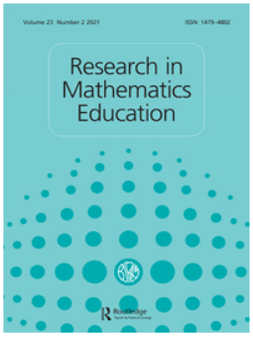

Routledge
Taylor & Francis Group

Research in Mathematics Education

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rrme20

Pattern in early years mathematics curriculum: a 25-year review of the status, positioning and conception of pattern in England

Catherine Gripton

To cite this article: Catherine Gripton (2022): Pattern in early years mathematics curriculum: a 25-year review of the status, positioning and conception of pattern in England, Research in Mathematics Education, DOI: 10.1080/14794802.2021.2010237

To link to this article: https://doi.org/10.1080/14794802.2021.2010237

Published online: 18 Apr 2022.

RESEARCH IN MATHEMATICS EDUCATION
https://doi.org/10.1080/14794802.2021.2010237

Routledge
Taylor & Francis Group

Pattern in early years mathematics curriculum: a 25-year review of the status, positioning and conception of pattern in England

Catherine Gripton

School of Education, University of Nottingham, Nottingham, UK

ABSTRACT

Pattern is fundamental to mathematical learning yet pattern has been conspicuously low key within early mathematics curriculum guidance in England despite evidence that it predicts later attainment in mathematics overall. Whilst recent curriculum changes have seen pattern afforded enhanced status, this is within a conception of pattern that marks a significant shift away from the evidence base. Through analysis of 25 years of early mathematics curriculum in England, this article identifies a curriculum landscape where pattern has been afforded little attention and narrowly framed. It finds that positioning pattern as a minor topic in mathematics potentially limits children's access to "powerful knowledge", in a critical realist sense, and to learning which is fundamental to deep, connected mathematical understanding. Recognition of pattern as powerful knowledge would position it as an essential, broad, central tenet of early childhood mathematics education and as an entitlement at the heart of national curricula.

ARTICLE HISTORY
Received 29 September 2020
Accepted 4 November 2021

KEYWORDS
pattern; early childhood mathematics; curriculum

Introduction

The early years curriculum in England has recently undergone a period of significant change. The Early Years Foundation Stage (EYFS) framework (DfE, 2020) for young children (birth to five years) has been rewritten with new early learning goals (expected development for each child by the end of this phase[1]). The revisions have been criticised for further risking the education of less advantaged and younger children with a developmentally inappropriate, school-like approach thereby baking in disadvantage early (Pascal, Bertram, & Rouse, 2019). For early mathematics, pattern has greater prominence than in the documents it replaces (DfE, 2017). This seems logical, given the indicators from research of early pattern knowledge as a predictor of later achievement in mathematics overall (Rittle-Johnson, Fyfe, Hofer, & Farran, 2017; Rittle-Johnson, Zippert, & Boice, 2019). Whilst we need to be cautious in using numbers-based attainment measures in this way (Ball, 2015), particularly in relation to early childhood education

CONTACT Catherine Gripton catherine.gripton@nottingham.ac.uk

(Roberts-Holmes & Bradbury, 2016), this strengthens the rationale for including pattern in early mathematics curricula following the example of other countries. Pattern runs through the algebra strand from foundation onwards in the Australian curriculum (ACARA, 2018) and is similarly ever present from three years in the Scottish curriculum as "patterns and relationships" (Education Scotland, 2017) and Welsh curriculum as "pattern" (DfES, 2015) in the previous or "algebra" in the new curriculum (Welsh Government, 2020). However, the type of pattern learning included within the new EYFS framework in England marks a significant divergence from earlier curricula (explored later) or those found in these other countries, so this warrants further scrutiny.

Tracing the place of pattern in curricula over time provides the context through which to critique the recent curriculum changes and a basis from which to move forwards with curriculum development. This article provides important understanding of the changing status, position and conception of pattern in 25 years of early mathematics curricula in England by considering it as powerful knowledge. The article is premised upon an assumption that early mathematics curriculum content should prioritise the mathematical knowledge that will be most powerful for the child. Whilst differing conceptions of powerful knowledge exist, there is general agreement that powerful knowledge is core disciplinary knowledge which is specialised (to mathematics in this case) and gives power to those who acquire it (Beck, 2013; Muller & Young, 2019). Whilst this esoteric knowledge takes the learner beyond the everyday knowledge they encounter within their communities (in a social realist sense), critical realists recognise that specialist powerful knowledge is developed through and within social and cultural structures. Wrigley explains that curriculum needs to harness this vernacular as well as the canonical disciplinary knowledge to produce knowledge that is truly powerful (2018). From a critical realist standpoint, powerful knowledge is contextualised and values-laden and this provides greatest potential to further social justice (Alderson, 2020). It is this theoretical understanding of powerful knowledge that is operationalised and applied to pattern in this study.

After summarising the nature and importance of pattern learning for young children, this article reports on an analysis of pattern in macro-level mathematics curriculum documentation in England across the 25-year period where the education of under-fives became a national policy agenda issue. I argue that pattern is given little attention relative to other areas of mathematics and is narrowly framed, concluding that pattern is powerful knowledge and should be afforded a more substantial and central role in early childhood mathematics curricula.

The nature and importance of pattern: the "what" and the "how"

Whilst debate naturally arises from differing epistemologies (for example, Platonism's universal truths or formalism's mathematical rules), what is largely uncontested amongst these is the importance of pattern in mathematics (Oliveri, 1997). Mathematics is the science of patterns according to Steen (1990), Devlin (1994), Resnik (1997) and others (see Orton, 2005). As such, these patterns are dynamic and invented as well as existing or static because science is the process of creating knowledge as well as the body of knowledge itself (Chevallard, 2005). These dynamic and invented patterns are the priority according to Wittmann (2005) who argues that these value process over

outcome. Clearly both are important and position a mathematician as someone who is both a *learner* of patterns (the "what") and a *maker* of patterns[2] (the "how"). In this section, I define "pattern" and summarise its role in early childhood mathematics education, finally arguing that this constitutes powerful knowledge in a critical realist sense.

What is "pattern"?

Whilst definitions of pattern vary, there is overall agreement that there are two key features, regularity and predictability (Wijns, Torbeyns, De Smedt, & Verschaffel, 2019). To have regularity, there must be an overarching order or organisation which continues or changes according to an underlying rule which creates predictability (Orton, 2005; Sarama & Clements, 2009). We can recursively predict using the relationships between elements, and can use this predictability to think more functionally (globally) and perceive an overall structure (Brownell, Chen, & Ginet, 2014; McGarvey, 2012; Wijns, Torbeyns, De Smedt, et al., 2019). Structure is sometimes separated from pattern in definitions to emphasise its importance, notably by Mulligan and Mitchelmore (2009), with their construct of "Attention to Pattern and Structure", but the term "pattern" can include structure. In this study, "pattern" is used in this broader sense.

Why is pattern important in learning mathematics?

A pattern is an overarching rule or set of rules which is predictable. This predictability enables generalisation which is "the heartbeat of mathematics" (Mason, 1996, p. 65), supporting the formation of conceptual understanding. Virtually all mathematics is based on pattern and abstracting these patterns is the goal of mathematics learning (Mulligan & Mitchelmore, 2009; Warren, 2005). In identifying a pattern, we determine which aspects are involved in the regularity and which are not (McGarvey, 2012), which seems related to cognitive flexibility (the ability to switch between two aspects of a stimulus) according to Bock et al. (2015). Learning specific patterns (the "what") and learning to look for and use patterns (the "how") are both important.

