

Increased Efficiency in the Second-Hand Tire Trade Provides Opportunity for Dengue Control

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Abstract

Dengue fever is increasing in geographical range, spread by invasion of its vector mosquitoes. The trade in second-hand tires has been implicated as a factor in this process because they act as mobile reservoirs of mosquito eggs and larvae. Regional transportation of tires can create linkages between rural areas with dengue and disease-free urban areas, potentially giving rise to outbreaks even in areas with strong local control measures. In this work we sought to model the dynamics of mosquito transportation via the tire trade, in particular to predict its role in causing unexpected dengue outbreaks through vertical transmission of the virus across generations of mosquitoes. We also aimed to identify strategies for regulating the trade in second-hand

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tires, improving disease control. We created a mathematical model which captures the dynamics of dengue between rural and urban areas, taking into account the movement and storage time of tires, and mosquito diapause. We simulate a series of scenarios in which a mosquito population is introduced to a dengue-free area via movement of tires, either as single or multiple events, increasing the likelihood of a dengue outbreak. A persistent disease state can be induced regardless of whether urban conditions for an outbreak are met, and an existing endemic state can be enhanced by vector input. Finally we assess the potential for regulation of tire processing as a means of reducing the transmission of dengue fever using a specific case study from Puerto Rico. Our work demonstrates the importance of the second-hand tire trade in modulating the spread of dengue fever across regions, in particular its role in introducing dengue to disease-free areas. We propose that reduction of tire storage time and control of their movement can play a crucial role in containing dengue outbreaks.

Keywords:

Aedes; vertical transmission; diapause; reservoirs; transportation; mobility; metapopulations

1 **Introduction**

2 Dengue fever is among the most widespread vector-borne diseases, with
3 approximately 2.5 billion people at risk and 50 million infections annually
4 (World Health Organization, 2009). Dengue is endemic in over 100 tropical

5 and subtropical countries (Gubler, 2002). It is also the fastest re-emerging
6 disease (Cook and Zumla, 2008), imposing an economic burden alongside the
7 impaired health of affected individuals. Two mosquito species are responsible
8 for transmission of the virus via infective bites. The most common vector is
9 *Aedes aegypti*, but the Asian tiger mosquito (*Aedes albopictus*) is increasingly
10 important due to a rapidly expanding global distribution encompassing most
11 tropical regions (Belli et al., 2015; Rezza, 2012). *Aedes albopictus* began
12 to spread worldwide in the 1970s thanks to marine transport of tires and
13 other goods, leading to colonization of many areas of the world (Eritja et al.,
14 2005). At a global scale *Aedes albopictus* continues to spread to naive regions
15 due to commercial transport of used tires and climate change; the species is
16 also showing signs of adaptation to colder climates (Benedict et al., 2007;
17 Bonizzoni et al., 2013; Rochlin et al., 2013). There are four dengue virus
18 serotypes (García-Rivera and Rigau-Pérez, 2006), and once an individual
19 has been infected by one serotype they are permanently immune to that
20 serotype but only temporarily immune to the others (García-Rivera and
21 Rigau-Pérez, 2006; Esteva and Vargas, 2003).

22 Second-hand tires are widely traded both locally and globally. In coun-
23 tries with *A. aegypti* mosquitoes these often contain standing rain water
24 and eggs (Rezza, 2012; Yee, 2008), providing excellent larval habitats which
25 are frequently infected with both species (Alves Honório et al., 2006; Higa
26 et al., 2010). Tires have been an important dispersal mechanism for both
27 mosquitoes and dengue virus. *A. albopictus* originated in Asia but invaded

28 the New World in the 1980s via imported used tires and bamboo plants (Belli
29 et al., 2015; Gubler, 2002). It is now present in 20 countries in the Americas
30 (Belli et al., 2015). International trade in used tires and bamboo has also
31 been implicated in the introduction of *A. albopictus* to Europe (Medlock
32 et al., 2012). There is also circumstantial evidence that the transportation
33 of second-hand tires between urban areas has led to the introduction or re-
34 emergence of dengue in areas previously free of disease (Belli et al., 2015;
35 Medlock et al., 2012; Kourí et al., 1998).

