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Direct and indirect influences of executive functions on mathematics achievement

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ABSTRACT

Achievement in mathematics is predicted by an individual's domain-specific factual knowledge, procedural skill and conceptual understanding as well as domain-general executive function skills. In this study we investigated the extent to which executive function skills contribute to these three components of mathematical knowledge, whether this mediates the relationship between executive functions and overall mathematics achievement, and if these relationships change with age. Two hundred and ninety-three participants aged between 8 and 25 years completed a large battery of mathematics and executive function tests. Domain-specific skills partially mediated the relationship between executive functions and mathematics achievement: Inhibitory control within the numerical domain was associated with factual knowledge and procedural skill, which in turn was associated with mathematical achievement. Working memory contributed to mathematics achievement indirectly through factual knowledge, procedural skill and, to a lesser extent, conceptual understanding. There remained a substantial direct pathway between working memory and mathematics achievement however, which may reflect the role of working memory in identifying and constructing problem representations. These relationships were remarkably stable from 8 years through to young adulthood. Our findings help to refine existing multi-component frameworks of mathematics and understand the mechanisms by which executive functions support mathematics achievement.

1. Introduction

A good understanding of mathematics is essential for success in modern society, leading not only to good job prospects but also a better quality of life (Gross, Hudson, & Price, 2009; OECD, 2013; Parsons & Bynner, 2005). Children develop an understanding of mathematics throughout their primary and secondary education. In order to ensure effective pedagogy that supports the needs of all learners it is critical to recognise the range of factors that contribute to mathematical achievement so that teaching practices can be targeted appropriately. One set of factors that play an important role in mathematics achievement are the cognitive resources that an individual can draw on. Here we evaluate the direct contribution of domain-general skills, in particular executive functions, the set of processes that control and guide our information processing, to mathematics achievement. In addition we explore to what extent the contribution of executive functions to mathematics achievement is mediated by domain-specific mathematical abilities, and whether this changes with age. Addressing these questions will refine our understanding of the ways in which executive functions support mathematics achievement, which can then inform intervention approaches that aim to capitalise on this relationship.

Attainment in mathematics rests on success in a number of underlying cognitive skills. Several researchers have proposed a multi-component model in which mathematics is underpinned by both domain-specific mathematical knowledge in addition to more general cognitive processes (Fuchs et al., 2010; Geary, 2011; LeFevre et al., 2010). For example, Le Fevre's Pathways Model of early mathematical outcomes includes linguistic and spatial attention pathways in addition to a quantitative pathway. Geary (2004; Geary & Hoard, 2005) outlined a hierarchical framework (see Fig. 1) in which achievement in any area of mathematics is underpinned by skill in applying the appropriate procedures, and an understanding of the underlying concepts. In turn, these domain-specific processes draw upon a range of domain-general skills, including language and visuospatial skills and in particular executive functions. This model therefore suggests that the

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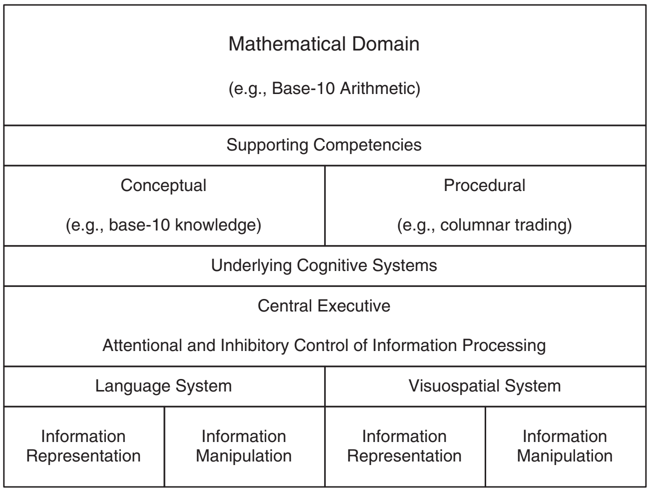

Fig. 1. Hierarchical framework of the skills underpinning mathematics. Taken from Geary (2004).

influence of executive function skills on mathematics achievement is mediated through its role in domain-specific mathematical competencies.

It is well established that an individuals' procedural skill and conceptual understanding contribute to their mathematical achievement, in addition to their factual knowledge: the ability to recall stored number facts from long-term memory (Baroody, 2003; Cowan et al., 2011; Dowker, 2005; Hiebert, 1986; LeFevre et al., 2006). More recently, a growing body of evidence has demonstrated a link between domain-general executive functions and mathematics achievement (see Bull & Lee, 2014; Cragg & Gilmore, 2014 for reviews). Executive functions, the skills used to guide and control thought and action, are typically divided into three main components following Miyake et al. (2000). These are (i) *updating* or *working memory*, the ability to monitor and manipulate information held in mind, (ii) *inhibition*, the suppression of irrelevant information and inappropriate responses, and (iii) *shifting*, the capacity for flexible thinking and switching attention between different tasks. Below we review the literature exploring the links between each of these components of executive functions and overall mathematics achievement before going on to consider its contribution to the underpinning skills of factual knowledge, procedural skill and conceptual understanding.

1.1. Executive functions and mathematics achievement

Across many studies working memory has been found to be a strong predictor of mathematics outcomes, both cross-sectionally (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013) and longitudinally (Fuchs et al., 2010; Hecht, Torgesen, Wagner, & Rashotte, 2001). According to the influential Baddeley and Hitch (1974) model of working memory, adopted by the majority of researchers in this field, working memory is made up of short-term stores for verbal and visuospatial information in addition to a central executive component that coordinates these storage systems and allows the manipulation and storage of information at the same time. Accordingly, tasks that simply require information to be stored for a short amount of time are used as an index of the capacity of the verbal and visuospatial stores, while tasks that require the simultaneous storage and manipulation of information are used to also tap into the central executive component of working memory. In general, tasks that tap into this executive working memory system show stronger relationships with mathematics achievement than those which simply measure the short-term storage of information. The results from a recent meta-analysis of 111 studies found that verbal executive working memory showed the strongest relationship with mathematics, followed by visuospatial executive working memory and short-term storage, which did not differ, and finally the short-term storage of verbal information (Friso-van den Bos et al., 2013). This suggests that it is the central executive component of working memory that is most important for mathematics.

The tasks that are typically used to tap into the central executive are not a pure measure of this process however, as the short-term storage and processing of information is also required. To try and isolate the exact components of working memory that contribute to mathematics achievement Bayliss and colleagues adopted a variance partitioning approach whereby they used a complex span combining the storage and processing of information, as typically used to index executive working memory, but also measured storage and processing independently. Using a series of regression models they were able to isolate the unique variance associated purely with the central executive, storage capacity and processing speed, as well as the shared variance between these processes. In one study with 7–9-year-olds, Bayliss, Jarrold, Gunn, and Baddeley (2003) found that the executive demands of combining verbal storage and processing explained significant variance in mathematics achievement, but that combining visuospatial storage and processing did not. Moreover, the executive working memory tasks involving verbal storage explained more variance in mathematics achievement than a short-term verbal storage task alone.

A follow-up study investigating developmental changes in working memory and cognitive abilities (Bayliss, Jarrold, Baddeley, Gunn, & Leigh, 2005) demonstrated that shared variance between age, working memory, storage and processing speed across both verbal and visuospatial domains contributed most to mathematics achievement across ages, explaining 38% of the variance. The central executive accounted for around 5% of unique variance, as did shared variance between age, working memory and storage. Storage alone accounted for 2.5% of the variance, which was attributed to variation in the ability to reactivate items in memory. Processing speed accounted for a small amount of variance both uniquely (1.3%) and shared with working memory and age (25%). Taken together, these findings suggest that all components of working memory play some role in successful mathematics achievement but that the demands of combining the storage of verbal information with additional information processing do seem to be particularly important for mathematics achievement in childhood.

The findings of Friso-van den Bos et al. and Bayliss et al. suggest that there may be some domain-specificity in the relationship between working memory and mathematics achievement, with verbal working memory playing a larger role than visuospatial working memory. Other researchers have argued for the opposite pattern however, with a stronger relationship between mathematics and visuospatial working memory than verbal working memory, particularly in children with mathematics difficulties but with typical reading and/or verbal performance (Andersson & Östergren, 2012; McLean & Hitch, 1999; Schuchardt, Maehler, & Hasselhorn, 2008; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013). In a comprehensive study which tested a large sample of typically developing 9-year-olds on an extensive battery of measures, Szűcs, Devine, Soltesz, Nobes, and Gabriel (2014) found that visuospatial short-term and working memory were significant predictors of mathematical achievement, while verbal short-term and working memory were not. Phonological decoding and verbal knowledge were found to be significant predictors however, which may have accounted for some of the variance associated with verbal short-term and working memory. These conflicting findings may be due to the type of mathematics under study, and could also be related to age. Li and Geary (2013) found that (verbal) central executive measures, but not visuospatial short-term memory measures predicted mathematics achievement in 7-year-olds, but that the children who showed the largest gains in visuospatial short-term memory from 7 to 11 years achieved a higher level of attainment in mathematics at 11 years of age. Age-related differences in these relationships could reflect either maturation-related changes in the involvement of working memory, or differences in the mathematical content of curriculum-based or standardised achievement test. For example, verbal working memory may be more important for basic topics such as arithmetic, whereas visuospatial working memory may be more important for more advanced topics, such as geometry. Research with adults also points to a greater role for visuospatial than verbal working memory (Hubber, Gilmore, & Cragg, submitted for publication; Webb, Lubinski, & Benbow, 2007). This suggests that visuospatial short-term (and potentially also working) memory becomes increasingly important for mathematics with age.

