Interval Type–2 Fuzzy Decision Making

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Abstract

This paper concerns itself with decision making under uncertainty and the consideration of risk. Type-1 fuzzy logic by its (essentially) crisp nature is limited in modelling decision making as there is no uncertainty in the membership function. We are interested in the role that interval type–2 fuzzy sets might play in enhancing decision making. Previous work by Bellman and Zadeh considered decision making to be based on goals and constraint. They deployed type–1 fuzzy sets. This paper extends this notion to interval type–2 fuzzy sets and presents a new approach to using interval type-2 fuzzy sets in a decision making situation taking into account the risk associated with the decision making. The explicit consideration of risk levels increases the solution space of the decision process and thus enables better decisions. We explain the new approach and provide two examples to show how this new approach works.

Keywords:
fuzzy decision making, interval type–2 fuzzy sets

1. Introduction

In this paper we are concerned with decision making under uncertainty. In particular, we are interested in the role that interval type–2 fuzzy sets might play in enhancing decision making. In part, this has been motivated by our recent work on the properties of type-2 defuzzification operators (Runkler et al., 2015) where we explored the role of defuzzification of type–2 fuzzy sets in decision making. In particular that work explored the semantic meaning of interval type–2 fuzzy sets from the perspective of opportunity or risk, in respect to defuzzification operators. This led us to explore how risk could be modelled using interval type–2 fuzzy sets. Most fuzzy logic based risk research relates to applications of risk (e.g. (Mays et al., 1997; Malek et al., 2015)). We are interested in the notion of risk from the perspective of how different individuals might make decisions with their own notions of risk.

In the context of this work, by decision making we mean where we have a goal(s) that is limited by some constraints. In the case of type–1 fuzzy sets the fuzzy decision making process finds an optimal decision when goals and constraints are specified by fuzzy sets (Zadeh, 1965). A type–1 fuzzy set is defined by a membership function $u : X \to [0, 1]$. So, they are by their very nature crisp and there is no uncertainty around the membership function. In this paper we will always consider fuzzy sets over one–dimensional continuous intervals $X = [x_{\min}, x_{\max}]$. An interval type–2 fuzzy set (Zadeh, 1975; Liang and Mendel, 2000; Mendel et al., 2006) $\tilde{A}$ is defined by two membership functions[1], a lower membership function $\underline{u}_{\tilde{A}} : X \to [0, 1]$ and an upper

[1]Interval type–2 fuzzy sets are known to be equivalent to interval–fuzzy sets (Gorzal-

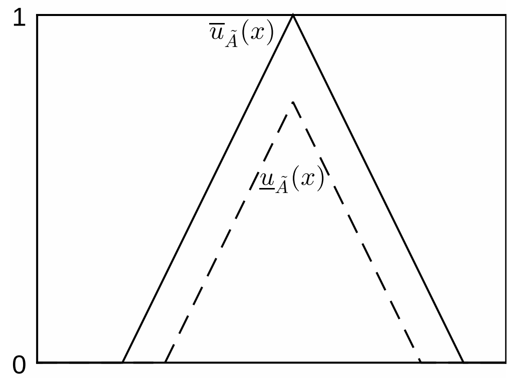

Figure 1: Interval type–2 fuzzy set.

membership function $\overline{u}_{\tilde{A}} : X \to [0, 1]$, where

$$\underline{u}_{\tilde{A}}(x) \leq \overline{u}_{\tilde{A}}(x) \tag{1}$$

for all $x \in X$. Fig. 1 shows an example of a triangular interval type–2 fuzzy set and its upper (solid) and lower (dashed) membership functions. Fuzzy decision making using type–1 fuzzy sets was introduced by Bellman and Zadeh (1970). Given a set of goals specified by the membership functions

$$\{u_{g_1}(x), \ldots, u_{g_m}(x)\} \tag{2}$$

and a set of constraints specified by the membership functions

$$\{u_{c_1}(x), \ldots, u_{c_n}(x)\} \tag{3}$$

the optimal decision x^* is defined as

$$x^* = \underset{x \in X}{\operatorname{argmax}} \Big(u_{g_1}(x) \wedge \ldots \wedge u_{g_m}(x) \ \wedge \ u_{c_1}(x) \wedge \ldots \wedge u_{c_n}(x) \Big) \tag{4}$$

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czany, 1987; Gehrke et al., 1996).

