Linear and Nonlinear Fractional Hereditary Constitutive Laws of Asphalt Mixtures

Gaetano Di Mino, Gordon Airey, Mario Di Paola, Francesco Paolo Pinnola, Giacomo D'Angelo, Davide Lo Presti

Abstract. The aim of this paper is to propose a fractional viscoelastic and viscoplastic model of asphalt mixtures using experimental data of several tests such as creep and creep recovery performed at different temperatures and at different stress levels. From a best fitting procedure it is shown that both the creep one and recovery curve follow a power law model. It is shown that the suitable model for asphalt mixtures is a dashpot and a fractional element arranged in series. The proposed model is also available outside of the linear domain but in this case the parameters of the model depend on the stress level.

Keywords: fractional calculus, asphalt mixture, viscoelasticity, viscoplasticity, rheology, mechanical models, creep test.

Introduction

The mechanical behaviour of asphalt mixtures is traditionally predicted by classical Kelvin-Voigt or Maxwell elements that are composed of spring and dashpot arranged in parallel or in series, respectively. In order to fit experimental data of asphalt mixture other more sophisticated models like Zener, Burger (Liu, You, 2009) or other arrangements of elementary units of springs and dashpots have been used (Werkmeier *et al*. 2013). Whatever the number and the arrangement of elementary units are, the kernel in the Boltzmann superposition principle has an exponential kernel. On the other hand Nutting (1921) observed that the creep or the relaxation test performed on any real material follow a power law kernel instead of an exponential one. Based on this observation the constitutive law of any real material, including rubber, glass, asphalt mixture, is ruled by a fractional operator (Koeller 1984; Bagley, Torvik 1983, 1986; Slonimsky 1967; Smit, de Vrie 1970; Soczkiewicz 2002; Di Paola *et al*. 2011). The characterization of the real material by means of fractional derivative and integrals of real order produces strong variations on the response with respect to the characterization involving derivatives (or integrals) of integer order. While the integer order derivative involves the knowledge of the state of the system at one or more previous instants, when the fractional operator appears the entire past history (hereditary materials) gives information at the given time instant (long tail memory). From recent literature on the subject for bitumen and asphalt mixture based upon fractional constitutive laws the readers may be referred to (Di Paola *et al*. 2009; Celauro *et al*. 2012; Stastna, Zanzotto 1994, 1996; Oeser *et al*. 2008). In previous papers the mechanical model was composed of a fractional element alone (usually termed as *springpot* in literature) or arranged in parallel with a spring. That is the Kelvin-Voigt fractional element.

In this paper, based upon experimental tests performed on asphalt mixtures at several stress levels and various temperatures, a quite different model is proposed. It is a *springpot* in series with a dashpot element. With this choice the creep function, obtained by a best fitting procedure, gives impressive match between experimental results and theoretical ones. This result is obtained for any temperature T and for any stress intensity. As experimental creep tests were performed at high stress levels, the nonlinear behaviour was found. This is confirmed by the fact that the characteristic parameters of both the dashpot and the *springpot* are different for different stress levels. This renders the constitutive law nonlinear in the sense that the characteristic coefficients of the dashpot and the *springpot* will depend not only on the temperature, but also on the stress level. The nonlinearity of response in terms of the strain also depends on the signum of the derivative of the stress ($sign(\dot{\sigma})$). It is shown that with this important modification on the dependence of the parameters from a physical point of view leads to a residual strain how it happens on the plasticity. Then at high stress level the behaviour of the asphalt mixture is viscoplastic.

It is shown that the proposed model is fully available for monotonic stress history. For non-monotonic ones other experimental creep recovery tests are necessary for the complete definition of the material at hands.

1. Fractional linear viscoelastic model

The constitutive law of any linear viscoelastic material may be obtained by starting from the creep test. Such a test is performed by putting a constant load at time $t = 0$ and by measuring the corresponding strain in time. Then the creep $J(t)$ function is the strain for the stress history $U(t)$, $U(t)$ being the unit step function.

Once $J(t)$ is known from the experimental test, the Boltzmann superposition principle gives the strain history $\varepsilon(t)$ for any stress distribution $\sigma(t)$ in the form:

$$\varepsilon(t) = \int_0^t J(t-\tau)\dot{\sigma}(\tau)d\tau. \tag{1}$$

Equation (1) is valid for $\sigma(0) = 0$. If $\sigma(0) \neq 0$, then in Eq. (1) the term $J(t)\sigma(0)$ has to be added.