The relationships between mathematics achievement and inhibition and shifting tend to be less consistent than the relationship with working memory, with significant correlations found in some studies (e.g., Blair & Razza, 2007; Clark, Pritchard, & Woodward, 2010; Thorell, 2007; Yeniad, Malda, Mesman, van IJzendoorn, & Pieper, 2013), but not others (Lee et al., 2012; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). One suggestion for this variety is that inhibition and shifting contribute unique variance when they are studied as sole predictors, but that if working memory is also included then this accounts for the variance otherwise explained by inhibition and shifting (Bull & Lee, 2014; Lee & Bull, 2015). Another possibility for the inconsistency is that inhibition and shifting make a lesser contribution to mathematics achievement than working memory, which in some studies reaches significance, while in others it does not. The results from the meta-analysis of Friso-van den Bos et al. (2013) suggest that inhibition and shifting are indeed less important for mathematics achievement than working memory. They found that inhibition and shifting explained a similar amount of variance in mathematics achievement, but significantly less than was explained by both verbal and visuospatial short-term storage and executive working memory.

1.2. Executive functions and the component processes underpinning mathematics achievement

Despite a growing focus on understanding the neurocognitive predictors of overall mathematics achievement, there has been relatively little research investigating the contribution of executive function skills to the component processes that underpin mathematics achievement; retrieving arithmetic facts from long-term memory, selecting and performing arithmetic procedures and understanding the conceptual relationships among numbers and operations. It is important to study the role of executive functions to each of these component processes separately as although they all contribute to successful mathematics achievement, children can show different patterns of strengths and weaknesses across these processes (Dowker, 2005; Gilmore, Keeble, Richardson, & Cragg, in press; Gilmore & Papadatou-Pastou, 2009), suggesting that the domain-general processes that support them may also differ. Moreover, it is currently unclear whether the relationship between executive functions and these components of mathematics may in fact mediate the relationship between executive functions and overall mathematics achievement, as Geary (2004) suggests. Most research to date has taken place within the domain of arithmetic, therefore below we review the role of executive functions in factual knowledge, procedural skill and conceptual understanding of arithmetic in turn before going on to compare the contribution that executive functions make to each component.

1.2.1. Factual knowledge

According to theoretical models, arithmetic facts are stored in an associative network in long-term memory (Ashcraft, 1987; Campbell, 1995; Verguts & Fias, 2005) in a verbal code (Dehaene, 1992). Many models of working memory propose that one of its roles is to activate information in long-term memory (Barrouillet, Bernardin, & Camos, 2004; Cowan, 1999; Engle, Kane, & Tuholski, 1999; Unsworth & Engle, 2007). Taken together, these models suggest that verbal working memory may be required to recall arithmetic facts and that inhibitory processes may be required to suppress the neighbouring solutions or alternative operations that are co-activated when a fact is retrieved. There is evidence that individuals with low verbal short term and working memory capacity are less likely to choose a retrieval strategy for solving simple arithmetic problems (Barrouillet & Lépine, 2005; Geary, Hoard, & Nugent, 2012), and are also likely to retrieve them less accurately (Andersson, 2010; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). In contrast, verbal and visuospatial working memory tasks have not always been found to uniquely predict performance on arithmetic fact fluency tasks in elementary school children over and above basic numerical skills and other domain-general skills (Cowan & Powell, 2014; Fuchs et al., 2005; Träff, 2013), although this may be due to the wide ranging influence of working memory in many of these processes. There is also recent evidence that inhibitory processes play a role in arithmetic factual knowledge in terms of suppressing co-activated but incorrect answers. De Visscher and Noël have demonstrated that a patient with an arithmetic fact retrieval deficit (2013), and 8–9-year-olds with poor arithmetic fact fluency (2014) all demonstrate difficulties in suppressing interfering items within memory (see also Geary, Hoard, & Bailey, 2012). There is therefore some evidence for a role of working memory and inhibition in the retrieval of arithmetic facts, although to date there has been little research in this area.

1.2.2. Procedural skill

The ability to accurately and efficiently select and perform appropriate arithmetic procedures is likely to rely on executive functions in order to represent the question and store interim solutions, select the appropriate strategy and inhibit less appropriate ones, as well as shift between operations, strategies and notations. Convincing evidence that working memory, in particular the central executive, plays a key role in using arithmetic procedures comes from experimental dual-task studies which have found that procedural strategies are impaired by a concurrent working memory load (Hubber, Gilmore, & Cragg, 2014; Imbo & Vandierendonck, 2007a, 2007b). Correlational studies have also demonstrated a relationship between working memory and procedural skill (Andersson, 2008; Cowan & Powell, 2014; Fuchs et al., 2010; Wilson & Swanson, 2001) (but see Fuchs et al., 2006; Träff, 2013), although there are mixed findings concerning whether simple storage or central executive processes play a larger role, and whether the storage of verbal or visuospatial information is more important. There is some evidence that children with better inhibitory control are better able to select the most efficient strategy (Lemaire & Lecacheur, 2011) and also perform better on tests of procedural skill (Clark et al., 2010). Similarly for shifting, children with better cognitive flexibility have been found to have better procedural skill (Andersson, 2010; Clark et al., 2010), although evidence that children with a mathematics difficulty show a significant deficit in shifting in comparison to typical controls (as measured by a trail-making task) has been mixed (Andersson, 2008; Andersson & Lyxell, 2007; McLean & Hitch, 1999).

The contribution of executive functions, in particular inhibition and shifting, to procedural skill may well depend on age- or schooling-related changes in mathematical content and strategies (Best, Miller, & Naglieri, 2011; Friso-van den Bos et al., 2013; Träff, 2013). These domain-general skills may play a greater role in younger, less-skilled children but become less important with age as procedural skills become more automatic and children begin to use fact retrieval and decomposition, breaking a problem down into smaller parts, to solve arithmetic problems (Ashcraft, 1982; Bailey, Littlefield, & Geary, 2012). In their meta-analysis Friso-van den Bos et al. (2013) found that the contribution of shifting and the visuospatial sketchpad decreased with age, while the contribution of visuospatial working memory increased with age. The role of verbal short-term and working memory and inhibition remained constant. The majority of these studies were based on measures of overall mathematics achievement. Given that procedural skills are required in most of these general mathematics measures, these findings suggest that, for at least some aspects of executive function, their role in procedural skills changes during childhood. This needs to be confirmed with a more specific measure of procedural skill however.

1.2.3. Conceptual understanding

Theoretical models suggest that executive functions may be required to switch attention away from procedural strategies to allow underlying conceptual numerical relationships to be identified (Siegler & Araya, 2005) and also to activate conceptual knowledge in long-term memory (Barrouillet et al., 2004; Cowan, 1999; Engle et al., 1999; Unsworth & Engle, 2007). Comparatively little empirical work has investigated the role of domain-general skills in conceptual understanding however. Robinson and Dubé (2013) found that 8–10-year-old children with poorer inhibitory control were less likely to use a conceptually-based shortcut than children with good inhibitory control when presented with problems where such a strategy was possible. They suggested that this may be because the children found it difficult to inhibit well-learned procedural algorithms. Empirical studies do not appear to support the role of working memory in conceptual understanding however, at least in the domain of fractions (Hecht, Close, & Santisi, 2003; Jordan et al., 2013).

1.2.4. Comparing the contribution of executive functions to different mathematical components

In summary, it can be seen that executive functions do seem to play a role in an individual's ability to recall arithmetic facts from long-term memory, select and perform arithmetic procedures and understand the conceptual relationships among numbers and operations. However much of this evidence is drawn across separate studies and thus it is not possible to directly compare the contribution of executive functions across these three core competencies of arithmetic. A direct comparison is important theoretically in order to be able to accurately refine multi-component models of arithmetic. It is also of practical importance in order to understand the mechanisms through which interventions aimed to enhance mathematics outcomes via executive function training might be operating, as well as provide some indication as to how they could be modified and improved.