36 According to the Pan American Health Organization and World Health
37 Organization (2014), *A. aegypti* was eliminated from the Americas in 1960.
38 Subsequently several countries interrupted control measures and the mosquito
39 began to spread again. Concurrently, social and economic changes in the
40 Americas, which increased trade and migration, permitted re-infestation
41 of the vector and dengue virus throughout South America. Due to economic
42 development, Briseño-García et al. (1996) suggest that in Mexico there was
43 correlation, if not causation, between the increase in the annual production of
44 tires from 1960 to 1990 and dengue incidence. A direct relation between tire
45 trade and dengue in Cuba was posited by Kourí et al. (1998). He mentions
46 that from 1981 to 1996, Cuba lacked any dengue transmission. Reintroduc-
47 tion has now occurred in some areas; the municipality of Santiago de Cuba
48 was reinfested in 1992 by *A. aegypti* transported in tires, followed by the
49 return of dengue.

50 Two processes play an important role in the transportation of *Aedes* and

51 dengue fever via tires. The first is the diapause phase in the mosquito life
52 cycle, enabling eggs to survive long periods of unfavorable conditions, includ-
53 ing desiccation (Thomas et al., 2012). Vertical or transovarial transmission
54 of dengue also occurs, with infected females passing the virus to their eggs
55 (Esteva and Vargas, 2000; Günther et al., 2007; Martins et al., 2012; Murillo
56 et al., 2014). Emerging adults are therefore able to transmit the disease with-
57 out first interacting with an infected host (Gubler, 1986; Cook and Zumla,
58 2008), potentially causing outbreaks in dengue-free areas.

59 There is a large tradition in ecology of studying the possible and viable
60 mechanisms of spread and colonization of species. Thus there is a broad liter-
61 ature describing different models and approaches to this problem red(Gotelli
62 and others, 1995; Levin et al., 2009; Loreau, 2010). For example, island-
63 mainland models assume constant migration of individuals MacArthur and
64 Wilson (2016) from an infinite mainland to an island. In contrast to this, in
65 metapopulation models red(Levins, 1969) there is no mainland but different
66 patches. In this work we develop an explicit metapopulation model describ-
67 ing the colonization of *A. aegypti* between two areas of different ecological
68 characteristics. In addition to this, we also analyzed the conditions that lead
69 to the emergence or re-emergence of dengue caused by mobile reservoirs for
70 disease. It should be noted that even if spread and colonization of species
71 has been related with re-emerging zoonoses Thompson (2000); Bengis et al.
72 (2004), there is little or none literature showing explicit dynamical models of
73 this phenomena, as we do here. We assess the potential role of transportation

74 of tires containing infected eggs in causing outbreaks in areas otherwise free
75 of both vectors and dengue. We consider the spread of dengue caused by a
76 single serotype.

77 Our mathematical model is based on two patches, representing a rural
78 area with endemic dengue and an urban area which begins as dengue-free.
79 We incorporate vertical transmission, diapause during transportation, and
80 the efficiency of tire processing. Through this we generate scenarios in which
81 (a) there is establishment of mosquitoes in an urban area from a rural area,
82 (b) these lead to a dengue outbreak occurring in the urban area, (c) a per-
83 sistent disease state is created in the urban area due to continuous influx of
84 infected eggs from rural area, and (d) an existing endemic infection is en-
85 hanced through additional input of infected vectors. In order to assess the
86 potential for management, we present a case study of implementing a man-
87 agement program to reduce tire processing times. Our work demonstrates
88 that, if effectively regulated, a reduction in the time that tires are stored
89 could aid in dengue control.

90 **Methods**

91 Our model aims to capture the dynamics of dengue fever in both humans
92 and female mosquitoes through tire movements at the landscape scale. Our
93 rationale for doing so is that, without taking this into account, other mea-
94 sures focused on disease treatment and migration control may prove to be
95 unexpectedly inefficient. We omit movement of infected mosquitoes, given

96 that newly hatched *A. aegypti* only fly around 20 m from their point of emer-
 97 gence (Christophers, 1960), and we also omit movement of infected humans
 98 as we are only interested in the particular effects of tire movement.

