

1 **Determining the presence of scour around bridge foundations using vehicle-induced**
2 **vibrations**

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4
5 **Abstract**

6 Bridge scour is the number one cause of failure in bridges located over waterways. Scour
7 leads to rapid losses in foundation stiffness and can cause sudden collapse. Previous research
8 on bridge health monitoring has used changes in natural frequency to identify damage in
9 bridge beams. The possibility of using a similar approach to identify scour is investigated in
10 this paper. To assess if this approach is feasible, it is necessary to establish how scour affects
11 the natural frequency of a bridge and is it possible to measure changes in frequency using the
12 bridge dynamic response to a passing vehicle. To address these questions, a novel Vehicle-
13 Bridge-Soil Interaction (VBSI) model is developed. By carrying out a modal study in this
14 model, it is shown that for a wide range of possible soil states, there is a clear reduction in the
15 natural frequency of the first mode of the bridge with scour. Moreover, it is shown that the
16 response signals on the bridge from vehicular loading are sufficient to allow these changes in
17 frequency to be detected.

18 **Keywords:** Scour, Vibrations, Frequency, Soil Stiffness, Bridges, SHM

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19 **Introduction**

20 **Bridge scour**

21 Bridge scour is the term given to the excavation and removal of material from the bed and
22 banks of rivers as a result of the erosive action of flowing water (Hamill 1999). Scouring of
23 bridge foundations is the primary cause of failure of bridges in the United States (Briaud et
24 al. 2001, 2005; Melville and Coleman 2000). One study of over 500 bridge failures which
25 occurred between 1989 and 2000 in the US deemed flooding and scour to be the primary
26 cause of 53% of failures (Wardhana and Hadipriono 2003). Another review claims that over
27 the past 30 years, 600 bridges in the US have failed due to scour problems (Briaud et al.
28 1999; Shirole and Holt 1991). As well as the risk to human life, these failures cause major
29 disruption and economic losses (De Falco and Mele 2002). Lagasse et al. (1995) estimate that
30 the average cost for flood damage repair of bridges in the United States is approximately \$50
31 million per annum. Scour is relatively difficult to predict and poses serious risks to the
32 stability of vulnerable structures. It typically results in a loss in foundation stiffness that can
33 compromise structural safety. With regard to scour, visual inspections involve the use of
34 divers to inspect the condition of foundation elements (Avent and Alawady 2005). These
35 types of inspections can be expensive and can have limited effectiveness as inspecting the
36 condition of the foundation can be dangerous in times of flooding, when the risk of scour is
37 highest. Due to the re-filling of scour holes as flood waters subside, visual inspections
38 undertaken after a flood event may fail to detect the loss in stiffness resulting from scour as
39 the backfilled material may be loose and therefore have significantly reduced strength and
40 stiffness properties. Many mechanical and electrical instruments have been developed that
41 aim to remotely detect the presence of scour. These include systems such as magnetic sliding
42 collars, float-out systems (Briaud et al. 2011), radar systems (Anderson et al. 2007; Forde et
43 al. 1999), vibration-based systems (Fisher et al. 2013; Zarafshan et al. 2012) and time-domain

44 reflectometry (Yankielun and Zabilansky 1999; Yu 2009) among others. A comprehensive
45 overview of the instrumentation available is given in Prendergast and Gavin (2014). The
46 primary drawback of both visual inspections and the use of mechanical scour depth
47 measuring instrumentation is that these typically cannot detect the distress experienced by a
48 structure due to the development of a scour hole around the foundation. Monitoring changes
49 in the modal properties of a structure can potentially provide insight into a structure's distress
50 due to a scour hole. Some background on previous research into this is provided in the next
51 section.

52

53 **Scour monitoring using structural dynamics**

54 The overall stiffness of a bridge is comprised of a combination of the mechanical properties
55 of the structural elements (e.g. deck, piers, abutments) and the properties of the foundation
56 soil. Detecting damage in a bridge superstructure by looking for changes in the dynamic
57 response has received much attention in the literature (Abdel Wahab and De Roeck 1999;
58 Doebling and Farrar 1996; Sampaio et al. 1999). Whilst scour will result in changes in the
59 stiffness and therefore the dynamic response of a structure, research on detecting scour using
60 vibration-based methods is relatively limited. In previous studies, properties such as natural
61 frequency, mode shapes, mode shape curvature, covariance of acceleration signals and
62 changes in the Root Mean Square (RMS) of acceleration signals have all been examined as
63 possible indicators of scour (Briaud et al. 2011; Chen et al. 2014; Elsaid and Seracino 2014;
64 Klinga and Alipour 2015).

65 Foti and Sabia (2011) describe a full-scale investigation undertaken on a five-span bridge
66 where one pier was adversely affected by scour during a major flood in 2000. The modal
67 parameters of the bridge deck spans (namely natural frequencies and mode shapes), were
68 identified from traffic-induced vibrations before and after replacement of the pier. Most of

69 the spans did not show a significant change. However, the span supported by the scoured pier
70 did exhibit a lower frequency than the others. The pier itself was analysed in a different
71 manner. It was recognised that scour affecting one side of the pier would result in asymmetric
72 dynamic behaviour, therefore to detect this behaviour an array of accelerometers was placed
73 along the foundation in the direction of flow. The method used to analyse the signals was the
74 creation of a covariance matrix of the signals, whereby the diagonal terms of this covariance
75 matrix coincide with the variances of single signals. The difference in magnitude of the
76 variance along the foundation showed that scour could be detected using this methodology.
77 Elsaid and Seracino (2014) describe a study undertaken into the effect of scour on the
78 dynamic response of a scaled model of a coastal bridge supported by piles. Both laboratory
79 testing and finite element modelling were undertaken. Horizontally displaced mode shapes
80 showed significant sensitivity to scour progression due to the reduction in the flexural rigidity
81 of the piles. Other indicators namely; mode shape curvature, flexibility-based deflection and
82 curvature were also investigated. It was concluded that these methods each showed promise
83 at detecting the location and extent of scour to varying degrees of accuracy. No soil-structure
84 interaction was considered in the study by Elsaid and Seracino (2014). Briaud et al. (2011)
85 describe a laboratory study into the effect of scour on the dynamic response of a model scale
86 bridge with a span of 2.06 m and a deck width of 0.53 m. Both shallow and deep foundations
87 were tested in a large hydraulic flume. Fast Fourier transforms were used to obtain the
88 frequency content of the acceleration signals measured in three directions for both foundation
89 types, namely the flow direction, the traffic direction and the vertical direction. The ratio of
90 Root-Mean Square (RMS) values of accelerations measured in two different directions
91 (traffic/vertical, flow/traffic or flow/vertical) was also calculated to ascertain if it could be
92 used as a scour indicator. The frequency response in the flow direction as well as the ratio of
93 RMS values for flow/traffic showed the highest sensitivity to scour. A full-scale deployment

94 of the methods by Briaud et al. (2011) on a real bridge proved unsuccessful due to a failure of
95 the logging system and the high energy required to store and transmit acceleration data. It
96 was concluded that accelerometers showed potential for detecting and monitoring scour but
97 would require significant further research. Ju (2013) investigated the effect of soil-fluid-
98 structure interaction using finite-element modelling in calculating scoured bridge natural
99 frequencies. A full-scale field experiment was undertaken to validate the numerical model
100 and it was concluded that frequency reduces with scour but the trend is non-linear due to non-
101 uniform foundation sections and layered soils. It was also concluded that although the
102 presence of fluid lowers the frequency value obtained, the fluid-structure effect is not obvious
103 and therefore it may be neglected in the bridge natural frequency analysis.

