6121	0.7504	0.8810	1.0028
Cen-NSFLC ($\sigma_F=0.75$)	1.8213×10^{-13}	0.0999	0.1885	0.2627	0.3500	0.4329	0.5442	0.6316	0.7289
Sta-NSFLC ($\sigma_F=1$)	1.6388×10^{-12}	0.1163	0.2403	0.3609	0.4867	0.6027	0.6915	0.8338	0.9023
Cen-NSFLC ($\sigma_F=1$)	1.7542×10^{-12}	0.0930	0.1672	0.2472	0.3381	0.3980	0.4648	0.5339	0.5957
Sta-NSFLC ($\sigma_F=1.25$)	1.3936×10^{-14}	0.1246	0.2356	0.3515	0.4582	0.5721	0.6941	0.8215	0.9192
Cen-NSFLC ($\sigma_F=1.25$)	1.4813×10^{-14}	0.1024	0.1777	0.2471	0.3133	0.4079	0.4598	0.5695	0.6125
Sta-NSFLC ($\sigma_F=1.5$)	1.4123×10^{-14}	0.1197	0.2143	0.3410	0.3929	0.5487	0.6837	0.8117	0.8989
Cen-NSFLC ($\sigma_F=1.5$)	2.3086×10^{-14}	0.0877	0.1503	0.2219	0.2976	0.3907	0.4384	0.5102	0.6427
Sta-NSFLC ($\sigma_F=1.75$)	1.5418×10^{-12}	0.1080	0.1975	0.3294	0.4249	0.5587	0.6395	0.7674	0.8269
Cen-NSFLC ($\sigma_F=1.75$)	1.2412×10^{-12}	0.0805	0.1630	0.2133	0.2793	0.3842	0.4314	0.5260	0.6289
Sta-NSFLC ($\sigma_F=2$)	1.7693×10^{-14}	0.1123	0.2111	0.3071	0.3767	0.5121	0.6545	0.6760	0.7853
Cen-NSFLC ($\sigma_F=2$)	1.7184×10^{-14}	0.0748	0.1462	0.2100	0.2846	0.3520	0.4063	0.5033	0.5608

TABLE IV: Average MAE of Yaw (Unit: Radian)

FLC / Noise Level (σ_N)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Singleton FLC	8.3566×10^{-16}	0.0157	0.0320	0.0491	0.0753	0.1152	0.1344	0.1629	0.2256
Sta-NSFLC ($\sigma_F=0.25$)	7.3227×10^{-13}	0.0144	0.0356	0.0515	0.0669	0.0707	0.0972	0.1042	0.1174
Cen-NSFLC ($\sigma_F=0.25$)	1.0855×10^{-12}	0.0309	0.0377	0.0626	0.0758	0.0871	0.0951	0.1030	0.1155
Sta-NSFLC ($\sigma_F=0.5$)	1.1355×10^{-12}	0.0237	0.0408	0.0488	0.0716	0.0702	0.0973	0.1090	0.1142
Cen-NSFLC ($\sigma_F=0.5$)	9.5998×10^{-13}	0.0136	0.0348	0.0398	0.0500	0.0670	0.0730	0.0782	0.0908
Sta-NSFLC ($\sigma_F=0.75$)	6.8491×10^{-13}	0.0177	0.0256	0.0418	0.0512	0.0610	0.0721	0.1019	0.0970
Cen-NSFLC ($\sigma_F=0.75$)	1.0515×10^{-13}	0.0142	0.0232	0.0309	0.0360	0.0437	0.0543	0.0583	0.0706
Sta-NSFLC ($\sigma_F=1$)	9.4595×10^{-13}	0.0199	0.0285	0.0448	0.0688	0.0651	0.0817	0.0957	0.1031
Cen-NSFLC ($\sigma_F=1$)	1.0128×10^{-12}	0.0148	0.0205	0.0322	0.0462	0.0501	0.0555	0.0608	0.0826
Sta-NSFLC ($\sigma_F=1.25$)	8.0445×10^{-15}	0.0188	0.0286	0.0355	0.0509	0.0550	0.0661	0.0819	0.0989
Cen-NSFLC ($\sigma_F=1.25$)	8.5523×10^{-15}	0.0140	0.0228	0.0271	0.0323	0.0387	0.0432	0.0479	0.0573
Sta-NSFLC ($\sigma_F=1.5$)	8.1205×10^{-15}	0.0112	0.0222	0.0347	0.0436	0.0532	0.0697	0.0809	0.0743
Cen-NSFLC ($\sigma_F=1.5$)	1.3331×10^{-14}	0.0092	0.0165	0.0241	0.0306	0.0314	0.0445	0.0490	0.0479
Sta-NSFLC ($\sigma_F=1.75$)	8.9016×10^{-13}	0.0107	0.0187	0.0346	0.0474	0.0579	0.0591	0.0767	0.0849
Cen-NSFLC ($\sigma_F=1.75$)	7.1659×10^{-13}	0.0081	0.0179	0.0231	0.0352	0.0328	0.0380	0.0501	0.0602
Sta-NSFLC ($\sigma_F=2$)	1.0358×10^{-14}	0.0121	0.0225	0.0304	0.0409	0.0542	0.0629	0.0740	0.0823
Cen-NSFLC ($\sigma_F=2$)	9.9232×10^{-15}	0.0094	0.0155	0.0213	0.0320	0.0312	0.0364	0.0442	0.0525

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