99 The landscape is divided in a rural and an urban patch. Each patch
 100 contains a local human population. One system (Fig. 1) is used to model
 101 disease dynamics in the rural area, while another (Fig. 2) applies to the urban
 102 area. The systems in the two patches differ due to the transfer of eggs from
 103 rural to urban areas and in the values of parameters. Table 1 summarizes
 104 the model parameters.

105 The system of differential equations that model the dynamics of dengue
 106 in human and mosquito populations in the rural area is given by:

$$\begin{aligned}
 \dot{S}_R &= \eta N_R - \alpha \frac{S_R}{N_R} M_{IR} - \eta S_R, \\
 \dot{I}_R &= \alpha \frac{S_R}{N_R} M_{IR} - (\eta + \gamma) I_R, \\
 \dot{R}_R &= \gamma I_R - \eta R_R, \\
 \dot{M}_{SR} &= \kappa \omega E_{SR} - \alpha \frac{I_R}{N_R} M_{SR} - \epsilon M_{SR}, \\
 \dot{M}_{IR} &= \kappa \omega E_{IR} + \alpha \frac{I_R}{N_R} M_{SR} - \epsilon M_{IR}, \\
 \dot{E}_{SR} &= \phi M_{SR} \left(1 - \frac{E_R}{C_r}\right) + (1 - \nu) \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) - \left(\pi + \omega + \frac{r}{\theta}\right) E_{SR}, \\
 \dot{E}_{IR} &= \nu \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) - \left(\pi + \omega + \frac{r}{\theta}\right) E_{IR}.
 \end{aligned} \tag{1}$$

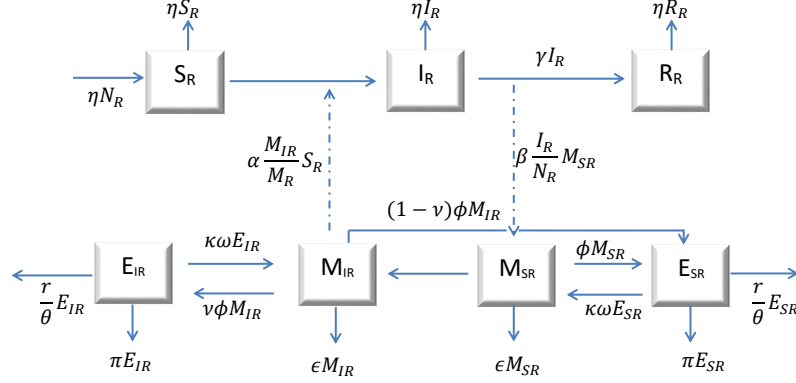


Figure 1: **Flowchart from rural dengue fever model.** Elements of the upper row refer to segments of the human population, susceptible (S), infected (I) and recovered (R). The lower row refers to adult mosquitoes (M) or their eggs (E). Arrows represent transition rates between stages. See Tables 1 and 2 for definitions of terms.

108 The differential equations that model the dynamics of dengue disease in
 109 human and mosquito populations in the urban area are given by:

$$\begin{aligned}
 \dot{S}_U &= \eta N_U - \alpha \frac{S_U}{N_U} M_{IU} - \eta S_U, \\
 \dot{I}_U &= \alpha \frac{S_U}{N_U} M_{IU} - (\eta + \gamma) I_U, \\
 \dot{R}_U &= \gamma I_U - \eta R_U, \\
 \dot{M}_{SU} &= \kappa \omega E_{SU} - \alpha \frac{I_U}{N_U} M_{SU} - \epsilon M_{SU}, \\
 \dot{M}_{IU} &= \kappa \omega E_{IU} + \alpha \frac{I_U}{N_U} M_{SU} - \epsilon M_{IU}, \\
 \dot{E}_{SU} &= \phi M_{SU} \left(1 - \frac{E_U}{C_u}\right) + (1 - \nu) \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) - (\pi + \omega) E_{SU} + \dots \\
 &\quad \frac{r}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d}\right) E_{SR}, \\
 \dot{E}_{IU} &= \nu \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) - (\pi + \omega) E_{IU} + \frac{r}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d}\right) E_{IR}.
 \end{aligned} \tag{2}$$

110 Where $N_R = S_R + I_R + R_R$, $N_U = S_U + I_U + R_U$, $M_R = M_{SR} + M_{IR}$,
 111 $E_R = E_{SR} + E_{IR}$, $M_U = M_{SU} + M_{IU}$, $E_U = E_{SU} + E_{IU}$.

