Q-Ball Superradiance

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(Received 21 December 2022; revised 24 February 2023; accepted 29 June 2023; published 11 September 2023)

Q-balls are nontopological solitons that coherently rotate in field space. We show that these coherent rotations can induce superradiance for scattering waves, thanks to the fact that the scattering involves two coupled modes. Despite the conservation of the particle number in the scattering, the mismatch between the frequencies of the two modes allows for the enhancement of the energy and angular momentum of incident waves. When the Q-ball spins in real space, additional rotational superradiance is also possible, which can further boost the enhancements. We identify the criteria for the energy and angular momentum superradiance to occur.

DOI: 10.1103/PhysRevLett.131.111601

Q-balls arise as solitonic solutions [1–3] in a variety of field theories that admit attractive nonlinear interactions [4]. The attractive nature of the interactions allows charges to condense into a localized object, which however is not a static field configuration, as the phase of the field evolves in time, so as to evade Derrick's theorem [5]. Q-balls may spin and become hollow in the center after acquiring some angular momentum [6–10], which may happen when forming from a system with nonzero angular momentum or in a Q-ball collision [11]. Q-balls may naturally arise in the early Universe [11–21] and are a candidate for dark matter [22–30]. In the presence of strong gravity effects, the Q-ball counterparts are known as boson stars [31–33], which are another candidate for dark matter [32,34]. Q-balls can also be made in laboratories [35,36] and can have composite charge structures [37–39].

Superradiance, coined by Dicke [40] originally for emission enhancement in a coherent medium, is a collection of phenomena where radiation is amplified during a physical process; see Refs. [41,42] for a review. The well-known Cherenkov radiation is an example of inertial motion superradiance [43]. Later, Zel'dovich pointed out that rotating objects, such as a radiation-absorbing cylinder or a Kerr black hole, can also superradiate [44,45] (also Ref. [46] independently for black holes). Black hole superradiance has since been extensively studied [42], thanks to its relevance to gravitational theories, astrophysics, and particle physics (see, e.g., Refs. [47–71]). Also, several novel superradiance effects have recently been observed in laboratories [72–76].

In this Letter, we shall point out that Q-balls can also superradiate, a property unknown so far, despite the long history of Q-balls. Q-ball superradiance originates from the fact that a Q-ball field is complex, having two components, and that the phase of the Q-ball solution evolves in time. Indeed, a Q-ball can be viewed as a localized Bose-Einstein condensate of particles that oscillate coherently and, as we shall see, enhances scalar waves incident on it, somewhat parallel to Dicke's original scenario. If the Q-ball acquires some angular momentum and spins, additional rotational superradiance can provide further enhancement. Interestingly, Q-ball superradiance occurs despite the fact that the particle number is conserved in the scattering. The mass gap required for a Q-ball to exist splits the superradiance spectrum into two separate parts.

We will take the U(1) symmetric theory

$$\mathcal{L} = -\eta^{\mu\nu}\frac{\partial\bar{\Phi}^*}{\partial\bar{x}^\mu}\frac{\partial\bar{\Phi}}{\partial\bar{x}^\nu} - V(|\bar{\Phi}(\bar{x})|) \tag{1}$$

with potential $V(|\bar{\Phi}|) = \mu^2|\bar{\Phi}|^2 - \lambda|\bar{\Phi}|^4 + \bar{g}|\bar{\Phi}|^6$ as the fiducial example. Requiring $\bar{\Phi} = 0$ to be the true vacuum imposes the following conditions: $\lambda > 0$ and $\mu^2\bar{g} \geq \lambda^2/4$ [3]. Upon using the dimensionless variables $x = \mu\bar{x}$, $\Phi = \lambda^{1/2}\bar{\Phi}/\mu$, and $g = \mu^2\bar{g}/\lambda^2$, the model can be rewritten as

$$\mathcal{L} = -\partial_\mu\Phi^*\partial^\mu\Phi - V(|\Phi|), \qquad V = |\Phi|^2 - |\Phi|^4 + g|\Phi|^6. \tag{2}$$

This may be viewed as a low energy truncation of an effective field theory expansion for a generic U(1) scalar field (and is renormalizable in $2+1$D). In the following, we will for simplicity mainly focus on the $2+1$D case, except toward the end where the $3+1$D case is briefly discussed.