Another function useful in the viscoelastic theory is the relaxation function, in the following denoted as $G(t)$. This function is the stress history for an imposed strain history $U(t)$. Using again the Boltzmann superposition principle we get:

$$\sigma(t) = \int_0^t G(t-\tau)\dot{\varepsilon}(\tau)d\tau. \tag{2}$$

If the initial condition $\varepsilon(0) \neq 0$, then the term $G(t)\varepsilon(0)$ has to be added in Eq. (2). By making the Laplace Transform of Equations (1) and (2) and equating the ratio between the Laplace Transform of $\sigma(t)$ and $\varepsilon(t)$, the following fundamental relation between the Laplace Transform of $J(t)$ and $G(t)$ is obtained as:

$$\hat{G}(s)\hat{J}(s) = \frac{1}{s^2}, \tag{3}$$

where $\hat{J}(s)$ and $\hat{G}(s)$ are the Laplace Transform of $J(t)$ and $G(t)$ respectively and s is the Laplace parameter (Christensen 1982; Flügge 1975).

Experimental creep tests on real materials like polymers, rubber, bitumen and so on are well fitted by a power law of the type:

$$J(t) = \frac{t^{\alpha}}{C_{\alpha}\Gamma(1+\alpha)}, \tag{4}$$

where $\Gamma(\cdot)$ is the Euler Gamma function and α, C_{α} are parameters obtained by best fitting on experimental data. By using Eq. (3), the corresponding relaxation function is readily found in the form:

$$G(t) = \frac{C_{\alpha}\, t^{-\alpha}}{\Gamma(1-\alpha)}. \tag{5}$$

By inserting Equations (4) and (5) into (1) and (2), respectively, we get:

$$\begin{aligned}\varepsilon(t) &= \frac{1}{C_{\alpha}\Gamma(1+\alpha)}\int_0^t (t-\tau)^{\alpha}\dot{\sigma}(\tau)d\tau = \\ &= \frac{1}{C_{\alpha}\Gamma(1+\alpha)}\left[(t-\tau)^{\alpha}\sigma(\tau)d\tau\right]_0^t + \\ &+ \frac{1}{C_{\alpha}\Gamma(1+\alpha)}\int_0^t (t-\tau)^{\alpha-1}\sigma(\tau)d\tau = \frac{1}{C_{\alpha}}\left(I_{0^+}^{\alpha}\sigma\right)(t), \end{aligned} \tag{6}$$

$$\sigma(t) = \frac{C_{\alpha}}{\Gamma(1-\alpha)}\int_0^t (t-\tau)^{-\alpha}\dot{\varepsilon}(\tau)d\tau = C_{\alpha}\left({}^{c}D_{0^+}^{\alpha}\varepsilon\right)(t). \tag{7}$$

Eq. (6) is valid provided $\sigma(0) = 0$. In Equation (6) the symbol $\left(I_{0^+}^{\alpha}\sigma\right)(t)$ is the Riemann-Liouville fractional integral and in Eq. (7) $\left({}^{c}D_{0^+}^{\alpha}\varepsilon\right)(t)$ is the Caputo's fractional derivative. Equations (6) and (7) are the constitutive laws of the viscoelastic material at hand. It is to be emphasized that $0 \leq \alpha \leq 1$. In particular if $\alpha = 0$ then the material is purely elastic, if $\alpha = 1$ then the material is a pure fluid and C_{α} is its viscosity coefficient. A more detailed discussion on this point may be found in (Di Paola, Zingales 2012; Di Paola *et al*. 2013).

More complex behaviour may be obtained by enriching the creep function by additional terms (Podlubny 1999; Mainardi 2010; Di Paola *et al*. 2013; Grzesikiewicz *et al*. 2013; Zbiciak 2013). Equations (6) and (7) are the starting points to characterize the viscoelastic properties of materials.

2. Experimental investigation on asphalt mixtures

In order to investigate the viscoelastic properties of the asphalt mixtures a number of uniaxial creep compression tests have been carried out at the laboratory of University of Nottingham. The tests have been performed according to *UNI EN 12697* (2005), on two typical bituminous mixtures (British Standards Institution 2003).

The first mixture is Hot Rolled Asphalt (HRA), generally used as wearing course in flexible pavements, which is characterized by a gap-graded mixture with very little medium-sized aggregate. Fig. 1 shows a typical aggregate grading curve and a cross section of a HRA sample.