104

105 **Development of Vehicle Bridge Soil Interaction (VBSI) model**

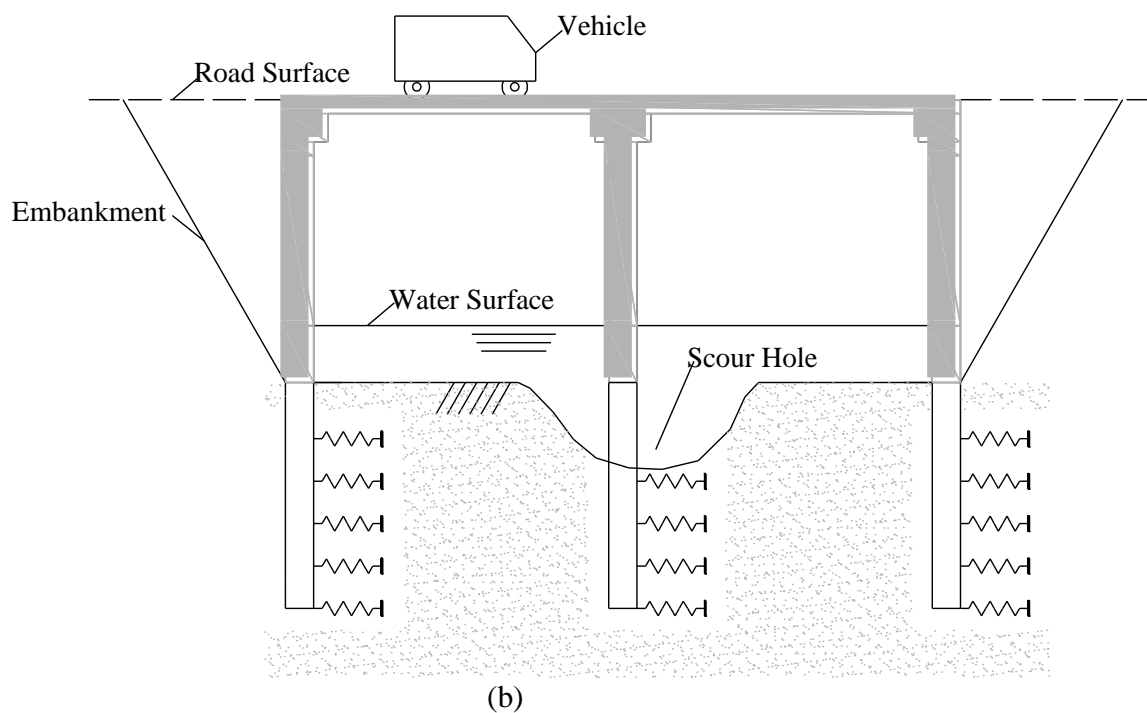
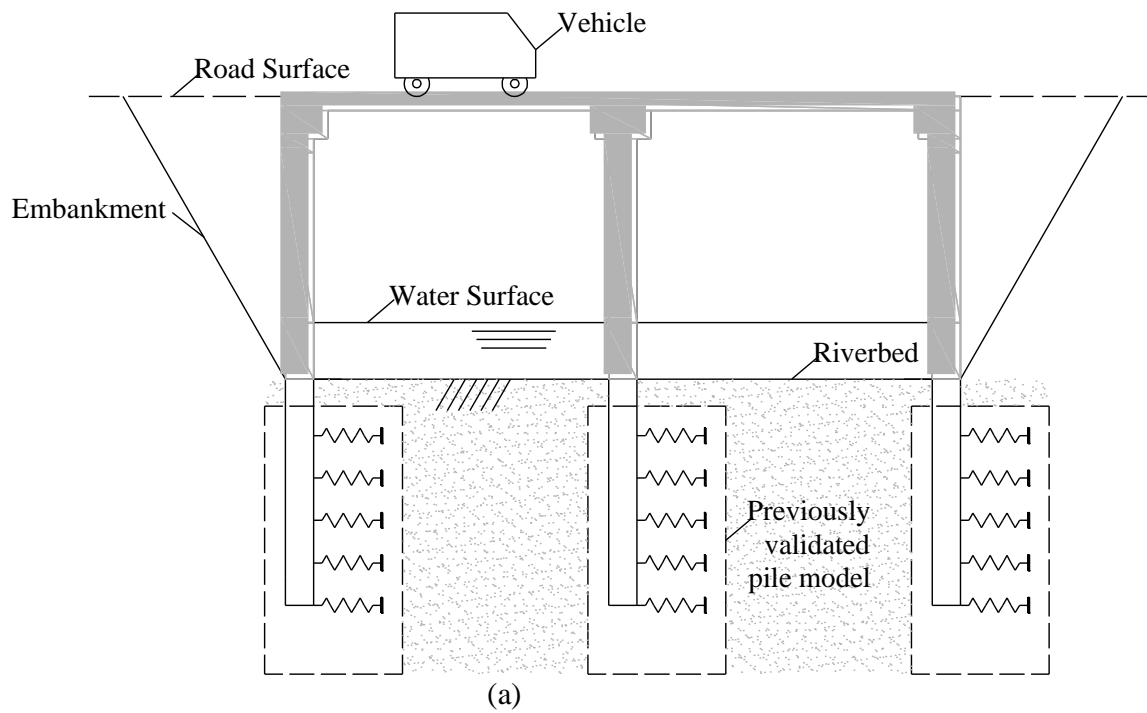
106 ***Background***

107 This paper builds on work presented by Prendergast et al. (2013) in which a numerical soil-
108 structure dynamic interaction model was developed to describe the change in natural
109 frequency of a pile foundation subjected to scour. The model was shown to be capable of
110 tracking the change in the natural frequency of a single pile affected by scour using input
111 parameters which included the structural properties of the pile and the small strain stiffness of
112 the soil. Experimental validation of the numerical model was undertaken both in a laboratory
113 model scale and full-scale field test on a 8.76 m long pile embedded in dense sand. The pile
114 geometry was typical of those used to support road and rail bridges. This validated numerical
115 model is represented by the pile/spring system shown boxed in Fig. 1(a).

116 ***Extended Model***

117 The work described by Prendergast et al. (2013) was validated for the case of a stand-alone
118 pile foundation with forced vibration being imposed through the use of a modal hammer. In

119 reality, pile foundations are used to provide vertical and/or lateral support for a structure (in
120 this case a bridge) the presence of which will have a significant effect on the natural
121 frequency response of the pile-soil interaction problem. In this paper, the previously
122 developed model is extended to consider the effect of a bridge superstructure. The structure
123 considered is an integral bridge, comprised of two abutments, and a central pier supported on
124 pile foundations. The purpose of extending the model to include a bridge superstructure is to
125 ascertain if it is possible to detect changes in the structure's natural frequency due to scour of
126 the foundation and moreover to investigate if it is practicable to detect these changes by
127 analysing the acceleration signals caused by traffic loading (i.e. when a truck crosses the
128 bridge). Figs. 1(a) and (b) show a schematic of the un-scoured and scoured situations
129 respectively. To make the simulated acceleration signals as realistic as possible, interaction
130 effects between the vehicle and the bridge are considered and external noise is added to the
131 signals. In this work, the change in natural frequency due to scour around the central pier
132 foundation is modelled, see Fig. 1(b). The possibility of detecting these changes by analysing
133 the acceleration response signals from vehicular loading is considered. Details of the model
134 are given below.



135

136

137

Fig. 1. Schematic of model. (a) un-scoured, (b) post scour.

138 **Bridge structure to be modelled**

139 The bridge modelled is a two-span concrete integral bridge. A Young's modulus of $E =$
140 $3.5 \times 10^{10} \text{ N m}^{-2}$ and a material density of $\rho = 2400 \text{ kg m}^{-3}$ are assumed for all bridge elements.

141 For this type of bridge the abutment is formed using a series of vertical concrete columns and
142 reinforced earth. The columns support the deck and the reinforced earth retains the
143 embankment fill, see Fig. 2. The bridge is not intended to represent any particular real-life
144 structure. However, the properties were chosen to be representative of bridges of this type.

145 The bridge deck is comprised of nine U10 concrete bridge beams (Concast 2014). Each beam
146 supports a 200 mm deep deck slab giving a total combined moment of inertia of $I = 2.9487$
147 m^4 and a cross-sectional area of $A = 9.516 \text{ m}^2$ for the bridge deck, which are typical values
148 for this type of bridge. The abutment consists of nine concrete columns supporting the bridge
149 deck, each column is 500 mm in diameter and the columns are at 1900 mm centres, see Fig.

150 2(c). This results in a total moment of inertia of $I = 0.0276 \text{ m}^4$ and a cross-sectional area of A

151 $= 1.7671 \text{ m}^2$ for the abutment elements. This type of bridge does not have a conventional
152 expansion joint so the thermal movements of the deck have to be accommodated by lateral
153 movements of the abutment columns. To facilitate this movement, the abutment columns are

154 cast in vertical sleeves so that there is a gap of 50 to 100 mm on all sides, i.e. the reinforced

155 earth provides no lateral restraint to the columns. These abutment columns are therefore

156 assumed free to move laterally. Two large concrete piers support the bridge at the centre and

157 have plan dimensions of 1375 mm x 2625 mm. This results in a total combined moment of

158 inertia of $I = 1.137 \text{ m}^4$ and a cross-sectional area of $A = 7.22 \text{ m}^2$ for the combined bridge pier

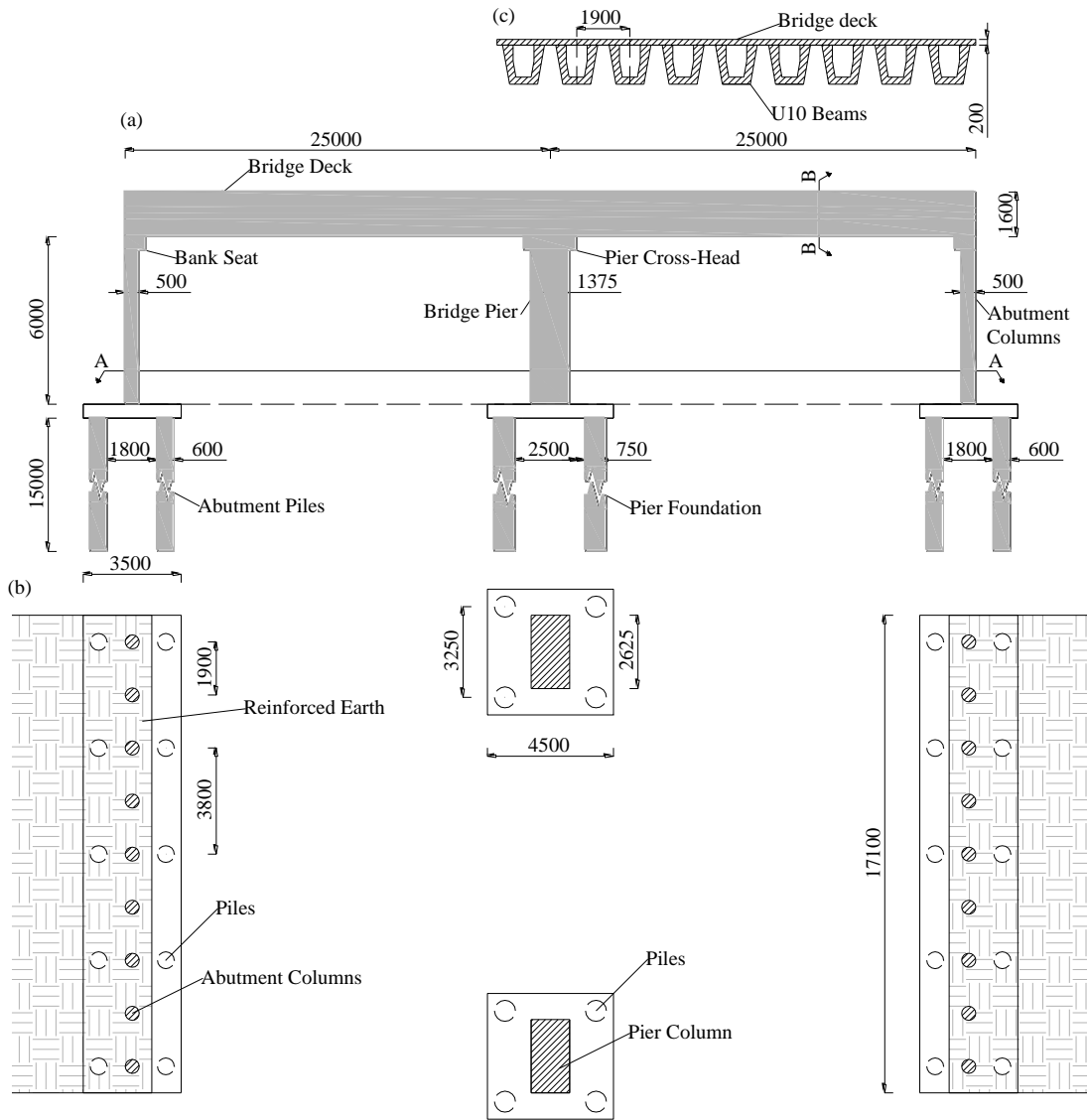
159 element. The piers are large stiff elements and they provide lateral restraint to the bridge

160 deck.

