# The impact of mesoscale textile architecture on the structural damping in composite structures

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## Abstract

In this article, a method allowing for prediction of the structural damping in composite structures at a mesoscale level is presented. The method enables a fast and accurate calculation of the loss factor function of the direction of the wave propagation and the frequency in any periodic woven architecture. The scheme combines two reduction methods, the first being the mode-based Component Mode Synthesis that reduces the size of the dynamic stiffness matrix of a unit cell, while the second is the Wave and Finite Element Method that employs Periodic Structure Theory to reduce the size of the problem by allowing to model only a unit cell of a periodic structure. The exhibited methodology can allow for structural damping prediction optimization through judicious design of the textile architecture. Numerical examples of models of different and complex architectures are shown.

*Keywords:* Damping Loss factor, Textile composites, Component Mode Synthesis, Periodic Structure Theory, Wave propagation

#### 1. Introduction

Textile composites are increasingly used in the modern industry due to their excellent mechanical properties: high specific stiffness and strength, superb fatigue strength, excellent corrosion resistance and most importantly a low density [1]. For these reasons, composite materials are seen as a good alternative to the traditionally used structural materials. In particular, low density allows for a great reduction of the energy consumption when used in the transport industry. In the aerospace industry, the most recent aircrafts are mainly made of composites (up to 60% of their weight). However, unlikely the traditional alloy materials used in the industry, composite materials display some very complex mechanical behaviour. They are for example prone to many modes of failures (fibre breakages, cracks, delamination etc.), some harder to detect than others. In order to provide the earliest possible detection of any damage, Structural Health Monitoring (SHM) techniques have been studied. A technique finding increasing popularity for 'on-line' inspection is ultrasound guided waves spectroscopy [2]. As guided wave propagation in thin plates is dispersive, the velocity of wave propagation (and thus the time-of-flight) and the damping depend on the frequency. The dispersion characteristics of undamped complex textile composites has been studied by the authors and the results published in recent manuscripts [3, 4].

However, damping is also an important parameter in the design of composite materials, especially for engineering applications in which the dynamic response often needs to be controlled. Numerous analytical models have been developed throughout the years, at both macro- and mesoscales. At macromechanical scale, the effect of the lay-up sequences of the composites on the damping are studied, while at a mesoscale, the fibers arrangements are of concern. The damping capacity in composite structures is generally higher than in a traditional material, mostly because of the viscoelastic properties of the matrix [5]. This capacity can be used to enhance the uses of composite materials. Indeed it is well known that composites can be tuned to fit particular stiffness and strength properties, but damping properties can be tuned as well modifying constitutive parameters such as the periodic sequence, the mesoscale arrangement of the fibre etc. [5]. It is also known that the inherent damping of the components of a composite is the main source of damping. Increasing the volume fraction of the matrix often results in damping increasing [5]. As damping of the propagating guided wave modes results in a reduction in inspection ranges, it is of the upmost importance to know these properties for Non Destructive Evaluation (NDE) purposes and thus for SHM [6].

Numerous studies have been carried out on the effect of the fibre properties on the damping. Early work was performed experimentally: in [7], Wright compares the loss factors of different fibre resin combinations by a resonant beam method, for glass and carbon fibres. Crane [8] investigated the effect of fibre and matrix properties as a function of frequency on the damping of composites for glass and graphite fibre composites. Ply orientation and layup are some other fundamental factors and thus their effect on the damping have been thoroughly investigated. In the first place, those investigations were mostly involving analytical and experimental work. Adams and Bacon [9] studied the effect of fibre orientation and laminate geometry on the dynamic properties of CFRP. They also stated, by separating the energy dissipations associated to the individual stress components, that shear is the factor that can give high damping. Using this concept of energy dispersion separation, Adams and Maheri studied the effect of stress level on the damping variation in CFRP [10] and showed that the fibre orientation and stacking has an effect on the damping in [11]. Berthelot [12] did a similar study, comparing the effect of fibre orientation in glass and Kevlar fibre composites on the damping. This subject has drawn a lot of experimental work since [13].

Thanks to the early work of Adams and Bacon and their damping criterion, some theoretical models for predicting damping emerged. Ni and Adams [14] developed a model both useful and accurate for predicting damping in composite laminates. Yim compared some damping prediction models (including Ni and Adams') in [15] for laminated composite beams and stated that the fibre orientation has a strong effect on the damping. Berthelot et al. [16] developed a synthesis of damping analysis of laminate material, comparing analytic method with experimental results. In [17] Maheri compare the damping in layered FRP panel under different boundary conditions and using various layup. It showed that the angle of the fibre has an influence on the modal damping.

Thanks to the enhancement of numerical methods in the last decades, the effect of the micromechanical arrangement of the fibers could be thoroughly investigated, using finite element methods (FEM) for example. Hwang and Gibson [18, 19] utilized a finite element (FE) approach for characterizing the effects of stress on damping in laminated composites. Tsai and Chi [20] compared different micromechanical arrangements of the fiber in fiber composites with a FEM analysis. Chandra et al. [21] have investigated the effect of fiber cross-section and fiber volume fraction on damping. FEM was used as well for establishing damping model at a macroscale. Mahi in [22] evaluates the energy dissipation for different fibre orientations, for composite materials. Guan and Gibson [23] used FEM to study the damping characteristics of woven fabric-reinforced for the first time. The method was compared with a closed-form solution and experiments in order to assess the validity of the method. However, the effect of undulate fiber bundles on the damping properties is not considered in that study. In [24], Yu and Zhou established a damping prediction approach for woven composites, taking the undulation of the fibers into account. This study shows once again a correlation between the decreasing of the loss factor with an increasing fiber volume fraction.