Pattern involves numerical, spatial or logical relationships (Mulligan & Mitchelmore, 2009) so children learn specific patterns from across the mathematics curriculum (the "what"). The same pattern can be represented in different contexts and patterns can occur within a single object, multiple objects or within abstract items or ideas (Papic, Mulligan, & Mitchelmore, 2011). There are three broad categories of pattern types that young children engage with, as identified by Papic and Mulligan (2007). For children under five years "spatial structure patterns" (arrangements) and "repeating patterns" (repeated unit) are most developmentally appropriate, along with growing patterns that fit into both categories and are typically more challenging (Papic & Mulligan, 2007; Warren & Cooper, 2008; Wijns, Torbeyns, Bakker, De Smedt, & Verschaffel, 2019).

Spatial structuring is crucial for all pattern development so is important for all pattern types (Mulligan, Woolcott, Mitchelmore, & Davis, 2018). Within the three types, spatial structure patterns are the foundations of more complex patterning, developing earlier with children exploring arrangements of objects that are often non-linear and involve proximity and spacing (Garrick, Threlfall, & Orton, 2005). Much development occurs in spatial patterning between three and four years where children typically become

more adept at copying a pattern (Lüken, 2018). These patterns include subitising (immediate recognition of number from their patterned arrangement), arrays or grids and symmetry (Papic & Mulligan, 2007). For repeating patterns, understanding the pattern as having a repeating rather than alternating structure is important (Papic, 2007) with identifying the "unit of repeat" as the overarching concept (Papic & Mulligan, 2007; Threlfall, 2005). This is a key assessment of pattern understanding which children can typically do by the age of six (Clements & Sarama, 2021) or earlier non-verbally (Papic et al., 2011). The development of children's patterning with one pattern type supports their patterning with other types. Instruction on repeating patterns has been found to support knowledge of growing patterns, for example (Papic et al., 2011). There are fewer research studies on young children's learning with growing patterns but there are indications that children find growing patterns easier to copy than continue and overall more challenging than repeating patterns (Wijns, Torbeyns, Bakker et al., 2019). Growing patterns require functional attention to rules governing *change* so seem more challenging than repeating patterns which ideally require functional thinking about the rules governing what *stays the same* but might also be continued using recursive understanding (Wijns, Torbeyns, Bakker et al., 2019). Growing patterns can be particularly helpful in supporting children to understand the number system.

The dispositional aspect of pattern (the "how") is more than "pattern spotting" but is an attention to structure, meaning making and algebraic thinking to support learning (Kieran, Pang, Schifter, & Ng, 2016; Orton, 2005). This pattern sensing is a mathematical behaviour or disposition – a mathematical habit of mind (Clements & Sarama, 2021; Cuoco, Goldenberg, & Mark, 1996) – and key within "mathematising reality" (Gravemeijer & Terwel, 2000, p. 782) where pattern brings order to solve problems from the real world (Freudenthal, 1971). The Erikson Institute explain that, "pattern is less a topic of mathematics than a defining quality of mathematics" (Brownell et al., 2014, p. 84).

"Seeing" a pattern is less to do with eyesight and more about cognitively grasping relationships (Mason, 1985) and "sense-making" (Schifter, 1999, p. 80; Schoenfeld, 1992, p. 335). It involves algebraic thinking, which supports understanding of areas such as counting, multiplicative reasoning and statistics (Ginsburg, Lee, & Boyd, 2008; Mason, 2018; Papic, 2007; Papic et al., 2011). Sharing and expressing patterns is about communicating generality which is crucial for the algebra learning they will engage with as older learners (Mason, 1985). The connection between pattern and algebraic thinking is well established (Kaput, 2008; Lee & Freiman, 2006; Warren, 2005) and a fundamental concept of early mathematics (as "algebraic reasoning") according to Perry & Dockett where "relationships of equality, sequence and argument" are developed through patterning activities (2008, p. 85). The role of pattern in early childhood mathematics is to support the child's current but also future mathematical development as pattern learning provides the foundations for later algebraic learning.

Success in patterning varies considerably from child to child (Mulligan & Mitchelmore, 2009; Papic et al., 2011) but it can be taught successfully to young children (Mulligan & Mitchelmore, 2009; Papic et al., 2011; Papic & Mulligan, 2007) in a way that is authentic and enjoyable as "entirely natural, pleasurable and a part of human sense-making" (Mason, Graham, & Johnston-Wilder, 2005, p. 2). Where pattern is perceived solely as a topic in mathematics education or as a set of existing patterns to be learned ("the what"), there is a risk of pattern learning without algebraic thinking (Lee

& Freiman, 2006), so without the development of attention to pattern as a mathematical behaviour or disposition (the "how").

Pattern as powerful knowledge

This dual understanding of pattern as both core mathematics content (the "what") and a key mathematical disposition or behaviour (the "how") aligns with a critical realist understanding of powerful knowledge as being core disciplinary knowledge (the "what"), learned within and through the child's context (the "how") where having this knowledge gives power to the child. In this section, I make the case that pattern fits this understanding of powerful knowledge and should be central within an early mathematics curriculum for social justice.

Mathematics has its own epistemology as a discipline (Young & Muller, 2015) where pattern is central (Törner & Sriraman, 2007). Although related to measures of general cognitive ability, pattern is not a proxy for it and is a unique contributor to children's general mathematical knowledge (Kidd et al., 2013; Rittle-Johnson et al., 2019; Zippert, Clayback, & Rittle-Johnson, 2019), fitting both social realist and critical realist criteria for powerful knowledge as differentiated and specialised (Alderson, 2020; Young & Muller, 2013). Pattern is core disciplinary knowledge and therefore important curriculum content ("what" is to be learned in mathematics). It is universal knowledge that can be applied across mathematics, potentially unlocking a whole range of mathematics topics for the child, giving them increased chance of success.

Whilst social realists define powerful knowledge as separate from values and contexts including the everyday knowledge of children and their communities (Muller & Young, 2019), critical realists embrace these arguing that these tether the knowledge to the child. They argue that it is through the child's context (including social and cultural structures) that they acquire powerful knowledge so "how" they learn is important as well as "what" they learn. Pattern is powerful in this way in that it involves a disposition ("how" children are mathematical through attention to pattern) and existing patterns within mathematics ("what" patterns need to be learned). Pattern is aligned with a critical realist understanding of powerful knowledge as it similarly values children's contexts and how they learn in addition to what they learn. So, teaching pattern as powerful knowledge in a critical realist sense, involves and values attention to structure and algebraic thinking.

The power of pattern lies in the universality of the content knowledge (the "what") and the impact of behaving mathematically (the "how"). Pattern supports children to perceive regularity and identify structures which provide conceptual understanding, flexibility, choices and efficiency in mathematics. The child is empowered with mathematical thinking which goes beyond power transference (from adult to child) with the teacher as the source of patterns to be learned by the child (Garrick et al., 2005). Pattern as powerful knowledge supports the child's agency within the social and cultural structures of school mathematics where pattern is ethical, emancipatory and purposeful for the child – it is useful to succeed in the child's context. This acknowledges relativity and context but simultaneously reduces the child's reliance upon these contextual structures (Alderson, 2020). Traditional mathematics teaching can require children to surrender

agency (Boaler & Greeno, 2000) but teaching children to perceive and construct patterns in mathematics shifts power to the child, equipping them to be mathematical with increasing autonomy and independence.

