Performance Analysis of PMSM for High-Speed Starter- Generator System

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Abstract— This paper presents the operating characteristics of surface-mounted permanent magnet synchronous machine taking into account the influence of stator resistance and inductance variation at high frequencies, whose effect are always neglected in the previous research papers which deal with the operating limits during the flux weakening. First, the machine dynamic equations in d-q reference frame considering the effect of high operating frequency on stator resistance and inductance is discussed based on experimental data. Then, the stator voltage constraint is deduced in the current plane as a function of the stator resistance to study the influence of parameters variation on the operating characteristics. Finally, results are compared with the case of neglecting the resistance and inductance variation at high frequencies.

Keywords — Flux weakening control, permanent magnet synchronous machines, high frequency operation, voltage and current constraints

NOMENCLARURE

| U_d, U_q | d-q axis component of stator voltage | |
|-------------------|--------------------------------------|--|
| i_d, i_q | d-q axis component of stator current | |
| R_s | Stator resistance | |
| φ_m | Permanent Magnet flux Linkage | |
| L_d, L_q | d-q axis component of stator self- | |
| • | inductance | |
| ω _e | Electric rotor speed in rad/s | |
| T_e | Electromagnetic Torque | |
| V _{smax} | Maximum phase stator voltage | |
| I _{max} | Maximum phase stator current | |
| | | |

I. INTRODUCTION

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High speed electric machines have attracted great attention in many industrial applications, particularly the mobile applications such as helicopter engine, racing engines and starter/generator systems. This is due to the benefits of the high speed technology such as saving in system weight for a given magnitude of power conversion which improves fuel efficiency and reduce the emissions. Also the machines which provide the high speed operation in certain applications improve the reliability as a result of the elimination of intermediate gearing. One of the high speed electrical machines which has great interest and promising future in these applications is the permanent magnet synchronous machine (PMSM). This is due to its attractive characteristics like high power density, high efficiency, free from maintenance and high torque/inertia ratio. These advantages are refer to the robust construction of the PMSM and low effective air gap [1-3]. These features permit control of the machine not only in constant torque region but also in constant power region up to a high speed.

According to the operating speed of PMSM, a suitable control strategy should be chosen to obtain satisfactory performance. Below base speed, constant torque region, the maximum torque is limited by the maximum allowable current of motor and inverter due to the thermal constraints. So that Maximum torque per Ampere (MTPA) control is always applied which leads to maximum motor-drive efficiency and minimum copper losses. When the speed rises and becomes equal to the base speed, the applied converter voltage to the machine equals to the maximum voltage. Above base speed, constant power region, the flux weakening control strategy must be applied to keep the equivalent back EMF compatible with the applied maximum converter output voltage. This is achieved by reducing the flux in the air gap by injecting direct current opposing the rotor flux produced by the permanent magnet. So, the maximum voltage and maximum current for the machine-drive system must be characterized to ensure safe operation at high speeds [4, 5].

These limits of voltage and current applied to the machine are depicted in the d-q current plane as circles by solving the steady state voltage equations of the machine. These constraints have been discussed in many papers when the feedforward method used for flux weakening control. The main defect in the previous studies about voltage and current limiting circles during the flux weakening is neglecting the influence of winding resistance to avoid the complexity in the equations [6-8]. However, this is only appropriate for machines which have relatively small stator resistances. This assumptions can't achieve optimal flux weakening control based on analytical model especially for high resistance machines [9, 10]. In [10] The effect of stator resistance has been considered to achieve optimal flux weakening strategy and emphasis to the actual exploitation of the motor torque-speed capability.

This paper presents the performance analysis of PMSM at high frequency operation which is used as starter-generator. The variation of the machine parameters due to high frequency operation has been discussed based on experimental measurements of machine parameters for wide range of operating frequency. Accordingly, The PMSM steady state performance characteristics with the applied stator current and voltage constraints and under the variation of machine parameters with operating frequency are studied and compared with the case of neglecting the winding resistance. The characteristics of the PMSM at high frequency operation are presented.