161 The abutment columns each rest on a pilecap, under which ten 15 m long concrete bored piles

162 are used as the foundation system, see Fig. 2. The pier columns each rest on a pilecap

163 supported by four piles. The scour action is assumed to be uniform along the transverse
164 length of a given support, so for modelling purposes, the structure shown in Fig. 2 is idealised
165 as the 2D frame shown in Fig. 3. (Note: scour is assumed to be equal on both sides of the
166 pier). The properties of each of the elements of the model in Fig. 3 are calculated by
167 summing the properties of the individual components shown in Fig. 2. For example, the
168 moment of inertia of the left abutment column shown in Fig. 3 is calculated by summing the
169 moment of inertia of the nine abutment columns shown in Fig. 2. Similarly the stiffness of the
170 two leaves of the pier shown in Section A-A of Fig. 2 is attributed to the central pier element
171 of Fig. 3. When apportioning stiffness to the pile elements shown in Fig. 3, a similar
172 philosophy was adopted. The abutment piles modelled have a combined cross-sectional area
173 of $A = 2.827 \text{ m}^2$ and a moment of inertia of $I = 0.0636 \text{ m}^4$ whereas the central pier piles have
174 $A = 3.534 \text{ m}^2$ and $I = 0.1243 \text{ m}^4$. Details on the spring stiffness coefficients used to model the
175 soil are given below and are summarised in Fig. 4.



176

177 **Fig. 2.** Bridge layout with all dimensions shown in mm (a) elevation; (b) section A-A; (c)

178

section B-B

179

180 Numerical modelling approach

181 The specific technical details of the model used in this paper have been published in

182 Prendergast et al. (2016b), therefore this section does not provide too much detail on the

183 model. However an overview of the modelling approach is provided, in particular the

184 philosophy for modelling the bridge, the vehicle, and the soil is briefly discussed. The model

185 is developed using MATLAB.

186

187 **Bridge model**

188 The elements used in the bridge model are 6 degree-of-freedom (DOF) Euler-Bernoulli frame
189 elements (Kwon and Bang 2000). Each frame element has two nodes and each node has an
190 axial, transverse and a rotational degree of freedom as shown in the insert in Fig. 3.

191 The global mass and stiffness matrices for the model are assembled together according to the
192 procedure outlined in Kwon and Bang (2000). Damping is modelled using a Rayleigh
193 damping approach, with a damping ratio of 2% being assumed for all simulations in this
194 paper. The dynamic response of the bridge is obtained by solving the second order matrix
195 differential equation shown in Eq. (1).

$$196 \quad \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t) \quad (1)$$

197 where \mathbf{M} , \mathbf{C} and \mathbf{K} are the ($nDOF \times nDOF$) global consistent mass, damping and stiffness
198 matrices respectively, and $nDOF$ is the total number of degrees of freedom in the system. The
199 vector $\mathbf{x}(t)$ describes the displacement of every degree of freedom for a given time step in
200 the analysis. Similarly, the vectors $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ describe the velocity and acceleration of
201 every degree of freedom in the model for the same time step. The vector $\mathbf{F}(t)$ describes the
202 external forces acting on each degree of freedom for a given time step in the analysis. Eq. (1)
203 is solved using a numerical integration scheme, the Wilson-theta method (Dukkipati 2009).

204 Mode shapes and natural frequencies were extracted from the model by performing an
205 eigenvalue analysis on the system. In order to verify that the model was operating correctly,
206 the static displacements, mode shapes and natural frequencies predicted by the model were
207 verified against those calculated by a commercially available finite-element package. Good
208 agreement was observed between the model and the commercial software.

209

210 **Vehicle model**

211 The vehicle model, used in this work is similar to the model described in Hester and
 212 González (2012) and González and Hester (2013). The vehicle model has four degrees of
 213 freedom, namely a vertical displacement for each of the two axles (y_1 and y_2), the body
 214 bounce (y_b) and body pitch (ϕ_p), see Fig. 3. The body has mass m_b and has rotational moment
 215 of inertia I_p (for pitch). The body is supported on a suspension/axle assembly. The mass of
 216 the wheel/axle assembly is m_w . The suspension has a stiffness K_s and a damping coefficient
 217 C_s . Finally, the tyre is modelled as a spring with stiffness K_t . Table 1 provides the parameters
 218 of the vehicle (Cantero et al. 2011; El Madany 1988). Using the properties given in Table 1,
 219 stiffness \mathbf{K}_v , mass \mathbf{M}_v and damping \mathbf{C}_v matrices for the vehicle can be populated. The natural
 220 frequencies of the vehicle for bounce, pitch, and front and rear axle hops are 1.43 Hz, 2.07
 221 Hz, 8.860 Hz and 10.22 Hz respectively.

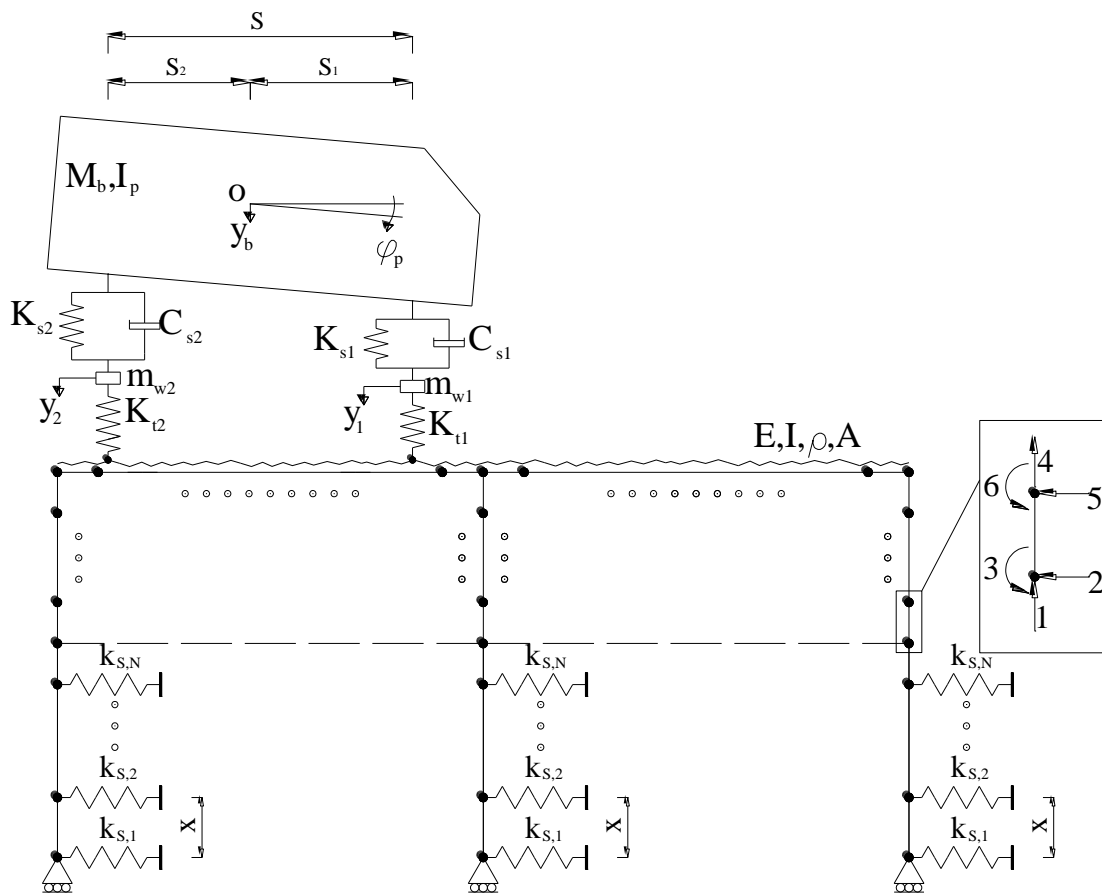
222 **Table 1.** Parameters of vehicle model.

Parameter	Property	Value
Dimensions (m)	Wheel base (S)	5.5
	Dist from centre of mass to front axle (S_1)	3.66
	Dist from centre of mass to rear axle (S_2)	1.84
Mass (kg)	Front wheel/axle mass (m_{w1})	700
	Rear wheel/axle mass (m_{w2})	1,100
	Sprung body mass (m_b)	13,300
Inertia (kg m^2)	Pitch moment of inertia of truck (I_p)	41,008
Spring stiffness (kN m^{-1})	Front axle (K_{s1})	400
	Rear axle (K_{s2})	1,000
Damping (kN s m^{-1})	Front axle (C_{s1})	10
	Rear axle (C_{s2})	10
Tyre stiffness (kN m^{-1})	Front axle (K_{t1})	1,750
	Rear axle (K_{t2})	3,500

223

224

225 Modelling the dynamic behaviour of a vehicle-bridge interaction system is complex as there
 226 are two sub-systems, namely the moving vehicle and the bridge/substructure. These two
 227 systems interact with each other via the contact forces that exist between the vehicle wheels
 228 and the bridge surface, therefore mathematically the problem is coupled and time dependant
 229 (Yang et al. 2004). It is necessary to solve both subsystems while ensuring compatibility at
 230 the contact points (González 2010). In this paper, an iterative approach was employed to
 231 implement the VBI model (Green and Cebon 1997; Yang and Fonder 1996), see Prendergast
 232 et al. (2016b) for more information.