Pattern is powerful in that it predicts future learning. Pattern understanding is a significant longitudinal predictor of arithmetic skills (Burgoyne, Malone, Lervag, & Hulme, 2019; Lüken & Kampmann, 2018) and mathematics skills in general (Rittle-Johnson et al., 2017; Rittle-Johnson et al., 2019) with particular benefits for multiplicative reasoning and calculation (MacKay & De Smedt, 2019; Papic et al., 2011). Children with access to this powerful knowledge have a clear advantage. Where children lack conceptual understanding in pattern, this might lead to later lower achievement in mathematics with children using inefficient or inflexible strategies without noticing underlying patterns and structures which would help them (Mulligan, 2011; Mulligan, English, Mitchelmore, Welsby, & Crevensten, 2011). Pattern teaching is particularly powerful for children from low-income families where it can lead to substantial learning gains in mathematics (Rittle-Johnson et al., 2017). In addition, pattern is closely linked to executive functioning (Burgoyne et al., 2019; Miller, Rittle-Johnson, Loehr, & Fyfe, 2016) with some indicators that patterning can also support early reading (Burgoyne et al., 2019; Kidd et al., 2014).

Pattern is powerful knowledge in that it can unlock current and future mathematical learning and such powerful knowledge should be an entitlement for all children through curriculum. The focus of this study is the status, positioning and conception of pattern evident within recent curriculum documents in England. The aim was to review these in the light of what is known about the nature of pattern and pattern learning, in order to find how pattern is prioritised, positioned and conceived at the macro (national) level and to reflect on how this marries with pattern as powerful knowledge, as described here. The methods for reviewing curriculum documents is outlined in the next section.

Methods

In order to analyse pattern in curriculum documents, there was a process of identification, selection and analysis of documents. The criteria for inclusion and process of analysis are outlined in this section. To investigate the status, positioning and conception of pattern in these documents, the research questions were:

(1) Is the pattern learning specified in curriculum content an entitlement (statutory) or recommendation (non-statutory guidance)?
(2) Where is pattern located in mathematics curriculum content?
(3) What pattern types are included in mathematics curriculum content?
(4) (a) Is pattern included as a separate mathematics topic or as a quality of mathematics? (b) What is implied or assumed about how children learn about pattern in mathematics? (c) What indications are there that pattern is relational and linked to mathematical thinking and communicating?

Identification of curriculum documents

There have been a substantial number of national curriculum documents relating to mathematics education of young children in recent years in England due to a period of what Margaret Brown describes as "continual curriculum meddling" (2011, p. 151) with regular changes made by successive Conservative and Labour governments which has continued under coalition and Conservation governments more recently. In addition to being a time of significant curriculum change, this period also saw early education become central to education and social change in government policy with new major policies effecting the early education sector at a rate of almost one per year (Nutbrown & Clough, 2014). The 25-year period 1994–2019 therefore provides a sufficient number of documents pertaining to mathematics education in early education settings in England for trends within the status, positioning and conception of pattern to be identified.

In this review, curriculum documents were selected based on inclusion criteria (Table 1) from broad searches of education policy documents using online archives: DERA and Education in England[3] as well as current and archived government education websites (e.g. The National Strategies). Documents for leaders and teachers/practitioners which provided guidance as to what children should learn in early years settings in England were included. Early years, for the purpose of selecting documents, was deemed birth to five years in line with government education policy of this era (although birth to seven or eight years is a more commonly accepted definition in early childhood education). Policy documents such as those related to non-curriculum requirements (safeguarding or behaviour, for example) as well as speeches and press releases were not included in the review. Curriculum reviews and reports were included as these shape interpretation of curriculum through exemplification of content and teaching

Table 1. Inclusion criteria.

Criteria type	Inclusion criteria	Examples of exclusion
Topic	Addresses learning and teaching, supporting child development or education and care	Documents about health or social care
Document focus	Includes content on curriculum (what should be taught or learned) to support cognitive development	Documents exclusively about social or physical development
Subject content	Includes mathematics curriculum	Documents exclusively focussed on literacy curriculum
Audience	Includes leaders and teachers/practitioners in settings for children birth to five years	Documents intended exclusively for local authority personnel or school inspectors
Publication dates	Published between 01/01/1994 and 31/12/2019 in final format	Draft documents or consultations released before 31/12/2019 but not published for use until after this date
Authorship	Official publications by government and government departments	Documents written by think tanks, charities and researchers (unless commissioned and published by government)
Document type	Curriculum requirements, curriculum guidance, advice for the interpretation of curriculum requirements, advice for assessment of curriculum, advice for organisation of curriculum, reviews of curriculum, curriculum reports	Consultation documents, responses or reports
Application	Includes national guidance for England	National guidance exclusively for countries other than England Regional guidance School curriculum documents

approaches as well as identification of priorities that provides direction to schools and settings. Similarly, documents on assessment were included as they interpret curriculum for leaders and practitioners (e.g. statutory assessment handbooks). It was important to review non-statutory documents as they have significant authority for early childhood educators, given their authorship (typically government and regulatory bodies), and because they frequently expand, explain and exemplify statutory requirements.

The research process

Fifty-five documents were identified using inclusion criteria (Table 1) and analysed chronologically. All sections where pattern was indicated were extracted. Based on the

Table 2. Review process.

Stage 1 – Identify documents using inclusion criteria (Table 1)

Stage 2 – Analyse documents, by date of publication. For each document:

- Read all mathematics content for birth to five years age range (including cross-curricular)
- Record all extracts relating to pattern which includes words or description indicating advice for pattern learning (both as specific patterns and attention to structure). Indicative list of terms to support this progress:
  - pattern, patterns, patterning
  - subitising
  - sequences
  - algebra or algebraic thinking
  - rules, relationships, connections, structures
  - arrange, arrangement, organise
  - copy, continue, create, spot, identify
  - spatial, growing, repeating, repeated, alternating
  - symmetry, symmetrical, odd/even

A principle of inclusion was applied, i.e. where content is vague but could potentially relate to pattern it was included for further analysis. Where guidance included content relating to the teaching of older children, only content relating to four- to five-year-olds was included.