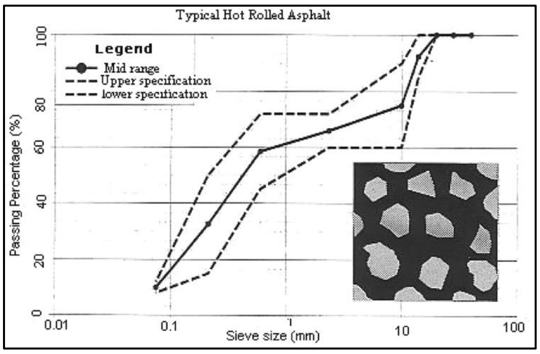

Fig. 1 A typical grading and cross section of a HRA30/14.

The second mixture is Dense Bitumen Macadam (DBM), generally used as a base layer in flexible pavements, which is characterized by a continuously graded mixture. A typical grading and an idealised section through this mixture are shown in Fig. 2.

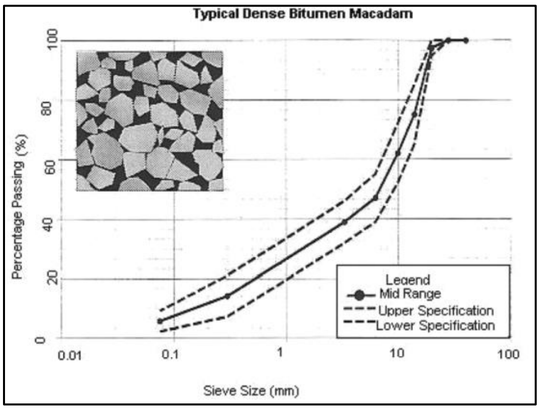

Fig. 2 A typical grading and cross section of a 20 mm DBM.

In order to study only the effect of different aggregate structures on the deformation behaviour of mixtures, the same bitumen (70/100 pen; softening point at 45°C) was used in both mixtures. A total of 28 specimens were prepared and tested. Uniaxial creep compression tests were performed at three temperatures (10, 20 and 40 °C) and various stress levels depending on the test temperature.

3. Results and best fitting procedure

In order to obtain the parameters of the model, a best fitting procedure was implemented using Wolfram Mathematica 9.0® software with the experimental data regarding creep and creep recovery tests.

Various candidate shapes of power law have been analysed. The optimum shape that has been obtained for both HRA and DBM is given as follows:

$$J(t) = U(t)\left(\frac{t}{c} + \frac{t^{\alpha}}{C_{\alpha}\,\alpha\,\Gamma(\alpha)}\right), \tag{8}$$

where c is the viscosity coefficient, C_{α} and α are parameters related to temperature, stiffness and viscosity.

In all cases Eq. (8) fitted the experimental data very well.

The mechanical equivalent of creep curve described in Eq. (8) is a dashpot (term t/c) and a so called *springpot* (term $t^{\alpha}/C_{\alpha}\,\alpha\,\Gamma(\alpha)$) in series as shown in Fig. 3. The results are contrasted with the experimental creep tests. Figs 4 and 5 summarize the results obtained by the best fitting procedure whose relevant parameters are reported in Table 1 (other Figures and Tables can be downloaded from the following link *: https://www.dropbox.com/sh/ju3263gzsui2znx/oAQfPPtxfI).

Once the creep law is fixed then as soon as we assume c, C_{α}, α independent of the stress level the linear law of viscoelastic asphalt mixture is given as:

$$\varepsilon(t) = \frac{1}{c}\left(I_{0^+}^{1}\sigma\right)(t) + \frac{1}{C_{\alpha}}\left(I_{0^+}^{\alpha}\sigma\right)(t)\,. \tag{9}$$

Eq. (9) states the strain history due to the assigned stress history $\sigma(t)$ is composed by two terms: a purely viscous part as the first term and a fractional elastic-viscous term ruled by the fractional operator. The latter is ruled by the order α of the fractional integral. If $\alpha = 0$ then the second term is purely elastic and Eq. (9) is a classical Maxwell element. If $\alpha = 1$ Eq. (9) reverts into two dashpots arranged in series. When $0 \leq \alpha \leq 1$ the mechanical model is that represented in Fig. 3, where the fractional term is a *springpot* that has an intermediate behaviour between elastic and viscous.