The vibro-acoustic and ultrasonic wave propagation properties of the structure is studied using the Wave and Finite Element Method (WFEM). This is an efficient tool to study waves in periodic structures. The one-dimensional WFEM has been applied in numerous previous works and for various type of structures such as beams like structures [25, 26], plates [27, 28], thin-walled structures [29] and curved layered shells [30] and then extended to two-dimensional by [31]. However, to apply the WFEM to textile composites, a fine mesh is needed, which implies large number of degrees of freedom and therefore a large CPU time. The Component Mode Synthesis (CMS) approach has been used to counteract this issue. The combining of the WFEM and CMS methods has been widely used in the literature [3, 32, 33, 34, 35]. The textile composite is modelled at a mesoscopic scale in order to establish the effects inherent to the micromechanics of the material. Both the mass and the stiffness matrix are computed using a FE software.

The principal novelty of this paper is the fast and accurate prediction of the damping properties associated to the dispersion characteristics of complex textile composites at a mesoscopic scale. The paper is organised as follows: in Sec.2 an overview of the modelling approach is presented. In the following section (Sec.3), some damping modelling methods are introduced, the one used in this paper is shown, as well as the loss factor determination method. Finally, four case studies (a laminate and three textile composites of different architecture) are displayed and a numerical validation of the methodology is proposed in Sec.4.

#### 2. Modelling textile composites through a full FE approach

Mechanical modelling of textile composites can be achieved at different scales [36]. A macroscale modelling implies homogenised mechanical properties along the unit cell, while a mesoscale modelling suggests a more detailed geometric representation of fabrics such as representing the yarns and the matrix (in this case, the assembly of fibre is homogenised). In this paper, the analysis is performed at a mesoscopic scale. This section summaries the mesoscale methodology developed and validated in recent publications from the same authors [3, 4] to study the dispersion char-

acteristics of undamped textile composites. The readers are invited to refer to these publications for some advanced details on the mesoscale modelling of complex textile composites.

The modelling of this textile composite is performed using TexGen, an open source software developed by the Composites Research Group at the University of Nottingham [37, 38]. It is used for modelling the geometry of textile structures such as 3D woven textiles or braid fabric for example. The model can then be exported on a FE software where the mass and stiffness matrices  $\mathbf{M}$  and  $\mathbf{K}$  of the unit cell can be extracted, as shown in Fig.1.



Figure 1: Methodology for obtaining the stiffness and mass matrices of the mesoscale model

The nodal DOFs of the unit cell are partitioned in the following way: bottom, top, left, right, left-bottom corner, right-bottom corner, left-top corner, right-top corner and internal DOFs, which gives:

 $\mathbf{q} = \left\{ \mathbf{q}_{B}^{T} \quad \mathbf{q}_{T}^{T} \quad \mathbf{q}_{L}^{T} \quad \mathbf{q}_{R}^{T} \quad \mathbf{q}_{LB}^{T} \quad \mathbf{q}_{RB}^{T} \quad \mathbf{q}_{LT}^{T} \quad \mathbf{q}_{RT}^{T} \quad \mathbf{q}_{I}^{T} \right\}^{T}.$  The equation of motion of the unit cell can be written as

$$\mathbb{M}\ddot{\mathbf{q}}(t) + \mathbb{C}\dot{\mathbf{q}}(t) + \mathbb{K}\mathbf{q}(t) = \mathbf{f}(t) \tag{1}$$

Assuming time-harmonic behaviour leads to  $\ddot{\mathbf{q}} = -\omega^2 \mathbf{q}$ , Eq.(1) can be rewritten as follow:

$$\left[\mathbf{K} + \mathrm{i}\omega\mathbf{C} - \omega^2\mathbf{M}\right]\mathbf{q} = \mathbf{F}$$
<sup>(2)</sup>

In order to reduce the complexity of the structural dynamic model, a Component Mode Synthesis (CMS) method is applied. It is mostly used to reduce the use of CPU time and memory.

The key to this method is the reduction of the relative DOFs (internal nodes), whereas the boundary DOFs are kept as physical coordinates [39]. In the fixed interface CMS, a set of 'fixed boundary modes', also called component modes are selected among a subset of the local modes of the unit cell when the boundary DOFs are fixed and no force is acting on the internal nodes. The reader is referred to Appendix A for details on the method.

Once this reduction method has been applied, the periodic structure theory (PST) is employed (details on the method can be found in Appendix B). The PST states that when a free wave travels along a waveguide, the displacements between two opposite boundary sides of a cell differ only by a propagation factor. The unit cell needs however to be meshed in a similar way on its opposite boundaries, so that continuity in displacements and forces equilibrium is respected [25], as shown in Fig.2.



Figure 2: Periodic structure theory

In our case, (see Fig.2) the wave motion is in the Oxy plan, which gives

$$\mathbf{q}_{\mathbf{R}} = \lambda_x \mathbf{q}_{\mathbf{L}}; \qquad \mathbf{q}_{\mathbf{T}} = \lambda_y \mathbf{q}_{\mathbf{B}}$$

(3)

 $\mathbf{q_{RB}} = \lambda_x \mathbf{q_{LB}}; \quad \mathbf{q_{LT}} = \lambda_y \mathbf{q_{LB}}; \quad \mathbf{q_{RT}} = \lambda_x \lambda_y \mathbf{q_{LB}}$ 

 $\lambda_x$  and  $\lambda_y$  being respectively the propagation factors along the axis x and y.

The propagation factors are calculated using the following formula

$$A_x = e^{-ik_x\Delta}; \quad \lambda_y = e^{-ik_y\Delta}$$
 (4)

and the direction of propagation  $\theta$  is computed using the following relations

$$k_x = k\cos(\theta); \quad k_y = k\sin(\theta)$$
 (5)

Combining the CMS reduction method with the periodicity relation, allows us to reduce drastically the number of unknown in the equations and thus the calculation time. Both methods are further described in the appendices.