To our knowledge, only a handful of studies have compared the role of domain-general skills across factual, procedural and conceptual components of arithmetic. Cowan and Powell (2014) examined the contribution of working memory to fact retrieval and procedural skill at written arithmetic in 7–10-year-olds, alongside other domain-general skills including reasoning, processing speed and oral language as well as measures of numerical representations and number systems knowledge. They found that domain-general factors accounted for more variation in procedural skill (43%) than in fact retrieval (36%) and that much of this variance was shared among the domain-general predictors. The unique predictors differed across tasks. While visuospatial short-term memory and verbal working memory predicted procedural skill (alongside reasoning, naming speed and oral language) only processing speed, naming speed and oral language emerged as a significant unique predictor of factual knowledge. Similarly, Fuchs et al. (2005) found that domain-general skills accounted for more variance in procedural skill than factual knowledge. Language and phonological processing were significant unique predictors of fact retrieval and phonological processing also predicted procedural calculation skill. Working memory was not a significant predictor for either component, however teacher ratings of attention uniquely predicted performance on both. Inattention is known to be strongly related to working memory capacity (Gathercole et al., 2008; Martinussen, Hayden, Hogg-Johnson, & Tannock, 2005), therefore it is possible that any variance associated with working memory is shared with this measure of attention.

The studies of Cowan and Powell (2014) and Fuchs et al. (2005) suggested that working memory and other domain-general skills play a larger role in procedural skill than factual knowledge. They did not include a measure of conceptual understanding however. Two studies have compared the contribution of working memory to procedural skill and conceptual understanding within the domain of fractions. Both Hecht et al., (2003) and Jordan et al. (2013) found that working memory was a significant predictor of procedural skill but not conceptual understanding. This adds further evidence that working memory makes a greater contribution to procedural skills than other components of mathematics.

Only one study to date has compared the contribution of working memory skills to all three components of mathematics; factual knowledge, procedural skill and conceptual understanding. Andersson (2010) tested a large sample of children with and without mathematics and reading difficulties three times between the ages of 10 and 12 years on a large battery of mathematics and cognitive tasks which include measures of factual, procedural and conceptual knowledge, as well as visuospatial working memory, verbal short-term memory, and shifting. Regression analyses revealed that executive functions accounted for more variance in procedural skills than in factual knowledge or conceptual understanding. Processing speed and verbal short-term memory were significant unique predictors of fact retrieval accuracy whereas shifting, but not the working memory measures, predicted procedural skill. Visuospatial working memory was a predictor of conceptual understanding however. This discrepancy with the studies of Hecht et al., (2003) and Jordan et al. (2013) may be because a visuospatial working memory task was used here, in contrast to the verbal working memory tasks used by Hecht et al. and Jordan et al.

Taken together, these findings indicate that working memory skills do play a different role in recalling arithmetic facts from long-term memory, selecting and performing arithmetic procedures and understanding the conceptual relationships among numbers and operations. Yet to date no studies have used a comprehensive battery of working memory and wider executive function tasks in order to gain a full picture of the contribution of executive functions to these three components of arithmetic. Moreover, given that performance in these three component skills underpins overall mathematics achievement it is likely that factual, procedural and conceptual understanding may mediate the overall relationship that has been found between executive functions and mathematics achievement. Revealing these subtleties will allow us to pinpoint the mechanisms by which executive functions support mathematics achievement and perhaps refine intervention approaches that build on this relationship.

1.3. The current study

The current study aimed to investigate the role of executive functions in factual, procedural and conceptual knowledge of arithmetic, ascertain how this might change with development, and determine whether these cognitive components of arithmetic mediate the relationship between executive function and overall mathematics achievement. A large sample of 8–9-year-olds, 11–12-year-olds, 13–14-year-olds and 18–25-year-olds were administered a battery of mathematics and executive function measures in addition to a standardised test of mathematics achievement. Three sets of analyses were conducted: The first used regression models to determine the relative contribution of working memory, inhibition and shifting to factual knowledge, procedural skill and conceptual understanding of mathematics and how this changes with age. The second used mediation analysis to ascertain if cognitive components of mathematics mediate the relationship between executive functions and overall mathematics achievement. The final set of analyses used a variance partitioning approach to explore which components of working memory are driving the relationships with mathematics.

In light of theoretical models of mathematical cognition and the available empirical evidence we predicted that executive functions would be significantly related to overall mathematics achievement, with working memory contributing more variance than inhibition and shifting. We predicted that all components of working memory would contribute to mathematics achievement but that verbal executive working memory would explain the most variance. We anticipated that the contribution of visuospatial working memory might increase with age.

We expected that executive functions would play a greater role in procedural skills than in factual knowledge and conceptual understanding. We predicted that factual knowledge would be demanding of cognitive resources, particularly verbal working memory and inhibition to suppress activated but incorrect answers. We anticipated that all aspects of executive function would be associated with procedural skill, but that the strength of this relationship would change with age, with stronger relationships in 8–9-year-olds in comparison to 11–12- and 13–14-year-olds. For conceptual understanding we anticipated that while working memory may be required to retrieve conceptual information from long-term-memory, inhibition and shifting would play an important part in suppressing procedural strategies in favour of conceptual ones, as well as rearranging problems into different formats in order to identify conceptual relationships.

2. Method

2.1. Participants

A total of eighty-four 8–9-year-olds (M = 8.9 years, SD = 0.28; 38 male), sixty-seven 11–12-year-olds (M = 12.2 years, SD = 0.37; 35 male), sixty-seven 13–14-year-olds (M = 14.2 years, SD = 0.30; 30 male) and seventy-five young adults (M = 21.4 years, SD = 1.80; 30 male) took part in the study. The young adults were students at the University of Nottingham and all spoke English as their first language. They gave written informed consent and received course credit or an inconvenience allowance for taking part. The 8–9-year-olds attended suburban primary schools and the 11–14-year-olds suburban secondary schools in predominantly White British, average socio-economic status neighbourhoods of Nottingham, UK. Primary schools in the UK are attended by pupils aged from 5 to 11 years. UK secondary schools are typically attended by pupils from 11 to 18 years. Parents of all children in the school year groups taking part in the study were sent letters about the study and given the option to opt out. All children were given a certificate for taking part. The study was approved by the Loughborough University Ethics Approvals (Human Participants) Sub-Committee.

2.2. Equipment and materials

The arithmetic and executive function tasks were created using PsychoPy software (Peirce, 2007) and presented on an HP laptop computer. For the mathematics tasks, the experimenters recorded response times for child participants by pressing a key immediately as participants began to give their answer.

2.3. Tasks

2.3.1. Mathematics achievement test

The Mathematics Reasoning subtest of the Wechsler Individual Achievement Test (Wechsler, 2005) was administered following the standard procedure. This test provides a broad assessment of curriculum-relevant mathematics achievement and is a good predictor of performance on the national school achievement tests used in the UK (Nunes, Bryant, Barros, & Sylva, 2012). It includes a series of verbally and visually presented word problems covering arithmetic, problem solving, geometry, measurement, reasoning, graphs and statistics. Raw scores were used as the measure of performance.

2.3.2. Arithmetic tasks

2.3.2.1. Factual knowledge task. This task assessed participants' knowledge of number facts. On each trial an arithmetic problem was presented on screen for 3 s and participants were asked to retrieve the result without mental calculation. The participants were instructed to give their answer verbally, at which point the experimenter pressed a key and inputted by the answer. Participants were instructed to say "I don't know" if they could not retrieve the answer.

Participants completed four practice trials and then 12 experimental trials in random order. An additional four easy 'motivational trials' were intermixed with the experimental trials. To ensure that performance was not at floor or ceiling level in any group we selected a different set of items for each age group (8–9 years, 11–12 years, 13–14 years, young adults). Following pilot testing, the problems given to the primary school students were composed of single-digit addition operations only, those given to the secondary school students also included subtraction operations. The problems for the 11–12 year olds involved single-digit numbers, and the problems for the 13–14 year olds were composed of one single-digit number and one double-digit number. The problems given to the young adults involved addition, subtraction, multiplication and division operations composed of one single-digit and one double-digit number. The measure of performance was the proportion of items answered correctly within the 3 s presentation time (Cowan & Powell, 2014; Jordan, Hanich, & Kaplan, 2003).

2.3.2.2. Procedural skills task. This task assessed the strategy choice and efficiency with which participants could accurately perform arithmetic procedures. Prior to starting the task participants were shown pictures representing different strategies (i.e., counting in your head, counting on fingers, decomposition, and retrieval) to ensure that younger participants understood that any strategy was acceptable in this task. The experimenter described the strategies and told participants that any of these strategies, or others, could be used to solve the task. Following this, on each trial an arithmetic problem was presented on screen and participants were instructed to solve it using any mental method they preferred.

Participants were given four practice trials and then 10 (8–9 years and 11–12 years) or 12 (13–14 years, young adults) experimental trials. The operations were designed to be age appropriate, and of a difficulty level where retrieval would be unlikely. The problems for all age groups involved a mix of single and double-digit numbers, with a greater proportion of double-digit numbers for the older groups. The trials given to 8–9-year-olds and 11–12-year-olds were composed of addition and subtraction operations and the trials given to 13–14-year-olds and young adults were composed of addition, subtraction, multiplication and division operations. The items in each version were presented in one of two orders counterbalanced across participants.