- Record the density (proportionally how much of the mathematics content relates to pattern) – description and estimated percentage
- Record near misses (extracts or sections which were considered but excluded) and non-mathematical references to pattern, for double checking

Stage 3 – Double check all near misses from 3 and move to include these, if needed

Stage 4 – Code documents for indicators of status, position and conception. For each document:

- Code the status – statutory or non-statutory
- Code the domains (if indicated) as specified in the document (e.g. algebra, number, calculation, shape, measures) – free coding, codes refined, codes applied
- Code the treatment of pattern – quality, topic or both/unclear
- Code pattern types – spatial, repeating, growing or none/unclear
- Record advice on pattern pedagogy (if this was included) – text extracts

Stage 5a – Create filtered groups of documents (based on research questions). Analyse the extracts for all documents with the group code to create summaries of the range and type of content in these code groups. Code groups:

- Statutory (question 1)
- non-statutory (question 1)
- Quality/both (question 4a, question 4c)
- Topic (question 4a)
- Pattern pedagogy (question 4b)

Stage 5b – create filtered groups of documents (based on research questions). Analyse extracts for all documents with group code and for the balance across the code groups within that area. Areas and code groups:

- Domains – number, shape, space, problem solving – (question 2, question 4c)
- Pattern types – spatial structural, repeating, growing – (question 3)

definition of pattern given earlier in this article, a list of terms which potentially indicate pattern content (stage 2 in Table 2) was used to support searching to try to mitigate accidental omission, as far as possible, but all mathematics content was reviewed, including descriptions and examples. A cautious approach was taken where all possible or borderline examples were extracted and "near misses" and non-mathematical references to pattern were recorded and doubled checked (stage 3). Everyday uses of terms such as pattern and sequence were not recorded, such as "the pattern of attainment for children". The researcher also recorded the density (proportion) of pattern content within the mathematics content overall (recording an estimated percentage). In the 21 documents pertaining to the education of four- to 11-year-olds, only content relating to four- to five-year-olds was included.

In stage 4 (Table 2), each document (and extracts) was coded for status, domain(s) (if indicated), the treatment of pattern, pattern types and suggestions about pattern pedagogy (if included). Treatment of pattern as a quality or topic followed Brownell et al.'s definition of pattern as a quality of mathematics (2014). Pattern types were coded using Papic & Mulligan's "key aspects of patterning" (2007) as spatial, repeating and growing with all types found in the document, recorded. In stage 5, filters were applied to the data set using the codes from stage 4 and extracts were reviewed from documents in the same code (status, treatment, pattern pedagogy, domains and pattern types) and for balance across codes (domain and pattern types). The outcome of this documentary review is summarised and discussed in the next section.

The place of pattern in early years curriculum and policy in England

The documentary review provided evidence of the status, position and conception of pattern and each are discussed in turn here.

The status of pattern

The status of pattern in curriculum was reasonably strong if viewed purely in terms of it featuring in curriculum documentation with statutory status. From 1994 to 2019 pattern featured consistently in every statutory curriculum framework in England[4] for children birth to five years (although it was not until 2008 that one curriculum document applied to the full birth to five age range). It had a stable single sentence in the six successive statutory curriculum documents reviewed. This was: "Talk about, recognise and (re)create simple patterns" (originally, "They recognise and recreate patterns" in DfEE, 1996, p. 5, developing to "They recognise, create and describe patterns" in DfE, 2017, p. 12). This was part of an early learning goal which all children were to achieve by the end of the school year in which they were five years old (statutory from 2002 to 2019, and beyond until 2021).

Whilst included as an entitlement for all children, the focus was quite narrow in the statutory documents. Pattern was situated solely within "shape, space and measures" with no suggestion of pattern in statutory goals for number and calculation. Statutory assessment documents afforded relatively little explanation, compared to other areas of mathematics, and tended to include examples of linear repeating patterns where these were provided (e.g. STA, 2012), discussed later.

One statutory document stood out from the others in this period. In the "Curriculum Guidance for the Foundation Stage" (DfEE, 2000), which was statutory from 2002 to 2008 for three- to five-year-olds, pattern was more prominent in the mathematical development programme. Here, children were to develop their understanding of pattern through a broad range of contexts and through everyday routines and experiences. Pattern was identified as a broad key skill running "across different aspects of mathematics" (DfEE, 2000, p. 73). This more fundamental conception of pattern was, however, undermined as it was not translated into the assessed goals for early learning and therefore the associated statutory assessment, the "Foundation Stage Profile" where pattern featured in only one of the 27 assessment scale points for mathematics, and then only as linear repeating patterns in "shape, space and measures" (QCA, 2003). The 15 statutory documents reviewed suggest that pattern learning was required in early mathematics provision throughout the 25-year period reviewed but overall was given a minor role, despite its inclusion as a statutory element.

The position of pattern

The positioning of pattern in geometry as part of "shape, space and measures" in documents with statutory status was balanced somewhat in the non-statutory guidance that accompanied the first statutory birth to five years framework in 2008. (DCSF, 2008a) It explained that children should be encouraged "to explore real-life problems, to make patterns and to count and match together" (DCSF, 2008b, p. 63) and learn through activities which focus upon mathematics as well as through activities where the mathematics can be drawn out (with pattern in block play provided as the example, p. 64). For the 2012 EYFS (DfE, 2012), the same statutory pattern sub-goal was retained but the accompanying non-statutory guidance (which then remain unchanged until 2021) no longer contained the contextualising statements about children exploring patterning or attending to pattern within other activities (Early Education, 2012). This positioned pattern more firmly as a discrete topic within the domain of geometry. Pattern in number was the second most common domain, included in 15 of the documents compared to 31 for geometry. Six of the documents including pattern in number were very brief mentions (despite being large mathematics focused documents) and only five were curriculum guidance for younger children (before the first year of schooling at four to five years). Only two documents included significant attention to number patterns and both were non statutory (DfEE, 1999, for four- five-year-olds and DCSF, 2009b for one- to six-year-olds).

Whilst the non-statutory documents were typically larger documents than their statutory counterparts, with increased scope for detail and exemplification, there was relatively little pattern content in the longer documents reviewed. This was the case even when these specifically related to mathematics and this age range, for example only extracts in 35 pages of guidance on mathematics with three- to four-year-olds in DfES (2002a). The highest proportion of text featuring pattern content was in this, the accompanying guidance for four- to five-year-olds (DfES, 2002b) and the "Numbers and patterns: laying the foundations in mathematics" (DCSF, 2009b). Pattern received minimal attention (a mention in a list of what children learn or one example of an activity) in government commissioned reports or those from the regulator (Ofsted)

despite their considerable attention to mathematics and extensive commentary on the quality of mathematics practice.

Whilst pattern was afforded relatively little attention within the documents reviewed, the proportion of content which included pattern reduced over the 25-year period reviewed. This is evident in the 19 documents pertaining to the non-statutory but highly influential national strategies (numeracy and then numeracy within primary national strategies), a Blair/Brown era Labour government policy project of the late 1990s and 2000s. These documents relate mainly related to four and five-year-olds in compulsory schooling. Pattern formed an important part of the fundamental "solving problems" strand in the main document as well as featuring in number (counting) and shape and space strands (DfEE, 1999). By the 2006 revision, this was reduced to listing pattern as a topic (included in the "using and applying" strand) and brief mentions in number (knowledge) and shape strands (DfES, 2006). The apparent reduction in focus on pattern in the documents across this period could be a result of a shift of focus in this particular policy agenda. Alternatively, it could be indicative of a more general trend within text reduction where pattern is a casualty of the slimming down of curriculum guidance, as exemplified by the removal of contextualising statements from DCSF (2008b) to Early Education (2012) discussed earlier in this section.