## 3. Damping calculation in composite structures using FE

There are several ways of modelling the damping using FE. One of the simplest way to model damping would be using a linear viscoelastic model, such as Kelvin-Voigt's one [40, 41], as used in [42]. However those models are simplistic and therefore might not represent well the viscoelastic behaviour of the material. Another method is the complex modulus approach, which consists of allowing complex components in the material's stiffness matrix. The complex modulus approach is widely used for modelling the damping in FE codes [43, 44, 45, 46] and this is the approach employed in this paper. The  $\mathbb{C}\dot{\mathbf{q}}(t)$  term in Eq.(1) is suppressed and the stiffness matrix  $\mathbf{K}$  is treated as complex [47]. These complex matrices are composed of a real part that represents the storage modulus referring to the elastic behaviour, while the imaginary part represents the loss modulus referring to the dissipative behaviour of the constituent elements is not the only damping mechanism occuring in composite structures: thermoplastic damping and Coulomb friction damping in the fiber/matrix interface regions are two other mechanisms to name a few. However, it has been identified as the dominant damping mechanism [50] and thus is the one studied here.

In this modelling method, the global stiffness matrix K is given in Eq.(6).

$$\mathbf{K} = \mathbf{K}' + \mathbf{i}\mathbf{K}'' = \sum_{k=1}^{n} (\mathbf{K}'_{k} + \mathbf{i}\mathbf{K}''_{k})$$
(6)

Where *n* is the total number of solid elements used in the FE discretisation.  $\mathbf{K'}_k$  and  $\mathbf{K''}_k$  are respectively the real and imaginary stiffness matrix contributions to the *k*<sup>th</sup> finite element of the global stiffness matrices  $\mathbf{K'}$  and  $\mathbf{K''}$ . Using the fact that the dissipated energy is the result of each component contributions (fibers and matrix in the case of composite materials), it can be concluded that the contributions of each component are divided based on the energy that the constituent material stores.

In this paper, structural damping is assumed: the viscoelastic properties of each material involved are characterised by complex component in the stiffness matrix. The imaginary part  $\mathbf{K}''$  of the global stiffness matrix  $\mathbf{K}$  is inspired from the strain energy method, stating that the contributions of each component are divided based on the energy the constituent material stores, and thus we use

$$\mathbf{K}^{\prime\prime} = \eta_{yarn} \mathbf{K}^{\prime}_{yarn} + \eta_{mat} \mathbf{K}^{\prime}_{mat} \tag{7}$$

With  $\eta_{yarn}$  and  $\eta_{mat}$  respectively the yarn and matrix loss factor.  $\mathbf{K}'_{yarn}$  and  $\mathbf{K}'_{mat}$  being respectively the stiffness matrix of the yarn and matrix elements. It should be noted that in this method, the loss factor data for the constituent elements must be pre-determined.

Once the damping has been modelled, the CMS and WFE methods can be applied to the problem. Several formulations of the WFE scheme, all leading to a different eigenvalue problem, can be employed. In this work, the adopted formulation is the one involving the wavenumbers being real and known [3]. In this case the frequencies of the wave that propagates in the structure can be calculated from the standard linear eigenvalue problem. The equation of motion is thus solved for complex angular frequencies  $\omega^2 = (\omega_{re} + i\omega_{im})^2$ . As depicted in [5], the ratio between the imaginary (representing the dissipated energy) and real coefficients (representing the stored energy) gives the loss factor of the material. The loss factor associated with a propagating wave is calculated as proposed in [46, 51]. The angular frequency is rewritten as follows

$$\omega^2 = (\omega_{re} + i\omega_{im})^2 = 2i\omega_{re}\omega_{im} + \omega_{re}^2 - \omega_{im}^2$$
(8)

The loss factor  $\eta$  is the ratio between the imaginary and real coefficients, which gives:

$$\eta(\omega,\theta) = 2 \frac{\omega_{re}\omega_{im}}{(\omega_{re}^2 - \omega_{im}^2)}$$
(9)

The loss factor is dependent of both the considered direction of propagation  $\theta$  and the frequency  $\omega$ . A numerical investigation on three different textile composites is presented bellow.

## 4. Numerical results

Four case studies are proposed. In the first subsection, a macroscale model of a laminate composite is studied, an eigenproblem formulation comparison is provided as well as a numerical validation. The next three subsections present three different textile composites modelled at a mesoscopic scale and a numerical validation is proposed for the last model.

#### 4.1. Case study 1: macroscale modelling of a composite laminate and numerical validation

## 4.1.1. Comparison of the eigenvalue problem formulations

The methodology presented in this paper is compared to the one presented in [46] for a simple case of a composite laminate made of two layers of the same lamina stacked in the following sequence [0/90]. The lamina is considered at a macroscopic scale, its orthotropic elastic properties are given in Table.C.1 in the Appendix. The model is a beam of 0.5 mm thickness and 1 mm width, it has 10 elements in the thickness and 20 in the width.

The adopted formulation for solving the eigenvalue problem shown in Eq.2 involves known and real wavenumber and the associated complex frequency vector is sought ( $\omega(k)$ ), this is the standard linear eigenvalue problem used in [3, 4] for textile composites. The damping characteristics are computed using Eq.9. Another formulation is a polynomial eigenvalue problem and it involves real prescribed frequency ( $k(\omega)$ ). The associated complex wavenumber vector is sought as presented in [31, 46]. A disadvantage of this second approach compared to the first is that the considered unit cell whose dispersion characteristics are sought cannot display a complex internal structure similarly to a textile composite. It can however deal with laminate composites and thus is used here for comparison.

This numerical example compares the methodology that can take into account the mesoscale architecture of a unit cell, presented in this publication ( $\omega(k)$  formulation) with the macroscale methodology presented in [46] ( $k(\omega)$  formulation). Using the second formulation methodology,  $\eta(\omega, \theta)$  is computed as follows:

$$\eta(\omega, \theta) = \frac{V_j^* \mathbf{K}''(\omega) V_j}{V_j^* \mathbf{K}'(\omega) V_j}$$
(10)

where \* denotes the conjugate transpose and  $V_i$  is the wave mode.

This particular comparison is not made to show the advantages of the mesoscale methodology over the macroscale one but rather to verify that both methodologies provide identical results for a same model.

In Fig.3 are shown the dispersion characteristics on the left and the loss factor on the right for both methodologies. The results are in excellent agreement.



Figure 3: Comparison of the loss factor computed for the four first modes using the  $k(\omega)$  and  $\omega(k)$  formulations. A very good agreement is observed.