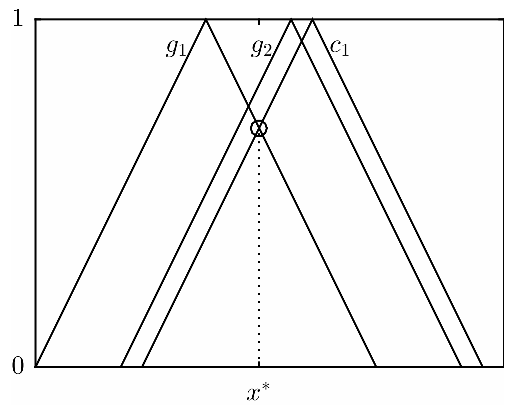

Figure 2: Type–1 fuzzy decision.

where $\wedge$ is a triangular norm such as the minimum or the product operator. In the experiments presented in section 4 we will use the minimum operator. Fig. 2 shows an example of a type–1 fuzzy decision with two type–1 triangular goals g_1, g_2 and one triangular constraint c_1. Notice that in fuzzy decision making goals and constraints are treated in the same way, so we do not need to explicitly distinguish between goals and constraints.

Successful applications of type-1 fuzzy decision making include environmental applications such as water resource planning (Afshar et al., 2011) or waste management (Kara, 2011), infrastructure planning applications such as energy system planning (Kaya and Kahraman, 2010) or location management (Guneri et al., 2009), logistic applications such as supplier selection (Bottani and Rizzi, 2008), transportation planning (He et al., 2012), fuzzy data fusion (Shell et al., 2010) or optimisation of logistic processes (Sousa et al., 2002).

In this paper we provide a new fuzzy decision making approach using interval type–2 fuzzy sets within the context of risk. Chen and Wang (Chen and Wang, 2013, 2011) deploy interval type-2 fuzzy sets to aid decision making through a ranking mechanism and fuzzy multiple attributes decision making. Multi-Criteria Group Decision Making and type-2 fuzzy sets are explored by Naim and Hagras (Naim and Hagras, 2015) in an extensive comparison of different approaches. They are interested in where groups make decisions. Lascio et al. (Di Lascio et al., 2007) take a formal mathematical approach to type-2 fuzzy decision making. Zhang and Zhang (Zhang and Zhang, 2012) extend so called soft sets to type-2 fuzzy sets and provide limited examples of type-2 fuzzy soft sets in decision making. An example application is that of using type-2 fuzzy sets in multi-criteria decision making for choosing energy storage (Ozkan et al., 2015).

The decision making research using type-2 fuzzy sets does not align the decision making with the notion of risk. When making a decision our attitude to risk affects our decision making. Our approach then is to consider risk and decision making and provide an interval type-2 fuzzy set approach to that.

The rest of the paper is structured as follows: Section 2 provides an overview of interval type-2 fuzzy decision making; Section 3 discusses the properties of this type of decision making; Section 4 provides examples of the use of the approach and Section 5 provides some closing remarks.

2. Interval Type–2 Fuzzy Decision Making

In type–1 fuzzy decision making the membership values of the goals and constraints quantify the degrees of utility of the different decision options. In interval type–2 fuzzy decision making the utility is subject to uncertainty.

The upper and lower membership values of each option quantify the lower bound (worst case) and upper bound (best case) of the corresponding utility, respectively. Hence, it is straightforward to define the worst case interval type–2 fuzzy decision as

$$x^{\underline{*}} = \operatorname*{argmax}_{x \in X} \Big(\underline{u}_{\tilde{g_1}}(x) \wedge \ldots \wedge \underline{u}_{\tilde{g_m}}(x) \ \wedge \ \underline{u}_{\tilde{c_1}}(x) \wedge \ldots \wedge \underline{u}_{\tilde{c_n}}(x) \Big) \tag{5}$$

and to define the best case interval type–2 fuzzy decision as

$$x^{\overline{*}} = \operatorname*{argmax}_{x \in X} \Big(\overline{u}_{\tilde{g_1}}(x) \wedge \ldots \wedge \overline{u}_{\tilde{g_m}}(x) \ \wedge \ \overline{u}_{\tilde{c_1}}(x) \wedge \ldots \wedge \overline{u}_{\tilde{c_n}}(x) \Big) \tag{6}$$