In polar coordinates, a general Q-ball configuration takes the form

$$\Phi_Q(t,r,\varphi)=\frac{1}{\sqrt{2}}f(r)e^{-i\omega_Q t+im_Q\varphi} \tag{3}$$

where ω_Q is the oscillation frequency of the Q-ball in field space, φ is the azimuth angle, and the (real space) angular phase velocity of this configuration is $\Omega_Q=\omega_Q/m_Q$ if m_Q is nonzero. The (2D) nonrotating Q-ball is the special case where $m_Q=0$, in which case $f(r)$ peaks at $r=0$ and falls off quickly to zero at spatial infinity. For $m_Q\neq 0$, $f(r)$ will peak at some finite r and asymptotes to zero both when $r\to 0$ and $r\to\infty$. The regularity condition at the origin requires that $f(r\to 0)\propto r^{|m_Q|}$. Without loss of generality, we assume that $\omega_Q>0$ (and also $m_Q>0$ for a spinning Q-ball). For a Q-ball to exist, ω_Q must be in the range of $[1-(4g)^{-1}]^{1/2}\lesssim\omega_Q<1$ (see Supplemental Material [77]). Q-balls in isolation are classically stable against small perturbations [3,8]. However, in this Letter, we will show that in "dirty" environments where waves are scattered around, energy can actually be extracted from a Q-ball via superradiant scattering.

To this end, let us look at the perturbative equations of motion around the Q-ball solution $\Phi=\Phi_Q+\phi(t,r,\varphi)$:

$$\Box\phi-U(r)\phi-e^{-2i(\omega_Q t-m_Q\varphi)}W(r)\phi^*=0, \tag{4}$$

where $U=\frac{1}{2}\{[(d^2V)/(df^2)]+[(1/f)(dV/df)]\}$ and $W=\frac{1}{2}\{[(d^2V)/(df^2)]-[(1/f)/(dV/df)]\}$. We see that the perturbative field ϕ interacts with the coherent background of the Q-ball that oscillates temporally and angularly. Indeed, as we shall see later, the Q-ball condensate can enhance the energy, angular momentum, and charge of waves incident on it, giving rise to *superradiance*. However, U and W depend on r, preventing a straightforward spatial Fourier decomposition.

To proceed, we restrict to the minimal case where the field has only two frequencies

$$\phi=\eta_+(\omega,m,r)e^{-i\omega_+t+im_+\varphi}+\eta_-(\omega,m,r)e^{-i\omega_-t+im_-\varphi}, \tag{5}$$

where $\omega_\pm=\omega_Q\pm\omega$ and $m_\pm=m_Q\pm m$. The general solution is a linear superposition of this case. The reason why two modes are needed is due to the coupling between ϕ and ϕ^*, unlike the case of superradiance with a real scalar where a single frequency suffices. The mode functions satisfy the following equations:

$$\eta_\pm''+\frac{1}{r}\eta_\pm'+\left(\omega_\pm^2-U-\frac{m_\pm^2}{r^2}\right)\eta_\pm-W\eta_\mp^*=0, \tag{6}$$

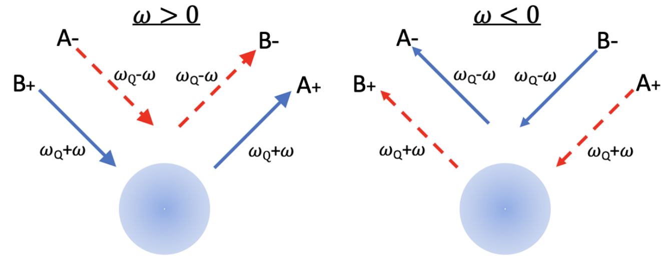

FIG. 1. Ingoing and outgoing waves scattering on and off a Q-ball [cf. Eqs. (4), (5), and (7)]. Solid (dashed) lines represent positive (negative) charge.