## 4.1.2. Numerical validation using guided waves in a composite laminate beam through transient FE analysis

In order to validate the method used in this paper, the dispersion relations and the loss factor of a longitudinal wave are investigated in a beam structure made of a composite laminate material by transient finite element analysis (FEA).

*Theoretical model description.* A beam made of a two layers composite laminate is modelled. Its width is of 1 mm, while its thickness is of 0.5 mm and its length is of 200 mm with a perfect absorbing layer at the far end so that the results are not affected by the wave reflection.

Longitudinal waves are chosen for the study as they are straightforward to induce to a beam model. A force envelope is applied on one end side of the beam in the direction of the beam length. The magnitude of the load is variable over time so that the signal carries a narrow band frequency, and has a short time pulse. To avoid the coupling of various modes, displacement in the width and thickness directions are set as null independently of time. This is also allowing for shorter computation time.

The signal is composed of a signal carrying the frequency of interest, mixed with a Hanning window function. The carrier signal is periodic sinusoidal and composed of eleven cycles. The displacement in the length direction is measured over time at a large set of positions along the length of the beam. Using these displacement amplitude over time data, the damping can be observed and the loss factor calculated for each studied frequency.

The method for computing the damping  $\xi$  from the transient FEA is presented here. The amplitude at a selected point *p* on the beam can be expressed as

$$A_p = A e^{i(\omega t - kx_p)} \tag{11}$$

with  $x_p$  the position of the point p along the length of the beam. The ratio of amplitude of two points placed at different positions is thus written

$$\frac{A_2}{A_1} = e^{i(kx_1 - kx_2)}$$
(12)

it follows

$$\ln(\frac{A_2}{A_1}) = -ik\delta_x \tag{13}$$

Assuming  $k = k_{re} + ik_{im}$  and that the dissipation is only due to the imaginary part

$$k_{im} = \frac{\ln(\frac{A_2}{A_1})}{\delta_x} \tag{14}$$

 $\xi$  is calculated as proposed in [52]

$$\xi = \left| \frac{k_{im}}{k_{re}} \right| \tag{15}$$

Combining Eq.(14) and Eq.(15),  $\xi$  is given by

$$\xi = \left| \frac{\ln(\frac{A_2}{A_1})}{k_{re}\delta_x} \right| \tag{16}$$

As stated in [53], the attenuation of propagating waves in a thin structure is mainly caused by four factors. One factor is the geometric spreading of the wave, which does not occur in a beam-like structure. Another factor is the wave dispersion, which does not occur either in our case as the carrier signal has a very narrow frequency band and is lower than the cut-off frequency. Another is the dissipation of the energy into an adjacent media, which does not occur here as there are no adjacent media. Lastly, the fourth factor of attenuation is the material damping which is introduced in the model using the loss factor. This last factor should be the only source of damping in this model.

*Transient FEA*. For an accurate comparison, the model mesh size is the same for the two compared methods (transient FEA and WFE/CMS). In this FE model, the damping is modelled using Rayleigh's proportional damping as proposed by Adhikari in [54], but neglecting the mass damping in order to leave us with the structural damping only. Using this damping model ensures an adequation with the one used in the WFE/CMS model presented in Sec.3.

$$\xi = \frac{\beta\omega}{2} \tag{17}$$

Thus, for each material and considered frequency, the  $\beta$  coefficient is calculated as follows

$$\beta = \frac{2\xi}{\omega} = \frac{\eta}{\omega} \tag{18}$$

From the transient FE simulations performed with a narrow band input signal at different frequencies, the loss factor is computed using Eq.(16). The maximum amplitudes are calculated using the envelope of the displacement amplitude over time. The loss factor is calculated by comparing the displacement amplitude over time at different positions along the beam (at least a thousand points). The transient analysis is firstly performed for a small set of 5 frequencies ranging from 300kHz to 500kHz with a unique loss factor for both layers of the laminate ( $\eta = 0.003$ ). Using an identic loss factor for every constituents of the material should produce an equal and constant loss factor for the whole material, independently of the frequency. However, a difference is observed in the transient FEA as the resulting loss factor is equal to 0.003 at low frequency but ever slightly increasing as the frequency grows, which the model formulation should prevent. It is believed that this difference is induced by the FE software in use, and these results are used to adjust the determination of the  $\beta$  coefficient for later computations.

The simulation is performed once again, this time using different input loss factor for the different layers orientations (see Table.C.1) and the adjusted  $\beta$  coefficient. The mean and mean-squared error are plotted on Fig.4 along with the results from the presented methodology.

A good agreement between the loss factor curve (plotted against frequency) computed with the WFE/CMS methodology presented in this paper and the one computed from the transient FEA is observed.



Figure 4: Loss factor in a two layers laminate beam, function of the frequency, propagating in the x direction, associated to the pressure mode. '+' the dispersion curve, 'x' the loss factor computed with the methodology introduced in this paper, 'o' the loss factor computed from the transient finite element analysis

## 4.2. Case study 2: mesoscale modelling of a 2D weave textile composite

As a first textile composite example, a unit cell of a 2D weave fabric is modeled (Fig.5) and the loss factor is calculated in function of the propagation angle and the frequency. The dimensions of the unit cell are 2x2x0.2 mm. This FE model is composed of 6250 elements (25x25x10), 3336 elements comes from the yarns while the 2914 remaining elements are matrix elements. This gives a fiber fraction volume of 0.5338.  $\eta_{yarn} = 0.0001$  and  $\eta_{mat} = 0.02$  are used as the pre-determined loss factor for respectively the yarns and matrix constituents.



Figure 5: Unit cell of a 2D weave fabric

The WFE method allows for obtaining the dispersion relation for the textile, as shown in Fig.6. From this relation can

be obtained the phase and group velocities. Also it allows for detecting the appearance of stop-bands such as for the flexural and pressure modes which both present a stop-band respectively around 190 and 270 kHz.