The worst case interval type–2 fuzzy decision maximizes the utility that is obtained under the worst possible conditions. This decision policy reflects a cautious or pessimistic decision maker. The best case interval type–2 fuzzy decision maximizes the utility that is obtained under the best possible conditions. This decision policy reflects a risky or optimistic decision maker. Fig. 3 shows an example of worst case and best case type–2 fuzzy decisions with two type–2 triangular goals $\tilde{g_1}$, $\tilde{g_2}$ and one triangular constraint $\tilde{c_1}$. We do not want to restrict the interval type–2 fuzzy decision to the worst case and best case decisions but we want to allow to specify the level of risk $\beta \in [0, 1]$ associated with the decision, where risk $\beta = 0$ corresponds to the worst case decision $x^{\underline{*}}$ and risk $\beta = 1$ corresponds to the best case decision $x^{\overline{*}}$. This leads us to define the interval type–2 fuzzy decision at risk level β as

$$x^*_\beta = \operatorname*{argmax}_{x \in X} \Big(((1-\beta) \cdot \underline{u}_{\tilde{g_1}}(x) + \beta \cdot \overline{u}_{\tilde{g_1}}(x))$$

$$\wedge \ldots \wedge ((1-\beta) \cdot \underline{u}_{\tilde{g_m}}(x) + \beta \cdot \overline{u}_{\tilde{g_m}}(x))$$

$$\wedge ((1-\beta) \cdot \underline{u}_{\tilde{c_1}}(x) + \beta \cdot \overline{u}_{\tilde{c_1}}(x))$$

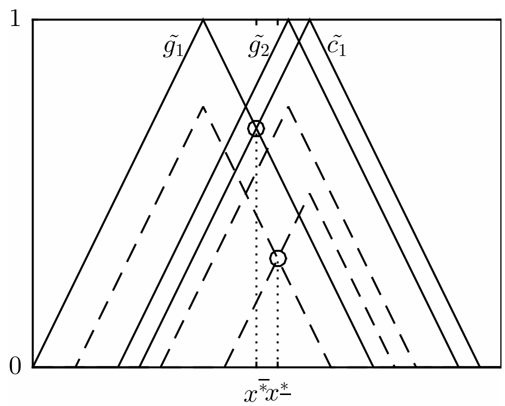

Figure 3: Interval type–2 fuzzy decisions.

$$\wedge \ldots \wedge \left((1-\beta) \cdot \underline{u}_{\tilde{c_n}}(x) + \beta \cdot \overline{u}_{\tilde{c_n}}(x)\right)\Big) \tag{7}$$

It is worth noting the relationship between equations (5) and (6) and the intersection operator. The worst case decision computed in equation (5) may also be calculated through the intersection operator when using the same t-norm as used by the $\wedge$ operator in equations (5), (6) and (7). Equations (5) and (6) find the maximum value across the domain X from the minimum of all the membership functions at a domain point x. This could equally be obtained by finding the highest membership grade across the domain of a fuzzy set which is the intersection of all goals and constraints. Let this fuzzy set f be calculated by equation (8) below.

$$\tilde{f} = \tilde{g_1} \cap \ldots \cap \tilde{g_m} \cap \tilde{c_1} \cap \ldots \cap \tilde{c_n} \tag{8}$$

Figure 4 depicts the intersection of a single goal and constraint with the points $x^{\underline{*}}$ and $x^{\overline{*}}$ highlighted by circles. The approach leads to equations (9) and (10) giving alternative ways of calculating the respective worst and best

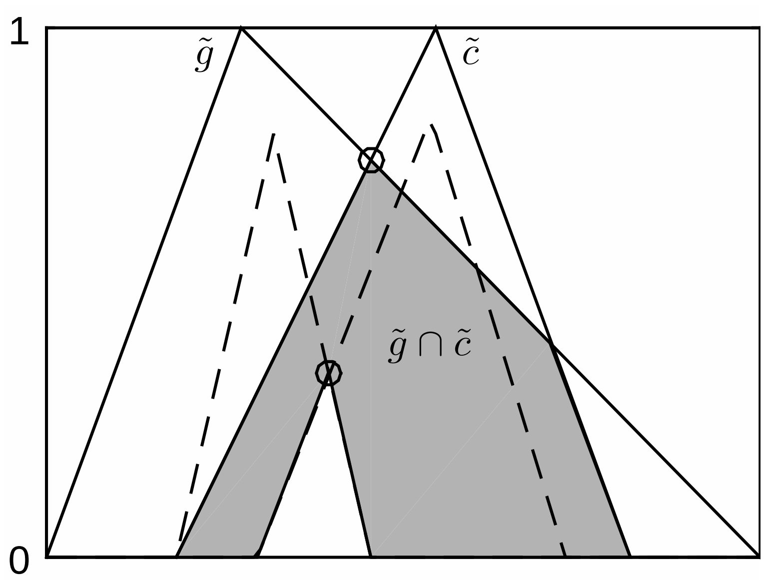

Figure 4: The intersection of an interval type-2 fuzzy goal and constraint

case decisions.

$$x^{\underline{*}} = \underset{x \in X}{\text{argmax}}(\underline{\tilde{f}}(x)) \tag{9}$$

$$x^{\overline{*}} = \underset{x \in X}{\text{argmax}}(\overline{\tilde{f}}(x)) \tag{10}$$

We can use equations (9) and (10) to calculate the decision for given risk value β using equation 11.

$$x^*_\beta = \text{argmax}\left((1-\beta) \cdot \mu_{\underline{\tilde{f}}}(x) + \beta \cdot \mu_{\overline{\tilde{f}}}(x))\right) \tag{11}$$

where $\beta \in [0, 1]$. The next section explores some properties of this approach.