II. MATHEMATICAL MODEL OF PMSM

The machine model for the IPMSM can be expressed in d-q synchronously rotating reference frame with electrical angular velocity ω_e as following:

$$U_d = i_d R_s + L_d \frac{di_d}{dt} - \omega_e L_q i_q \tag{1}$$

$$U_q = i_q R_s + L_q \frac{di_q}{dt} + \omega_e L_d i_d + \omega_e \varphi_m \qquad (2)$$

Where U_d and U_q are the components of the stator voltage in direct and quadrature axes, i_d and $i_q are$ direct and quadrature axes stator current components, φ_m is the permanent magnet flux linkage, L_d and L_q are the self-inductances components in d-q axes, R_s is the resistance of the stator winding.

$$T_e = \frac{3}{2} P \left[\varphi_m i_q + (L_d - L_q) i_d i_q \right]$$
(3)

Where T_e is the PMSM electromagnetic torque, P is the number of poles pairs. This mathematical model refer to the interior PMSM type where the saliency part can be observed from the torque equation due to the inequality between the inductance in the d& q axes. In case of the surface type ($L_d \cong L_q = L_s$), the reluctance torque will be removed from the torque equation.

III. PMSM PARAMETERS VARIATION AT HIGH FREQUENCY OPERATION

In AC machines, the values of resistance and leakage inductance change with the operating frequency[11]. This change can be neglected at low frequency machines. This assumption can't be considered at high frequencies due to the non-uniform current distribution across wire cross section area caused by increased skin and proximity effects. This leads to increasing in the AC resistance and reduction in the self-inductances [11-13].

The resistance and leakage inductances in d &q axes are measured experimentally at high operating frequencies for the AEGART machine which is designed to run as a starter/generator. For further description on the electrical machine details and design can be found in [14, 15]. These measurements are shown in Fig.1 and Fig.2.



Fig .1 Variation of stator resistance with frequency



Fig.2 Variation of d & q inductances with frequency

The machine parameters are measured at high frequencies from about 280 Hz up to 1600 Hz where this machine is designed to be suitable for high speed applications. It can be shown from Fig.1 & Fig.2 that the resistance increases proportionally with the frequency whereas the inductance has slightly decrease during this range of frequency .Also, the difference between the two inductances is very small so that we can assume that the values of two inductances are equal at each operating frequency $(L_d \cong L_q = L_s)$.

IV. EFFECT OF THE FREQUENCY VARIATION ON THE OPERATING RANGE UNDER V&I CONSTRAINTS

Operating the PMSM drive system at higher operating speeds in constant power operation mode requires special control algorithm. As the back-EMF is proportional to the motor speed and air gap flux, this will lead to higher back-EMF values. Once the back-EMF becomes larger than the maximum output voltage of the drive, the PMSM will be incapable of drawing current and hence incapable of developing torque. Thus, when the back-EMF reaches the voltage threshold of a drive, the rotor speed of such a motor cannot be increased unless the air gap flux can be weakened. This strategy is called flux weakening control to get speeds higher than the base speed without applying excessive voltage than the maximum converter output voltage.

During the Flux Weakening operation, the voltage and current limits at each speed must be taken into consideration to avoid machine current larger than the maximum value. These limits are responsible for determining the safe operating region for the machine at high operating speeds.

A. The Operating current limits

The current constraint can be expressed in the d-q rotating reference frame in the following equation

$$\{\sqrt{i_d^{*2} + i_q^{*2}} \le I_{max}\}$$
(4)

Where the max current (I_{max}) can be determined according to the inverter and machine ratings. By using equation (4), the current constraint can be depicted as a circle as shown in Fig (3) called the current limiting circle.

B. The Operating Voltage Limits

The maximum voltage of the machine (V_{smax}) supplied from the inverter depends on the DC link voltage (V_{dc}) and the type of PWM control technique used. For instance, if the SVPWM will be used, then the maximum voltage will be calculated as following:-

$$V_{smax} = \frac{V_{dc}}{\sqrt{3}} \tag{5}$$

The voltage constraint can be expressed in the d-q rotating reference frame in the following equation:-

$$\{\sqrt{V_d^{*2} + V_q^{*2}} \le V_{smax}\}$$
(6)