Figure 6: Dispersion curves for a 6250 elements model of a 2D weave fabric, propagating in the x direction

Thanks to the method presented in Sec.3, the loss factor associated to the first flexural mode can be calculated, in function of the wave direction of propagation and the frequency, as shown in Fig.7. A mirror symmetry to the  $45^{\circ}$  direction of propagation plane can be observed for the loss factor values. This is in agreement with the geometry of the unit cell, itself presenting a symmetry to the  $45^{\circ}$  axis. For the sake of clarity, the following results are displayed in two dimensions.



Figure 7: Loss factor in a 6250 elements model of a 2D weave fabric, function of the wave direction of propagation and the frequency, associated to the flexural mode of the anisotropic plate

In Fig.8, the loss factor, associated to the flexural mode, as function of the frequency, propagating in the *x* direction is displayed for a larger frequency range. The dispersion curve for this mode is shown as well on the figure allowing

for comparison. A stop-band is present in the dispersion curve, and the loss factor seem to have an asymptotic behavior next to it.



Figure 8: Loss factor in a 6250 elements model of a 2D weave fabric, function of the frequency, propagating in the x direction, associated to the flexural mode of the anisotropic plate

Three parametric studies are subsequently presented. The first to compare the effect of the fiber volume fraction. The second to show the impact of different pre-determined component loss factors values and lastly the effect of the wave direction of propagation on the loss factor.

## Parametric study 1: fiber volume fraction.

Two new models have been created based on the one described above. The difference between these models is the width of the yarns (and thus the number of fibers), which has an impact on the fiber volume fraction (see Fig.9), and therefore on the structural damping performance. The external dimensions of the unit cells are the same for these three models.



Figure 9: Parametric study: change of the fibers width (the model in the middle of the figure is the reference model used in the previous part of the subsection)

The results of this study are displayed in Fig.10-12. As expected, the lower the fiber volume fraction is, the more

effective the model is in dissipating energy, and this is true for the first flexural, shear and pressure modes. All three models present similar loss factor curve shapes for the three first modes.

The loss factor curves of the three models present an asymptotic behavior when the frequency reaches the stop-band for the first flexural mode (see Fig.10). The loss factor has a lower value on the higher frequency part of the figure, after the stop-band.

In Fig.11, the dissipative characteristics slightly decrease until reaching a frequency of around 300 kHz. The loss factor then increases until overcoming its initial value.

The loss factor associated to the first pressure wave slightly grows (see Fig.12), then the growing rate suddenly increases and the loss factor seems to tend to a high value around the stop-band. On the right side of the stop-band, the same phenomenon can be observed. It seems to tend to a high value around the stop-band and then settles after the frequency of 350 kHz. The loss factor at 500 kHz is more than the double of the loss factor at 50 kHz. This parametric study clearly shows that the fiber volume fraction has an impact on the structural damping, and it also shows the strong influence of the frequency over this same parameter.



Figure 10: Loss factor displayed for three 6250 elements models of a 2D weave fabric as a function of the frequency, associated to the flexural mode of the anisotropic plate (angle of propagation null: x direction): structure with larger fibers (–), reference structure (\_), thinner fibers (-.)

#### Parametric study 2: pre-determined components loss factor.

Another parametric study has been conducted on the original model of the 2D weave fabric, by altering the initial loss factor of the matrix ( $\eta_{mat}$ ) and yarn ( $\eta_{yarn}$ ) components. The loss factor function of the frequency is displayed in Fig.13. It appears that a relation of proportionality exists between the loss factor of the assembly and the loss factor of the constituent components and that this relation is independent from the initial loss factors chosen for each components.



Figure 11: Loss factor displayed for three 6250 elements models of a 2D weave fabric as a function of the frequency, associated to the shear mode of the anisotropic plate (angle of propagation null: x direction): structure with larger fibers (–), reference structure (\_), thinner fibers (-.)



Figure 12: Loss factor displayed for three 6250 elements models of a 2D weave fabric as a function of the frequency, associated to the pressure mode of the anisotropic plate (angle of propagation null: x direction): structure with larger fibers (–), reference structure (\_), thinner fibers (-.)

In Fig.14 is displayed the same parametric study with the loss factors adjusted to a scale going from the fiber loss factor as the low limit to the matrix loss factor as the high limit. It can be observed that all curves are superposed, and it is the case also for the loss factors of the other two first modes. This shows that the structural damping of the structure is mostly independent from the initial choice of the constituent components loss factors.



Figure 13: Loss factor displayed for a 6250 elements model of a 2D weave fabric (with changing components loss factor), function of frequency, associated to the flexural mode of the anisotropic plate (propagation along the x direction)



Figure 14: Scaled loss factor displayed for a 6250 elements model of a 2D weave fabric (with changing components loss factor), function of frequency, associated to the flexural mode of the anisotropic plate (propagation along the x direction)

# Parametric study 3: direction of propagation.

Finally, the effect of the direction of propagation is studied. As the model presents a symmetry axis, the dispersion curves and loss factors are symmetric as well. The directions of propagation with an angle superior to  $45^{\circ}$  will not be studied. However, directions of propagation such as  $[0^{\circ}, 30^{\circ}, 45^{\circ}]$  can be studied (see Fig.15). The results of this study are displayed in Fig.16.

It can be observed that the higher the angle of propagation is (among the studied ones), the more the first flexural wave is attenuated, and yet, both values of the loss factor for the  $30^{\circ}$  and  $45^{\circ}$  angles of propagation start decreasing for a frequency around 100 kHz until reaching and following the curve representing an angle of propagation of  $0^{\circ}$ . In the right part of the figure, the same order is followed. The angle of propagation with the most damping is of  $45^{\circ}$ , followed by  $30^{\circ}$  and then  $0^{\circ}$ . This was indeed expected as it is easier for the energy to propagate in the direction of the fibers as they act as waveguides. It can be concluded from this paragraph that the damping is clearly affected by the direction of propagation in a complex textile composite.