3. Properties of Interval Type–2 Fuzzy Decision Making

In this section we investigate in some detail the properties of the interval type–2 fuzzy decision at risk level β defined by (7).

It is easy to see that $x_0^* = x^{\underline{*}}$ and $x_1^* = x^{\overline{*}}$. It seems reasonable to require that for any risk level $\beta \in [0,1]$ the decision should be in the interval bounded by the worst case decision $x^{\underline{*}}$ and the best case decision $x^{\overline{*}}$, so

$$\min\left(x^{\underline{*}}, x^{\overline{*}}\right) \leq x_\beta^* \leq \max\left(x^{\underline{*}}, x^{\overline{*}}\right) \tag{12}$$

for arbitrary t–norms $\wedge$.

We now consider whether equation (12) holds for all fuzzy sets, placing no constraints on the membership functions. For simplicity consider a decision with only one goal and no constraint, so for the decision we consider only one single type–2 fuzzy set, and we don't have to worry about the t–norm $\wedge$. Fig. 5 shows an example of such a type–2 fuzzy set where the maximum of the upper membership function (solid) is at $x^{\underline{*}} = 0.5$, the maximum of the lower membership function (dashed) is at $x^{\overline{*}} = 1$, but where for the risk level $\beta = 0.5$ (dotted) we obtain the decision $x_{0.5}^* = 0$, which is outside the interval between the worst case and the best case, i.e. $x_{0.5}^* \notin [x^{\underline{*}}, x^{\overline{*}}]$. This example proves that (12) does not hold in general.

We will now consider equation (12) for interval type-2 fuzzy set whose membership functions are convex. By convex we mean both the upper and lower membership functions are convex. Consider the two convex interval type-2 fuzzy sets $\tilde{g}_1$ and $\tilde{c}_1$ over the domain X. We know that taking the minimum or the product of two convex functions will always yield a convex function. Therefore $\underline{\tilde{g}_1} \cap \underline{\tilde{c}_1}$ and $\overline{\tilde{g}_1} \cap \overline{\tilde{c}_1}$ must yield convex functions when using the product or minimum t-norm. Let $\tilde{f} = \tilde{g}_1 \cap \tilde{c}_1$ as with equation(8). It is obvious that the lower membership function of $\tilde{f}$ is contained by the upper membership function of $\tilde{f}$ i.e. $\overline{\tilde{f}}(x) \geq \underline{\tilde{f}}(x)$, $\forall x \in X$. We can now show that (12) holds for convex sets when using the minimum and product

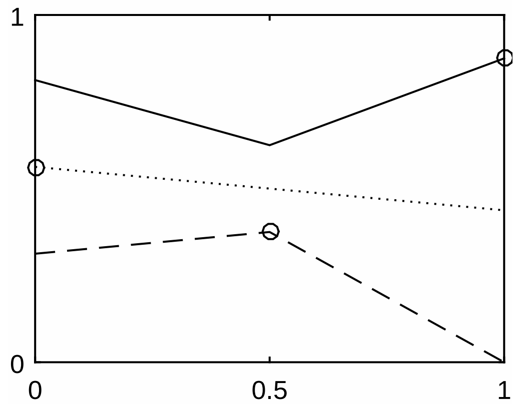

Figure 5: Nonconvex example for an interval type–2 fuzzy decision.

t-norms. First divide the domain X into three distinct regions.

- Region I: $\min\left(x^{\underline{*}}, x^{\overline{*}}\right) \leq x \leq \max\left(x^{\underline{*}}, x^{\overline{*}}\right)$
- Region II : $x < \min\left(x^{\underline{*}}, x^{\overline{*}}\right)$
- Region III : $\max\left(x^{\underline{*}}, x^{\overline{*}}\right) < x$

These regions are depicted in Figure 6. For any value of x^*_β to be outside region I it must be in either region II or III. For x^*_β to be in region II the derivative of either function must negative with respect to x. Since both functions are convex this is impossible. For x^*_β to be in region III the derivative of either function must positive with respect to x. Since both functions are convex this is impossible. Therefore any value of x^*_β must be in region I. This completes the proof.

There is a caveat we must add to this discussion which is that x^*_β is only non zero when x is in the support of the intersection of all the goals and constraints. If the intersection is an empty set we have no decision agreement.