The voltage constraints can be dedicated in the current plane by substituting of steady state voltage equation for the IPMSM into equation (6) as following:-

$$\left(i_d R_s - \omega_e L_q i_q \right)^2 + \left(i_q R_s + \omega_e L_d i_d + \omega_e \varphi_m \right)^2$$

$$\leq V_{smax}^2$$

The previous equation can be simplified to represent a circle equation and can be depicted in current plane as following:-

$$\left(i_d + \frac{a}{Z^2}\right)^2 + \left(i_q + \frac{b}{Z^2}\right)^2 \le \frac{V_{smax}^2}{Z^2}$$
 (7)

Where

$$Z = \sqrt{R_s^2 + \omega_e^2 L_s^2}$$
$$a = \omega_e^2 L_s \varphi_m$$
$$b = \omega_e R_s \varphi_m$$

In the case of neglecting the machine resistance the equation (7) can be simplified into:-

$$\left(i_d + \frac{\varphi_m}{L_s}\right)^2 + i_q^2 \le \frac{V_{smax}^2}{\omega_e^2 L_s^2} \tag{8}$$

The voltage limiting circles can be drawn in two cases. The first case, when the parameters variation (R_s, L_s) at high frequencies is considered using equation (7). The second case, when the resistance is neglected and the inductance is frequency independent using equation (8). Voltage circles for the two cases can be shown in Fig (3).



Results refer to the SPM machine whose data are shown in Table I. whereas the voltage circles case at variable resistance and inductance, is based on the data mentioned in section (II).

Table I DATA OF THE TEST MAHCINE

| DATA OF THE LEST MAILENE | | | |
|--------------------------|-------|---------|--|
| Number of poles pairs | 3 | | |
| Stator resistance (1 | nohm) | 1.058 | |
| Stator inductance (| μH) | 100 | |
| Magnet Flux Linkage | (Wb) | 0.03644 | |
| Maximum Voltage (| volt) | 155.9 | |
| Maximum Current | (Amp) | 360 | |

According to equation (7), it can be noticed that the voltage circle dimensions depend on the operating speed and the machine parameters (R_s, L_s, φ_m) . Fig (3) shows that the variation of (R_s, L_s) with the frequency causes asymmetry between motoring and generating regions. Also the maximum feasible torque, determined by the intersection of current circle with the voltage circles is different for the two quadrants of operation. As for case-2, it can be noticed from equation (8) that voltage circle is symmetrical around q-axis so that motoring and generating regions of the machine are symmetrical which can be seen in Fig (3). Also the maximum feasible torque is the same in motoring and generating modes.





Fig (4) current and voltage constraints for motoring and generating mode

The difference between the voltage circles in case-1 and case-2 can be explained using equation (7) and assuming $R_s^2 \ll \omega_e^2 L_s^2$ so that R_s^2 can be neglected compared to $\omega_e^2 L_s^2$. Therefore, the voltage limit model can be simplified to:

$$\left(i_d + \frac{\varphi_m}{L_s}\right)^2 + \left(i_q + \frac{R_s \varphi_m}{\omega_e L_s^2}\right)^2 \le \frac{V_{smax}^2}{\omega_e^2 L_s^2} \qquad (9)$$

From equations (8) & (9) we can deduce formula for the i_d current to apply in flux weakening as a function of i_q as:

$$i_{d}' = \sqrt{\frac{V_{smax}^{2}}{\omega_{r}^{2}L_{s}^{2}} - \left(i_{q} + \frac{R_{s}\Psi_{m}}{\omega_{r}L_{s}^{2}}\right)^{2}} - \frac{\Psi_{m}}{L_{s}} = \sqrt{A'} - B$$

Case (1) (10)

$$i_d = \sqrt{\frac{V_{smax}^2}{\omega_r^2 L_s^2} - i_q^2} - \frac{\Psi_m}{L_s} = \sqrt{A} - B$$

Case (2) (11)

From equations (10) & (11), the d-axis current value required for SPMSM to operate in flux weakening mode can be generated according to the operating speed and the load torque current. Also by using equation (10), effect of machine parameters variation can be considered easily to apply the optimal flux weakening controller for SPMSM.

In motoring mode, when the motor is loaded by certain torque consequently there is a certain value for drawn i_q . From equations (10) and (11) we can notice that:-

Always $B > \sqrt{A}$ and $B > \sqrt{A'}$ to produce negative i_d for flux weakening.

As A' < A So that $|i_d'| > |i_d|$.