233 **Fig. 3.** Schematic of the Vehicle-Bridge-Soil Interaction (VBSI) model.
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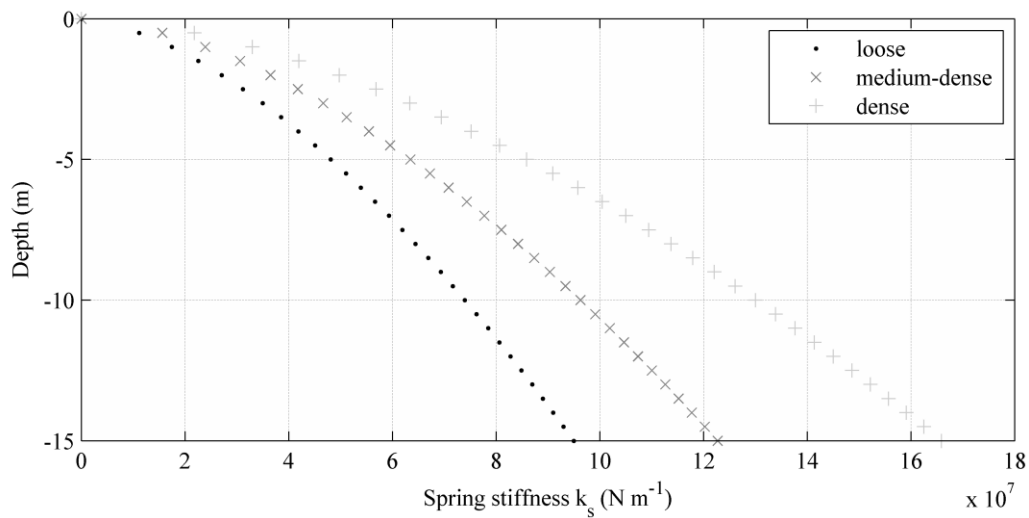
235

236 **Calculating soil spring stiffness**

237 Soil-structure interaction is incorporated into the model by means of the Winkler method.

238 The soil is modelled as a system of discrete, mutually independent and closely-spaced lateral

239 springs (Dutta and Roy 2002; Winkler 1867). The method for developing spring stiffness
 240 values is based on Prendergast et al. (2013) who derived spring stiffness values using the
 241 small-strain shear modulus (G_0) profile from their experimental site. Full details on
 242 calculating soil spring stiffness coefficients is available in Prendergast and Gavin (2016a) and
 243 Prendergast et al. (2015). The spring stiffness profiles used in this paper are shown in Fig. 4
 244 for loose, medium-dense and dense sand. The individual spring stiffness moduli are shown by
 245 the data markers on the plot. These profiles are for the central pier foundation piles.



246
 247 **Fig. 4.** Postulated soil spring stiffness profiles for a loose, medium-dense and dense sand
 248 around the central pier piles (N m^{-1}) for the analysis.

249
 250 **Analysis & results**

251 In the analyses performed using the model described previously, a moving vehicle excites the
 252 bridge. The lateral response of the bridge is excited by the vehicle moving over the bridge,
 253 inducing moments at the head of the abutments and the pier causing lateral sway. Horizontal
 254 vehicle forces that would be induced by vehicle acceleration and braking are not included in
 255 the model. However, these may contribute to the lateral response on a real system. The
 256 vehicle crosses the bridge at typical highway speed and the horizontal acceleration from the

257 top of the pier is recorded and analysed. The effect of (initial) soil stiffness on the frequency
258 changes with scour was examined for the three soil stiffness profiles. To aid in choosing
259 appropriate locations to place accelerometers on the structure and to ascertain a baseline for
260 the expected change in natural frequency due to scour, an eigenvalue modal analysis was
261 conducted in the first instance.

262

263 **Eigen frequencies and mode shapes**

264 An eigenvalue analysis was conducted in the model to extract the fundamental frequency of
265 (lateral) vibration for different depths of scour. A maximum scour depth of 10 m was
266 considered in the model and the difference in frequency between zero scour and this
267 maximum value is shown in Table 2. The results indicate that a scour depth of 10 m produced
268 a change in fundamental frequency of $\approx 40\%$ for the three soil stiffness profiles considered.

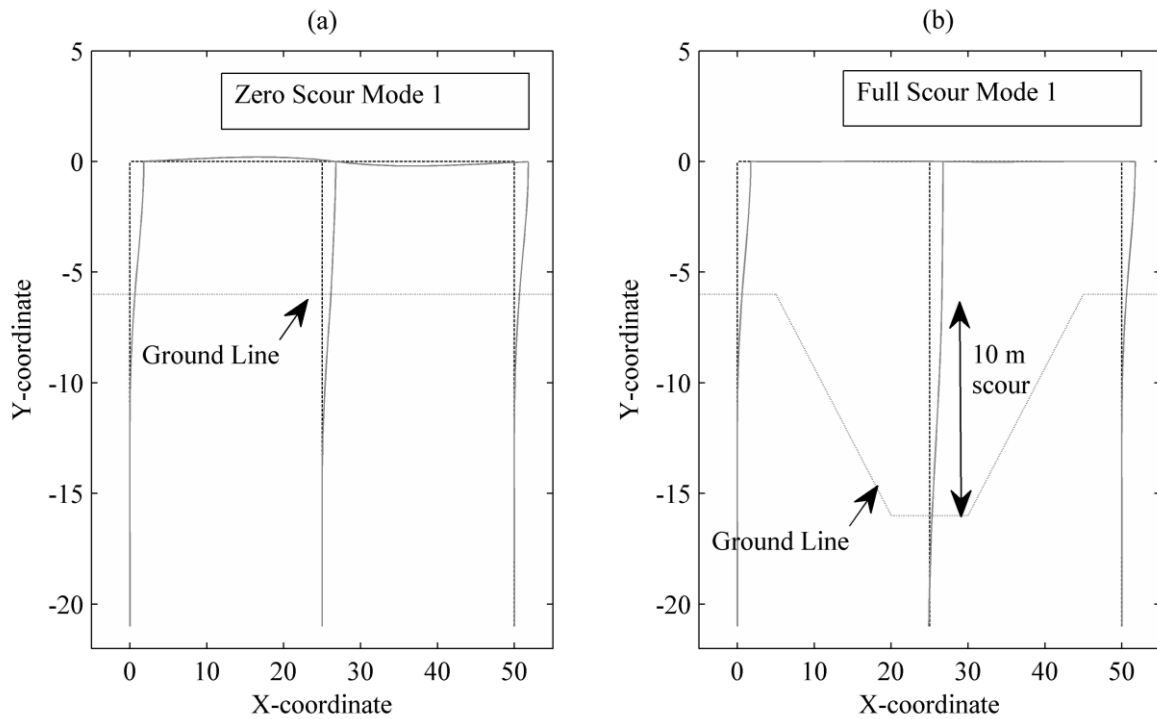
269 Once the expected shift in frequency due to scour was established, the next step was to
270 determine the optimum points on the structure to record accelerations to give the best
271 opportunity to capture the first mode of vibration of the integral bridge. By plotting the first
272 mode shape of the structure for zero scour and full pier scour, it is possible to obtain a
273 pictorial view of the locations showing the highest modal displacements for the fundamental
274 mode. Fig. 5 shows that the first mode shape for both zero scour and maximum pier scour
275 (10 m) is a global sway mode. The data shown in Fig. 5 was for the analysis performed in
276 loose sand. However, the shape was the same for all three soil stiffness profiles considered.
277 From the figure, it can be seen that the maximum modal amplitude occurs at deck level. In
278 this study the top of the bridge pier is used as the location to measure acceleration as it assists
279 in identifying the frequency when using signal processing and also aids with signal to noise
280 ratio (SNR) issues.

281

Table 2. Eigenvalue analysis of the scour effect.

Scour depth (m)	Frequency (loose sand) (Hz)	Frequency (medium dense sand) (Hz)	Frequency (dense sand) (Hz)
0 m	1.5643	1.6481	1.7357
10 m (full)	0.9386	0.9772	1.017
% Difference	-39.99%	-40.708%	-41.4%

282



283

284 **Fig. 5.** Fundamental mode shapes in loose sand – global sway. (a) zero scour (b) full scour.

285

286 **Response of structure to moving half-car vehicle model**

287 *Simulation of noise free pier accelerations due to the passage of a vehicle*

288 From the eigenvalue analysis in the previous section, it was observed that significant

289 reductions in natural frequency occurred due to scour of the central pile foundation system.

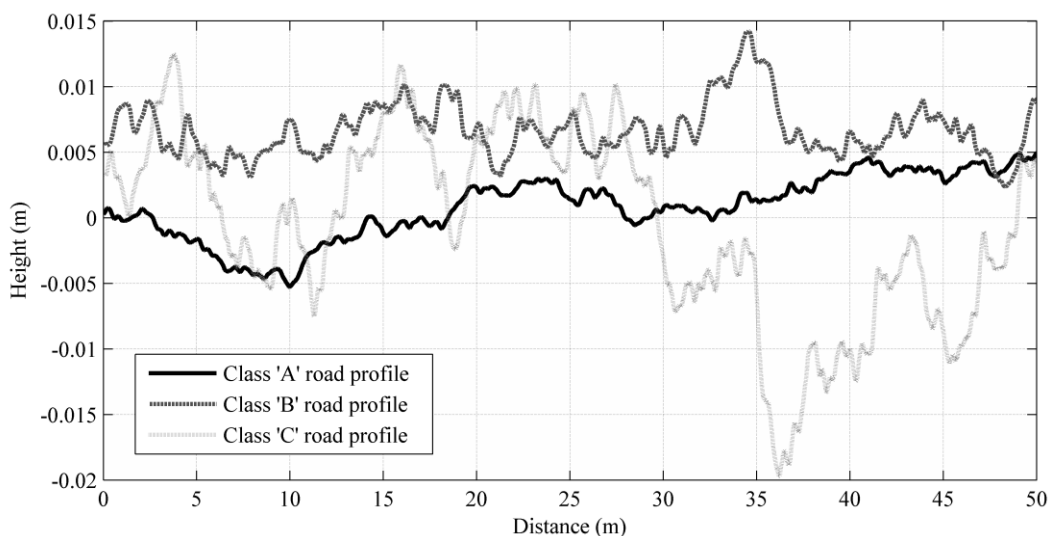
290 However, the fact that frequency changes will occur is of little use if the relevant mode is not

291 excited in the structure. The most practical way to excite a rail / highway bridge is to use

292 ambient traffic (Farrar et al. 1999). Therefore in this section the aim is to ascertain if it is

293 possible to detect these frequency changes by analysing the bridge acceleration response to a

294 moving sprung vehicle. In this analysis, accelerations generated at a lateral degree of freedom
295 near the top of the bridge pier are analysed using a fast Fourier transform to obtain the
296 frequency content. The vehicle modelled is a 15 tonne two-axle truck (see Table 1), and to
297 make the model as realistic as possible interaction between the vehicle and the bridge is
298 allowed for. The bridge is excited by the sudden arrival of the vehicle on the bridge deck,
299 which effectively acts as an impulse load.
300 The vehicle is a four-degree-of-freedom system that moves along the bridge deck. The
301 vehicle is excited by the presence of a road profile which causes the body to pitch and bounce
302 and this in turn means that the forces that the vehicle applies to the bridge are not constant. In
303 the model the vehicle commences movement at an approach distance of 100 m from the start
304 of the bridge so that the initial vehicle motion conditions (axle displacements and body
305 displacement / pitch) when the vehicle meets the bridge are more realistic. The road profile
306 used in the current analysis is a Class 'A' profile (well-maintained road surface, see Cebon
307 (1999)), and the part of the road profile on the bridge is reproduced in Fig. 6. This figure also
308 shows a Class 'B' and a Class 'C' road profile, in order of degrading quality.