Figure 15: Angles of propagation in the unit cell of a 2D weave fabric



Figure 16: Loss factor displayed for a 6250 elements model of a 2D weave fabric (with changing direction of propagation), function of frequency, associated to the flexural mode of the anisotropic plate

## 4.3. Case study 3: mesoscale modelling of a 3D weave textile composite

The second example of a textile composite is a unit cell of a 3D weave fabric which was modeled by TexGen (see Fig.17). The dimensions of the unit cell are 2x1.5x0.6 mm. The loss factor was calculated in function of the propagation angle and the frequency. This FE model is composed of 15625 elements (25x25x25), 7834 elements represent the yarns while the 7791 remaining elements are matrix elements. This gives a fiber fraction volume of 0.5014.



Figure 17: Unit cell of a 3D weave fabric

In Fig.18, the loss factor, associated to the flexural mode, as function of the frequency, propagating in the *y* direction is displayed. The dispersion curve for this mode is shown as well on the figure allowing for comparison. The results obtained here are comparable to the ones obtained with the 2D weave model. The dispersion curve presents a stopband and the loss factor has an asymptotic behavior around it once again.



Figure 18: Loss factor in a 15625 elements model of a 3D weave fabric, function of the frequency, propagating in the *y* direction, associated to the flexural mode

Fig.19 shows the loss factor, associated to the shear mode, as function of the frequency, propagating in the y direction.

The dispersion curve for this mode is shown as well on the figure allowing for comparison. The dispersion curve presents a stop-band and the loss factor has an asymptotic behavior around it.



Figure 19: Loss factor in a 15625 elements model of a 3D weave fabric, function of the frequency, propagating in the *y* direction, associated to the shear mode

At 380 kHz, a light peak is observed. This is caused by the interaction of this mode with the shear mode, causing both modes to veer away.

A parametric study, showing the bending and the pressure modes, veering away for low pre-determined components loss factors and crossing for higher ones is presented in Fig.20 for the same model, only this time for wave propagating in the *x* direction. The modes dispersion curves and their assigned loss factor curves are displayed for four couples of pre-determined components loss factors. It can be observed that when the modes veer away, the damping ratios are swapped, while this does not happen when they cross. When they cross, a peak in the loss factor curve can be observed but the higher the pre-determined components loss factors are, the more faded this local peak will be. This phenomenon is known and other examples where veering is influenced and sometimes suppressed by the effect of damping are presented in [6] and [55].

Fig.21 shows the loss factor, associated to the pressure mode, as function of the frequency, propagating in the *y* direction. It is presented for both this 3D weave model and for the braid fabric model from the previous subsection as well. The dispersion curves for this modes are shown as well on the figure allowing for comparison. It can be observed that the loss factor for the braid fabric is higher than for the 3D weave, even though the fiber volume fraction is higher for the braid fabric model. It is surprising as in this model, it is the matrix material that has the highest component loss factor. It shows that the mesoscale architecture of the textile composite has a strong effect on the damping.



Figure 20: Loss factor in a 15625 elements model of a 3D weave fabric, function of the frequency, propagating in the x direction, associated to the flexural and pressure modes for four different configurations of pre-determined components loss factors. '+' dispersion curves, 'x' loss factors.



Figure 21: Comparison of the loss factor in a 15625 elements model of a 3D weave fabric and in a 12000 elements model of a triaxial braid fabric, function of the frequency, propagating in the *y* direction, associated to the pressure mode

## 4.4. Case study 4: mesoscale modelling of a triaxial braid textile composite and numerical validation

As a third textile composite case study, a unit cell of a triaxial braid fabric is modeled (Fig.22) and the loss factor of the flexural wave is calculated in function of the propagation angle and the frequency. The dimensions of the unit

cell are 2x0.6x0.4 mm. This FE model is composed of 12000 elements (40x20x15), 6665 elements comes from the yarns while the 5335 remaining elements are matrix elements. This gives us a fiber fraction volume of 0.5554.



Figure 22: Unit cell of a triaxial braid fabric

In Fig.23, the loss factor, associated to both the flexural and the shear modes, as function of the frequency, propagating in the *x* direction is displayed. The dispersion curves for this modes are shown as well on the figure allowing for comparison. Both have been displayed on the same figure as there is a strong coupling between these two modes (around 150 kHz and again around 750 kHz). The coupling is creating local stop-bands as the dispersion curves veer away (when displayed in the Brillouin zone) instead of crossing and take each other trajectory. This is the result of the two eigenvalue loci approaching closely and it causes the properties of the two modes to be swapped, including eigenvectors or the damping ratios [56, 55] as can be seen on this same figure. Other stop-bands can be observed around 300 kHz and 850 kHz for the flexural mode and around 600 kHz and 1.1 MHz for the shear mode. These are Bragg stop-bands, intrinsic to the periodic properties of the structure. Around these stop-bands, the loss factor seems to have a asymptotic behavior once again. Finally, it can be noted that the loss factor curves are largely located bellow the loss factor weighted mean, which would indicates that the loss factor of the yarns has a stronger influence on the final loss factor in that direction of propagation. The same phenomenon does arise with the pressure mode as well which is displayed in Fig.24.

In Fig.24 are displayed the loss factors, associated to the pressure modes of the braid fabric, propagating in both the x and the y directions. It can be observed that the loss factor is lower for the pressure wave propagating in the y direction until it reaches 800 kHz and takes over on the loss factor of the pressure wave propagating in the x direction.

Finally, a numerical validation using guided the transient FE method presented in 4.1.2 is performed for 13 frequency points ranging from 200 to 800 kHz. A beam made of a triaxial braid fabric is modelled, its width is of 0.6 mm, while its thickness is of 0.4 mm.

The WFE/CMS method shown in this paper allows for computing the dispersion relations and the loss factor over frequency and it is used for the described beam model as shown in Fig.25.