The conception of pattern

A topic not a quality

In the status section, I identified the "Curriculum Guidance for the Foundation Stage" (DfEE, 2000) as an outlier in the statutory documents reviewed. Although undermined in the assessment requirements (discussed in the status section), this document includes an understanding of pattern as a quality of mathematics rather than a topic, following Brownell et al. (2014)'s definition of pattern:

> The idea of "pattern" runs through the different aspects of mathematics. Children might notice repeating patterns of colours or shapes on a favourite tee-shirt, for example, or they might help to create a repeating pattern with beads, Children begin to appreciate symmetry, and this may feature in some of their drawings. They might also notice patterns when working with numbers of objects, for example, "You get three and I get four. Three, four!" (DfEE, 2000, p. 73)

In this document pattern seeking is deemed a key mathematical skill (DfEE, 2000, p. 68) suggesting a conception of pattern as relational and structural. The definition of mathematics in this statutory curriculum document includes pattern: "pattern seeking, making connections, recognising relationships" and is reproduced later in the non-statutory Primary National Strategy (DfES, 2006) and EYFS guidance (DCFS, 2008b) but without the clarity of the 2000 document, that it underpins learning across the mathematics and is not included in the statutory EYFS framework (DCFS, 2008a). This conception of pattern as a quality of mathematics is evident in only these three of the 55 documents reviewed. In others, it is treated as a separate area of mathematics; a relatively minor one of several topics within geometry or is so briefly mentioned that the conception of pattern is unclear. Manyukhina and Wyse (2019) point out the knowledge-based approach to curriculum in England (instead of skills- or learner-orientated) and

this can perhaps mitigate against the inclusion of dispositional aspects of disciplinary learning such as pattern. It is challenging to fit such a fundamental concept into topic-based curriculum with modular units of content (Davis & Sumara, 2000), where the mathematical focus is upon single concepts such as counting (Mulligan, 2011).

A limited range of pattern types

Where detail on pattern was very limited, it was unclear which types of pattern children should be learning about from the curriculum documents. Whilst this could be due to the limited scope of these documents, they often contained more detail on other areas of the mathematics curriculum. In the documents reviewed pattern types, if indicated, were typically repeating patterns, occasionally spatial structural patterns (e.g. subitising dice patterns in DfEE, 1999 and DCSF, 2009a) and rarely growing patterns (e.g. a "one more" sequence staircase number pattern in Ofsted, 2012).

Statutory assessment guidance sometimes provided clarification of the pattern types required for the statutory pattern statement (none were provided in STA, 2013 or STA, 2014). Only examples of AB repeating patterns were provided in QCA (2003) and QCA (2008). The most detailed guidance on pattern types was in the exemplification of the "shape, space and measures" early learning goal which was non-statutory guidance for 2012–2019 where 13 of the 17 example patterns are linear repeating patterns, seven of these are AB patterns with all but one being of alternating colours (STA, 2012; STA, 2014). These documents all relate to the end of phase assessment (at four to five years) and there are fewer examples of pattern types for younger children or guidance as to how children progress towards this.

An adult-led, talk-based pedagogy

The perception of pattern as knowledge that is learned from an adult using a language-based pedagogy is implied within the limited curriculum guidance for younger children in particular but for also for older children (in the birth to five age range). Whilst national curricula does not necessarily include pedagogy, much of the non-statutory documentation for this age range does contain such guidance. With little detail on pattern learning in mathematics, it is difficult to draw conclusions but "talk" does feature in statutory curriculum frameworks (as "talk about" 2002–2012 and "describe" 2012–2019) and the associated practice guidance where pattern is mentioned. This contradicts the findings of the Leeds Pattern Project that found that on the whole, three- to four-year-olds "only occasionally try to verbalize their pattern-related activities and intentions during play activities, and so for the most part children's thinking about pattern is shown only by what they do" (Garrick et al., 2005, p. 1). Statements about exploring patterns in fabric, pictures or routines are included in guidance for two- to four-year-olds, if these include pattern at all (e.g. DCSF, 2008b; DfES, 2002c; Early Education, 2012). These statements focus upon adults showing, drawing children's attention to or talking to children about patterns in these contexts, which demonstrates a conception of pattern as accessed only through the practitioner. Where there is advice that children should make patterns, there is a narrow focus upon recreating patterns provided by the adult until children are four to five years old (apart from in the non-statutory guidance for Nursery: DfES, 2002a). Whilst there is only limited evidence, this does suggest an apparent adult-led model of pattern learning in the curriculum.

Conclusion

Overall, pattern was afforded little attention and was narrowly framed in national early years curriculum documentation in England over 25 years. The implications for education policy, explored below, are that pattern needs repositioning, reconceptualising and affording enhanced status across the mathematics curriculum to entitle all children to have access to pattern as powerful knowledge.

The status, positioning and conception of pattern in early childhood curriculum has not been that of powerful knowledge

Pattern was afforded relatively little attention in curriculum documentation across the 25-year period studied. It had a minor role with other areas of mathematics afforded much greater explanation with pattern rarely included within these other areas or included as a mathematical disposition ("how" we are mathematical). Whilst pattern was afforded a statement in the statutory curriculum for birth to five years, it was generally positioned as a separate, more minor, geometry topic. This involved a narrow range of pattern types ("what" was to be learned), mainly linear repeating patterns of two or three elements (usually colours) but this was only on the few occasions where further detail was provided.

Taking all 55 reviewed documents into account, pattern was not a curriculum priority at national level across the 25 years and indeed became less of a priority over this period. As curricular detail was reduced then pattern, positioned as a minor topic, was afforded relatively less attention. Whilst many high level documents do not include guidance on specific teaching approaches, where pattern pedagogy was suggested it isolated "the teacher as the only source of pattern" (Garrick et al., 2005, p. 16). This underplays the mathematical richness of pattern-making and pattern-creating (Fox, 2005) and the value of attention to pattern as a mathematical behaviour that builds understanding across all areas of mathematics. The place of pattern across 25 years of early childhood mathematics curriculum is far from a critical realist conception of powerful knowledge.

Limitations

In this study, macro-level national curriculum documentation was reviewed. These do not evidence how the guidance was operationalised in educational settings with young children and practitioners had no duty to follow the content of the 40 non-statutory documents, although many linked directly to statutory ones. Pattern may be characterised quite differently in practice to curriculum guidance, although stepping away or outside of national guidance is a risk that requires practitioner confidence, courage and subject knowledge (Ball, 1996). Home learning is another of many factors which shape children's mathematical learning so it is not possible to draw conclusions about children's attainment based on the status, positioning and conception of pattern in national curriculum documents. Nevertheless, research on pattern in school curriculum and planning documents would build upon this study and provide understanding of how the national level curriculum guidance was interpreted at local level.

There are further limitations to the findings of this study. It is possible that a document might have been missed in performing searches. Whilst the whole period is covered in terms of statutory documents, a more minor non-statutory document may have been missed. Researcher judgement was used in the identification and interpretation of document extracts and this shaped the findings. Whilst pattern-related terms and a principle of inclusion were used to help minimise human error, this would not eradicate it. Indeed, the caution exercised may have led to an overrepresentation of the attention given to pattern in early childhood curriculum with more extracts identified using a generous criteria for identification of pattern extracts.

The review is limited by the categories used including text density of pattern content as a broad indicator of how pattern is being positioned within that document (prioritisation of pattern). It is also important to note that curriculum documents differ significantly in scope. For statutory documents in particular, the dispositional nature of pattern might not be included because how children should be learning might be beyond the scope or intention of the document, which might aim to describe outcomes and not processes. Nevertheless, perhaps this signifies a missed opportunity to include content that could be enormously powerful as an entitlement for all young children.