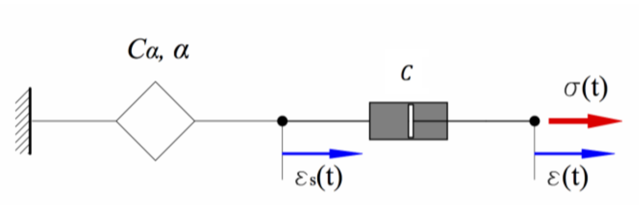

Fig. 3. Springpot element in series with a dashpot.

It has to be emphasized that if $c \to \infty$, then $J(t) = \frac{t^{\alpha}}{C_{\alpha}\,\Gamma(1+\alpha)}$ is a purely viscoelastic term and is the well-known Scott-Blair model. In Figs 4-9 in dashed line the results obtained by best fitting procedure are reported, showing that the experimental date are better fitted with Eq. (8).

Once the constitutive laws have been derived from best fitting procedure based on experimental data some considerations on the physical properties of the material can be drawn, in particular on the role played by the parameters c, C_{α} and α.

As the temperature increases the value of c highly decreases. This means that the deformation is mainly related to the fluid phase. As in fact the smaller c the higher the deformation of the dashpot element represented in Fig. 3 is.

As the temperature increases the values of C_{α} and α moderately decrease. That is the relative weight of the viscoelastic component (springpot in Fig. 3) of the asphalt mixture, is less sensitive to the temperature. In any cases since the smaller C_{α} the higher the deformation is, we can state that also the viscoelastic component gives an increasing value to the total deformation (as we expect). On the other hand α decreases as the temperature increases.

As a conclusion we can state that as the temperature increases the viscous phase, coming in part from dashpot and in part from the fractional element, prevails on the elastic component.

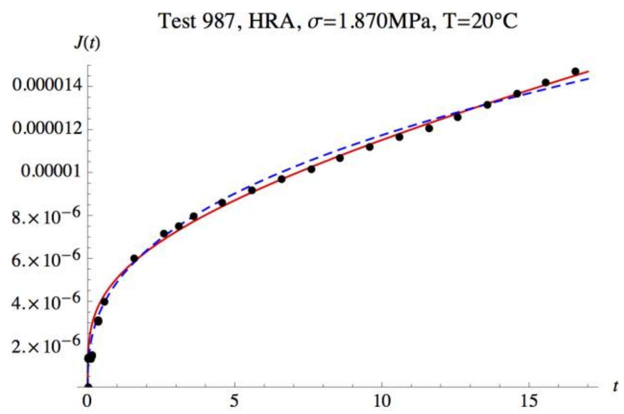

Fig. 4. HRA Experimental data (dots) contrasted with best-fitting curves of the proposed model (continuous line) and the Scott-Blair model (dashed line), 20°C.

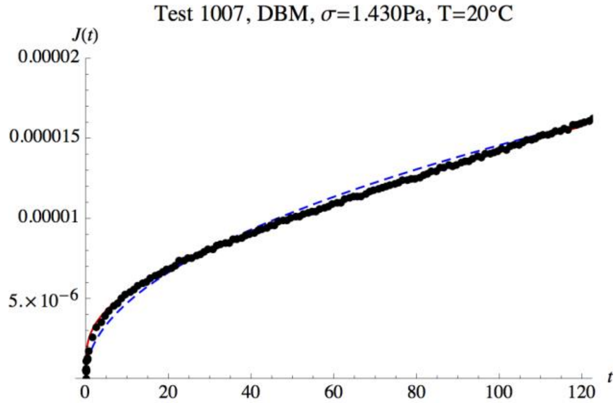

Fig. 5. DBM Experimental data (dots) contrasted with best-fitting curves of the proposed model (continuous line) and the Scott-Blair model (dashed line), 20°C.

Table 1. Best fitting parameters of creep tests 987 (HRA) and 1007 (DBM).

Test	A.M.	Air Voids	Stress	t1	T	Fractional proposed model		
						c*10^{-3}	C_α*10^{-3}	α
		[%]	[kPa]	[s]	[°C]	[Mpa s]	[Mpa s^α]	[-]
987	HRA	4.6	1870	0.126	20	3800	230.0	0.266
1007	DBM	5	1430	0.097	20	18500	425.0	0.265

In Table 1 the symbol t_1 is the time at which the assigned stress level is reached by the test machine.

From data obtained from all creep tests some considerations can be drawn: i) for fixed stress level and temperature the best fitting parameters showed very close values to each other; ii) all $c,\ C_\alpha, \alpha$ parameters are very sensitive to the temperature variations; iii) other test conditions being equal the DBM's C_α was higher than HRA's one, as confirmed from dynamic tests carried out at Nottingham.