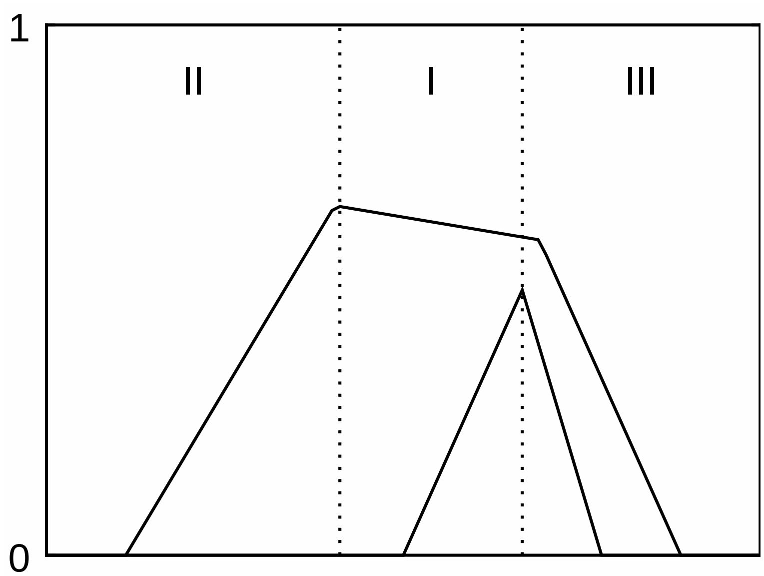

Figure 6: Regions in a pair of convex functions.

The next section looks at two examples as to how this decision making approach works.

4. Application Examples

In this section we illustrate our proposed interval type–2 fuzzy decision making approach with two application examples: optimization of the room temperature and choosing optimal travel times with low road congestion.

For the first application example assume you have invited two guests, A and B, and wonder to which room temperature you should set the heater. You know that A will be completely happy with 17 degrees, and will be completely unhappy at less than 16 degrees or more than 19 degrees. And B will be completely happy with 20 degrees, and will be completely unhappy for less than 18 degrees or more than 22 degrees. This can be modeled using

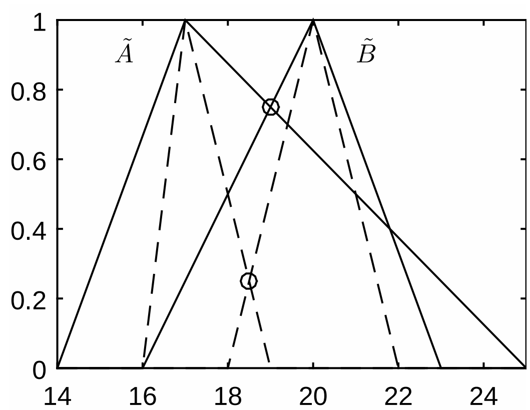

Figure 7: Interval type–2 fuzzy decision for the temperature example.

the interval type–2 fuzzy sets shown in Fig. 7, where the upper membership functions for $\tilde{A}$ and $\tilde{B}$ are shown as solid triangles and the lower membership functions for $\tilde{A}$ and $\tilde{B}$ as dashed triangles. Now a cautious decision maker will set the temperature to 18.5 degrees (lower circle, at the intersection of the lower membership functions, dashed), because then none of the guests will be less happy than 25%. And a risky decision maker will set the temperature to 19 degrees (upper circle, at the intersection of the upper membership functions, solid), because in the best case both guests will be 75% happy. Intermediate levels of risk between $\beta = 0$ and 1 will yield optimal temperatures between 18.5 and 19 degrees.

For the second application example assume that we want to drive to work at some time between 6 and 12 o'clock, work for 8 hours, and then drive back. From a traffic reporting system we have obtained the traffic density curves for the 10 previous work days that are shown in Fig. 8. These curves represent, in our view, a sensible view of typical daily traffic density. Note they are non

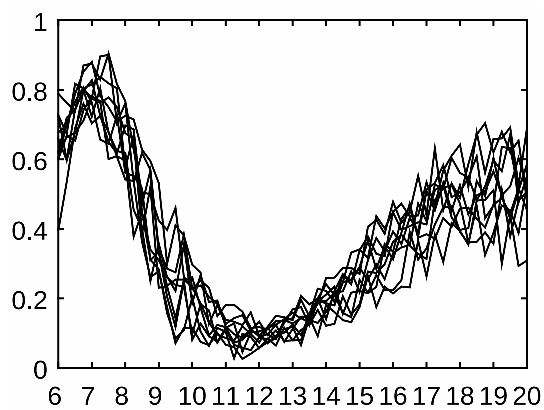

Figure 8: Traffic example: Observed traffic densities.

convex and that, as is typical, the uncertainty at the beginning and end of the day is larger.