where the prime is a derivative with respect to r. Since $f(r\to\infty)=0$, we have $U(r\to\infty)=1$, $W(r\to\infty)=0$. At large r, the above equations reduce to $\eta_\pm''+\frac{1}{r}\eta_\pm'+(\omega_\pm^2-1)\eta_\pm\to 0$, which are solved by

$$\eta_\pm(\omega,m,r\to\infty)\to\frac{A_\pm}{\sqrt{k_\pm r}}e^{ik_\pm r}+\frac{B_\pm}{\sqrt{k_\pm r}}e^{-ik_\pm r}, \tag{7}$$

where wave numbers $k_\pm=(\omega_\pm^2-1)^{1/2}$. Here we are interested in waves scattering on and off the Q-ball. Assuming both of the two modes are propagating waves imposes the reality conditions on $k_\pm$: $|\omega_Q\pm\omega|>1$. The 1 on the rhs of this inequality is due to the mass gap in the scalar theory (essentially in our units the scalar mass $\mu=1$). Since we have assumed that $\omega_Q>0$, this implies that the physical boundary of ω is

$$|\omega|>\omega_Q+1. \tag{8}$$

As shown in Fig. 1, if $\omega>0$, the A_- and B_+ terms represent ingoing waves and the A_+ and B_- terms outgoing waves, where the subscript $+$ ($-$) indicates positive (negative) charge; if $\omega<0$, the wave directions flip, and $-$ ($+$) indicates positive (negative) charge.

With these set up, we solve Eq. (6) by treating it as an initial value problem in r. That is, we can prepare the values of η_+ and η_- near $r=0$ and evolve Eq. (6) to a large r to obtain $A_\pm$ and $B_\pm$. Regularity near the origin requires that $\eta_\pm(\omega,m,r\to 0)\to F_\pm(k_\pm r)^{|m_\pm|}$, $F_\pm$ being complex constants, which provides the "initial" conditions to solve the system. Additionally, since the system is linear, if (η_+,η_-) is a solution, so is $(\zeta\eta_+,\zeta^*\eta_-)$, where ζ is a complex constant. This means that we can scale one of the constants (say F_+) to unity, and choose the following "initial condition" as $r\to 0$:

$$\eta_+(m,r)=(k_+r)^{|m_+|},\qquad \eta_-(m,r)=F_-(k_-r)^{|m_-|}. \tag{9}$$

For every F_-, the initial value problem gives a set of A_+, A_-, B_+, B_-. Thus, generically, we have two waves coming in and two going out. If we want to have one ingoing wave, we can use a shooting method to tune F_- so as to make

A_- vanish if $\omega > 0$ or to make B_- vanish if $\omega < 0$. (Making the ingoing η_+ mode vanish at large r does not produce new results due to the symmetry of the equations.) The numerical schemes used to solve the equations are detailed in the Supplemental Material [77].

To monitor how the Q-ball alters incident waves, let us define a few quantities. Firstly, we can look at the average energy in an annular region (from r_1 to r_2) far away from the origin ($r \to \infty$):

$$E_{\odot} = \frac{1}{r_2 - r_1}\int_{r_1}^{r_2} r dr \langle |\partial_t \phi|^2 + |\partial_r \phi|^2 + |\phi|^2 \rangle,$$
$$= 2\frac{\omega_+^2}{k_+}(|A_+|^2 + |B_+|^2) + 2\frac{\omega_-^2}{k_-}(|A_-|^2 + |B_-|^2), \quad (10)$$

where $r_2 - r_1$ includes at least a full spatial oscillation of the longest wave, $\langle \rangle$ is the time average over a few oscillations, and we have only kept the leading order terms. We used the perturbative field to calculate the energy because the Q-ball profile falls off exponentially at large r. An amplification factor may be defined as the ratio of energy in the outgoing field, compared to the ingoing,

$$\mathcal{A}_E = \left(\frac{\frac{\omega_+^2}{k_+}|A_+|^2 + \frac{\omega_-^2}{k_-}|B_-|^2}{\frac{\omega_+^2}{k_+}|B_+|^2 + \frac{\omega_-^2}{k_-}|A_-|^2}\right)^{\mathrm{sign}(\omega)}. \quad (11)$$