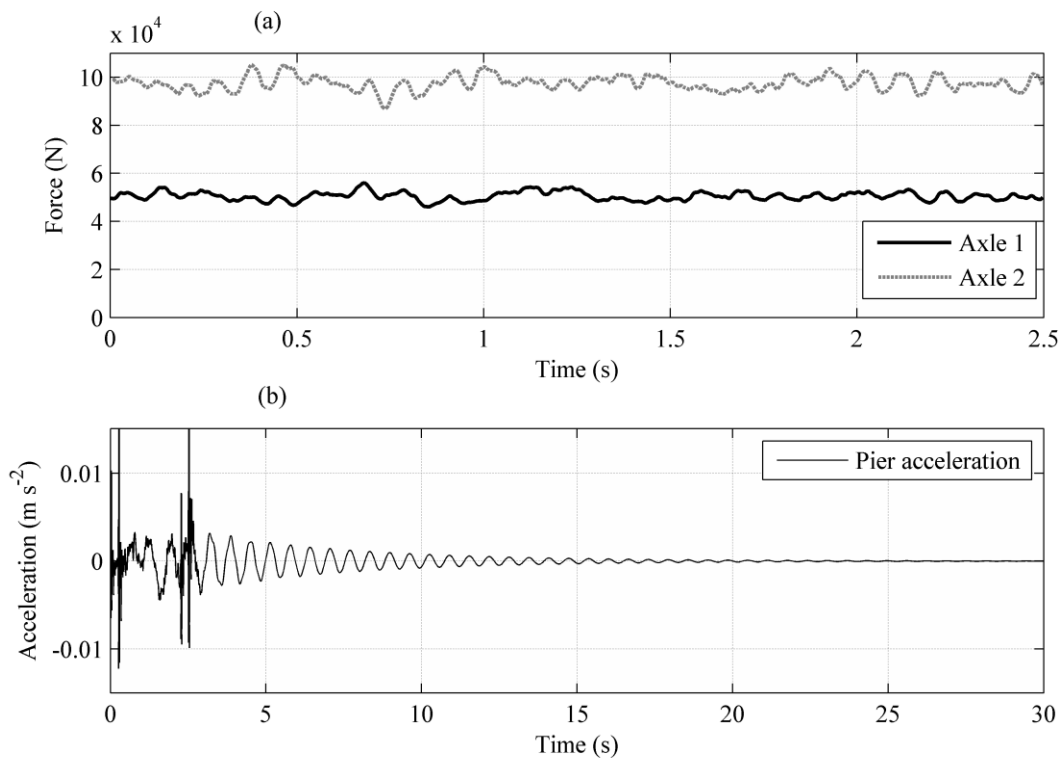


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Fig. 6. Road profiles on the bridge.

312 The vertical forces generated by the vehicle moving over the Class ‘A’ road profile are
313 shown in Fig. 7(a) for a vehicle speed of 80 km hr^{-1} , the loose sand soil profile and the case of
314 zero scour. In Fig. 7(a) it can be seen that the rear axle is significantly heavier than the front
315 axle and this is typical of a fully loaded 2 axle truck.

316 Fig. 7(b) shows the lateral acceleration response of the top of the pier when the truck crosses
317 the bridge (for the loose sand profile). The large peaks in acceleration at 0 seconds and 2.5
318 seconds correspond to the vehicle entering and leaving the bridge. After the vehicle leaves
319 the bridge there is a logarithmic decay in the acceleration signal over the following 27.5
320 seconds. This is to be expected as a damping ratio of 2% is used in the simulations.



321 **Fig. 7.** Results for vehicle crossing bridge for zero scour level and loose sand profile (a) axle
322 contact forces (b) lateral acceleration response at top of pier.

323

324

325

326 *The effect of noise on determining the frequency of the pier vibrations*

327 Real data will contain noise, so in this study, noise was added to the simulated signal. In
 328 order to check if the (scour detection) method was sensitive to the level of noise in the signal,
 329 signals with different levels of noise are analysed. The method used to add noise is based on
 330 the signal-to-noise ratio (SNR), given in Eq. (2) (Lyons 2011).

$$331 \quad SNR = 10 \log_{10} \frac{\text{Signal Power}}{\text{Noise Power}} \quad (2)$$

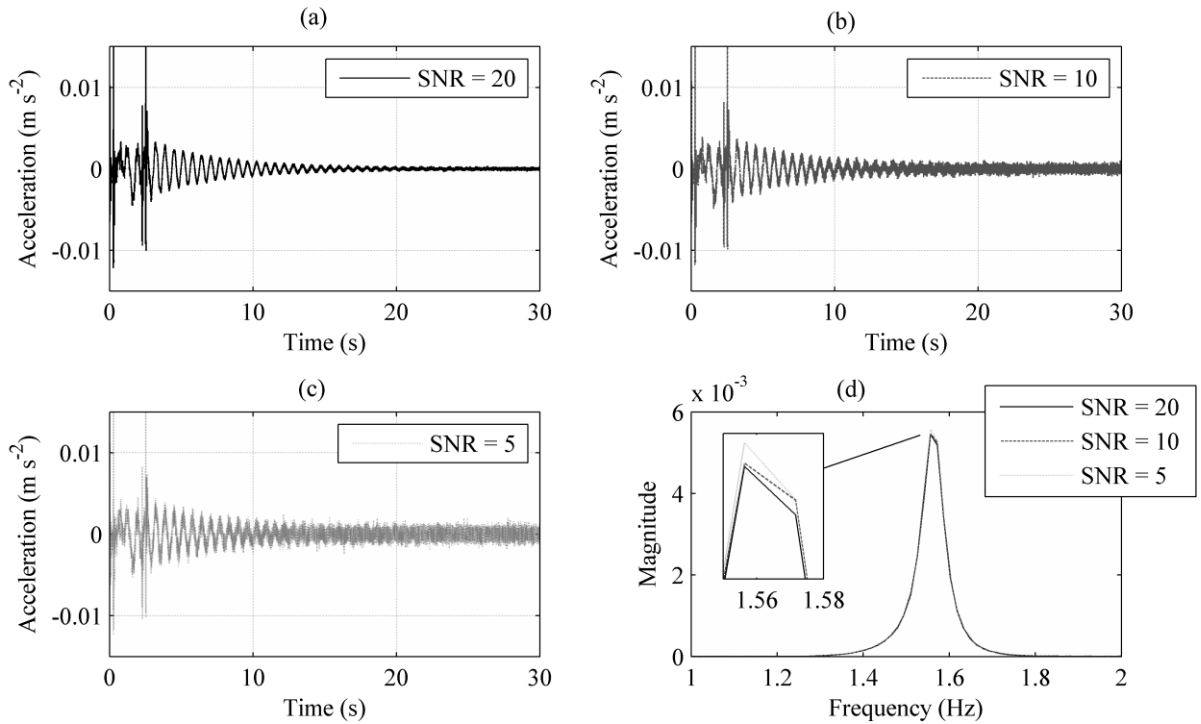
332 where SNR is the ratio of the strength of a signal carrying information equating to that of
 333 unwanted interference. Eq. (2) is rearranged to give Eq. (3).

$$334 \quad \sigma_N = \sqrt{\text{Noise Power}} = \sqrt{\frac{\text{Signal Power}}{\exp\left(\frac{SNR \cdot \log_e(10)}{10}\right)}} \quad (3)$$

335 where σ_N is the noise variance. Using Eq. (3), noise signals with different signal-to-noise
 336 ratios were added to the original clean signal. This process is shown in Eq. (4).

$$337 \quad \text{Sig}_{\text{NOISE}} = \sigma_N [\text{rand}] + \text{Sig}_{\text{CLEAN}} \quad (4)$$

338 In this study, three noise levels were examined, namely SNRs of 20, 10 and 5. Figs. 8(a-c)
 339 show the result of adding noise to the signal shown in Fig. 7(b). Fig. 8(d) shows the
 340 frequency content of the signals in Fig. 8(a-c). It can be seen in the figure that for all levels of
 341 noise the frequency plot is practically identical which proves that the method will not be
 342 particularly sensitive to noise. For the purpose of completeness, the figure has an insert which
 343 shows a zoomed in view of the frequency peak. In the insert it can be seen that that there are
 344 small differences in the frequency peak for the different levels of noise. However, in relative
 345 terms these differences are insignificant. Since noise does not impede the ability of the
 346 method to detect the frequency accurately, all analysis from this point will contain a SNR =
 347 20 as it is easier for the reader to interpret the remaining time domain plots for lower values
 348 of noise.



349

350 **Fig. 8.** Sensitivity of frequency content to noise. (a) signal from bridge pier with SNR = 20,

351 (b) signal with SNR = 10, (c) signal with SNR = 5, (d) frequency content of signals shown in

352

Figs. 8(a)-(c).

