

Parity-Time Symmetric Chain Resonators

S. Phang^{1,2}, A. Vukovic¹, S. C. Creagh², G. Gradoni², P. Sewell¹ and T. M. Benson¹

¹George Green Institute for Electromagnetics Research, University of Nottingham, UK

²School of Mathematical Sciences, University of Nottingham, UK

*corresponding author: sendy.phang@nottingham.ac.uk

Abstract- A simple tight-binding model to study the band-structure of an infinite length Parity-time (PT) symmetric chain of resonators is presented in this paper. In the talk, we will investigate the impact of having a structure of finite length and consider the effect of a modulation of the real part of the refractive index on the complete band-structure of the PT-chain system. For a finite PT-chain structure under certain modulation, we observe the existence of a localised mode which is either lasing or dissipating.

Photonic structures with combined gain and loss, known as Parity-Time (PT) symmetric structures, have been shown to exhibit unique properties [1,2]. Within the context of PT-resonator structures, we have reported an analytical approach for calculating the threshold point of such structures using a Boundary Integral Equation method[1]. Furthermore we also analysed the impact of unbalancing gain/loss in the PT-resonator structure in which we observed a laser operation in a loss dominated structure [1]. In this contribution, we report our recent study of a PT-symmetric chain of resonator structures (see Fig. 1).

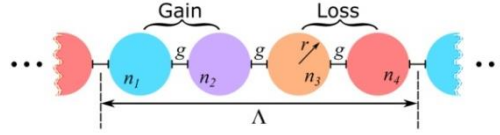


Figure 1. Schematic illustration of the PT-chain resonator structure.

The PT-chain considered is schematically illustrated in Fig. 1; each cell is comprised of 4 cylindrical resonators of the same radius r and separated by a fixed gap g . The refractive index within a unit cell is given by:

$$\begin{cases} n_1 = n_0 + \Delta n' + jn'' \\ n_2 = n_0 - \Delta n' + jn'' \\ n_3 = n_0 - \Delta n' - jn'' \\ n_4 = n_0 + \Delta n' - jn'' \end{cases} \quad (1)$$

where, n_0 , $\Delta n'$ and n'' are the average refractive index, modulation of the real part of the refractive index and modulation of the imaginary part of the refractive index (gain/loss), respectively.

In this work, we investigate the impact of having a structure of finite length and consider the effect of a modulation of the real part of the refractive index on the complete band-structure of the PT-chain system. A simple tight-binding model of the *infinite* chain for the case of no real modulation, i.e. $\Delta n' = 0$ is given by,

$$H = \begin{pmatrix} \omega_0 & \kappa & & e^{j\beta\Lambda\kappa} \\ \kappa & \omega_0 & \kappa & \\ & \kappa & \omega_0^* & \kappa \\ e^{-j\beta\Lambda\kappa} & & \kappa & \omega_0^* \end{pmatrix} \quad (2)$$

where ω_0 is the complex resonant frequency of the resonator, κ is the coupling strength between resonators

and $\beta\Lambda$ is the Bloch phase. The Hamiltonian H has multiple degeneracies even when the non-Hermitian parameter, $\gamma = \Im(\omega_0)/\kappa$ is zero, see Fig. 2(a); thus a PT-chain structure without modulation $\Delta n' = 0$ will spontaneously break the PT-symmetry for a non-zero γ , i.e. $n'' > 0$, as shown in Fig. 2(b).

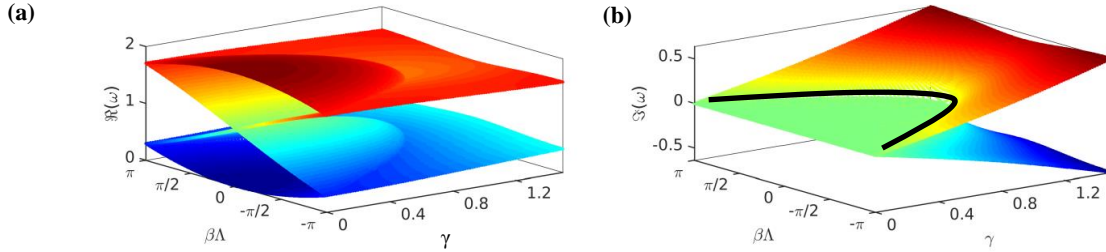


Figure 2. Band-structure of an infinitely long PT-chain with no real part modulation $\Delta n' = 0$

Meanwhile, by introducing a small real part modulation $\Delta n' \neq 0$, the resonant frequencies of the resonator are shifted, i.e. they decrease if $\Delta n' < 0$ and increase if $\Delta n' > 0$; now the Hamiltonian is given by,

$$H = \begin{pmatrix} \omega_1 & \kappa & e^{j\beta\Lambda}\kappa \\ \kappa & \omega_2 & \kappa \\ e^{-j\beta\Lambda}\kappa & \kappa & \omega_1^* \end{pmatrix} \quad (3)$$

where $\omega_1 - \omega_2 = 2\delta$ and $\omega_0 = (\omega_1 + \omega_2)/2$. The real part and the imaginary part of the eigenfrequencies of (3) are plotted in Fig. 3(a,b) respectively for $\delta = 0.25\omega_0$. It can be seen in Fig. 3(a) that by introducing real modulation $\Delta n'$, the eigenfrequencies at the edge of the Brillouin zone are no longer degenerate. As such there exists a minimum threshold of the γ parameter which causes PT-symmetry breaking.

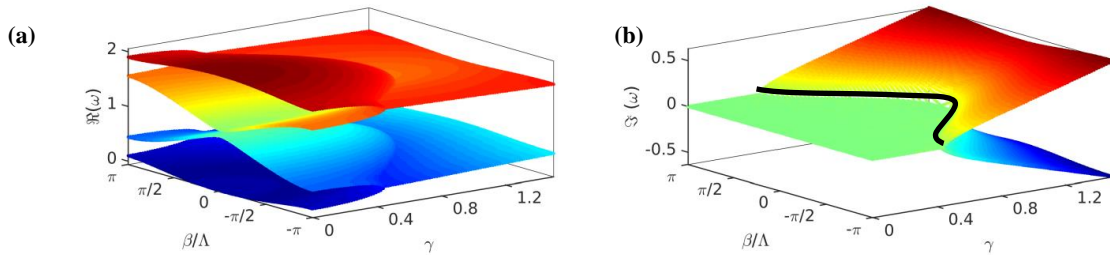


Figure3. Band-structure of an infinitely long PT-chain with real modulation

In the talk, we will discuss the effect of having a finite length of chain in which we observed the presence of localised broken-PT-symmetry mode, which has a characteristic of either lasing or decaying and is localised at either end of the chain structure.

REFERENCES

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2. Sendy Phang, Ana Vukovic, Stephen C. Creagh, Trevor M. Benson, Phillip D. Sewell, and Gabriele Gradoni, "Localized Single Frequency Lasing States in a Finite Parity-Time Symmetric Resonator Chain," *Scientific Reports* 6, 20499 (2016).