Implications for policy

Not all knowledge is equally valuable to the learner so it is important to differentiate and prioritise that which is potentially most beneficial to the child (Young, 2009). For critical realists, this means a curriculum based on ethics, as well as epistemology, and of what is most beneficial to the individual as well as to society (Manyukhina & Wyse, 2019). Pattern underpins virtually all mathematics (Brownell et al., 2014, p. 83; Mulligan & Mitchelmore, 2009, p. 33) and is the basis for learning all areas of mathematics curriculum content (Iliada, Baccaglini-Frank, Levenson, & Matsuo, 2021). Reimagining and recalibrating the place of pattern within early mathematics curricula in England is necessary in order to reflect both its importance in mathematics and its value to children. Pattern should be positioned prominently in national curriculum documents as an entitlement for every child. Following the example of Mulligan and Mitchelmore's (2009) "Awareness of Mathematical Pattern and Structure" (AMPS) approach, a broad fundamental conception of pattern would draw attention to the importance of pattern within mathematical thinking (as "quality" of mathematics), helping educators shift the focus away from outcomes and towards mathematical processes (how children understand the mathematics).

Implications in the context of recent curriculum developments – new EYFS

Recent early childhood curriculum development in England demonstrates a step shift in pattern content in mathematics. There is a recognition of the importance of pattern as a mathematical disposition in the new EYFS, (statutory from September 2021) with the mathematics curriculum programme including mention of children looking for patterns and relationships as a mathematical disposition (DfE, 2020). There is also a new early learning goal (expected development for four- five-year-olds) with pattern in the title (replacing the previous shape, space and measures goal). Whilst this might seem a

forwards step, this new goal replaces the previous broader requirement to "recognise, create and describe patterns" with a limited focus on "numerical patterns" (DfE, 2020), excluding spatial structure and repeating pattern types. This narrower goal is concerned more with learning specific number sequences than the regularity and predictability of pattern as functional thinking. This is potentially limiting in the same way that pattern was limited previously to "shape, space and measures" in successive learning goals. This represents a missed opportunity to set pattern as fundamental across all areas of mathematics in the high profile early learning goals.

The focus on just numerical growing patterns by the end of the EYFS is interesting given that they are typically considered more appropriate for older children (e.g. Warren & Cooper, 2008, with eight-year-olds), although this is an area in need of more research. Papic and Mulligan (2007) reported no success on their growing pattern tasks from four- to five-year-olds who had not received a patterning intervention and Wijns, Torbeyns, Bakker et al. (2019) used non-numerical growing patterns with four-year-olds which children found more challenging than repeating patterns. Research suggests that identification of the "unit of repeat" in repeating patterns is a more fundamental threshold concept (Papic & Mulligan, 2007; Threlfall, 2005; Wijns, Torbeyns, Bakker et al., 2019).

In the non-statutory guidance (DfE, 2021), eight of the 43 statements include pattern. Whilst this is more than its predecessor's 6 out of 60 (Early Education, 2012), it is still relatively few. This is in contrast with the wealth of research about the importance of children's patterning at this age (e.g. Rittle-Johnson et al., 2017; Wijns, Torbeyns, Bakker et al., 2019). This does not position pattern as central in mathematics and as powerful knowledge, as discussed in this article.

Implications for practice

For pattern in early childhood mathematics curricula to realise its potential as powerful knowledge, clearly classroom practice is crucial. It seems sensible to suggest that a broad approach to pattern teaching is desirable, based upon strong traditions of early years practice. Explicit teaching of pattern should be included, as should child-initiated play as much valuable patterning occurs within this (Garrick et al., 2005; McCluskey, Mulligan, & Van Bergen, 2018). The content of pattern teaching should include patterning more generally rather than a sole focus upon narrow pattern duplication and extension skills (Garrick et al., 2005; Miller et al., 2016). It should include attention to pattern as a mathematical behaviour or disposition as well as learning existing patterns across the mathematics curriculum. Pattern learning can be developed through a range of contexts, including children's literature (for example Mattone, 2007), block play (within Johnson's stages of block play, Johnson, 1933) and music (Geist, Geist, & Kuznik, 2012) and as well as through other activities such as dance and art.

Practitioner's understanding of pattern (Houssart, 2000) and identification of pattern types (McGarvey, 2012) are varied in their sophistication and complexity and pattern within pedagogical content knowledge is key. Existing research indicates that pattern teaching should be underpinned by knowledge of a developmental progression in pattern (Frye et al., 2013). This can help prevent low achievement early by supporting accurate and specific assessment of a child's patterning, suggesting whether abstraction

has developed beyond known patterns and enabling this to be addressed through practice (Björklund & Pramling, 2014).

Nevertheless recognising conceptual development in patterning is not straightforward and simple inclusion within national statutory and non-statutory curriculum is not enough to support teachers in this endeavour. Further research is needed to ascertain effective approaches to professional development for pattern teaching, perhaps utilising Thouless and Gifford's (2019) sustained collaborative approach where shared pedagogy led to professional ownership of a trajectory for border pattern development. Possible priorities might include appropriate pedagogies for pattern (for example the PASMAP approach, Mulligan, 2011) and how to support children through the process of discerning governing rules in patterns (Björklund & Pramling, 2014). Professional development alongside curriculum entitlement would support the powerful nature of pattern to be realised, positioning algebraic thinking as "a natural birthright of all human beings" (Mason, 2008, p. 85) within a more socially just approach to mathematics education.

Notes

1. The end of phase assessment against the early learning goals occurs at a fixed point in the school year so children will be either four- or five-years-old depending on their month of birth.
2. Hardy (1940) argues that a mathematician is a maker of patterns of ideas in the same way that a painter makes patterns with paint and a poets makes patterns with words.
3. Digital Education Research Archive https://dera.ioe.ac.uk/ and Education in England http://www.educationengland.org.uk/index.html.
4. 1994–1996 no statutory curriculum for birth to five-year-olds; 1996–2002 statutory curriculum for four- to five-year-olds; 2002–2008 statutory curriculum for three- to five-year-olds; and 2008–2019 statutory curriculum for birth to five-year-olds.

Disclosure statement

No potential conflict of interest was reported by the author(s).

ORCID

Catherine Gripton http://orcid.org/0000-0003-1832-9480

References

ACARA. (2018). *Australian curriculum: F-10 (v 8.4)*. Sydney: ACARA.
Alderson, P. (2020). Powerful knowledge and the curriculum: Contradictions and dichotomies. *British Educational Research Journal*, *46*(1), 26–43.
Ball, D. L. (1996). Teacher learning and the mathematics reform: What we think we know and what we need to learn. *Phi Delta Kappan*, *77*(7), 500.
Ball, S. J. (2015). Education, governance and the tyranny of numbers. *Journal of Education Policy*, *30*(3), 299–301.
Beck, J. (2013). Powerful knowledge, esoteric knowledge, curriculum knowledge. *Cambridge Journal of Education*, *43*(2), 177–193.