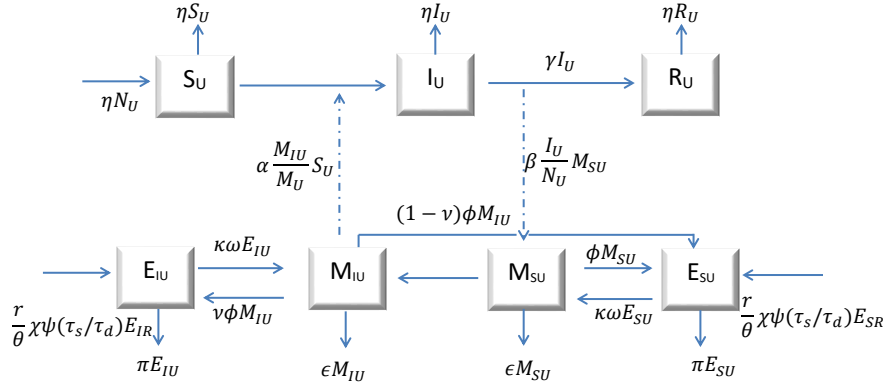


Figure 2: **Flowchart from urban dengue fever model.** Elements of the upper row refer to segments of the human population, susceptible (S), infected (I) and recovered (R). The lower row refers to adult mosquitoes (M) or their eggs (E). Arrows represent transition rates between stages. See Tables 1 and 2 for definitions of terms.

111

112 The total human populations in the rural and urban areas (N_R , N_U)
 113 are constant, given that the characteristic timescale of the disease is small
 114 (weeks) relative to that of human demographic processes (years). The sus-
 115 ceptible human class S increases by the per-capita birth rate η multiplied
 116 by the overall population size N . Individuals become infectious (class I) ac-
 117 cording to the bite rate α of infected vectors M_I . The rate at which humans
 118 recover from infection, whereupon they become permanently immune (class
 119 R), is γ . The per-capita death rate η is identical for all classes, and to the
 120 birth rate. The consequence of this, is that the total population is constant
 121 (Brauer et al., 2008).

122 Mosquito populations increase through egg eclosion at the development

123 rate ω and the total population is limited by a carrying capacity C_r in rural
 124 and C_u in urban areas. This is because mosquitoes exhibit density-dependent
 125 growth red(Juliano, 2007) and at the same time it is ensured the stability of
 126 the model for a broad range of entomological parameters. Adult mosquitoes
 127 die with a rate ϵ . Female mosquitoes oviposit at a rate ϕ and the eggs have
 128 an intrinsic mortality rate π . If a female mosquito is already infected, a
 129 fraction ν of its oviposited eggs are infected (vertical transmission). Vectors
 130 become infected by biting infectious hosts (I) at the contact rate α . In
 131 contrast to humans, mosquitoes never recover from the disease. Our model
 132 only considers the fraction κ of mosquitoes that are female, as males do not
 133 transmit the disease.

134 The number of tires transported from the rural to the urban area per unit
 135 time is r and θ is the mean number of tires in the rural area. Hence rE_{IR}/θ
 136 is the rate of infected egg movement from rural to urban areas and rE_{SR}/θ is
 137 the rate of susceptible egg movement also from rural to urban areas. During
 138 transportation a fraction χ of eggs survive. τ_s is the storage time before tire
 139 processing and τ_d represents the egg development time. As we assume that
 140 eggs stay in diapause stage during transportation and start its development
 141 once the tires arrive to the storage places, the fraction of eggs in the tires that
 142 are able to hatch as adults before being killed by tire recycling should be a
 143 function of τ_s/τ_d , i.e. $\psi(\tau_s/\tau_d)$. ψ should be a function such that when $\tau_s = 0$
 144 then $\psi = 0$, while when τ_s is greater than the development time τ_d , ψ should
 145 approach one. The total number of tires in the rural area remains constant,

146 but this is not true of the urban area. Thus, we would expect the effect of
147 tire transportation on disease dynamics in the rural area to be limited. We
148 also assume no substantive changes in the tire trade at the timescale of the
149 model dynamics.