Secondly, we can also monitor how the angular momentum of the wave changes in the scattering. The angular momentum density is $T^t{}_\varphi = -\partial_t \Phi^* \partial_\varphi \Phi - \partial_\varphi \Phi^* \partial_t \Phi$. After taking the average over an annular region at large r and the time average, to leading order we have

$$L_{\odot} = \frac{-1}{r_2 - r_1}\int_{r_1}^{r_2} r dr \langle \partial_t \phi^* \partial_\varphi \phi + \partial_\varphi \phi^* \partial_t \phi \rangle,$$
$$= 2(|A_+|^2 + |B_+|^2)\frac{\omega_+ m_+}{k_+} + 2(|A_-|^2 + |B_-|^2)\frac{\omega_- m_-}{k_-}. \quad (12)$$

The amplification factor for the angular momentum is then

$$\mathcal{A}_L = \left(\frac{\frac{\omega_+ m_+}{k_+}|A_+|^2 + \frac{\omega_- m_-}{k_-}|B_-|^2}{\frac{\omega_+ m_+}{k_+}|B_+|^2 + \frac{\omega_- m_-}{k_-}|A_-|^2}\right)^{\mathrm{sign}(\omega)}. \quad (13)$$

One may also want to see how the particle flux changes during the scattering. The particle flux averaged over time and an annular region at large r is given by $N_{\odot} = 2(|A_+|^2 + |B_+|^2 + |A_-|^2 + |B_-|^2)$, and the amplification factor is $\mathcal{A}_N = ((|A_+|^2 + |B_-|^2)/(|B_+|^2 + |A_-|^2))^{\mathrm{sign}(\omega)}$. However, due to a U(1) symmetry for the η_+ and η_- mode in Eq. (6), we always have $\mathcal{A}_N \equiv 1$. To see this, note that the equations of motion [Eq. (6)] can be obtained from the Lagrangian

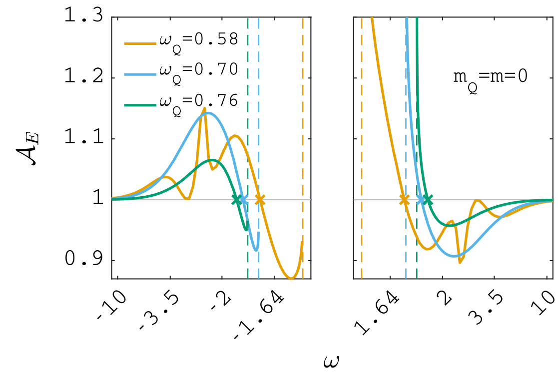

FIG. 2. Amplification of the energy in the scattering for a nonspinning Q-ball with only $\eta_+ (m = 0)$ as the ingoing mode. The coupling is $g = 1/3$. The vertical dashed lines indicate the boundary values of the reality conditions [Eq. (8)], due to the mass gap. The threshold frequency ω_E for energy superradiance is given by Eq. (18).

$$L(\eta, \eta') = \sum_{s=\pm}\left[|(\sqrt{r}\eta_s)'|^2 - r\left(\omega_s^2 - U - \frac{4m_s^2 - 1}{4r^2}\right)|\eta_s|^2\right]$$
$$+ rW(\eta_+^* \eta_-^* + \eta_+ \eta_-) \quad (14)$$

if r is viewed as “time.” This Lagrangian is invariant under the U(1) symmetry

$$\eta_+ = e^{i\alpha}\eta_+, \qquad \eta_- = e^{-i\alpha}\eta_-, \qquad \alpha = \mathrm{const}. \quad (15)$$