Björklund, C., & Pramling, N. (2014). Pattern discernment and pseudo-conceptual development in early childhood mathematics education. *International Journal of Early Years Education*, *22*(1), 89–104.
Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. *Multiple Perspectives on Mathematics Teaching and Learning*, *1*, 171–200.
Bock, A., Cartwright, K. B., Gonzalez, C., O'Brien, S., Robinson, M. F., Schmerold, K., & Pasnak, R. (2015). The role of cognitive flexibility in pattern understanding. *Journal of Education and Human Development*, *4*(1), 19–25.
Brown, M. (2011). Going back or going forward? Tensions in the formulation of a new national curriculum in mathematics. *The Curriculum Journal*, *22*(2), 151–165.
Brownell, J. O. N., Chen, J.-Q., & Ginet, L. (2014). *Big ideas of early mathematics: What teachers of young children need to know*. New York: Pearson.
Burgoyne, K., Malone, S., Lervag, A., & Hulme, C. (2019). Pattern understanding is a predictor of early reading and arithmetic skills. *Early Childhood Research Quarterly*, *49*, 69–80.
Chevallard, Y. (2005). Steps towards a new epistemology in mathematics education. *Proceedings of the 4th Conference of the European Society for Research in Mathematics Education (CERME 4)*, 21–30.
Clements, D. H., & Sarama, J. (2021). *Learning and teaching early math: The learning trajectories approach*. Oxon: Routledge.
Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *The Journal of Mathematical Behavior*, *15*(4), 375–402.
Davis, B., & Sumara, D. J. (2000). Curriculum forms: On the assumed shapes of knowing and knowledge. *Journal of Curriculum Studies*, *32*(6), 821–845.
Department for Children, Schools and Families. (2008a). *Statutory framework for the early years foundation stage*. Nottingham: DCSF.
Department for Children, Schools and Families. (2008b). *Practice guidance for the early years foundation stage*. Nottingham: DCSF.
Department for Children, Schools and Families. (2009a). *Children thinking mathematically: PSRN essential knowledge for early years practitioners*. Nottingham: DCSF.
Department for Children, Schools and Families. (2009b). *Numbers and patterns: Laying foundations in mathematics*. Nottingham: DCSF.
Department for Education. (2012). *Statutory framework for the early years foundation stage*. London: DfE.
Department for Education. (2017). *Statutory framework for the early years foundation stage*. London: DfE.
Department for Education. (2020). *Statutory framework for the early years foundation stage: Setting the standards for learning, development and care for children birth to five*. London: DfE.
Department for Education. (2021). *Development matters: Non-statutory curriculum guidance for the early years foundation stage*. London: DfE.
Department for Education and Employment. (1996). *Desirable outcomes for children's learning on entering compulsory education*. London: DfEE/SCAA.
Department for Education and Employment. (1999). *The National numeracy strategy: Framework for teaching mathematics from reception to year 6*. Cambridge: DfEE.
Department for Education and Employment. (2000). *Curriculum guidance for the foundation stage*. London: QCA.
Department for Education and Skills. (2002a). *Mathematical activities for the foundation stage: Nursery*. London: DfES.
Department for Education and Skills. (2002b). *Mathematical activities for the foundation stage: Reception*. London: DfES.
Department for Education and Skills. (2002c). *Birth to three matters*. London: DfES.
Department for Education and Skills. (2006). *Primary national strategy: Primary framework for literacy and mathematics*. London: DfES.
Department for Education and Skills. (2015). *Curriculum for Wales: foundation phase framework*. Cardiff: DfES.

Devlin, K. (1994). *Mathematics: The science of patterns: The search for order in life, mind and the universe*. New York: Scientific American Library.
Early Education. (2012). *Development matters in the early years foundation stage*. London: Early Education.
Education Scotland. (2017). *Curriculum for excellence: Numeracy and mathematics*. Livingstone: Education Scotland.
Fox, J. (2005). Child-initiated mathematical patterning in the pre-compulsory years. *International Group for the Psychology of Mathematics Education*, *2*, 313–320.
Freudenthal, H. (1971). Geometry between the devil and the deep sea. *Educational Studies in Mathematics*, *3*(3), 413–435.
Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children: A practice guide [NCEE 2014-4005]*. Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE).
Garrick, R., Threlfall, J., & Orton, A. (2005). Pattern in the nursery. In A. Orton (Ed.), *Pattern in the teaching and learning of mathematics* (pp. 1–17). London: Continuum.
Geist, K., Geist, E. A., & Kuznik, K. (2012). The patterns of music. *Young Children*, *2*, 74–79.
Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report*, *22*(1), 1–24.
Gravemeijer, K., & Terwel, J. (2000). Hans freudenthal: A mathematician on didactics and curriculum theory. *Journal of Curriculum Studies*, *32*(6), 777–796.
Hardy, G. H. (1940). *A mathematician's apology*. Cambridge: Cambridge University Press.
Houssart, J. (2000). Perceptions of mathematical pattern amongst primary teachers. *Educational Studies*, *26*(4), 489–502.
Iliada, E., Baccaglini-Frank, A., Levenson, E., & Matsuo, N. (2021, July 11-18). *Early childhood mathematics education [ICME research survey]*. The 14th international congress on mathematical Education (ICME 14), Shanghai, China.
Johnson, H. M. (1933). *The art of block building*. New York: The John Day Company.
Kaput, J. J. (2008). What is algebra? What is algebraic reasoning? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 5–18). Oxon: Routledge.
Kidd, J. K., Carlson, A. G., Gadzichowski, K. M., Boyer, C. E., Gallington, D. A., & Pasnak, R. (2013). Effects of patterning instruction on the academic achievement of 1st-grade children. *Journal of Research in Childhood Education*, *27*(2), 224–238.
Kidd, J. K., Pasnak, R., Gadzichowski, K. M., Gallington, D. A., McKnight, P., Boyer, C. E., & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. *Early Education & Development*, *25*(1), 134–151.
Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (2016). *Early algebra. Research into its nature, its learning, its teaching*. Hamburg: Springer.
Lee, L., & Freiman, V. (2006). Developing algebraic thinking through pattern exploration. *Mathematics Teaching in the Middle School*, *11*(9), 428–433.
Lüken, M. M. (2018). Repeating pattern competencies in three- to five-year old kindergartners: A closer look at strategies. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank, & C. Benz (Eds.), *Contemporary research and perspectives on early childhood mathematics education* (pp. 35–53). Cham: Springer.
Lüken, M. M., & Kampmann, R. (2018). The influence of fostering children's patterning abilities on their arithmetic skills in grade 1. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank, & C. Benz (Eds.), *Contemporary research and perspectives on early childhood mathematics education* (pp. 55–66). ICME-13 monographs. Cham: Springer.
MacKay, K. J., & De Smedt, B. (2019). Patterning counts: Individual differences in children's calculation are uniquely predicted by sequence patterning. *Journal of Experimental Child Psychology*, *177*, 152–165.
Manyukhina, Y., & Wyse, D. (2019). Learner agency and the curriculum: A critical realist perspective. *The Curriculum Journal*, *30*(3), 223–243.
Mason, J. (1985). *Routes to algebra, roots of algebra*. Berkshire: Open University Press.