The full FE computation is performed once with a broadband signal as suggested in Sec.4.1.2 and the displacement is measured for a set of positions along the length of the beam (B-scan). Post-treating these results with the 2D FFT



Figure 23: Loss factor in a 12000 elements model of a triaxial braid fabric, function of the frequency, propagating in the x direction, associated both to the flexural and shear modes



Figure 24: Comparison of the loss factor in a 12000 elements model of a triaxial braid fabric, function of the frequency, associated to the pressure mode, propagating in the x direction and in the y direction

allows for obtaining the dispersion relations as depicted in Fig.25. The comparison is made with the dispersion curves obtained with the WFE/CMS on the same figure. A very good agreement is observed, the loss factors can now be compared. It can also be noted that symmetries and translations appear in the dispersion curves computed from the B-scan data. These are due to the periodicity of the structure, the length of the translation vectors corresponds to  $k = \frac{2\pi}{\Lambda}$  with  $\Delta$  the length of a unit cell.

From the transient FE simulations performed with a narrow band input signal at different frequencies, the loss factor is computed using Eq.(16). The maximum amplitudes are calculated using the envelope of the displacement amplitudes over time. The loss factor is calculated by comparing the displacement amplitude over time at more



Figure 25: Dispersion curves for the triaxial braid beam with blocked displacement in the second and third directions. The background pixelized image results from the two-dimensional Fast Fourier Transform of the B-scan, while the dots result from the WFE/CMS methodology presented in [3]. A perfect agreement is observed.

than two thousand points along the beam. The mean is plotted on Fig.26 along with the results from the presented methodology. The mean-squared error is, however, not plotted because very low and thus barely visible.



Figure 26: Loss factor in a triaxial fabric beam, function of the frequency, propagating in the x direction, associated to the pressure mode. '+' the dispersion curve, 'x' the loss factor computed with the methodology introduced in this paper, 'o' the loss factor computed from the transient finite element analysis

A very agreement between the loss factor curve computed with the methodology presented in this paper and the one computed from the transient finite element analysis is observed once again. The calculation of these dispersion and damping properties using the WFE/CMS methodology presented in this paper took 30 min on a 1 core and 8GB RAM system for a wide frequency range, while it took around 5 hours on a 8 cores and 160 Go of RAM on a HPC system to get the loss factor for only one frequency point using transient FEA.

## 5. Conclusions

In this paper a method allowing for prediction of the structural damping in textile composite at a mesoscopic scale is presented. Three examples are presented: a 2D fabric weave, a triaxial braid fabric and a 3D fabric weave. For all three models, the dispersion relations are computed as well as the variation of the loss factor versus the frequency for the first modes.

- It can be seen that the damping is strongly affected by the direction of propagation of the waves and dependent of the frequency in a textile composite.
- It appears that when the textiles show stop-bands, the damping loss factor has a characteristic behavior depending on the stop-bands property.
- It can be seen that the damping is affected by the fiber volume fraction of the textile. But even though the numerical values of the damping loss factor change, the shape of the curves remain quite similar if the mesoscale architecture is similar as well.
- The shape of the loss factor curves is mostly independent from the pre-determined loss factors of the components.
- The mesoscale architecture of the textile composite has a strong influence over the damping.
- It should be possible to improve the model by inserting an orientation dependent damping ratio to each yarn component. It would also be of interest to model the friction at yarn/matrix interface to have a better estimation of the loss factor.

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#### Appendix A. Craig-Bampton Method

The boundary nodal DOFs are written as  $\mathbf{q}_{\mathbf{b}} = \{\mathbf{q}_{\mathbf{B}}^{\mathrm{T}} \ \mathbf{q}_{\mathbf{T}}^{\mathrm{T}} \ \mathbf{q}_{\mathbf{L}}^{\mathrm{T}} \ \mathbf{q}_{\mathbf{R}\mathbf{B}}^{\mathrm{T}} \ \mathbf{q}_{\mathbf{R}\mathbf{T}}^{\mathrm{T}} \ \mathbf{$ 

$$\begin{pmatrix} \begin{bmatrix} \mathbf{K}_{\mathbf{b}\mathbf{b}} & \mathbf{K}_{\mathbf{b}\mathbf{I}} \\ \mathbf{K}_{\mathbf{I}\mathbf{b}} & \mathbf{K}_{\mathbf{I}\mathbf{I}} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_{\mathbf{b}\mathbf{b}} & \mathbf{M}_{\mathbf{b}\mathbf{I}} \\ \mathbf{M}_{\mathbf{I}\mathbf{b}} & \mathbf{M}_{\mathbf{I}\mathbf{I}} \end{bmatrix} \begin{pmatrix} \mathbf{q}_{\mathbf{b}} \\ \mathbf{q}_{\mathbf{I}} \end{pmatrix} = \begin{cases} \mathbf{f}_{\mathbf{b}} \\ \mathbf{0} \end{cases}$$
(A.1)

In the fixed interface CMS, a set of 'fixed boundary modes', also called component modes  $\Phi_{\rm C}$  are selected among a subset of the local modes of the unit cell when the boundary DOFs are fixed and no force is acting on the internal nodes. Those local modes are the eigenvector  $\Phi_{\rm I}$  of Eq.(A.2). The sus-mentioned subset is selected based on the lower resonance frequencies of the clamped model. That is why the modes are selected into the frequency range  $[0, 3f_{max}]$ , with  $f_{max}$  being the maximum frequency of interest for the wave dispersion analysis [34, 33].

$$\begin{bmatrix} \mathbf{K}_{\mathbf{I}\mathbf{I}} - \omega^2 \mathbf{M}_{\mathbf{I}\mathbf{I}} \end{bmatrix} \mathbf{\Phi}_{\mathbf{I}} = \mathbf{0}$$
(A.2)

 $\Phi_b$  is the static condensation and is calculated as follow

$$\Phi_{\mathbf{b}} = \mathbf{K}_{\mathbf{II}}^{-1} \mathbf{K}_{\mathbf{Ib}} \tag{A.3}$$