150 The model explicitly takes into account the movement and storage time
151 of tires. Our study focuses on the necessary conditions for four possible
152 outcomes. This conditions are obtain by the analysis of the stationary state.
153 Scenario *I* considers the establishment in the urban area of mosquitoes from a
154 rural area where both areas are disease-free. In Scenario *II* a dengue outbreak
155 emerges in the urban area as a consequence of the joint introduction of the
156 mosquito and the virus in infected eggs. Scenario *III* induces or enhances a
157 persistent disease state in the urban area through the constant introduction of
158 infected mosquito eggs. Finally, in Scenario *IV*, we consider how regulation
159 of the market in second hand tires could act as a dengue control measure.
160 To demonstrate the impacts on dengue spread we calculate the secondary
161 dengue cases generated in the urban area as the result of a single case in rural
162 area. This quantity can be used as a preliminary measure of the impact of
163 controlling the movement of tires during dengue outbreaks. Finally we apply
164 our model to a specific study of the tire management system in Puerto Rico
165 using data from the Solid Waste Authority (A.D.S., 2014).

Parameter	Description
η	Per-capita birth and natural mortality rates in humans
γ	Per-capita recovery rate
α	Effective biting rate, per day
C_a	Carrying capacity of hatcheries, where $a \in \{r, u\}$, and r is rural and u urban area
ϕ	Number of eggs laid per day for every female mosquito
ϵ	Per-capita mortality rate of adult mosquitoes
π	Per-capita mortality rate of immature stage mosquitoes
ν	Proportion of eggs that are infected by vertical transmission
ω	Development rate of immature to mature stages
κ	Fraction of mosquitoes that are female
$\frac{\tau}{\theta}$	Per-tire transportation rate
χ	Fraction of eggs that survive the transportation
$\psi(\tau_s/\tau_d)$	Fraction of eggs in tires that were able to continue their development before tire processing
τ_s, τ_d	Tire storage time and egg development time

Table 1: Model parameters

166 **Results**

167 *Scenario I: Establishment of mosquitoes in an urban area*

168 Initial state: Mosquitoes only present in rural area; no disease.

169 If initially there were no mosquitoes in the urban area, the transportation
 170 of a single batch of tires can lead to the introduction of mosquito eggs from
 171 rural to urban areas. In order to obtain the conditions when establishment
 172 of an adult population of mosquitoes in an urban area might occur, we deter-
 173 mine the *urban net reproductive rate* (derived in Appendix A) R_M^u by means
 174 of the next generation matrix (Diekmann et al., 1990). The next generation
 175 matrix is a main element used in the formal mathematical procedure to ob-
 176 tain R_M^u . This quantity may change in different environmental and ecological
 177 conditions due to the change of insect development. Thus, if $R_M^u > 1$, then
 178 the population of mosquitoes is able to establish itself from a small number
 179 of eggs, while if adverse environmental conditions cause $R_M^u < 1$, then the
 180 mosquito population will eventually become extinct.

181 We are interested in the conditions that allow establishment of mosquitoes
 182 in a disease-free area. First, we find the condition that allows the
 183 immigration of viable eggs and then the condition in the urban area to sustain
 184 a mosquito population. In a single batch of N_T tires, the number of viable
 185 eggs that arrive in the urban area is given by

$$\frac{\omega}{\pi + \omega} \psi \left(\frac{\tau_s}{\tau_d} \right) \chi N_T \frac{E_R^*}{\theta} = \frac{\omega}{\pi + \omega} \psi \left(\frac{\tau_s}{\tau_d} \right) \chi N_T \frac{R_M^r - 1}{R_M^r} \frac{C_r}{\theta}$$