353

354 *Effect of vehicle properties, driving speed and road profile on detecting the frequency of the pier*
 355 *vibrations*

356 In the previous section, it was established that artificially added noise does not significantly

357 impede the method of detecting the first natural frequency of the structure (global sway) from

358 the pier accelerations due to a passing vehicle. However, the analysis in the previous section

359 only considers one set of vehicle properties, one driving speed and a Class ‘A’ road profile.

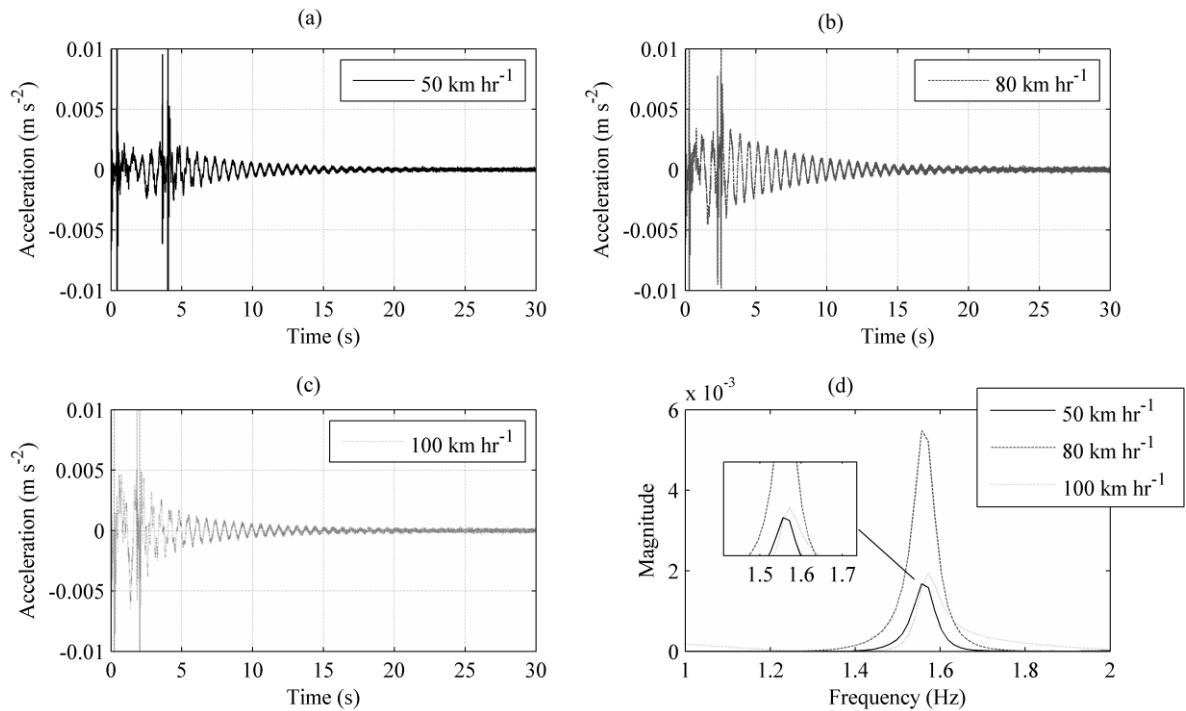
360 In this section, the effect of varying the driving speed, vehicle properties and road roughness

361 condition on the resilience of the method is investigated. Fig. 9 shows the effect of varying

362 the vehicle driving speed on the detected first natural frequency of the bridge. In this figure,

363 the vehicle traverses the bridge at 50, 80 and 100 km hr⁻¹ and the lateral acceleration signal

364 generated at the pier head is analysed for its frequency content. The vehicle traverses a Class
365 'A' road profile and the soil is assumed as loose sand with zero scour affecting the structure.
366 Fig. 9(a) shows the lateral acceleration of the pier head generated due to a vehicle passing at
367 50 km hr⁻¹. Similarly, Figs. 9(b) and (c) show the lateral acceleration of the pier head
368 generated due to a vehicle passing at 80 km hr⁻¹ and 100 km hr⁻¹ respectively. All signals
369 contain a SNR = 20. Fig. 9(d) shows the frequency content of the signals in Figs. 9(a-c). As is
370 evident, the frequency of the three signals is broadly in agreement (with minute differences
371 arising due to frequency resolution issues due to different signal lengths). The magnitude of
372 the frequency response differs between the three signals. This is as a result of interaction
373 effects between the vehicle travelling speed and the bridge's own dynamic motion. In short,
374 the rate at which the vehicle traverses the two-span bridge can either magnify or diminish the
375 bridge response depending on where the vehicle is on the bridge relative to the oscillation
376 cycle of the bridge itself, more information on this phenomenon is available in Prendergast et
377 al. (2016b). Overall, the frequency detection method is not particularly sensitive to vehicle
378 travelling velocity; therefore all further analyses in this paper are undertaken for a highway
379 speed of 80 km hr⁻¹.



380

381 **Fig. 9.** Sensitivity of frequency content to vehicle speed. (a) signal from bridge pier with
 382 vehicle speed = 50 km hr⁻¹, (b) signal with vehicle speed = 80 km hr⁻¹, (c) signal with vehicle
 383 speed = 100 km hr⁻¹, (d) frequency content of signals shown in Figs. 9(a)-(c).

384

385 The vehicle modelled in the simulations undertaken previously is a two axle truck, the
 386 properties of which are shown in Table 1. In order to assess if the vehicle properties have any
 387 noticeable effect on the ability of the method to detect the bridge's first frequency from
 388 vehicle induced lateral motion, a brief analysis is conducted herein. For this analysis, the
 389 vehicle whose properties are outlined in Table 1 (Veh 1) is run across the bridge and
 390 compared to a modified vehicle (Veh 2), which includes an altered front axle stiffness and
 391 gross body mass. The relevant properties of both vehicles are outlined in Table 3.

392

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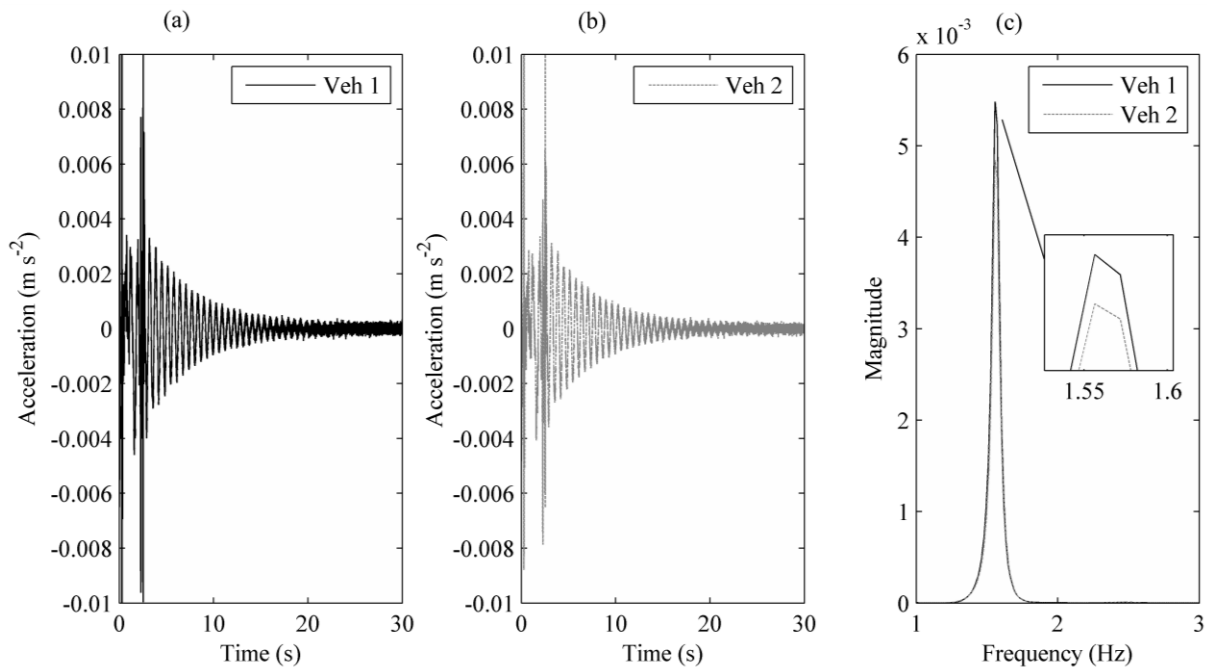
395

Table 3. Veh 1 and Veh 2 properties for sensitivity analysis.

	Veh 1	Veh 2
Gross body mass (kg)	13,300	9,000
Front axle stiffness (kN m⁻¹)	400	600
Body bounce frequency (Hz)	1.43	1.84

396

397 The result of running both vehicles over the bridge is shown in Fig. 10. Both vehicles traverse
 398 at 80 km hr⁻¹ over a bridge with zero scour, a loose sand profile and a Class ‘A’ road surface.
 399 Signals contain a SNR = 20. Fig. 10(a) shows the lateral pier head acceleration due to the
 400 passage of the original vehicle (Veh 1). Fig. 10(b) shows the lateral pier head acceleration
 401 due to the passage of the modified vehicle (Veh 2). Fig. 10(c) shows the frequency content of
 402 the signals in (a) and (b). As is evident, altering the vehicle properties does not significantly
 403 affect the frequency detection method, as the frequency is identical with only a minor change
 404 in magnitude. The analysis conducted here only considers a two-axle truck, however, so the
 405 effect for other vehicle types is not considered.