The participants were instructed to give their answer verbally at which point the experimenter pressed a key and inputted the answer. The measure of performance on this task was the mean response time (RT) for correctly answered trials.

2.3.2.3. Conceptual knowledge task. This task assessed participants' understanding of conceptual principles underlying arithmetic. As with the other arithmetic tasks, a different set of problems was used for each age group. The operations were designed to be difficult to solve mentally, to discourage the participants from attempting to do so. The 8–9-year-olds watched a puppet solve a double-digit addition or subtraction problem using counters and were shown the example problem (including the answer) written in a booklet (e.g., 23 + 24 = 47). They were then shown four probe problems that were presented without answers and asked whether the puppet could use the example (completed) problem to solve each probe problem, or if he would need to use the counters to solve it. Of the four probe problems, one of the related problems was identical (e.g., 23 + 24 =), one was related by commutativity (e.g., 24 + 23 =), one was related by inversion (e.g., 47 – 23 =) and one was unrelated (e.g., 32 + 24 =). The children were first asked to decide whether or not the example problem could help the puppet solve each probe problem, and asked to explain how. The children completed two practice example problems, with feedback, followed by 24 experimental trials (six example problems each with four probe problems). The items were presented in one of two orders counterbalanced across participants.

The conceptual task for the 11–12-year-olds, 13–14-year-olds and young adults was presented on a computer. On each trial an arithmetic problem with the correct answer was presented on the screen. Once this was read, the experimenter pressed 'return' on the computer keyboard and a second, unsolved operation appeared below the first problem. The participants were asked to state whether or not the first problem could help solve the second problem, and then were asked to explain how. Participants were given four practice trials and thirty experimental trials. Eighteen of the thirty problem pairs were related. The pairs of problems were related by the subtraction-complement principle (e.g., 113 − 59 = 54 and 113 − 54 =), inverse operations (e.g., 74 + 57 = 131 and 131 − 74 =), and associative operations (e.g., 87 − 54 = 33 and 87 − 34 − 20 =). The trials given to the 11–12-year-olds were composed of addition and subtraction problems involving two operands of two and three digit numbers. The trials given to the 13–14-year-olds were composed of addition and subtraction problems involving two or three operands of double-digit numbers, as well as some multiplication and division problems involving single and double-digit numbers. The trials for the young adults were composed similarly but they also included some division problems including two double-digit numbers. The items in each task version were presented in one of two orders counterbalanced across participants.

All participants gave their response verbally and the experimenter recorded this. Accuracy measures were calculated for how many relationships were correctly identified, and for how many accurate explanations each participant provided. The measure of performance used here was the proportion of trials for which the presence or absence of a relationship was correctly identified. Higher scores indicated better performance.

2.3.3. Executive function tasks

2.3.3.1. Verbal short-term and working memory. All participants completed separate verbal short-term memory and verbal working memory tasks. Verbal short-term memory was assessed via a word span task. Participants heard a list of single syllable words and were asked to recall them in order. There were three lists at each span length, beginning with lists of two words, and the participants continued to the next list length if they responded correctly to at least one of the trials at each list length. The total number of words correctly recalled was used as the dependent variable.

Verbal working memory was assessed via a sentence span task. Participants heard a sentence with the final word missing and had to provide the appropriate word. After a set of sentences they were asked to recall the final word of each sentence in the set, in the correct order. Participants first completed an initial practice block with one trial with one item and two trials with two items. The practice trials could be repeated if necessary. They then continued to the test trials where they received three trials at each span test length, starting with a test length of two items. Provided they recalled at least one trial correctly, the sequence length was increased by a single item and three further trials were administered. Participants' performance on the processing task was also assessed separately in two blocks (one before the sentence span task and one after) of 20 trials each. In these blocks they only had to provide the final word of the sentence, without the need to recall the words. Response times were measured for the processing trials and the total number of words correctly recalled was calculated for the storage element of the sentence span task.

2.3.3.2. Visuospatial short-term and working memory. The participants completed separate visuospatial short-term memory and visuospatial working memory tasks. In the visuospatial short-term memory task participants saw a 3 × 3 grid on the screen. They watched as a frog jumped around the grid and after the sequence finished they had to point to the squares he jumped on in the correct order, which was recorded by the experimenter using the mouse. There were three trials at each sequence length, beginning with sequences of two jumps, and participants continued to the next sequence length if they responded correctly to at least one of the sequences at each length. The total number of correctly recalled locations was used as the dependent variable.

Visuospatial working memory was assessed via a complex span task. Participants saw a series of 3 × 3 grids each containing three symbols and they had to point to the 'odd-one-out' symbol that differed from the other two. After a set of grids children were asked to recall the position of the odd-one-out on each grid, in the correct order. Participants first completed an initial practice block with one trial with one item and two trials with two items. The practice trials could be repeated if necessary. For the test trials there were three trials at each span length, beginning with a test length of two items, and children continued to the next span length if they responded correctly to at least one of the trials at each span length. Participants' performance on the processing task was also assessed separately in two blocks (one before the complex span task and one after) of 20 trials each. In these blocks they only had to identify the location of the odd-one-out, without the need to recall the position. Response times were measured for the processing trials and the total number of locations correctly recalled was calculated for the storage element of the complex span task.

2.3.3.3. Non-numerical inhibition task. To assess participants' ability to inhibit irrelevant information in a non-numerical context we used an animal-size stroop task (based on Szűcs et al., 2013). On each trial two animal pictures were presented on the screen. One animal was selected from a set of large animals (e.g., a bear, gorilla, and giraffe) and the other animal was selected from a set of small animals (e.g., an ant, rabbit, and mouse). The participants' task was to identify which animal was the larger in real life. On each trial, one animal image was presented with an area on screen four times larger than the other image. On congruent trials the animal that was larger in real life was also the larger image on the screen, and on incongruent trials the animal that was smaller in real life was the larger image on the screen. Participants were required to ignore the size of the images on the screen and to respond based on the size in real life only. On each trial the images were presented on screen and participants responded as quickly as possible by pressing one of two buttons on the keyboard that corresponded to the side of the screen with the larger animal.

Participants completed four experimental blocks each containing 48 trials in random order. The time taken to complete each block was recorded and presented to participants at the end of each block to encourage them to respond quickly. In the first two experimental blocks 75% of the trials were incongruent and 25% were congruent and in the second two experimental blocks 75% of the trials were congruent and 25% were incongruent. Participants had the opportunity to take breaks during the task as needed.

Prior to commencing the task participants were shown each of the animal images in one size and asked whether the animal was large or small in real life to ensure they had the necessary real-world knowledge to perform the task. All participants completed this without problem.

Median RTs for correctly-solved trials were calculated for the congruent and incongruent trials (collapsing across blocks). Inhibition score was the difference in RT for congruent and incongruent trials. Larger differences indicate lower levels of inhibitory control.

2.3.3.4. Numerical inhibition task. To assess participants' ability to inhibit irrelevant information in a numerical context we used a dot comparison task. On each trial the participants were shown two sets of white dots on a black screen and were instructed to identify which set had the highest number of dots. The dots were created using an adapted version of the matlab script provided by Gebuis and Reynvoet (2011). This method produced four types of trials, of which two were analysed. On fully congruent trials (n = 20) the more numerous array has larger dots and the array encompasses a larger area. On fully incongruent trials (n = 20) the more numerous array has smaller dots and the array encompasses a smaller area. Participants were required to ignore the size of the dots and the array on the screen and to respond based on the number of dots only. The number of dots in each array ranged from 5 to 28 and the ratio between the number of dots ranged from 0.5 to 0.8.

Participants completed 6 practice trials and 80 experimental trials in random order. They were given breaks during the task as needed. Mean accuracy was calculated for the fully congruent and incongruent trials. Inhibition score was the difference in accuracy for fully congruent and incongruent trials. Larger differences indicate lower levels of inhibitory control.

2.3.3.5. Set shifting task. To assess participants' ability to formulate basic concepts and shift from one concept to another we used the Animal Sorting subtest from the NEPSY-II. The task requires participants to sort eight cards into two groups of four using self-initiated sorting criteria. The cards are coloured blue or yellow and include pictures of animals (e.g., a cat, 2 fish, an elephant, etc.), and can be sorted in 12 different ways, for example blue vs. yellow cards, one animal vs. two animals, pictures with sun vs. pictures with rain. Following a teaching example the participants were given 360 s of cumulative sort time to sort the cards in as many different ways as they could. The test was discontinued before 360 s if the participant stated they had finished, or if 120 s elapsed without a response. Sorts were recorded using correct sort criteria and a raw score of the total number of correct sorts was calculated (maximum of 12). A larger score indicates better performance.

2.4. Procedure

Each participant was tested individually in a 2 h session. The tasks were presented in one of two orders, counterbalanced across participants, with executive function and mathematics tasks intermixed. The children were all tested in their school in a quiet room away from the classroom. Young adults were tested in a lab at their University.