In the next section a detailed description on the nonlinear behaviour of the asphalt mixture will be presented.

4. Nonlinear behaviour

It is widely recognized that linear viscoelasticity holds at a low stress level. In this range parameters $c,\ C_\alpha, \alpha$ remain constant, whereas as soon as the stress level increases in creep tests (at fixed temperature), the various parameters exhibit variations and the Boltzmann superposition principle fails. Detailed discussion on this point may be found in (Airey *et al*. 2003; Airey, Rahimzadeh 2004; Findley, Onaran 1976). For the particular case of asphalt mixture presented here the linearity strain range has to be of order 10^{-4} (Airey 2004), that is much lower than the strain levels corresponding to the laboratory tests. This is due to the fact that the goal of these tests was the study of steady-state deformation behaviour (Taherkhania 2011) or to evaluate mechanical parameters of the Burger generalized model (Huang 2004). From the experimental tests reported in Tables 1-2, we may affirm that the coefficients to describe the power law of the mixtures under study depend on temperature as well stress intensity. Because of the dependence of the stress level as first attempt we may suppose that the three parameters $c,\ C_\alpha, \alpha$ depend on the stress intensity and on the temperature, that is:

$$c = c(T,\sigma);\ C_\alpha = C_\alpha(T,\sigma);\ \alpha = \alpha(T,\sigma) \quad (10a,b,c)$$

From the data exploited in Tables 1-2, we may suppose that for fixed value of T, the relevant parameter may be expressed in the form:

$$\frac{1}{c} = a_c + b_c\sigma^{\beta};$$

$$C_\alpha = a_{C_\alpha} + b_{C_\alpha}\sigma^{\gamma};$$

$$\alpha = a_\alpha + b_\alpha\sigma^{\eta} \quad (11a,b,c)$$

where the various parameters a_c, b_c, a_{C_α}, b_{C_α}, a_α, b_α, β, γ, η are determined by the creep test at the various stress intensities. The limitation on β, γ, η is $\beta, \gamma, \eta > 1$. This limitation is due to the fact that at low stress level the material will behave linearly and then the various parameters will remain constant in proximity of $\sigma = 0$. It follows that the slope of the coefficients in Eq. (10) has to be zero as $\sigma \to 0$, that is:

$$\left.\frac{\partial c(T,\sigma)}{\partial \sigma}\right|_{\sigma=0} = 0;\ \left.\frac{\partial C_\alpha(T,\sigma)}{\partial \sigma}\right|_{\sigma=0} = 0;\ \left.\frac{\partial \alpha(T,\sigma)}{\partial \sigma}\right|_{\sigma=0} = 0 \quad (12a,b,c)$$

Conditions (12) are satisfied only if $\beta, \gamma, \eta > 1$. In Fig. 6 the only trend of $1/c$ for HRA is reported (all data are contained in *). The other limitations on the various coefficients are $c > 0, C_\alpha > 0, 0 \le \alpha \le 1$.

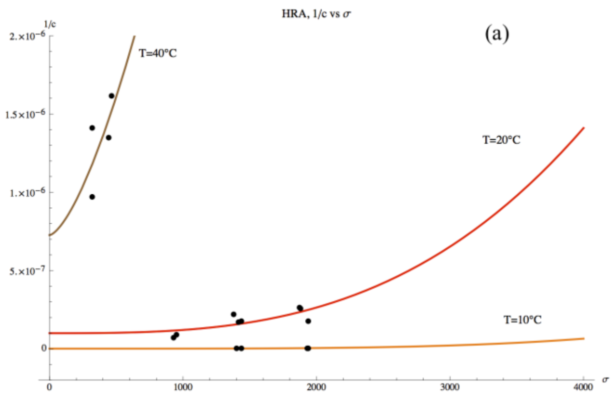

Fig. 6. Experimental trend of $1/c$ parameter at different temperatures (HRA).

In Tables 2 and 3 the various coefficients in Eq. (11a,b) are reported for both HRA and DBM.