Mason, J. (1996). Expressing generality and roots of algebra. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra* (pp. 65–86). Cham: Springer.
Mason, J. (2008). Making use of children's powers to produce algebraic thinking. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 57–94). Oxon: Routledge.
Mason, J. (2018). How early is too early for thinking algebraically? In C. Kieran (Ed.), *Teaching and learning algebraic thinking with 5- to 12-year-olds: The global evolution of an emerging field of research and practice* (pp. 329–350). Cham: Springer.
Mason, J., Graham, A., & Johnston-Wilder, S. (2005). *Developing thinking in algebra*. Berkshire: Open University Press.
Mattone, L. (2007). "I know an old lady": Using children's literature to explore patterns. *Teaching Children Mathematics*, *14*(4), 202–205.
McCluskey, C., Mulligan, J., & Van Bergen, P. (2018). Noticing mathematical pattern and structure embodied in young children's play. *Proceedings of the 41st Annual Conference of the Mathematics Education Research Group of Australasia*, 535–542.
McGarvey, L. M. (2012). What is a pattern? Criteria used by teachers and young children. *Mathematical Thinking and Learning*, *14*(4), 310–337.
Miller, M. R., Rittle-Johnson, B., Loehr, A. M., & Fyfe, E. R. (2016). The influence of relational knowledge and executive function on preschoolers' repeating pattern knowledge. *Journal of Cognition and Development*, *17*(1), 85–104.
Muller, J., & Young, M. (2019). Knowledge, power and powerful knowledge re-visited. *The Curriculum Journal*, *30*(2), 196–214.
Mulligan, J. (2011). Towards understanding the origins of children's difficulties in mathematics learning. *Australian Journal of Learning Difficulties*, *16*(1), 19–39.
Mulligan, J., English, L. D., Mitchelmore, M., Welsby, S., & Crevensten, N. (2011). Developing the pattern and structure assessment (PASA) interview to inform early mathematics learning. *Proceedings of the 34th Annual Conference of the Mathematics Education Research Group of Australasia and 23rd Biennial Conference of the Australian Association of Mathematics Teachers*, 1022–1030.
Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, *21*(2), 33–49.
Mulligan, J., Woolcott, G., Mitchelmore, M., & Davis, B. (2018). Connecting mathematics learning through spatial reasoning. *Mathematics Education Research Journal*, *30*(1), 77–87.
Nutbrown, C., & Clough, P. (2014). *Early childhood education: History, philosophy and experience* (2nd ed.). London: Sage.
Ofsted. (2012). *Mathematics: Made to measure*. Manchester: Ofsted. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/417446/Mathematics_made_to_measure.pdf
Oliveri, G. (1997). Mathematics. A science of patterns? *Synthese*, *112*(3), 379–402.
Orton, A. (Ed.). (2005). *Pattern in the teaching and learning of mathematics* (2nd ed.). London: Continuum.
Papic, M. (2007). Promoting repeating patterns with young children – More than just alternating colours!. *Australian Primary Mathematics Classroom*, *12*(3), 8–13.
Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education*, *42*(3), 237–268.
Papic, M., & Mulligan, J. (2007). The growth of early mathematical patterning: An intervention study. *Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia*, *2*, 591–600.
Pascal, C., Bertram, T., & Rouse, L. (2019). *Getting it right in the early years foundation stage: A review of the evidence*. https://www.early-education.org.uk/sites/default/files/Getting%20it%20right%20in%20the%20EYFS%20Literature%20Review.pdf

Perry, B., & Dockett, S. (2008). Young children's access to powerful mathematical ideas. In L. D. English, M. B. Bussi, G. A. Jones, R. A. Lesh, B. Sriraman, & D. Tirosh (Eds.), *Handbook of international research in mathematics education* (2nd ed., pp. 81–112). Abingdon: Routledge.
Qualifications and Curriculum Authority. (2003). *Foundation stage profile handbook*. London: QCA.
Qualifications and Curriculum Authority. (2008). *Early years foundation stage profile handbook*. London: QCA.
Resnik, M. D. (1997). *Mathematics as a science of patterns*. Oxford: Oxford University Press.
Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development*, *88*(5), 1727–1742.
Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*, *46*, 166–178.
Roberts-Holmes, G., & Bradbury, A. (2016). Governance, accountability and the datafication of early years education in England. *British Educational Research Journal*, *42*(4), 600–613.
Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Oxon: Routledge.
Schifter, D. (1999). Reasoning about operations: Early algebraic thinking in grades K-6. In L. Stiff & F. R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12* (pp. 62–81). Virginia: National Council of Teachers of Mathematics.
Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). Virginia: National Council of Teachers of Mathematics.
Standards & Testing Agency. (2012). *Early years foundation stage profile handbook*. STA.
Standards & Testing Agency. (2013). *Early years foundation stage profile handbook*. STA.
Standards & Testing Agency. (2014). *Early years foundation stage profile handbook*. STA.
Steen, L. A. (1990). Pattern. In L. A. Steen (Ed.), *On the shoulders of giants: New approaches to numeracy* (pp. 1–10). Washington, DC: National Academy Press.
Thouless, H., & Gifford, S. (2019). Paper plate patterns: Pre-school teachers working as a community of practice. *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education*, *1*, 375–381.
Threlfall. (2005). Repeating patterns in the early primary years. In J. Orton (Ed.), *Pattern in the teaching and learning of mathematics* (pp. 18–30). London: Continuum.
Törner, G., & Sriraman, B. (2007). A contemporary analysis of the six "theories of mathematics education" theses of Hans-Georg Steiner. *ZDM*, *39*(1), 155–163.
Warren, E. (2005). Patterns supporting the development of early algebraic thinking. *Building Connections: Research, Theory and Practice*, *2*, 759–766.
Warren, E., & Cooper, T. (2008). Generalising the pattern rule for visual growth patterns: Actions that support 8 year olds' thinking. *Educational Studies in Mathematics*, *67*, 171–185.
Welsh Government. (2020). *Curriculum for Wales guidance*. Education Wales. https://hwb.gov.wales/storage/b44ad45b-ff78-430a-9423-36feb86aaf7e/curriculum-for-wales-guidance.pdf
Wijns, N., Torbeyns, J., Bakker, M., De Smedt, B., & Verschaffel, L. (2019). Four-year olds' understanding of repeating and growing patterns and its association with early numerical ability. *Early Childhood Research Quarterly*, *49*, 152–163.
Wijns, N., Torbeyns, J., De Smedt, B., & Verschaffel, L. (2019). Young children's patterning competencies and mathematical development: A review. In K. M. Robinson, H. P. Osana, & D. Kotsopoulos (Eds.), *Mathematical learning and cognition in early childhood: Integrating interdisciplinary research into practice* (pp. 139–161). Cham: Springer.
Wittmann, E. C. (2005). Mathematics as the science of patterns–a guideline for developing mathematics education from early childhood to adulthood. *Plenary Lecture at Mathematical Learning from Early Childhood to Adulthood*, 1–25.
Wrigley, T. (2018). 'Knowledge', curriculum and social justice. *The Curriculum Journal*, *29*(1), 4–24.

Young, M. (2009). Education, globalisation and the 'voice of knowledge'. *Journal of Education and Work*, *22*(3), 193–204.
Young, M., & Muller, J. (2013). On the powers of powerful knowledge. *Review of Education*, *1*(3), 229–250.
Young, M., & Muller, J. (2015). *Curriculum and the specialization of knowledge: Studies in the sociology of education*. Oxon: Routledge.
Zippert, E. L., Clayback, K., & Rittle-Johnson, B. (2019). Not just IQ: Patterning predicts preschoolers' math knowledge beyond fluid reasoning. *Journal of Cognition and Development*, *20*(5), 752–771.