We start with a type–1 fuzzy approach to model this situation and find an optimal decision. Based on the observed traffic densities we estimate the average traffic densities using a mixture of two Gaussian membership functions as

$$u(x) = 0.775 \cdot e^{-\left(\frac{x-7\cdot 60}{133}\right)^2} + 0.525 \cdot e^{-\left(\frac{x-19\cdot 60}{290}\right)^2} \tag{13}$$

Fig. 9 left shows a plot of this membership function which may be associated with the linguistic label "traffic", so for example at 7 o'clock we have 0.775 traffic. We want to drive to work some time between 6 and 12 o'clock, so for the morning traffic we consider the part of the membership function for the time between 6:00 and 12:00 (solid curve in Fig. 9 right). We want to drive back after 8 hours of work, so for the evening traffic we consider the part of the membership function for the time between 14:00 and 20:00, shifted 8 hours to the left (dashed curve in Fig. 9 right). If we do the morning trip

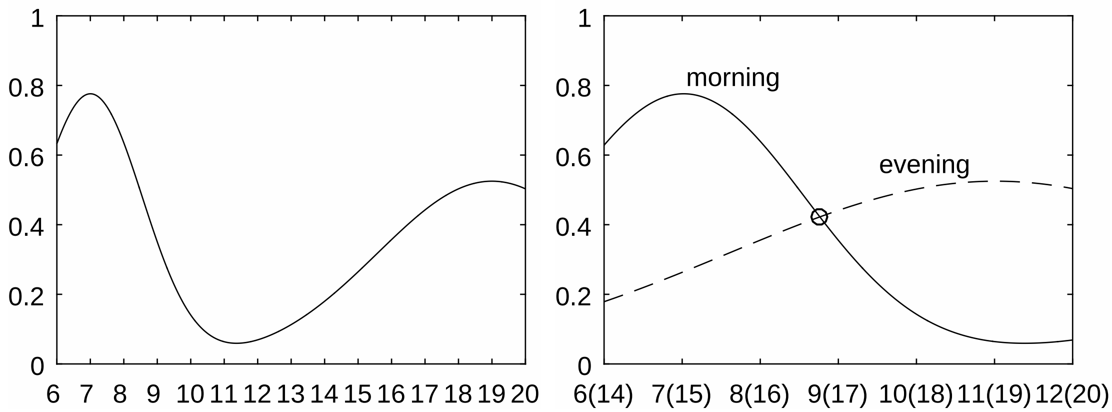

Figure 9: Traffic example: Type–1 fuzzy membership function of the traffic (left) and type–1 fuzzy decision (right).

at 7:00 and the evening trip at 15:00, for example, then we will have 0.775 traffic in the morning and about 0.26 traffic in the evening. Our goal is to find a travel time, where the traffic in the morning is low and the traffic in the evening is low. This yields a fuzzy decision with two goals that correspond to the two membership functions shown in in Fig. 9 right. In contrast to the first example we are looking for the minimum, not the maximum memberships, so we replace the argmax in the decision function by argmin. The optimal type–1 fuzzy decision (marked by a circle) is at 8:46 (return 16:46) with a traffic of 0.42 for both the morning and the evening trips.

Next, we consider a type–2 fuzzy approach for this problem. We estimate the minimum and maximum bounds of the traffic densities as

$$\overline{u}(x) = 0.95 \cdot e^{-\left(\frac{x-7\cdot 60}{3\cdot 60}\right)^2} + 0.75 \cdot e^{-\left(\frac{x-19\cdot 60}{3\cdot 60}\right)^2} \tag{14}$$

$$\underline{u}(x) = 0.6 \cdot e^{-\left(\frac{x-7\cdot 60}{1.5\cdot 60}\right)^2} + 0.3 \cdot e^{-\left(\frac{x-19\cdot 60}{4.5\cdot 60}\right)^2} \tag{15}$$