Table 2. Coefficients in Eq. (12a,b) for HRA

HRA		T [°C] 10	20	40
1/c	ac	1.36E-09	1.00E-07	7.27E-07
	bc	2.49E-22	2.05E-17	7.91E-11
	β	4	3	1.5
C_α	ac	247.7	170.6	27.3
	bc	6.65E-10	1.15E-08	9.81E-03
	γ	3.5	3	1.3

Table 3. Coefficients in Eq. (12a,b) for DBM

DBM		T [°C] 10	20	40
1/c	ac	1.14E-10	9.70E-10	5.95E-09
	bc	1.12E-25	3.35E-22	2.50E-16
	β	5	4.5	3
C_α	ac	309.0	251.6	35.3
	bc	6.43E-06	1.00E-04	4.30E-03
	γ	2.5	2	1.5

If we suppose that c, C_α, α at a given temperature depend only on the stress amplitude σ_0, then Eq. (9) has to be modified in the form:

$$\varepsilon(t) = \int_0^t \frac{\sigma(\tau)}{c(\sigma(\tau))}\, d\tau + \int_0^t \frac{(t-\tau)^{\alpha(\sigma(\tau))-1}\, \sigma(\tau)}{C_\alpha(\sigma(\tau))\cdot\Gamma(\alpha(\sigma(\tau)))}\, d\tau\,, \quad (13a)$$

because in $0 \div t$ we suppose that the stress $\sigma(\tau) \equiv \sigma_0$, then the corresponding strain is given as:

$$\varepsilon(t) = \frac{1}{c(\sigma_0)}\left(I_{0^+}^{1}\sigma\right)(t) + \frac{1}{C_\alpha(\sigma_0)}\left(I_{0^+}^{\alpha(\sigma_0)}\sigma\right)(t). \quad (13b)$$

Unfortunately Eq. (13) remains valid only for constant or monotonic stress history. In order to highlight this concept an experimental creep recovery test was contrasted with the solution obtained from Eq. (13). In Fig. 12a we may observe that during the creep phase the results predicted by Eq. (13) perfectly match the experimental curve, but for $t > t^*$, t^* being the time of which the load was removed, the result of Eq. (13) is drastically different from experimental curve during the recovery phase. This means that the hypothesis that the coefficients depend only on σ and T is not true when $\dot{\sigma} < 0$. Therefore when the stress history is not monotonic the dependence on the various parameters has to be highlighted outside of the viscoelastic behaviour, that is:

$$c = c(T,\sigma,\dot{\sigma});\ C_\alpha = C_\alpha(T,\sigma,\dot{\sigma});\ \alpha = \alpha(T,\sigma,\dot{\sigma}). \quad (14a,b,c)$$

Moreover the dependence of the parameters on $\dot{\sigma}$ has to be neglected if $\dot{\sigma} > 0$. In order to achieve this result the function $sign(\dot{\sigma})$ will present what it happens in plasticity. It follows that the new attempt for modelling the coefficient is:

$$c(T,\sigma,\dot{\sigma}) = c^{(1)}(T,\sigma) + sign(\dot{\sigma})c^{(2)}(T,\sigma);$$

$$C_\alpha(T,\sigma,\dot{\sigma}) = {C_\alpha}^{(1)}(T,\sigma) + sign(\dot{\sigma}){C_\alpha}^{(2)}(T,\sigma);$$

$$\alpha(T,\sigma,\dot{\sigma}) = \alpha^{(1)}(T,\sigma) + sign(\dot{\sigma})\alpha^{(2)}(T,\sigma). \quad (15a,b,c)$$

under the conditions that in the creep phase:

$$c^c(T,\sigma) = c^{(1)}(T,\sigma) + c^{(2)}(T,\sigma);$$

$${C_\alpha}^c(T,\sigma) = {C_\alpha}^{(1)}(T,\sigma) + {C_\alpha}^{(2)}(T,\sigma);$$

$$\alpha^c(T,\sigma) = \alpha^{(1)}(T,\sigma) + \alpha^{(2)}(T,\sigma). \quad (16a,b,c)$$

where the apex “c” stands for creep in the sense that they are already evaluated during the creep test.