186 where $\frac{\omega}{\pi+\omega}$ is the probability of an egg hatching into an adult mosquito, E_R^*
 187 is the stationary number of eggs in the rural area (see Appendix B), R_M^r is
 188 the *rural net reproductive rate* (see Appendix B), and $\chi\psi(\frac{\tau_s}{\tau_d})$ is the fraction
 189 of eggs that survive before the tire processing cycle completes. redLike the
 190 urban net reproductive rate, the rural net reproductive rate is an indicator for
 191 the long-term persistence of mosquitoes in the rural area, that is, the average
 192 number of offspring that a female mosquitoes produces during her lifetime.
 193 Thus, $\frac{E_R^*}{\theta}$ is the number of eggs per tire, which multiplied by the batch size
 194 N_T determines the number of transported eggs. Then $\chi\psi(\frac{\tau_s}{\tau_d})N_T\frac{E_R^*}{\theta}$ is the
 195 number of eggs that survive tire transportation and processing, which is then
 196 multiplied by probability of hatching $\frac{\omega}{\pi+\omega}$ to obtain the number of emerging
 197 adult mosquitoes in the urban area. Then, the introduction of the species
 198 happens if

$$\frac{\omega}{\pi + \omega} \frac{N_T}{\theta} \chi\psi \left(\frac{\tau_s}{\tau_d} \right) \frac{R_M^r - 1}{R_M^r} C_r > 1 \quad (3)$$

199 The establishment of the mosquito population in the urban area will occur
 200 if in addition to the previous condition, the following is also met:

$$R_M^u = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega)} > 1 \quad (4)$$

201 This condition indicates that an urban mosquito population is sustain-
 202 able. R_M^u can be interpreted in terms of the model parameters as follows: ϕ/ϵ
 203 is the average number of eggs laid by a single female mosquito, $1/(\pi + \omega)$ is
 204 the average time of survival of an immature mosquito, and $1/\omega$ is the average

205 time spent in development, then $\kappa\omega/(\pi+\omega)$ is the probability that an egg will
 206 succeed to become an adult mosquito, and finally ϕ/ϵ is the average number
 207 of eggs oviposited by a single female mosquito. Expression (3) shows the
 208 importance of limiting the batch size and not only the tire transport rate.
 209 As a single big batch of tires could be enough to introduce a species even if
 210 the average tire transportation rate tend to zero.

211 On the other hand, if tire recycling becomes an established market
 212 with a constant flux of tires from the rural to the urban area, then the
 213 expected waiting time T_M before the introduction of a mosquito species from
 214 the rural to the urban population is given by the inverse of the rate of egg
 215 introduction:

$$T_M = \left[\frac{\omega}{\pi + \omega} r \chi \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{R_M^r - 1}{R_M^r} \frac{C_r}{\theta} \right]^{-1} \quad \text{for} \quad R_M^r > 1 \quad (5)$$

216 The expression inside the square parenthesis is similar to the second ex-
 217 pression of (3) but N_T is replaced by the tire introduction rate r . Thus this
 218 expression represents the rate of introduction of successful eggs and its in-
 219 verse is the average time before introduction of a single egg. This two
 220 different cases, single batch and constant rate, give different insights about
 221 two important aspects of tire transportation.

222 *Scenario II: A dengue outbreak occurs*

223 Initial state: Mosquitoes and dengue only present in the rural area, but
 224 urban environmental conditions suitable for an outbreak.

225 If continuous introduction of tires takes place from a dengue-endemic
 226 rural area to the urban area, a dengue outbreak might be precipitated by
 227 transportation of infected eggs. In order for this to happen the conditions
 228 in equations (3) and (4) must be met, also *basic reproductive number with-*
 229 *out vertical transmission must be redgreater than one*, that is, $R_0^u > 1$. Its
 230 value is given by $R_0^u = \sqrt{\frac{\alpha}{\epsilon} \frac{\beta N}{(\eta + \gamma) M^*}}$ (derived in Appendix C). In this case
 231 vertical transmission is omitted as its effect is negligible at the beginning
 232 of an outbreak (see Adams and Boots, 2010). Thus, in addition to (3), the
 233 condition

$$\sqrt{\frac{\alpha}{\epsilon} \frac{\beta N}{(\eta + \gamma) M^*}} > 1 \quad (6)$$

234 must also be meet.

235 The characteristic waiting time before the introduction of infected female
 236 mosquitoes T_o , is given by

$$T_o = \left[\frac{\kappa \omega}{\pi + \omega} r \chi \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{E_{IR}^*}{\theta} \right]^{-1} \quad (7)$$

237 where $r \chi \psi \left(\frac{\tau_s}{\tau_d} \right)$ is the number of successfully imported eggs per unit time,
 238 $\frac{\kappa \omega}{\pi + \omega}$ is the probability of an egg hatching into a female mosquito before death
 239 by natural causes, and $\frac{E_{IR}^*}{\theta}$ is the fraction of infected eggs in the tires. Thus
 240 the expression represents the effective rate of introduction of infected female
 241 mosquitoes.