which represent the lower (dashed) and upper (solid) membership functions

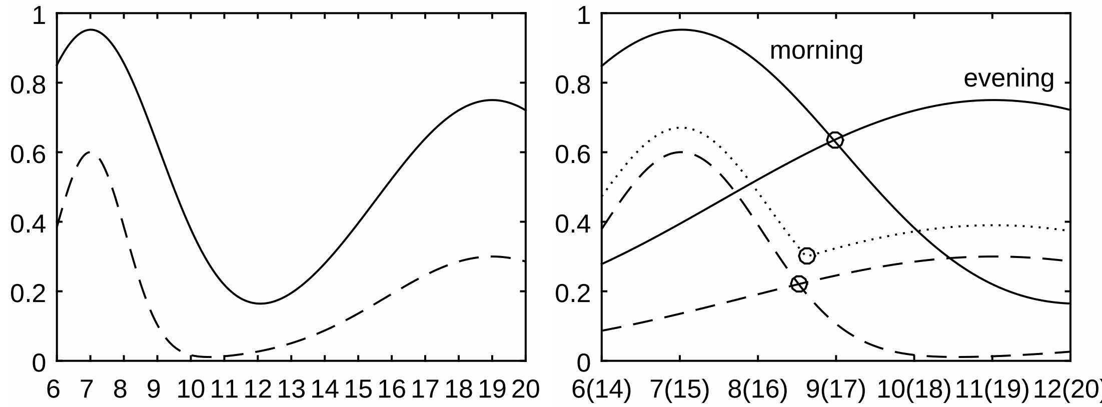

Figure 10: Traffic example: Interval type–2 fuzzy membership function of the traffic (left) and type–2 fuzzy decision (right).

of the interval type–2 fuzzy membership function of the traffic, as shown in Fig. 10 left. The lower and upper membership functions for both the morning and evening trips are shown in Fig. 10 right. The three circles show three type–2 fuzzy decisions at different risk levels. A cautious decision maker will drive to work at 8:59 and back at 16:59 (upper circle), because the worst case traffic is about 0.64. A risky decision maker will drive to work at 8:32 and back at 16:32 (lower circle), because the best case traffic is about 0.22. For intermediate levels of risk the optimal decision will be to leave between 8:32 and 8:59 and return 8 hours later. For example, for risk level $\beta = 0.8$ we obtain the dotted curve which is minimized for leaving at 8:37 and returning at 16:37 with a traffic of about 0.3.

A comparison of the type–1 and type–2 fuzzy decisions is shown in Fig. 11. The two almost linear solid curves show the worst case and best case traffic for the morning trip times between 8:30 and 9:00, corresponding to evening

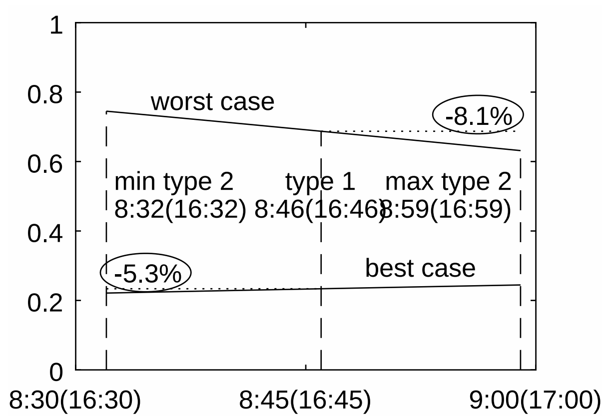

Figure 11: Traffic example: Comparison of the worst case and best case traffic for the type–1 and type–2 fuzzy decisions.

trip times between 16:30 and 17:00. The middle dashed line at 8:46(16:46) corresponds to the type–1 fuzzy decision, where the worst case traffic is about 0.69 and the best case traffic is about 0.23. The left dashed line at 8:32(16:32) corresponds to a risky decision maker who picks the minimum type–2 fuzzy decision, where the best case traffic is 5.3% lower than the best case for the type–1 fuzzy decision. The right dashed line at 8:59(16:59) corresponds to a cautious decision maker who picks the maximum type–2 fuzzy decision, where the worst case traffic is 8.1% lower than the worst case for the type–1 fuzzy decision. So if we specify a risk level that we are willing to accept, then type–2 fuzzy decision making can take this risk level into account and may therefore yield better results than type–1 fuzzy decision making.

5. Conclusions

Existing approaches supporting decision making using type-2 fuzzy sets ignore the risk associated with these decisions. In this paper we have presented a new approach to using interval type–2 fuzzy sets in decision making with the notion of risk. The method extends the work of Bellman and Zadeh (1970) by replacing the type–1 fuzzy sets with interval type–2 fuzzy sets. This brings an extra capability to model more complex decision making, for example, allowing trade-offs between different preferences and different attitudes to risk. The explicit consideration of risk levels increases the solution space of the decision process and thus enables better decisions. In a traffic application example, the quality of the obtained decision could be improved by 5.3–8.1%.

The paper explores some of the properties of this new approach and with two examples shows how it works. We will follow on this work by tackling larger, more complex, problems as well as investigating the properties in more detail.

References

Afshar, A., Mariño, M. A., Saadatpour, M., Afshar, A., 2011. Fuzzy TOPSIS multi-criteria decision analysis applied to Karun reservoirs system. Water Resources Management 25 (2), 545–563.