These conditions guarantee that if the load history is monotonic then we came back to Eqs. (10-13), whereas when $\dot{\sigma} < 0$ $(sign(\dot{\sigma}) = -1)$ the various parameters change. In order to identify the quantities $c^{(1)}$, $c^{(2)}$, ${C_\alpha}^{(1)}$, ${C_\alpha}^{(2)}$, $\alpha^{(1)}$, $\alpha^{(2)}$ we need other conditions that may be derived from creep recovery test. When the recovery phase at time t^* at which the stress is removed ($\dot{\sigma} = -\infty$, $sign(\dot{\sigma}) = -1$) we may identify the new set of parameters $c^r(T,\sigma)$, ${C_\alpha}^r(T,\sigma)$ and $\alpha^r(T,\sigma)$, where the apex “r” stands for recovery, with a best fitting performed in $(t^* \div \infty)$. For the creep recovery test depicted in Fig. 7 and for many other tests not reported here for brevity, the parameters significantly depend on $\dot{\sigma}$ are c and C_α. For the test in Fig. 7b they assumed the following values:

$$c^r(1\ MPa, 20°C) = 9140\ MPa\ s;$$

$${C_\alpha}^r(1\ MPa, 20°C) = 920\ MPa\ s^\alpha$$

Once the recovery coefficient is found from experimental data the values of $c^{(1)}$, $c^{(2)}$, ${C_\alpha}^{(1)}$ and ${C_\alpha}^{(2)}$ in Eq. (15a,b) may be derived in the form:

$$c^{(2)} = \frac{c^c - c^r}{2};\ c^{(1)} = \frac{c^c + c^r}{2} \quad (17a)$$

$$C_\alpha{}^{(2)} = \frac{C_\alpha{}^c - C_\alpha{}^r}{2};\ C_\alpha{}^{(1)} = \frac{C_\alpha{}^c + C_\alpha{}^r}{2} \qquad (17b)$$

$$\alpha^{(2)} = \frac{\alpha^c - \alpha^r}{2};\ \alpha^{(1)} = \frac{\alpha^c + \alpha^r}{2} \qquad (17c)$$

In Fig. 7b the deformation history evaluated with Eq. (13) in which $c(T, \sigma, \dot{\sigma})$ and $C_\alpha(T, \sigma, \dot{\sigma})$ are evaluated as in Eq. (15a,b) while $c^{(1)}$, $c^{(2)}$, $C_\alpha{}^{(1)}$ and $C_\alpha{}^{(2)}$ as in Eq. (17a,b), is contrasted with the experimental creep recovery test.

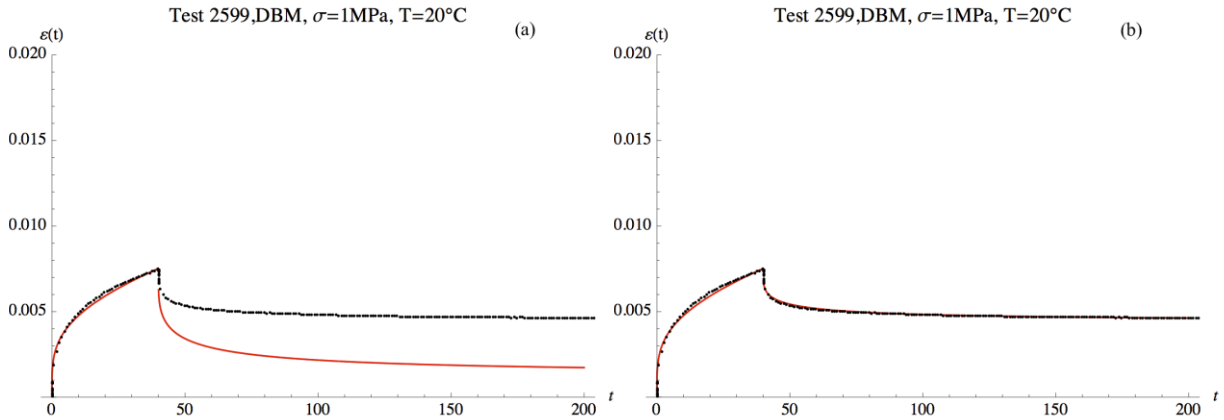

Fig. 7. Creep Recovery test (2599); a) parameters evaluated by Eq. (11); b) parameters evaluated by Eqs. (16,18).

Other creep recovery tests carried out in Nottingham were fitted using these equations. In Fig. 8 three creep recovery tests carried out on the same stress level and temperature and different loading time t^* are depicted; in Table 4 the respective relevant parameters are reported.