The projection matrix **B** can be determined as follow

$$\left\{ \begin{array}{c} \mathbf{q}_{\mathbf{b}} \\ \mathbf{q}_{\mathbf{I}} \end{array} \right\} = \left[ \begin{array}{c} \mathbf{I} & \mathbf{0} \\ \mathbf{\Phi}_{\mathbf{b}} & \mathbf{\Phi}_{\mathbf{C}} \end{array} \right] \left\{ \begin{array}{c} \mathbf{q}_{\mathbf{b}} \\ \mathbf{p}_{\mathbf{\Phi}} \end{array} \right\} = \mathbf{B} \left\{ \begin{array}{c} \mathbf{q}_{\mathbf{b}} \\ \mathbf{p}_{\mathbf{\Phi}} \end{array} \right\}$$
(A.4)

The mass and stiffness matrix M and K can then be projected on the B basis

$$\tilde{\mathbf{K}} = \mathbf{B}^{\mathrm{T}} \mathbf{K} \mathbf{B}, \qquad \tilde{\mathbf{M}} = \mathbf{B}^{\mathrm{T}} \mathbf{M} \mathbf{B}$$
(A.5)

# Appendix B. Wave and finite element method (WFEM)

By using the Periodic Structure Theory (PST), the WFE method, allowing to find wave numbers of all the propagating waves by modelling only a period of the structure with standard FE, has been developed. This method has been extended by Manconi and Mace in [31] for free wave propagation in homogeneous structures in both x and ydirections but whose properties may vary in the third direction through the thickness. The author applied this method in particular to isotropic, orthotropic and composite laminated plates and cylinders [31, 57].

In our case, the wave motion is in the Oxy plan, which gives

$$\mathbf{q}_{\mathbf{R}} = \lambda_{x} \mathbf{q}_{\mathbf{L}}; \qquad \mathbf{q}_{\mathbf{T}} = \lambda_{y} \mathbf{q}_{\mathbf{B}}$$
(B.1)

 $\mathbf{q_{RB}} = \lambda_x \mathbf{q_{LB}}; \quad \mathbf{q_{LT}} = \lambda_y \mathbf{q_{LB}}; \quad \mathbf{q_{RT}} = \lambda_x \lambda_y \mathbf{q_{LB}}$ 

And thus

$$\begin{array}{c} \mathbf{q}_{B} \\ \mathbf{q}_{T} \\ \mathbf{q}_{L} \\ \mathbf{q}_{R} \\ \mathbf{q}_{RB} \\ \mathbf{q}_{LB} \\ \mathbf{q}_{RT} \\ \mathbf{q}_{I} \end{array} \right\} = \left[ \begin{array}{ccccc} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I}\lambda_{y} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{x}\lambda_{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{x}\lambda_{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{x}\lambda_{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{z}\lambda_{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\lambda_{z}\lambda_{z} & \mathbf{0} \\ \end{array} \right] \right]$$

$$(B.2)$$

Equilibrium at the right top corner nodes gives (subscript bd stands for boundary):

$$\Lambda_{\mathbf{L}}(\lambda_x, \lambda_y) \left\{ \begin{array}{c} \mathbf{f}_{\mathbf{bd}} \\ \mathbf{0} \end{array} \right\} = \mathbf{0}$$
(B.3)

with  $\Lambda_{\mathbf{L}}(\lambda_x, \lambda_y)$  the conjugate transpose of  $\Lambda_{\mathbf{R}}(\lambda_x, \lambda_y)$ 

The following eigenvalue problem appears

$$\mathbf{\Lambda}_{\mathbf{L}}(\mathbf{K} - \omega^{2}\mathbf{M})\mathbf{\Lambda}_{\mathbf{R}} \begin{cases} \mathbf{q}_{\mathbf{B}} \\ \mathbf{q}_{\mathbf{L}} \\ \mathbf{q}_{\mathbf{LB}} \\ \mathbf{q}_{\mathbf{I}} \end{cases} = 0$$
(B.4)

Combining the CMS reduction method Eq.(A.4) with the periodicity relation Eq.(B.2), we obtain

$$\begin{cases} \mathbf{q}_{B} \\ \mathbf{q}_{T} \\ \mathbf{q}_{L} \\ \mathbf{q}_{L} \\ \mathbf{q}_{R} \\ \mathbf{q}_{LB} \\ \mathbf{q}_{RB} \\ \mathbf{q}_{LT} \\ \mathbf{q}_{RT} \\ \mathbf{q}_{L} \\ \mathbf{q}_{R} \\ \mathbf{q}_{RB} \\ \mathbf{q}_{LT} \\ \mathbf{q}_{RT} \\ \mathbf{q}_{L} \\ \mathbf{q}_{RT} \\ \mathbf{q}_{R} \\ \mathbf{q}_{RT} \\$$

The eigenvalue problem can be solved with a given  $(k_x, k_y)$  formulation [58].

## Appendix C. Material properties

C <sub>11</sub> (GPa)	C <sub>12</sub> (GPa)	<b>C</b> <sub>22</sub> (GPa)	C <sub>13</sub> (GPa)	C <sub>23</sub> (GPa)	C <sub>33</sub> (in GPa)
80.7	12.9	18.7	3.6	5.1	1.5
C <sub>44</sub> (GPa)	C <sub>55</sub> (GPa)	C <sub>66</sub> (GPa)	density $(kg/m^3)$	$\eta_x$ (%)	$\eta_y$ (%)
3.0	4.9	5.4	3212	0.118	0.62

Table C.1: Nine independent elastic stiffness parameters defining orthotropic elasticity of the material in Sec.4.1, its density and damping properties

E <sub>1</sub> (GPa)	<b>E</b> <sub>2</sub> (GPa)	E <sub>3</sub> (GPa)	<i>v</i> <sub>12</sub>	<i>v</i> <sub>13</sub>
200	10	10	0.3	0.4
v <sub>23</sub>	G <sub>12</sub> (GPa)	G <sub>12</sub> (GPa)	G <sub>23</sub> (GPa)	density $(kg/m^3)$
0.4	5	5	5	4600

Table C.2: Elastic properties of the yarn material used in Sec.4.2-4.4

E (GPa)	ν	<b>density</b> $(kg/m^3)$
3	0.2	1600

Table C.3: Elastic properties of the matrix material used in Sec.4.2-4.4