The “Noether charge” associated with this U(1) symmetry is

$$M_\eta = ir(\eta_+^{*\prime}\eta_+ - \eta_+^* \eta_+') - ir(\eta_-^{*\prime}\eta_- - \eta_-^* \eta_-'), \quad (16)$$

which satisfies $\partial_r M_\eta = 0$, meaning that M_η is independent of r. At large r, plugging in the asymptotic solution [Eq. (7)], we get, to leading order, $M_\eta = 2(|A_+|^2 - |B_+|^2 - |A_-|^2 + |B_-|^2)$. On the other hand, since $\eta_\pm$ are regular at the origin $r = 0$, we have $M_\eta = 0$. This implies conservation of the particle number in the scattering for generic ingoing and outgoing waves:

$$|A_+|^2 + |B_-|^2 = |B_+|^2 + |A_-|^2. \quad (17)$$

Although the particle number is conserved in the scattering, we can still have amplification or absorption of the wave energy, depending on the frequency ω as well as the Q-ball frequency ω_Q. In fact, combining the constraint [Eq. (17)] and Eq. (11), we see that the threshold frequency ω_E that delineates the amplification and the absorption of the energy, i.e., the case of $\mathcal{A}_E = 1$, is given by $\omega_+^2/k_+ = \omega_-^2/k_-$, or explicitly,

$$\frac{(\omega_Q + \omega_E)^2}{\sqrt{(\omega_Q + \omega_E)^2 - 1}} = \frac{(\omega_Q - \omega_E)^2}{\sqrt{(\omega_Q - \omega_E)^2 - 1}}. \quad (18)$$

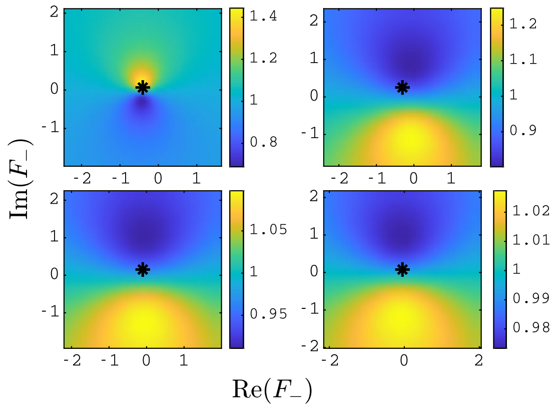

FIG. 3. Amplification of the energy $\mathcal{A}_E$ for a nonspinning Q-ball with both η_+ and η_- as the ingoing modes ($m_Q = m = 0$). For the F_- "initial" parameter, see Eq. (9). The other parameters are $g = 1/3$ and $\omega_Q = 0.7$; $\omega = 1.71$(top left), 2.2(top right), 4.2 (bottom left), 8.0(bottom right). Asterisks denote the case with only η_+ as the ingoing mode.

That is, this is the criterion for energy superradiance to appear at ω_E. Similarly, from Eq. (13), we see that, for an angular momentum mode m_L, the threshold frequency ω_L that delineates the amplification and the absorption of the angular momentum, i.e., the case of $\mathcal{A}_L = 1$, is given by

$$\frac{(\omega_Q + \omega_L)(m_Q + m_L)}{\sqrt{(\omega_Q + \omega_L)^2 - 1}} = \frac{(\omega_Q - \omega_L)(m_Q - m_L)}{\sqrt{(\omega_Q - \omega_L)^2 - 1}}. \tag{19}$$

Nonspinning Q-ball.—Let us see how the energy amplification factor $\mathcal{A}_E$ varies for incident frequency ω and for a few background ω_Q for the case with only an ingoing $\eta_+(m = 0)$ wave; see Fig. 2. (If we only have an ingoing η_- wave, the amplification curves will just be the $\omega \to -\omega$ flip of Fig. 2.) From the criterion [Eq. (18)], we know that superradiance can occur when

$$\omega < -|\omega_E| \quad \text{or} \quad \omega_Q + 1 < \omega < |\omega_E| \tag{20}$$