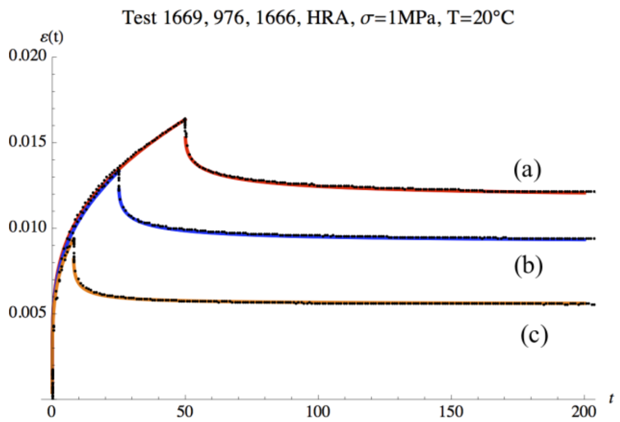

Fig. 8. Experimental curves matched with theoretical laws for creep recovery tests carried out at different loading time: $t^* = 50s$ (a); $t^* = 25s$ (b); $t^* = 8s$ (c).

Table 4. Best fitting parameters (coefficients of Eq. 17) of 1669, 976 and 1666 creep recovery tests

Test	Asphalt Mixture	Air Voids	Stress	t1	t*	T	Fractional proposed model		
							cr	Cαr	α
		[%]	[kPa]	[s]	[s]	[°C]	[Mpa s]	[Mpa s$^\alpha$]	[-]
1669	HRA	5.3	1000	0.058	50	20	4280	529.0	0.215
976	HRA	4.8	1000	0.06	25	20	2735	492.0	0.215
1666	HRA	5.7	1000	0.036	8	20	1444	423.0	0.215

From Fig. 8 some considerations may be drawn. First of all the residual strain at $t \rightarrow \infty$ is evidenced; this fact is easily explained by observing the mechanical model described in Fig. 3. During the creep phase the total deformation is composed by two terms: the first one is $\varepsilon_s(t)$, that is the deformation of the springpot; the second one is $\varepsilon(t) - \varepsilon_s(t)$, that is the relative strain due to the viscous fluid in the dashpot. The latter cannot be given back during the recovery phase, and that confirms the goodness of the mechanical model here presented. A second observation is: the longer the t^*, at a parity of stress level and temperature, the higher the residual strain. This is due to the fact that during the creep phase the dashpot move according to the equation:

$$\sigma(t) = c[\dot{\varepsilon}(t) - \dot{\varepsilon}_s(t)], \tag{18}$$

and $\varepsilon(t^*) - \varepsilon_s(t^*)$ is the residual strain that cannot be given back during the recovery phase.

With these observations in mind we may solve separately the two equilibrium equations during the creep phase:

$$\varepsilon_s(t) = \frac{1}{C_\alpha(\sigma_0)} \left(I_{0^+}^{\alpha}\sigma\right)(t), \tag{19}$$

where $\sigma(t) = \sigma_0\, U(t)$ and

$$\varepsilon(t) - \varepsilon_s(t) = \frac{1}{c}\sigma_0\, U(t), \tag{20}$$

In order to fully validate the model here proposed another test campaign will be necessary with more complex time histories (cyclic stress controlled).

Conclusions

In this paper a proper mechanical model of hereditariness on asphalt mixtures is presented. It is shown that the creep test, performed on two different asphalt mixtures at various stress levels and temperatures, is overlapped by the response of a mechanical model composed by a Maxwell element in which the spring is substituted by a fractional element (*springpot*). The constitutive law of the *springpot* is the Riemann-Liouville fractional integral of order $\alpha \in (0,1)$. That is the element exhibits an intermediate behaviour between elastic ($\alpha = 0$) and purely viscous Newtonian fluid ($\alpha = 1$).

The dependence of the three different parameters necessary for the definition of the constitutive law on the temperature and on the stress level is studied in detail so obtaining the nonlinear constitutive laws for the asphalt mixture. By the light of the experimental tests it has been observed that, in order to match the experimental data on the recovery phase ($\sigma = 0$), the parameters c and C_α have to be split in two parts. In the second part the dependence on $\dot{\sigma}$ has to be present. This part is very important in order to get a residual strain when the stress is removed.

For practical applications the parameter α ruling the power law trend of *springpot* may be assumed depending only on the temperature and this drastically simplifies the subsequent analysis since in the constitutive law the fractional term remains a simple linear (fractional) operator.

As a concluding remark, in the authors' opinion the correct way to define the proper constitutive law of a complex material like an asphalt mixture has to be validated by experimental tests. The latter have be performed at various temperature levels and at different stress intensity, starting from very small stress level in order to properly define the linear behaviour. Moreover in case of cyclic load history the only creep (or relaxation) test is not enough to fully characterize the viscoplastic behaviour.

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