242 In the case of introduction of a single batch of N_T tires, in addition to

243 conditions (3) and (6), the following must be satisfied:

$$\frac{\kappa\omega}{\pi + \omega} N_T \chi \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{E_{IR}^*}{\theta} > 1 \quad (8)$$

244 This is similar to T_o (equation 7) but r is replaced by the batch size N_T .
 245 Thus condition (8) represents the requirement that the number of eggs that
 246 hatch must exceed one.

247 *Scenario III: Persistent dengue states can be induced and enhanced*

248 Initial state: Mosquitoes and dengue only present in the rural area, and
 249 urban environmental conditions unfavorable for an outbreak.

250 There may be situations in which $R_0^u < 1$, and therefore dengue infes-
 251 tation in the urban area is not self-sustaining, but where continuous intro-
 252 duction of infected eggs in tires from an endemic rural area can induce a
 253 persistent disease state in the urban area. This state is not maintained by
 254 the intrinsic dynamics of the disease in the urban area and will cease if the
 255 introduction of infected eggs is interrupted (see Fig.s 3 and 4).

256 In this situation, the expected number of active dengue cases in the urban
 257 area is given by the stationary state I_U^* that is determined as

$$I_U^* = \frac{M_{IU}^*}{M_{IU}^* - N_U \frac{\eta}{\alpha}} \quad (9)$$

258 where M_{IU}^* represent the stationary state of the infected mosquitoes in the
 259 urban area (see Appendix H).

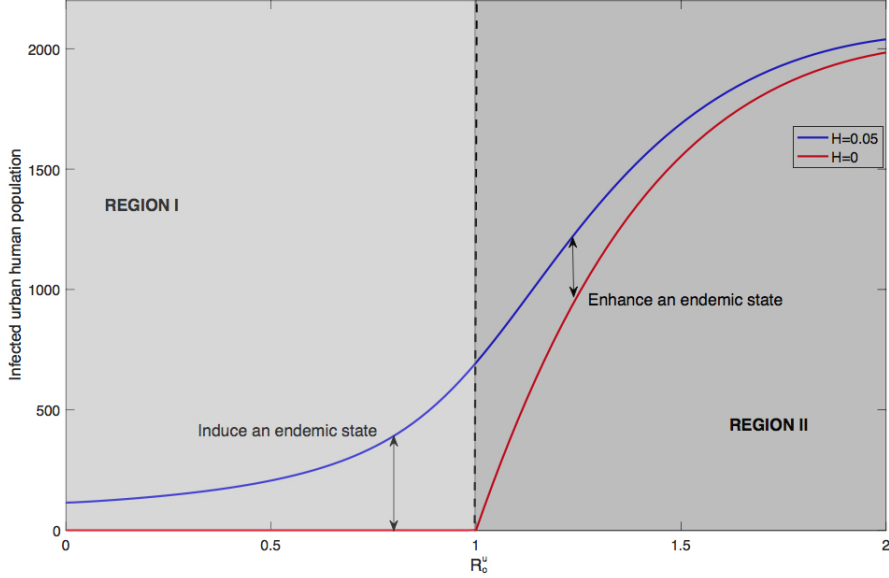


Figure 3: **Input of tires can enhance a dengue endemic state or induce a persistent one.** Steady-state number of infected humans as a function of basic reproductive number R_0^u . It is possible to induce a dengue-persistent state even though $R_0^u < 1$ (Region I) if there is a continuous flow of tires from an endemic rural area. If the disease is already endemic, tire transport of eggs will enhance the endemic state (Region II). $H = \frac{r}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d} \right)$ represents variation in the flow of tires.

260 Where dengue is already endemic in the urban area, the continuous im-
 261 portation of tires can enhance the number of infected people (see Figs. 3
 262 and 4). The number of infections at any given time is given by equation (9)
 263 when $R_0^u > 1$ in equation (H.1).