2.5. Data preparation

Nine participants; four 8–9-year-olds, four 11–12-year-olds and one 13–14-year-old failed to complete either one or two measures from the full battery of tests. Their missing data (0.3%) was replaced using the multiple imputation option in SPSS. Six participants; two 8–9-year-olds, one 11–12-year-old and three young adults, were classed as multivariate outliers using Mahalanobis distance and excluded from the study. A further three 8–9-year-olds were excluded for floor performance on the procedural skills task. This left a final sample of seventy-nine 8–9-year-olds, sixty-six 11–12-year-olds, sixty-seven 13–14-year-olds and seventy-two young adults. The content of the arithmetic tasks varied for each age group to prevent floor or ceiling effects on any tasks. As a result it was not appropriate to use raw scores in analyses involving multiple age groups. We therefore transformed raw scores on all measures to z-scores within each age group and used these in the subsequent analyses. For measures where a lower score indicated better performance the z scores were multiplied by -1 so that for all measures a higher z score indicated better performance. The consequence of using z scores was that overall age differences in mathematics or executive functions between the groups were not assessed, only how the relationships between executive functions and mathematics may differ with age.

3. Results

Descriptive statistics for raw performance on the mathematics and executive function tasks are presented in Table 1. There was a good range of performance on all of the tasks, with no evidence of floor or ceiling effects. Four sets of analyses were conducted. First, we established that the mathematics component skills were related to overall mathematics achievement. Second, regression models were used to determine the relative contribution of working memory, inhibition and shifting to overall mathematics achievement as well as factual knowledge, procedural skill and conceptual understanding of mathematics and establish how this changes with age. Third, a mediation analysis was performed to ascertain if cognitive components of mathematics mediate the relationship between executive functions and overall mathematics achievement. Finally, a variance partitioning approach explored which components of working memory are driving the relationships with mathematics.

Table 1
Descriptive statistics for all tasks.

Domain	Task	8–9-year-olds		11–12-year-olds		13–14-year-olds		Young adults	
		Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Mathematics Achievement	WIAT Mathematics Reasoning (raw score)	34.00	5.91	47.35	5.01	50.27	6.25	58.65	5.17
	WIAT Mathematics Reasoning (standard score)	91.43	13.35	96.97	9.76	92.66	13.26	105.65	10.07
Arithmetic	Number fact knowledge (accuracy)	0.51	0.19	0.72	0.22	0.74	0.21	0.86	0.13
	Arithmetic strategy efficiency (RT, seconds)	8.94	3.72	10.05	3.65	9.64	3.13	8.11	2.28
	Conceptual understanding (accuracy)	0.75	0.14	0.76	0.17	0.81	0.15	0.90	0.07
Executive Functions	Verbal short term memory (total score)	36.58	11.11	45.77	13.00	53.55	14.85	65.4	15.6
	Verbal processing (median RT, ms)	836.1	168.3	691.9	135.0	618.1	140.9	613.8	196.3
	Verbal working memory (total score)	13.04	5.68	21.12	6.44	24.27	9.04	39.15	17.01
	Visuospatial short term memory (total score)	39.62	16.60	60.33	19.64	70.94	25.72	81.15	18.28
	Visuospatial processing (median RT, ms)	1937.1	196.4	1785.5	306.5	1645.1	260.0	1519.7	115.3
	Visuospatial working memory (total score)	33.32	12.56	48.53	17.24	54.88	23.07	64.8	24.5
	Non-numerical inhibition (difference in median RT, ms)	142.85	70.39	104.95	55.63	95.57	49.34	86.20	42.70
	Numerical inhibition (difference in accuracy)	0.37	0.18	0.20	0.22	0.23	0.18	0.16	0.16
	Set shifting (number of correct sorts)	2.44	1.77	2.20	2.11	1.52	1.97	4.25	3.59

3.1. Relationships among the mathematics tests

To establish that factual knowledge, procedural skill and conceptual understanding all independently contribute to mathematics achievement, we conducted a hierarchical linear regression predicting WIAT mathematics reasoning scores from our measures of factual knowledge, procedural skill and conceptual understanding. To determine if the contribution of these three components changes during development, we also included interaction terms with two nested dummy coded contrasts. The first of these, D1, compared the young adults to all groups of children. The second contrast, D2, compared the primary school pupils to the two groups of secondary school pupils, i.e., the 8–9-year-olds to both the 11–12- and 13–14-year-olds. The age contrasts were entered in the first step of the model along with the measures of factual knowledge, procedural skill and conceptual understanding. The interaction terms were entered in the second step. As shown in Table 2 the three components of arithmetic all explained unique independent variance in mathematics achievement and there were no interactions with age.

3.2. The role of executive functions in components of arithmetic

To assess the role of executive functions in mathematics achievement as well as factual knowledge, procedural skill and conceptual understanding we carried out a series of hierarchical regressions. The dummy coded age contrasts and executive function measures were entered in the first step. For these analyses only the combined storage and processing verbal and visuospatial working memory tasks were included. Interaction terms between the executive function measures and age contrasts were entered in the second step. As shown in Table 3 the executive function measures alone explained 34% of the variance in mathematical achievement, 12% of the variance in factual knowledge, 15% of the variance in procedural skill and 5% of the variance in conceptual understanding. No further variance was explained when interaction terms were added to the model for any of the outcome measures. Verbal working memory was a unique independent predictor of factual knowledge, procedural skill and conceptual understanding as well as mathematics achievement. Visuospatial working memory was also a unique independent predictor of all of the outcome variables with the exception of conceptual understanding. Shifting and non-numerical inhibition did not independently predict any of the outcome variables, while numerical inhibition was a unique independent predictor of factual knowledge and procedural skill.

Table 2
Hierarchical linear regression predicting mathematical achievement by factual knowledge, procedural skill and conceptual understanding.

Predictor	Model 1β	Model 2β
D1	0.00	0.00
D2	0.00	0.00
Number fact knowledge (%)	0.18**	0.19**
Arithmetic strategy efficiency (RT)	0.40**	0.36**
Conceptual understanding (%)	0.15**	0.14**
D1 * Number fact knowledge (%)		0.04
D1 * Arithmetic strategy efficiency (RT)		0.01
D1 * Conceptual understanding (%)		0.01
D2 * Number fact knowledge (%)		0.10
D2 * Arithmetic strategy efficiency (RT)		−0.11
D2 * Conceptual understanding (%)		0.02
R^2	0.35	0.36
F for change in R^2	30.23**	0.736

Note. DV = WIAT Mathematics Reasoning. RT = reaction time. D1 = dummy contrast comparing young adults to all groups of children. D2 = dummy contrast comparing 8–9-year-olds to 11–12-year-olds and 13–14-year-olds.
* $p < 0.05$.
** $p < 0.01$.

3.3. Direct and indirect effects of working memory on mathematics achievement

The results so far indicate that working memory skills are related to mathematics achievement and also to the component arithmetic skills of factual knowledge, procedural skill and conceptual understanding. This raises the possibility that these component arithmetic skills mediate the relationship between working memory and mathematics achievement. In order to explore this possibility mediation analyses were performed using the Process macro for SPSS (Hayes, 2013). This calculates bias-corrected 95% confidence intervals (CIs) using bootstrapping with 10,000 resamples. A confidence interval that does not straddle zero represents an effect that is statistically significant. Two separate models were run for verbal and visuospatial working memory respectively. In both models, the mathematics achievement measure was the dependent variable. Factual knowledge, procedural skill and conceptual understanding were included as potential mediators and all other executive function measures were included as covariates.

There were small but significant indirect effects of verbal working memory on mathematics achievement through all three component arithmetic skills; factual knowledge ($\beta = 0.020$, 95% CI: 0.005–0.051), procedural skill ($\beta = 0.045$, 95% CI: 0.014–0.089) and conceptual understanding ($\beta = 0.015$, 95% CI: 0.002–0.041). The size of these indirect paths did not differ significantly from each other. There remained a substantial direct effect of verbal

Table 3
Hierarchical linear regression predicting mathematical achievement, factual knowledge, procedural skill and conceptual understanding by executive functions.