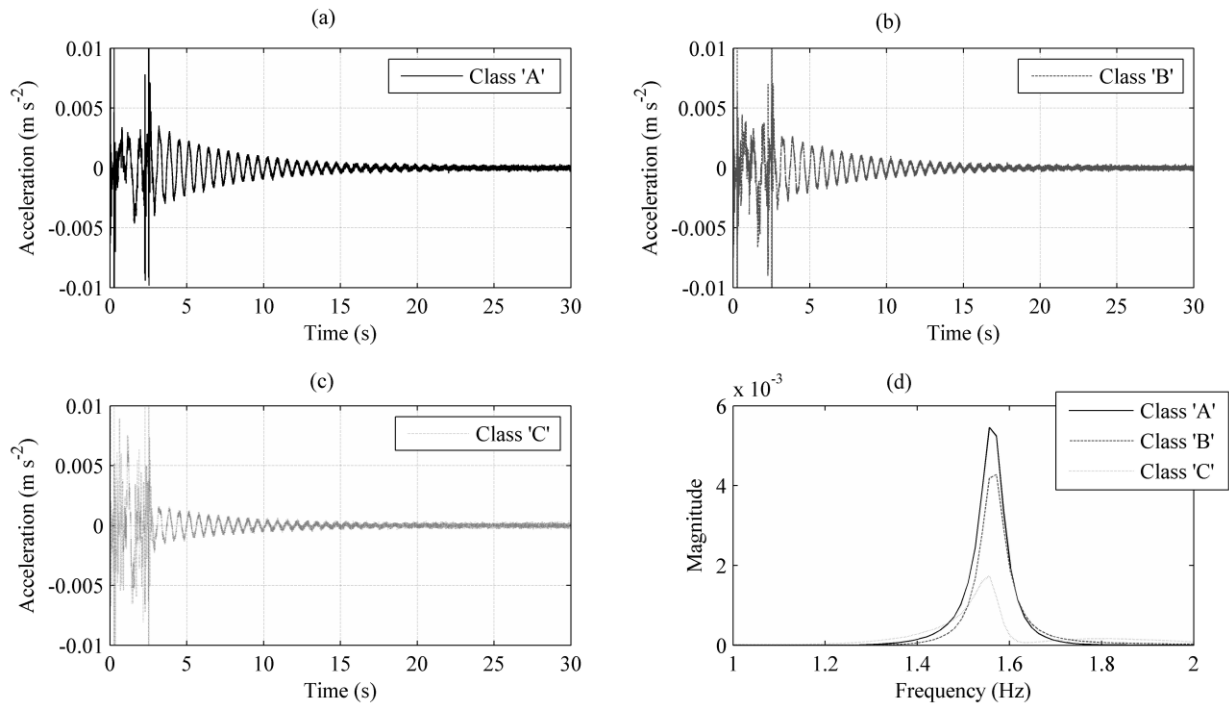


406

407 **Fig. 10.** Sensitivity of frequency content to vehicle mass and axle stiffness. (a) signal from
 408 bridge pier with original vehicle properties, (b) signal with modified vehicle properties (c)
 409 frequency content of signals shown in Figs. 10(a) and (b).

410

411 Finally, it is of interest to assess if a degrading road surface will impede the ability for the
412 first natural sway frequency of the bridge to be detected from vehicle induced vibrations. For
413 this analysis, the original vehicle (Veh 1) traverses the bridge over a Class ‘A’, ‘B’ and ‘C’
414 profile at 80 km hr^{-1} for the case of zero scour (see Fig. 6 for road profiles). All signals
415 contain a $\text{SNR} = 20$. The results are shown in Fig. 11. Fig. 11(a) shows the lateral pier head
416 acceleration due to the vehicle traversing a Class ‘A’ road profile. Similarly, Figs. 11(b) and
417 (c) show the lateral pier head accelerations measured due to a vehicle traversing Class ‘B’
418 and ‘C’ profiles respectively. Fig. 11(d) shows the frequency content of the signals presented
419 in parts (a) to (c) of the figure. The frequency peak corresponding to the first natural
420 frequency of the bridge is clearly detected in all three signals, with differences in magnitude
421 occurring for each road roughness profile. From this figure, it is clear that the presence of a
422 road roughness profile up to Class ‘C’ does not significantly impede the ability for the bridge
423 frequency peak to be detected (only very minor differences in frequency are detected due to
424 resolution of frequency bins). As a result, all analyses from here will utilise a Class ‘A’
425 profile, equivalent to a well-maintained highway surface. In the next section, the detection of
426 scour from pier head lateral accelerations is investigated.



427

428 **Fig. 11.** Sensitivity of frequency detection to road profile. (a) signal from bridge pier with
 429 Class 'A' road surface, (b) signal from bridge pier with Class 'B' road surface, (c) signal
 430 from bridge pier with Class 'C' road surface, (d) frequency content of signals in (a) to (c).

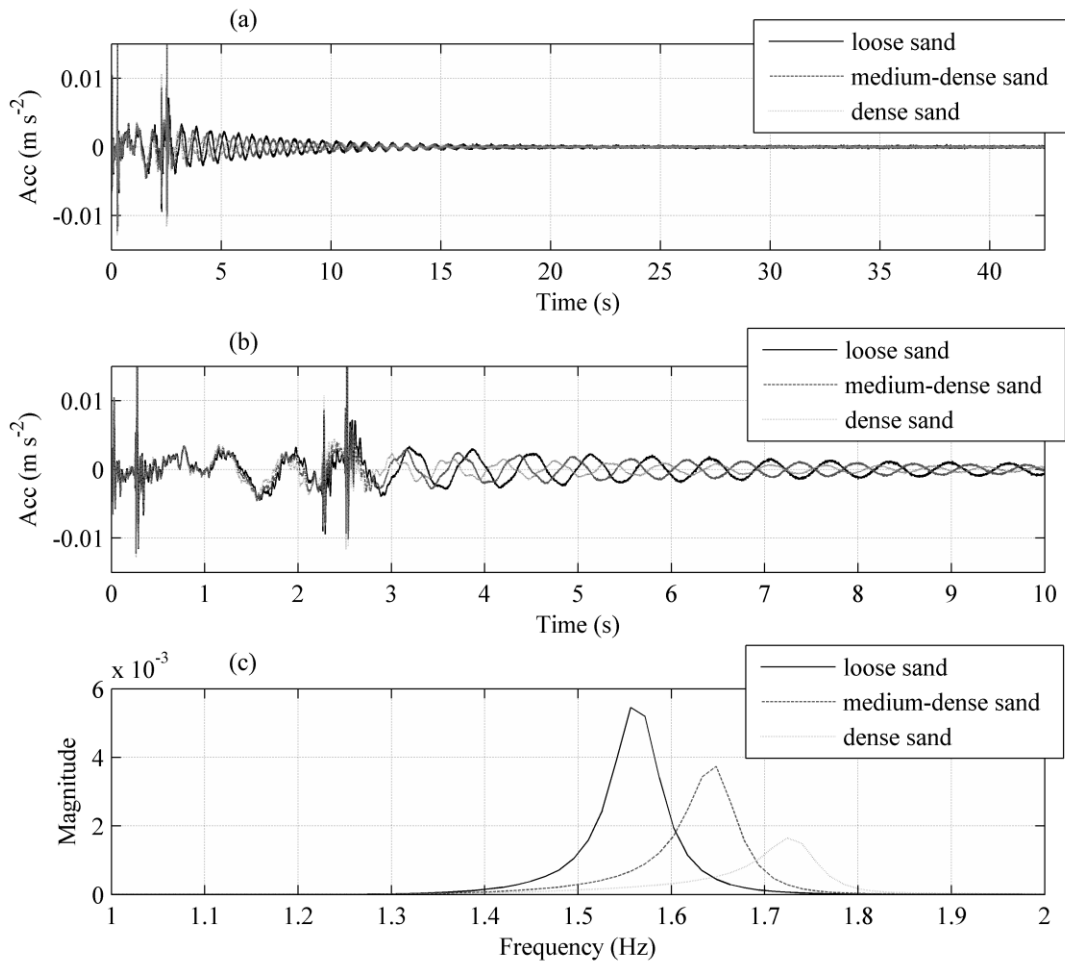
431

432

433 ***Identifying the presence of scour by analysing pier acceleration signals***

434 Fig. 12(a) shows the acceleration signal measured at the top of the bridge pier due to the
 435 passing vehicle for the three soil stiffness profiles considered for the case of zero scour. The
 436 vehicle traverses at 80 km hr⁻¹ over a Class 'A' road surface and the signals contain a SNR =
 437 20. The three signals in Fig. 12(a) are difficult to distinguish so Fig. 12(b) shows only the
 438 first 10 seconds of data. Fig. 12(c) shows the frequency content of the signals shown in Fig.
 439 12(a). From this figure, it is clear that it is possible to detect the first natural frequency of the
 440 bridge (which is lateral sway) for each of the soil stiffness profiles modelled. The difference
 441 in magnitude between each frequency peak is due to the relative stiffness of the soil impeding
 442 the lateral sway motion. The loose sand profile allows more movement than the dense sand

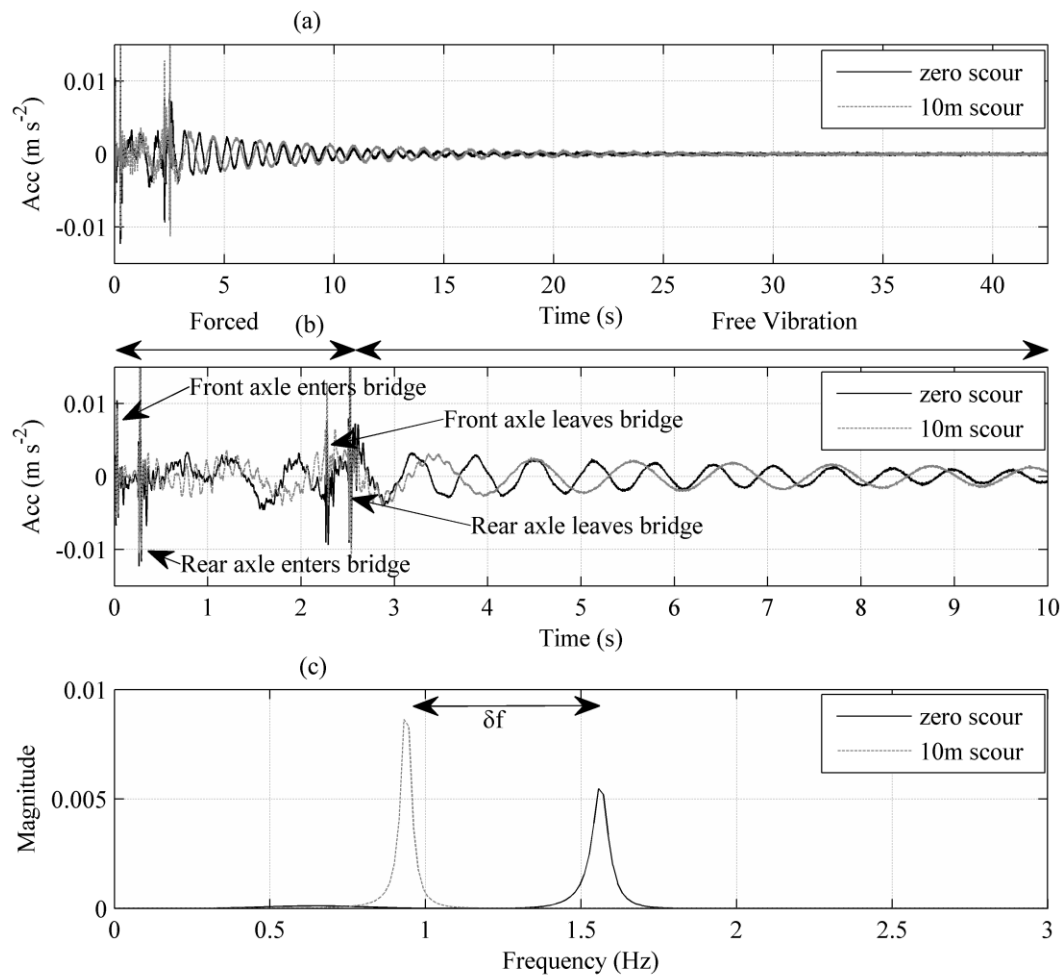
443 profile (due to the difference in spring stiffness); hence a higher peak was observed for the
444 loose sand.