with $|\omega_E| = [1 + \omega_Q^2 + (1 + 4\omega_Q^2)^{1/2}]^{1/2}$, which is consistent with Fig. 2 (see Fig. 8 in Supplemental Material [77] for a more careful verification). The gaps between the positive and negative ω curves originate from the fact that the scalar has a nonzero mass, and they are different for different ω_Q [see Eq. (8)]. Since we send in an η_+ wave, the positive (negative) ω branch of the amplification curve is when the frequency of the ingoing wave $\omega_+ = \omega_Q + \omega$ has the same (opposite) sign as the Q-ball frequency ω_Q (remember $\omega_Q > 0$). We see that when the sign of ω_+ is the same as ω_Q, greater superradiance can be achieved, and the greatest superradiance is obtained when the incident wave has the lowest frequency or longest wavelength, that is, when the frequency ω approaches the mass gap in that branch. Typically, the peak amplification factor increases as we lower the value of ω_Q, which corresponds to a bigger Q-ball. Another interesting observation is that for some ω_Q (say $\omega_Q = 0.58$) there is an intriguing multipeak structure in the amplification spectrum.

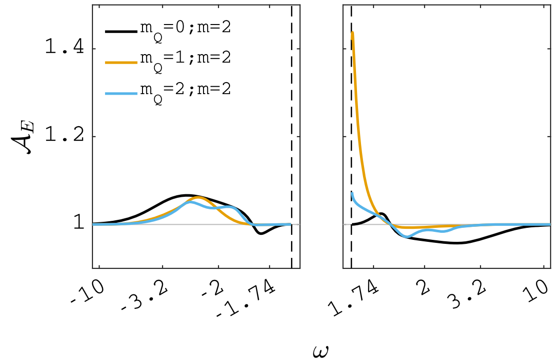

FIG. 4. Amplification of the energy with only η_+ as the ingoing mode. All the Q-balls have frequency $\omega_Q = 0.7$, and the coupling is $g = 1/3$. The threshold frequency ω_E is still given by Eq. (18).

Generally, we may have both the η_+ and η_- wave ingoing, which presumably is a typical case in a "dirty" environment. This mixing can be parametrized by the F_- parameter. The amplifications of the energy for different mode mixings are shown in Fig. 3. We see that allowing both ingoing modes in the scattering often significantly enlarges the energy amplification factor. As mentioned, this might be what one would expect, as Q-ball superradiance really arises from the interplay between the two modes of the complex scalar.

Spinning Q-ball.—When the Q-ball spins in real space, the additional component of rotational superradiance can also be activated, which can often enhance the energy amplification when the η_+ mode rotates in the same direction as the Q-ball [i.e., $\text{sign}(\omega_+/m_+) = \text{sign}(\omega_Q/m_Q)$], as shown in the rhs of Fig. 4. On the other hand, for the opposite cases, we can see slight reductions in energy enhancement. Note that the

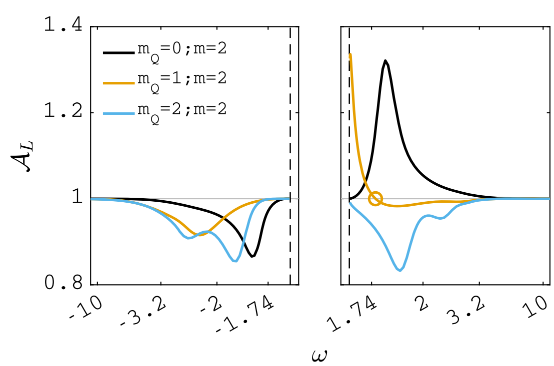

FIG. 5. Amplification of the angular momentum with only η_+ as the ingoing mode. The frequencies of Q-balls are $\omega_Q = 0.7$ and the coupling is $g = 1/3$. The threshold frequency ω_L for angular momentum superradiance is given by Eq. (19), as marked by the small circle.