Dependent variable	Mathematics achievement		Factual knowledge		Procedural skill		Conceptual understanding	
Predictor	Model 1β	Model 2β	Model 1β	Model 2β	Model 1β	Model 2β	Model 1β	Model 2β
D1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
D2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Verbal working memory (total score)	0.32**	0.28**	0.15*	0.10	0.16*	0.10	0.13*	0.07
Visuospatial working memory (total score)	0.34**	0.36**	0.20**	0.20**	0.20**	0.21**	0.07	0.06
Non-numerical inhibition (RT difference, ms)	0.05	0.03	−0.06	−0.05	0.01	−0.01	0.05	0.03
Numerical inhibition (difference in accuracy)	0.08	0.09	0.13*	0.14*	0.17**	0.19**	0.05	0.07
Set shifting (number of correct sorts)	−0.02	0.01	−0.07	−0.06	0.00	0.04	−0.09	−0.06
D1 * Verbal working memory (total score)		0.12		0.08		0.13		0.14
D1 * Visuospatial working memory (total score)		−0.06		−0.06		−0.08		0.03
D1 * Non-numerical inhibition (RT difference, ms)		0.11*		0.06		0.10		−0.05
D1 * Numerical inhibition (difference in accuracy)		−0.06		0.03		0.02		−0.02
D1 * Set shifting (number of correct sorts)		−0.13*		−0.12		−0.16*		−0.19*
D2 * Verbal working memory (total score)		−0.03		−0.07		−0.07		−0.03
D2 * Visuospatial working memory (total score)		0.04		0.00		0.01		0.01
D2 * Non-numerical inhibition (RT difference, ms)		−0.02		0.06		0.00		−0.05
D2 * Numerical inhibition (difference in accuracy)		0.01		−0.01		0.02		−0.00
D2 * Set shifting (number of correct sorts)		−0.02		−0.08		−0.02		−0.03
R^2	0.34	0.37	0.12	0.15	0.15	0.19	0.05	0.08
F for change in R^2	28.84**	1.07	7.65**	0.76	9.87**	1.09	2.85*	0.98

Note. RT = reaction time. D1 = dummy contrast comparing young adults to all groups of children. D2 = dummy contrast comparing 8–9-year-olds to 11–12-year-olds and 13–14-year-olds.
* $p < 0.05$.
** $p < 0.01$.

working memory on mathematics achievement however ($\beta = 0.24$, 95% CI: 0.15–0.34). For visuospatial working memory there were small indirect effects on mathematics achievement through factual knowledge ($\beta = 0.028$, 95% CI: 0.009–0.061), procedural skill ($\beta = 0.057$, 95% CI: 0.024–0.102) but not conceptual understanding ($\beta = 0.007$, 95% CI: −0.004 to 0.029). The indirect path via procedural skill was significantly larger than the non-significant path via conceptual understanding ($\beta = 0.050$, 95% CI: 0.015–0.094). There was also a substantial direct effect of visuospatial working memory on mathematics achievement ($\beta = 0.25$, 95% CI: 0.15–0.34).

3.4. Pinpointing the contribution of working memory in components of arithmetic

These findings demonstrate that working memory supports mathematics achievement directly, but also indirectly through factual knowledge, procedural skill and conceptual understanding. The measures used to index working memory in these analyses required participants to undertake concurrent storage and processing. Coordinating these two activities is thought to rely on the central executive, however the task is not a pure measure of the central executive and therefore it is possible that the lower-level storage and processing demands of the task are contributing to the relationships with mathematics achievement and components of arithmetic, in addition to the central executive demands of combining the two tasks. In order to investigate this, linear regression modelling was used to partition the variance between the storage, processing and central executive components of verbal and visuospatial working memory. This method helps disentangle the unique contributions each component makes as well as commonalities between them. (e.g., Salthouse, 1994). This allowed us to determine whether it was simply storing information in mind, processing information, or the executive demands of combining the two that accounted for variability in the different components of mathematics as well as overall mathematics achievement. This was done separately for the verbal and visuospatial domains.

The proportion of unique and shared variance explained by each combination of the working memory variables for each of the outcome measures is presented in Fig. 2. The first thing to note is that the pattern was largely similar across verbal and visuospatial domains. Both the verbal and visuospatial working memory tasks accounted for unique variance in mathematical achievement, factual knowledge and procedural skill even once simple storage and processing speed were controlled for. This contribution was largest for mathematics achievement (9%) followed by procedural skill (2.5–3.5%) and then factual knowledge (1.5%). Verbal but not visuospatial working memory also accounted for unique independent variance in conceptual understanding (1.3%). A similar pattern was found for shared variance between the working memory and short-term memory tasks. It contributed the largest amount to mathematics achievement (10–11%) with broadly similar contributions for procedural skill (3.6–5.0%) and factual knowledge (3.1–3.5%). The shared variance between verbal short-term and working memory was also linked to conceptual understanding (1.4%).

Unique variance associated with the verbal and visuospatial short-term memory and processing speed tasks differed slightly in the contribution that they made to mathematics outcomes. The verbal short-term memory task accounted for a small amount of unique variance in mathematics achievement and factual knowledge only whereas verbal processing speed did not explain variance in any of the mathematics outcomes. The visuospatial short-term memory task accounted for a small amount of unique variance in mathematics achievement, factual knowledge and procedural skill, whereas visuospatial processing speed accounted for unique variance in mathematics achievement, factual knowledge and conceptual understanding.

To summarise, the verbal and visuospatial working memory tasks contributed both unique variance as well as shared variance with short-term storage to mathematics achievement, factual knowledge, procedural skill and conceptual understanding (verbal only). The unique variance associated with verbal and visuospatial short-term storage differed across components of mathematics, and whereas visuospatial processing contributed unique variance to some mathematical processes, verbal processing did not.

4. Discussion

This study investigated the role of executive functions in factual knowledge, procedural skill and conceptual understanding as well

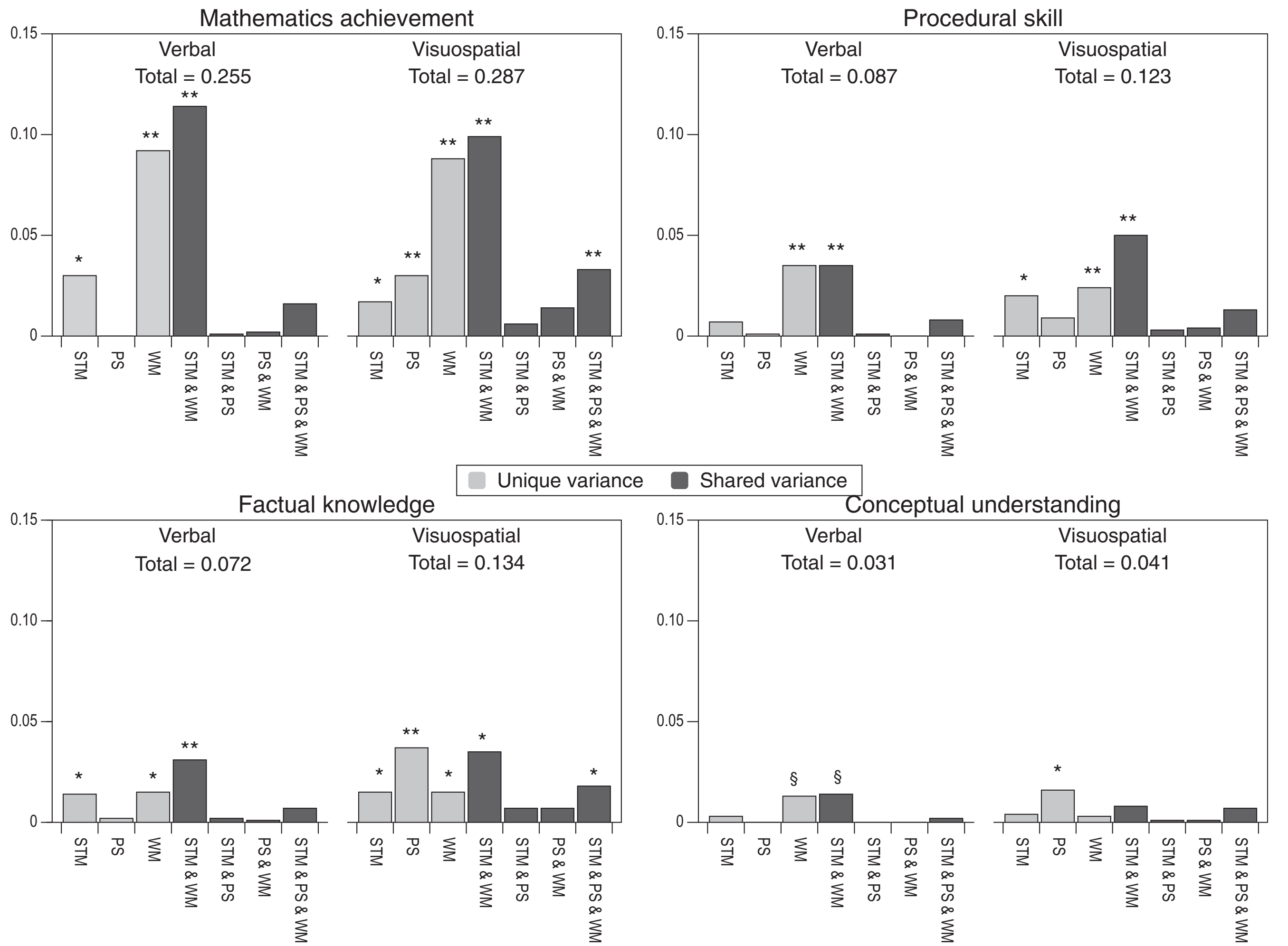

Fig. 2. Proportion of variance in mathematics achievement, factual knowledge, procedural skill and conceptual understanding accounted for by unique and shared contributions of verbal and visuospatial short-term memory (STM), processing speed (PS) and working memory (WM). Light grey bars indicate unique contributions and dark grey bars indicate shared contributions. $^{*}p < 0.05$, $^{**}p < 0.01$, $^{\S}p < 0.06$.

as overall mathematics achievement in individuals aged between 8 and 25 years of age. The findings support a modified version of a hierarchical framework for mathematics (Geary, 2004; Geary & Hoard, 2005) in which domain-general executive function skills, in particular working memory, support domain-specific mathematical processes, which in turn underpin overall mathematics achievement. We extended previous models by demonstrating that working memory also directly contributes to mathematical achievement (Fig. 3). This pattern of relationships between domain-general and domain-specific skills was found to be remarkably stable from 8 years of age through to young adulthood. Below we discuss the contribution of executive functions to mathematics, and the resulting theoretical implications, in more detail. We begin with a discussion of the role of executive functions in overall mathematical achievement, factual knowledge, procedural skill and conceptual understanding separately before moving on to compare across components and consider how executive functions contribute to mathematics achievement both directly and indirectly.