445
446 **Fig. 12.** Bridge response due to passing vehicle and subsequent free vibration. (a)
447 acceleration response from bridge pier for loose, medium-dense and dense sand profiles with
448 40 seconds of free vibration; (b) acceleration response from bridge pier for loose, medium-
449 dense and dense sand profiles with 7.5 seconds of free vibration; (c) frequency response of
450 signals shown in (a).

451
452 Fig. 12 demonstrates that the natural frequency of mode 1 can be accurately determined by
453 analysing the acceleration response of the pier with a Fourier transform for all three soil

454 densities. The next step is to induce scour in the analysis and observe the change in
455 frequency. An example of this analysis is shown in Fig. 13. The analysis involved running the
456 vehicle over the bridge to generate an acceleration signal at the top of the bridge pier and
457 adding noise. This signal was then analysed with a fast Fourier transform to determine the
458 frequency content of the signal. A scour depth of 10 m is induced by removing springs from
459 around the central pier foundation and the process is repeated to generate a scoured signal.
460 The solid and dashed plots in Fig. 13(a) shows the acceleration signals generated at the top of
461 the bridge pier for the case of zero scour and the 10 m scour depth respectively, (for a loose
462 sand profile). For ease of visualising the signals, Fig. 13(b) shows just the first 10 seconds of
463 the pier acceleration responses. On the left hand side of this plot, a total of four impulses in
464 the acceleration signals (between $t = 0$ and $t = 2.5$ s) are visible. This corresponds to the front
465 and rear axles entering and leaving the bridge. The front axle enters the bridge at $t = 0$ s and
466 the rear axle leaves the bridge at $t = 2.5$ s. Fig. 13(c) shows the frequency content of the
467 signals shown in Fig. 13(a). It can be seen in Fig. 13(c) that the natural frequency for zero
468 scour was 1.556 Hz. It can also be seen in Fig. 13(c) that the natural frequency at the
469 maximum scour depth of 10 m was 0.9308 Hz. Therefore, a significant and measureable
470 reduction in natural frequency was observed.



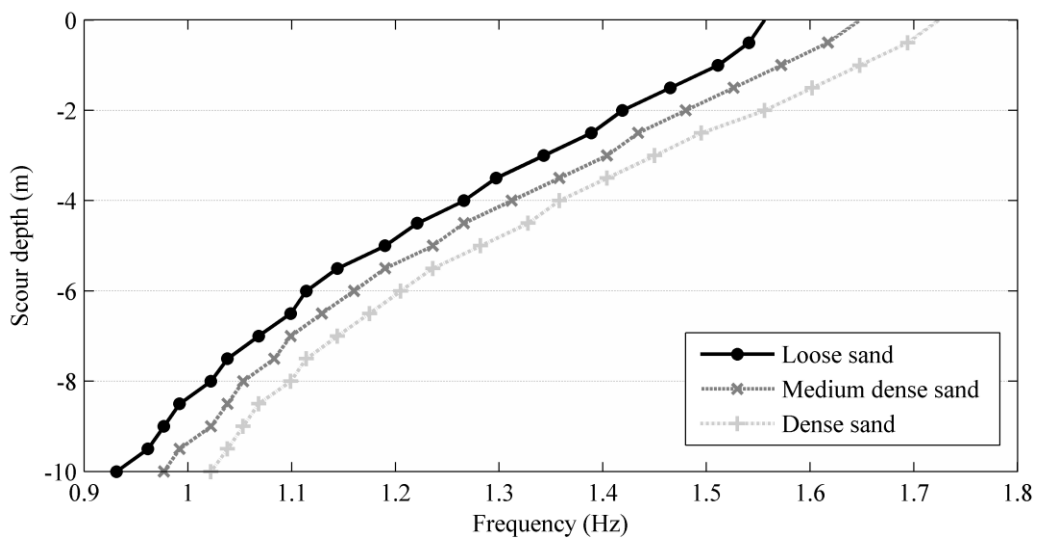
471

472 **Fig. 13.** Effect of 10 m of scour on the pier acceleration response for loose sand profile. (a)
 473 acceleration response (laterally) at top of bridge pier for zero and 10 m scour due to passage
 474 of vehicle, including 40 seconds of free vibration; (b) acceleration response of bridge pier
 475 with 7.5 seconds of free vibration; (c) frequency content of signals shown in (a).

476

477 By repeating the analysis for scour depths ranging from 0.5 m to 10 m, the natural frequency
 478 for each scour depth was determined. Scour was induced around the central pier piled
 479 foundation by removing springs iteratively from the model, this corresponds to an increase in
 480 scour depth and a loss of associated soil stiffness. A spring is removed and the vehicle is re-
 481 run across the bridge to generate a new acceleration signal, which is analysed for its
 482 frequency content. The variation in natural frequency with scour depth for the 'loose sand' is

483 shown by the solid plot with circular data markers in Fig. 14. Fig. 14 also shows the change
484 in the natural frequency plotted against the depth of scour for the ‘medium-dense sand’ and
485 ‘dense sand’ stiffness profiles. It is clear from this figure that for the three soil stiffness
486 profiles simulated, it was possible to detect a change in the natural frequency of the bridge
487 due to scour using vehicle induced vibrations. It is worth noting that the method was not
488 sensitive to soil stiffness (loose, medium-dense or dense) i.e. for all soil densities considered,
489 there is a clear reduction in natural frequency with increasing scour. Not surprisingly, the
490 magnitude of the frequency for a given scour depth varies with the soil stiffness. However,
491 the variation with soil stiffness is significantly less than the variation with scour depth. This
492 basically implies that the increase in effective length resulting from scour had a much larger
493 effect on the frequency response of the structure than changes in the stiffness of the soil
494 supporting the foundation.



495

496 **Fig. 14.** Frequency change with scour for all three soil stiffness profiles.

497

498

499 **Conclusion**

500 A field-validated model developed by the authors which is capable of tracking the change in
501 the natural frequency of a single pile affected by scour was extended in this paper to consider
502 the case of a full bridge subjected to traffic loading. A novel Vehicle-Bridge-Soil Interaction
503 (VBSI) model was developed to explore the potential frequency changes due to scour of an
504 integral bridge structure for a range of soil stiffnesses typically found in the field.

505 In the first instance, it was necessary to establish how scour affects the natural frequency of
506 the bridge and if the changes in frequency would be sufficiently large to warrant further
507 exploration of this method as a potential scour monitoring tool. A numerical modal study was
508 conducted to address this question. The aim of this study was to assess the magnitude of
509 frequency changes that can be expected for a typical bridge structure subjected to scour of the
510 central piles. From this study, the expected magnitude of the frequency shift was established
511 and deemed sufficiently large ($\approx 40\%$) to warrant an investigation into the feasibility of
512 detecting scour by analysing the bridge's response to a moving vehicle. The VBSI model was
513 used to generate realistic acceleration signals from the structure due to a two-axle truck
514 passing at typical highway speeds (80 km hr^{-1}). The lateral acceleration response at the top of
515 the bridge pier was analysed. Results indicate that for all three soil stiffness profiles modelled
516 (loose, medium-dense and dense sand) the response signals generated from this vehicular
517 loading are sufficient to allow the changes in natural frequency caused by scour to be
518 detected. Moreover, the shape of the scour depth vs frequency plot was the same for all three
519 soil stiffness profiles which shows that the method is not sensitive to soil stiffness.

520 Limitations in the analysis include the fact that only one type of vehicle was modelled,
521 namely a two-axle truck. Therefore the conclusions of the present study may only be relevant
522 for this vehicle type. Also, since the method relies on frequency changes of the bridge being
523 detected to infer the presence of scour, this method would be sensitive to other forms of

524 damage to the superstructure such as crack formation, thermal effects etc. Establishing the
525 exact mechanism causing the changes in frequency requires further study, and is not
526 addressed in this paper. The current paper serves as a feasibility study to detect the presence
527 of scour from vehicle-induced vibrations.

528 The method developed in this paper shows promise in terms of use as part of an infrastructure
529 management framework incorporating real-time low maintenance scour monitoring. The
530 advantage of the method is that it does not require complex underwater installations and
531 negates the requirement for dangerous diving inspections to monitor scour. The results
532 indicate that accelerometers fixed to the structure above the waterline may possibly be used
533 as a continuous scour monitoring solution. Real-time analysis of signals from a structure of
534 interest could be monitored for frequency changes or signals could be analysed before and
535 after major flood events to attempt to detect losses of stiffness caused by scour. Whilst this
536 appears promising, a full-scale application of the method on a real bridge is recommended as
537 future work.

538

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546

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