Bellman, R., Zadeh, L., 1970. Decision making in a fuzzy environment. Management Science 17 (4), 141–164.

Bottani, E., Rizzi, A., 2008. An adapted multi-criteria approach to suppliers and products selection an application oriented to lead-time reduction. International Journal of Production Economics 111 (2), 763–781.

Chen, S.-M., Wang, C.-Y., 2013. Fuzzy decision making systems based on interval type-2 fuzzy sets. Information Sciences, Volume 242, 1 September 2013, Pages 1-21, ISSN 0020-0255.

Chen, S.-M., Wang, C.-Y., 2011. A new method for fuzzy decision making based on ranking generalized fuzzy numbers and interval type-2 fuzzy sets. Machine Learning and Cybernetics (ICMLC), 2011 International Conference on, Guilin, 2011, pp. 131-136.

Di Lascio, L., Fischetti, E., Gisolfi, A., Gisolfi, A., and Nappi, A., 2011. Type-2 fuzzy decision making by means of a BL-algebra. IEEE International Fuzzy Systems Conference, London, 2007, pp. 1-6.

Guneri, A. F., Cengiz, M., Seker, S., 2009. A fuzzy ANP approach to shipyard location selection. Expert Systems with Applications 36 (4), 7992–7999.

He, T., Ho, W., Man, C. L. K., Xu, X., 2012. A fuzzy AHP based integer linear programming model for the multi-criteria transshipment problem. The International Journal of Logistics Management 23 (1), 159–179.

Gehrke, M., Walker, C., and Walker, E., 1996. Some comments on interval valued fuzzy sets. Int. J. Intell. Syst., vol. 11, pp. 751–759.

Gorzalczany, M. B., 1987. A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy Sets and Systems Volume 21, Issue 1, January 1987, Pages 1–17.

Kara, S. S., 2011. Evaluation of outsourcing companies of waste electrical and electronic equipment recycling. International Journal of Environmental Science & Technology 8 (2), 291–304.

Kaya, T., Kahraman, C., 2010. Multicriteria renewable energy planning using an integrated fuzzy VIKOR & AHP methodology: The case of Istanbul. Energy 35 (6), 2517–2527.

Liang, Q., and Mendel, J. M., 2000. Interval type-2 fuzzy logic systems: Theory and design. IEEE Trans. Fuzzy Syst., vol. 8, no. 5, pp. 535–550.

Malek, M., Tumeo, M., and Saliba, J., 2015. Fuzzy logic approach to risk assessment associated with concrete deterioration. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 1(1):04014004, 2015.

Mays, M. D., Bogardi, I., and Bardossy, A., 1997. Fuzzy logic and risk-based soil interpretations. Geoderma Volume 77, Issues 2–4, June 1997, Pages 299–315.

Mendel, J. M., John, R. I., and Liu, F., 2006. Interval type-2 fuzzy logic systems made simple. Fuzzy Systems, IEEE Transactions on, 14(6):808–821.

Naim, S., and Hagras, H., 2015. A Type-2 Fuzzy Logic Approach for Multi-Criteria Group Decision Making. Granular Computing and Decision-Making: Interactive and Iterative Approaches, Springer International Publishing Editor Pedrycz, W., and Chen, S.-M., 123–164

Özkan, B., Kaya, İ., Cebeci, U., and Başlıgil, H., 2015. A Hybrid Multi-criteria Decision Making Methodology Based on Type-2 Fuzzy Sets For Selection Among Energy Storage Alternatives. International Journal of Computational Intelligence Systems Vol. 8, Iss. 5.

Runkler, T. A., Coupland, S., and John, R., 2015. Properties of interval type-2 defuzzification operators. IEEE International Conference on Fuzzy Systems, pp 1–7.

Shell, J., Coupland, S., and Goodyer, E., 2010. Fuzzy data fusion for fault detection in wireless sensor networks. Computational Intelligence (UKCI), 2010 UK Workshop on, pages 1–6.

Sousa, J. M., Palm, R., Silva, C. A., Runkler, T. A., 2002. Fuzzy optimization of logistic processes. In: IEEE International Conference on Fuzzy Systems. Honolulu, pp. 1257–1262.

Zadeh, L. A., 1965. Fuzzy sets. Information and Control 8, 338–353.

Zadeh, L. A., 1975. The concept of a linguistic variable and its application to approximate reasoning. Information Science 8, 199–250, 301–357, 9:42–80.

Zhang, Z., and Zhang, S., 2012. Type-2 Fuzzy Soft Sets and Their Applications in Decision Making, Journal of Applied Mathematics, vol. 2012, Article ID 608681, 35 pages, 2012.