In line with a large body of literature we found a significant relationship between verbal and visuospatial working memory and overall mathematics achievement. This indicates that the ability to store and manipulate information in mind in the face of ongoing processing is strongly linked to the aptitude to do well in mathematics. The predicted relationship between inhibition and shifting and overall mathematics achievement was not found

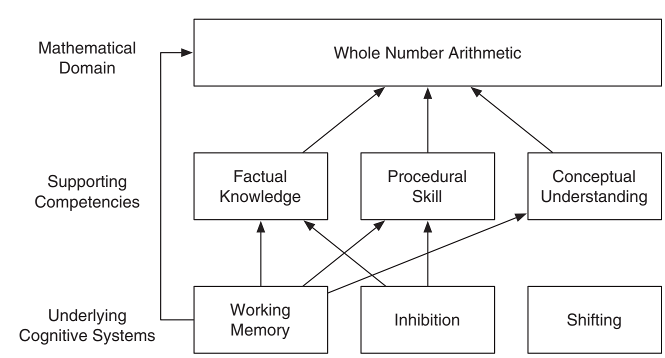

Fig. 3. Refined hierarchical framework of the executive functions underpinning mathematics.

however. This is partially consistent with evidence that inhibition and shifting account for less variance in mathematics achievement (Friso-van den Bos et al., 2013). It also provides support for the suggestion that inhibition and shifting may contribute unique variance to mathematics achievement when they are studied independently, but not when working memory is also included in the model (Bull & Lee, 2014; Lee & Bull, 2015).

The executive functions that contributed to factual knowledge and procedural skill were very similar, with verbal and visuospatial working memory as well as numerical inhibition accounting for unique variance in both components. This is consistent with theories that propose that working memory is required to activate and retrieve mathematical facts stored in long-term memory, and also that inhibitory processes are needed to suppress co-activated but incorrect answers. It also highlights the role of working memory in representing a problem and storing interim solutions in procedural mathematics skills, and suggests that inhibitory control may be required in order to select and employ the appropriate procedural strategy. In this study we did not find a relationship between shifting and procedural skill. This conflicts with findings from other studies that have examined the extent to which performance on a cognitive flexibility task predicts performance on a test of procedural skill (Andersson, 2010; Clark et al., 2010). Some of these positive findings were found in pre-schoolers (Clark et al., 2010) indicating that the role of shifting in mathematics may be greater earlier in childhood, as we suggested. Other positive relationships were found when a trailmaking task that involved numerical stimuli was used (Andersson, 2010). Relationships between working memory and mathematics have been found to be stronger when numerical stimuli are used within a working memory task (Raghubar, Barnes, & Hecht, 2010) and it is plausible that this could also be the case for measures of shifting. Similarly, we found that inhibitory control measured in a numerical context, but not including Arabic digits, was related to mathematics achievement as well as factual and procedural knowledge, but non-numerical inhibition was not. This is in line with previous research (Bull & Scerif, 2001; Szűcs et al., 2013) and provides evidence in support of the proposal that there are multiple domain-specific inhibitory control systems, rather than a single inhibitory system which applies across all domains (Egner, 2008).

The predicted relationship between working memory and conceptual understanding was found, albeit only in the verbal domain. This again is consistent with the idea of working memory being necessary to activate information stored in long-term memory. The fact that only verbal working memory was related to the retrieval of conceptual information, whereas both verbal and visuospatial working memory were implicated in the retrieval of mathematical facts could be because conceptual information is stored in a verbal code, whereas mathematical facts perhaps also contain a visuospatial component, related to the way that sums are often presented or the use of visual aids, such as times tables squares, at time of encoding. The predicted relationship between shifting and inhibition and conceptual understanding was not found. This may be because we used a task that required participants to apply conceptual knowledge that they already have. It may be that suppressing procedural strategies and rearranging problems into different formats in order to identify conceptual relationships are more important when conceptual information is being learnt rather than once it has been acquired.

This is the first study that has directly compared the contribution of executive function skills to factual knowledge, procedural skill and conceptual understanding across both children and adults using a comprehensive battery of executive function tasks. Together the executive function measures predicted more variance in factual knowledge and procedural skill than conceptual understanding, consistent with the findings of Hecht et al., (2003) and Jordan et al. (2013). We found that executive functions explained a similar amount of variance in both factual knowledge (12%) and procedural skill (15%), which is inconsistent with the findings of Cowan and Powell (2014) and Fuchs et al. (2005) who found that domain-general factors accounted for more variance in procedural skill than factual knowledge. The amount of variance explained was also much lower in our study than that of Cowan and Powell, where domain-general factors accounted for 43% variation in procedural skill and 36% variation in factual knowledge. This difference is likely due to the fact that Cowan and Powell included other domain-general factors in their model, such as visuospatial reasoning, processing speed and oral language. This may also explain the difference in variance explained between factual knowledge and procedural skill. It may be that while the contribution of executive functions is similar in both, other domain-general skills such as reasoning and language are more important for procedural skill than for factual knowledge. Similarly, the role of IQ in explaining variance in each mathematics components has yet to be fully explored. For example, it is possible IQ may explain more variance in conceptual understanding than executive functions.

The relationships between executive functions and factual knowledge, procedural skill and conceptual understanding were assessed across four different age groups; 8–9-year-olds, 11–12-year-olds, 13–14-year-olds and 18–25-year-olds. We predicted that executive functions would be more strongly related to procedural skill in the youngest age group in comparison to the older children and adults on the basis that executive functions may be required less with age as procedural skills become more automatic. Contrary to our predictions we found that the relationships between executive functions and all components of mathematics were the same from 8 years of age through to adulthood. There are two possible reasons for this. The first may be due to the nature of the mathematical measures that were used. Raghubar et al. (2010) distinguished between whether a skill is in the process of being acquired, consolidated or mastered, and suggested that the role of working memory in a particular mathematic process may differ depending on which of these stages the learner is at. By selecting separate age-appropriate content for the mathematics measures for each group it is possible that we were in fact assessing the role of executive functions in performing and applying already mastered mathematical skills and knowledge in all age groups, and that the role of executive functions in doing this is the same at all ages. This is consistent with another recent study that found little variation in the relationship between working memory and mathematics between the ages of 8 and 15 years (Lee & Bull, 2015). Further evidence that executive functions, in particular working memory, are required when individuals of all ages apply already mastered mathematical knowledge and procedures comes from dual-task studies in which solving relatively simple mathematical problems using factual and procedural strategies is impaired by a concurrent working memory load (Hubber et al., 2014; Imbo & Vandierendonck, 2007a, 2007b).

A second possibility for the stable relationship between executive functions and mathematics across age groups is that executive functions are particularly important for early skill acquisition leading to individual differences in learning arithmetic early in childhood, but that these individual differences remain and are still evident later in life. This would imply that executive functions play a greater role in learning new mathematical skills and knowledge compared to executing already mastered mathematical material. Further research directly comparing how executive functions are involved in mathematics at different levels of skill acquisition, for example when facts, procedures and concepts are first being learned compared to when they are mastered, is required to test these two possibilities, although they may not be mutually exclusive (Lee & Bull, 2015).