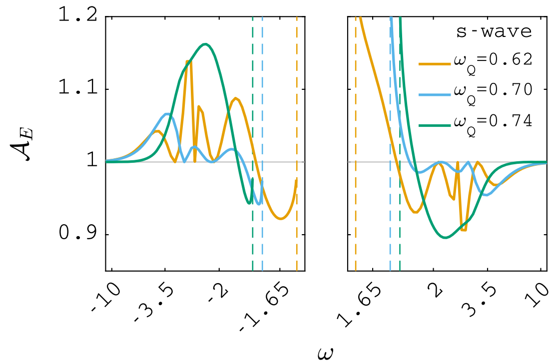

FIG. 6. Amplification of the energy for a 3D spherical Q-ball with only s-wave η_+ as the ingoing mode. The coupling is $g = 1/3$. We see that the 3D case tends to have more multipeak structures.

superradiance criterion for the energy is still Eq. (18), and a nonspinning Q-ball can also induce angular momentum superradiance.

In Fig. 5, we plot the superradiance of the angular momentum with only the η_+ ingoing mode. We see that the amplification criterion is not $\omega < m\Omega_Q$. This is not surprising as there are two coupled modes involved in the scattering and there is Q-ball coherent superradiance on top of the rotational effects. Instead, as we have pointed out, Eq. (19) is the correct criterion for the angular momentum amplification. This is consistent with the case of $m_Q = 1$, $m = 2$ in Fig. 5; for the other two cases, the angular momentum is either superradiantly enhanced or reduced across the whole positive or negative ω branch, without crossing the line of $\mathcal{A}_L = 1$, reflected in Eq. (19) by ω_L not having a real solution. See the Supplemental Material [77] for additional results.

The $3+1$D *case.*—Finally, we shall present some first results about Q-ball superradiance in $3+1$D, focusing on spherical symmetry for both the Q-ball and the scattering waves, which can be easily obtained by slightly modifying the $2+1$D equations above. In Fig. 6, we plot how the energy amplification factor varies with the frequency ω. We see that the $3+1$D case is completely analogous to the $2+1$D case, except that the $3+1$D case has more multipeak structures in the superradiance spectra. The results of spinning Q-balls, due to its numerical complexity, will be presented elsewhere.

In summary, we have found that Q-balls can induce superradiance for waves incident on them, thanks to the coherent rotation of the Q-ball in field space. For spinning Q-balls, the additional rotation in real space can further enhance superradiant emissions. An important feature in a Q-ball scattering is that it involves two coupled modes, which is essential for Q-ball superradiance to occur. Because of the presence of two coupled modes, the patterns of the superrandiance spectra are rather rich. Both the energy and angular momentum of the waves can be enhanced in the scattering, drawing energy and angular momentum from the Q-ball. However, the energy and angular momentum superradiance do not always occur simultaneously. When the ingoing waves contain both positive and negative frequency modes, the charge can also have superradiant enhancements (see the Supplemental Material [77]). Thanks to the particle number conservation in the Q-ball scattering, we have analytically identified the superradiance criteria for the energy and angular momentum.

These results imply that superradiant amplification can take place in a "dirty" environment with scalar waves scattering around Q-balls. It is an interesting question whether this can be turned into spontaneous superradiant instabilities. In any case, Q-balls have long been proposed as a dark matter candidate [22–30], for which longevity is a prerequisite. Q-balls have also been suggested to play other interesting roles in cosmology. The existence of Q-ball superradiance begs the question of how it will affect these scenarios.

Also, note that boson stars are essentially Q-balls in the presence of gravitational attractions. The finding in this Letter hints that boson stars can also superradiate, the implications of which are worth investigating in the era of gravitational wave astronomy and other accurate gravitational observations. Indeed, Ref. [78] has shown that the same amplification mechanism also works for boson stars. Additionally, since Q-balls have been made in laboratories, it would be interesting to observe Q-ball superradiance in condensed matter systems.

We would like to thank Stephen Green and Silke Weinfurtner for helpful discussions. S. Y. Z. acknowledges support from the National Natural Science Foundation of China under Grants No. 12075233, No. 11947301, and No. 12047502 and from the National Key R&D Program of China under Grant No. 2022YFC220010, and is also supported by the Fundamental Research Funds for the Central Universities under Grant No. WK2030000036. The work of P. M. S. was funded by STFC Consolidated Grant No. ST/T000732/1.

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