This shows that machine draws more d-current as absolute value when R is considered. This complies with the circles in motoring mode as shown in Fig (4)a. Increasing direct current drawn at the same load mean that more stator current will be drawn. This is reversed in generation mode due to the negative sign of i_q as seen in Fig (4) b.

Also, operating at the current circle contour where the machine operate at I_{max} with more d-axis current due to effect of machine parameters mean less q-axis current. Consequently the maximum torque produced by the machine will reduce when machine parameters is considered in motoring mode and the vice versa in generate mode.

V.OPERATING CHARACTERISTICS OF PMSM AT HIGH OPERATING SPEED UNDER V&I CONSTRAINTS

The flux weakening area can be divided into two regions. The first region when the operating point lies on the voltage circle arc to keep the current vector following certain value of constant power.

The machine in this region run under constant voltage constant power control. The second region when the operating point is determined by the intersection between the two circles where the machine run at its maximum feasible torque.

A. constant voltage constant power control

If the PM machine operate at certain power (45 Kw) at different frequencies, the electromagnetic torque can be determined from the following equation:-

$$T = \frac{P_{const}}{\omega_m} \tag{12}$$

According to the electromagnetic torque calculated, the q-axis current can be calculated from the torque in section (II) for SPMSM. Accordingly, the d-axis current can be determined from the voltage constraints at each operating speed. The change of $|i_d|$ and stator current (I_s) at constant power for the two cases of voltage circles can be shown in Fig (5) and (6) for motoring mode and Fig (7) and (8) for generating mode.



Fig (5) Absolute value of direct current variation with frequency at constant power in motoring mode



Fig (6) Stator current variation with frequency at constant power in motoring mode

In motoring mode, the stator current and the absolute value of the d-axis current drawn, when parameters variation is considered, are higher than the other case. Also the difference in currents between two cases is small and it decreases by very small amount with increasing the frequency as shown in Fig (5) and (6).



Fig (7) Absolute value of direct current variation with frequency at constant power in generating mode



Fig (8) Stator current variation with frequency at constant power in generating mode

In generating mode, the currents in case of considering parameters variation are less than in the other case. Also this difference will increase with rising the operating frequency as seen in Fig (7) and (8).

B. voltage and current limited maximum torque control

Once the operating points of machine reach to current circle, this mean that machine run at maximum feasible torque corresponding to each speed. These points are computed by the system of the respective equations (4) and (7). By eliminating d current component, a quadratic equation for q current is obtained as following:-

$$c \, i_a^2 + d \, i_a + e = 0 \tag{13}$$

Where

$$c = \frac{4b^2 + 4a^2}{z^4} , \ d = \frac{4mb}{z^2} , e = m^2 - \frac{4a^2 J_{max}^2}{z^4} ,$$
$$m = \frac{b^2}{z^2} + \frac{a^2}{z^4} - \frac{V_{max}^2}{z^2} + I_{max}^2$$

Solving the previous equation, two distinct roots are obtained: one positive root corresponding to motoring (i_{q1}) and the other is negative for generating (i_{q2}).the torque equation is used to evaluate the corresponding maximum torque produced by the machine in motoring and generating. This maximum torque is function of the operating speed and machine parameters. So the effect of parameters variation can be studied easily. The change of the maximum torques produced by the machine when parameters are frequency dependent and independent are shown in Fig (10).



Fig (10) Maximum torque versus Frequency capability computed accounting or neglecting parameters effect

Fig (10) shows that if the resistance and inductance variations are considered, the maximum available torque will be under predicted value when resistance is neglected in motoring and over in generating mode. Also the difference of maximum feasible torque when parameters are considered in motoring and generating mode increase at high frequency.

VI. CONCLSION

In this paper, Effect of stator parameters variation at high frequency operation on the operating regions of SPMSM are presented. Variation of stator parameters with the frequency reduce the operating region in motoring and increase it in generating. Also the current of machine is over and under predicted in motoring and generating respectively. The maximum possible torque of machine is higher in motoring and less in generating where the difference in maximum torque between the two cases of study will be slightly increase with the frequency in both operating mode. These differences lead to a non-similarity between the simulation and the reality. This study help to understand the effect of high speed operation for SPMSM on the stator parameters and how the variation for these parameters will affect the performance characteristics Also, considering the effect of frequency variation and resistance during flux weakening lead to optimal control design.

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