The regression analyses demonstrated that working memory contributed unique variance to overall mathematics achievement and also to factual knowledge, procedural skill and conceptual understanding. We subsequently carried out a mediation analysis to determine if, in line with hierarchical models of mathematics, performance on the domain-specific mathematics skills of retrieving mathematical facts, applying procedures and understanding concepts mediated the relationship between working memory and overall mathematics achievement. We found that verbal and visuospatial working memory do indeed contribute to mathematics achievement indirectly through factual knowledge, procedural skill and conceptual understanding, but that there is also a substantial pathway directly from working memory to mathematical achievement. A similar mediation analysis was conducted by Hecht et al., (2003) who compared the contribution of working memory to basic procedural arithmetic and conceptual understanding of fractions, and in turn to performance on tests of fraction word problems, estimation and computation. Hecht and colleagues found that working memory was a direct predictor of performance on fraction word problems, but not fraction computation. We have already discussed how working memory may support the different factual, procedural and conceptual components of mathematics, but what is its additional direct role in mathematics achievement? One suggestion is that an additional demand of mathematics achievement tests is the need to identify the mathematical problem that's presented within a verbal or visual description, construct a problem representation and then develop a solution for the problem (Andersson, 2010). It is likely that working memory plays a key role in these processes in terms of maintaining and manipulating these problem representations in mind. In keeping with this, studies have found that working memory is related to performance on word problems (Cowan & Powell, 2014; Fuchs et al., 2010). It remains to be established whether this relationship holds once the role of working memory in performing the appropriate arithmetic operation is taken into account.

The regression and mediation analyses demonstrated that working memory plays a key role directly in mathematics achievement, but also indirectly through its contribution to factual knowledge, procedural skill and conceptual understanding. These analyses could not reveal which components of working memory are driving these relationships however, and whether they differ depending on the mathematical process involved. This was because the verbal and visuospatial working memory tasks included in these analyses involved short-term storage as well as the executive demands of maintaining that storage in the face of concurrent processing. To that end, a variance partitioning approach was used to isolate the independent contribution of the central executive, short-term storage and processing as well as the shared variance between them. This was done separately for both verbal and visuospatial working memory. Consistent with previous findings (Bayliss et al., 2005) the working memory measures accounted for a moderate amount of unique variance in mathematical achievement as well as a smaller amount of variance in factual knowledge, procedural skill and conceptual understanding (verbal only) once short-term storage and processing had been accounted for. This is indicative of the contribution of the central executive and adds further evidence that it has a strong link with mathematics performance (Bayliss et al., 2003; Friso-van den Bos et al., 2013). A similar amount of variance in the mathematics tasks was explained by the shared variance between the working memory and short-term storage task however. This is likely to measure the ability to hold information in mind for a short amount of time given that this is a requirement of both the storage only and combined storage and processing tasks. This suggests that simply being able to hold information in mind is as important for mathematics as being able to hold that information while undertaking additional processing.

Within the literature there have been mixed findings suggesting that either verbal or visuospatial working memory plays a larger role in mathematics performance. Some previous evidence, including a meta-analysis of 111 studies, indicates that verbal working memory is more important for mathematical achievement in children compared to visuospatial working memory (Bayliss et al., 2003; Friso-van den Bos et al., 2013). In contrast, other researchers have suggested that it is in fact visuospatial working memory that plays a greater role (Andersson & Östergren, 2012; McLean & Hitch, 1999; Schuchardt et al., 2008; Szűcs et al., 2013, 2014), and that its importance may increase with age (Li & Geary, 2013). Very few studies have directly compared the role of verbal and visuospatial working memory using tasks that require storage alone and combined storage and processing in both domains however. In doing so we found that the contribution of verbal and visuospatial working memory was in fact very similar, both across different components of mathematics and also across age groups. The only major difference was that verbal, but not visuospatial, working memory contributed unique variance to conceptual understanding. Overall, these findings suggest that the ability to store both verbal and visuospatial information in mind in the face of ongoing processing is important for successful mathematics achievement. The domain-general central executive skills of monitoring and manipulating information play an important role, as do the domain-specific skills of holding both verbal and visuospatial information in mind. This is consistent with multi-component models of mathematics achievement which include both linguistic and spatial pathways (Geary, 2004; Geary & Hoard, 2005; LeFevre et al., 2010).

In addition to the variance shared with the working memory tasks, the verbal and visuospatial short-term memory tasks also both contributed unique variance to mathematical achievement and factual knowledge. The visuospatial short-term memory task also accounted for unique variance in procedural skill. This reflects a process that is not shared between the storage only and combined storage and processing tasks. One possibility is that this reflects the rehearsal of verbal items and visuospatial locations as there was more opportunity for this in the storage only tasks. It has also been proposed that this reflects the ability to reactivate items in memory (Bayliss et al., 2005). Visuospatial, but not verbal, processing accounted for unique variance in all of the mathematics tasks except for procedural skill, consistent with a large body of evidence demonstrating links between spatial skills and mathematics (see Mix & Cheng, 2012 for a review). Taken together, the results from the variance partitioning approach support our hypothesis that all components of working memory; storage, processing and the central executive, contribute to mathematics achievement. We did not find that the central executive was the most important component however. This is not inconsistent with previous findings as many studies use a combined storage and processing working memory task as a measure of the central executive when in fact it involves both the short-term storage components of working memory in addition to the central executive. Their results would perhaps be better interpreted as showing that both the short-term stores and central executive are important for mathematics, which is exactly what we found.

The results from this study provide further evidence that working memory capacity is linked to mathematics achievement, but indicate that the mechanisms by which working memory influences mathematics achievement might be varied and complex. This has important implications for current intervention approaches that aim to improve academic outcomes by training working memory capacity. To date, many studies have failed to show any improvement on standardised tests of mathematics achievement following working memory training (see meta-analysis by Melby-Lervåg & Hulme, 2013). Our results suggest that an intermediary approach may be beneficial to first ascertain whether working memory training can successfully enhance factual knowledge and procedural skill, or whether it has any impact on constructing problem representations. Such an approach has the potential to evaluate current interventions but would also further test our theoretical model (Fig. 3). More broadly, our findings support multi-component frameworks of mathematics which highlight that there are a wide range of skills, both domain-general and domain-specific, that contribute to successful mathematics achievement. A further corollary of multi-component models is that there are a range of reasons why children might struggle with maths. In terms of interventions it is important to identify the reasons children might be having difficulties, be it problems with factual knowledge, procedural skill, conceptual understanding or underlying working memory or inhibitory control problems such that interventions can be tailored accordingly. However, given that these processes are likely to interact (Gilmore et al., in press) training them in isolation may not be the most beneficial approach.

In conclusion, this study has shown that working memory plays a direct role in mathematics achievement in terms of identifying and constructing problem representations as well as an indirect role through factual knowledge, procedural skill and, to a lesser extent, conceptual understanding. Inhibitory control within the numerical domain also supports mathematics achievement indirectly through factual knowledge and procedural skill. Perhaps surprisingly, these relationships appear to be stable from 8 years through to adulthood. The results from this study support hierarchical multi-component models of mathematics in which achievement in mathematics is underpinned by domain-specific processes, which in turn draw on domain-general skills (Geary, 2004; Geary & Hoard, 2005). These findings begin to help us to comprehend the mechanisms by which executive functions support mathematics achievement. Such an understanding is essential if we are to create targeted interventions that can successfully improve mathematics outcomes for all learners.

Supplementary material

The full dataset for this study are available to download at http://reshare.ukdataservice.ac.uk/852106/.

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Appendix A

See Table A1.

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Table A1
Zero-order correlations between variables (all variables standardised within year group).

	1	2	3	4	5	6	7	8	9	10	11	12
1. Mathematics reasoning												
2. Number fact knowledge	0.494**											
3. Arithmetic strategy	0.550**	0.665**										
4. Conceptual understanding	0.308**	0.346**	0.241**									
5. Verbal STM	0.402**	0.232**	0.225**	0.137*								
6. Verbal processing	0.139*	0.108	0.101	0.037	0.244**							
7. Verbal WM	0.473**	0.232**	0.280**	0.168**	0.532**	0.205**						
8. Visuospatial STM	0.393**	0.274**	0.294**	0.140*	0.385**	0.127*	0.310**					
9. Visuospatial processing	0.289**	0.262**	0.172**	0.159**	0.144*	0.198**	0.160**	0.213**				
10. Visuospatial WM	0.484**	0.274**	0.302**	0.138*	0.324**	0.040	0.369**	0.554**	0.212**			
11. Numerical inhibition	0.244**	0.212**	0.260**	0.122*	0.191**	0.037	0.253**	0.179**	0.070	0.224**		
12. Non-numerical inhibition	0.162**	0.005	0.092	0.077	0.088	0.123*	0.172**	0.076	0.099	0.139*	0.185**	
13. Set shifting	−0.010	−0.094	−0.019	−0.084	−0.046	.054	.107	−0.048	−0.102	−0.067	−0.122*	.122*

Mathematics Reasoning = WIAT mathematics reasoning raw score; Number fact knowledge = accuracy; Arithmetic strategy = reversed RT; Conceptual understanding = accuracy; Verbal STM (short-term memory) = total score; Verbal processing = reversed RT; Verbal WM (working memory) = total score; Visuospatial STM (short-term memory) = total score; Visuospatial processing = reversed RT; Visuospatial WM (working memory) = total score; Numerical inhibition = difference in accuracy congruent – incongruent; Non-numerical inhibition = reversed difference in RT congruent – incongruent; Set shifting = number of correct sorts.

* $p < 0.05$.
** $p < 0.01$.

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