264 *Scenario IV: Regulation of the second hand tire market as a dengue control*
 265 *measure*

266 We now analyze the response of the infected human population with
 267 respect to tire movement. If tires are processed immediately, or at least soon

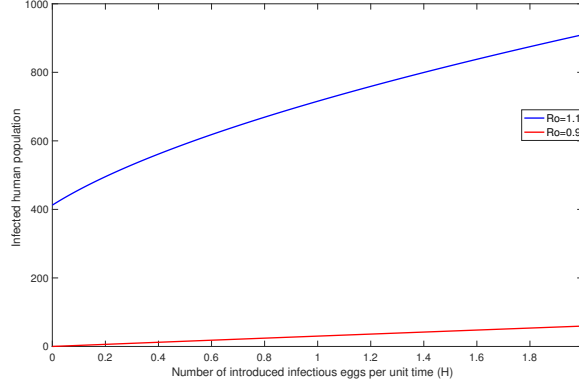


Figure 4: **Stationary level of infection in the urban population with increasing introduction of eggs.** Dengue cases increase as the number of introduced infected eggs per unit time H is increased. Red line $R_0^u < 1$ (intrinsically non-endemic state) and blue line $R_0^u > 1$ (endemic state). The parameter H is given by $\frac{r}{\theta} \chi \psi \left(\frac{\tau_s}{\tau_d} \right)$

268 after arrival to the urban area such that $\tau_s \ll \tau_d$, introduction of dengue
 269 fever does not occur.

270 In the rural area, diminishing the number of eggs by removal from the
 271 rural area reduces the basic reproductive number in rural area R_0^r (see Ap-
 272 pendix D); this means that the number of dengue cases is reduced. This is
 273 due to the increase in r while holding all other parameters constant.

274 In addition to this, if a large enough number of exported tires is main-
 275 tained, then the *net reproductive rate* could shift from $R_M^r > 1$ to $R_M^r < 1$,
 276 meaning that the rural mosquito population could no longer sustain itself.
 277 If it takes redtoo long before the mosquito population cease to exist, dengue
 278 can remain for some period of time before vanishing. If dengue remains in
 279 this situation in rural areas then the risk of introducing infected eggs into the
 280 urban area will persist unless tires are processed immediately. The maximum

281 storage time τ_s which still prevents dengue introduction can be estimated by
 282 ensuring that the expected latency before introduction of an infected female
 283 mosquito (equation 7) is greater than the extinction time of the vector in the
 284 rural area. In this work we have assumed that diapause ends when the eggs
 285 enter the urban area; the eggs thence continue their development, allowing
 286 some time before hatching.

287 When $R_M^r < 1$ we can use the Jacobian matrix of the vector demography
 288 (Appendix B.1) to obtain the expected extinction time. This is the time
 289 taken for the linearized system describing the dynamics of the mosquitoes to
 290 reach a population size of zero. The variable R_M^r can also be interpreted as
 291 the number of successful offspring that a female mosquito produces during
 292 its lifespan. The inverse of the smallest absolute value from its eigenvalues
 293 is an estimator of extinction time. Thus, the following condition should be
 294 met to reduce the risk of dengue dispersal in an established market where
 295 tires are continuously imported to the urban area:

$$T_o < \left| \frac{1}{2}(\gamma + \sqrt{\xi}) \right|^{-1} \quad \text{and} \quad R_M^r = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega + r/\theta)} < 1$$

296 where $\gamma = -(\epsilon + \pi + \omega + r/\theta)$, $\xi = \gamma^2 - 4\Xi$ and $\Xi = \epsilon(\pi + \omega + r/\theta)(1 - R_M^r)$
 297 (see Appendix B). This simultaneously works as a control measure in the
 298 rural area.

299 In order to assess the impact of interventions in the tire trade on disease
 300 dynamics, we can calculate the secondary human infections in the urban

301 disease free area caused by human infections in the rural area at the beginning
 302 of an outbreak $R_{r \rightarrow u}$ (see Appendix F). There will be one initial case of
 303 dengue virus in the urban area related to tire transportation for each $1/R_{r \rightarrow u}$
 304 cases in the rural area, where

$$R_{r \rightarrow u} = \frac{\alpha \kappa \omega r \chi}{\epsilon (\omega + \pi) (\theta (\omega + \pi) + r)} \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{\nu \phi}{\epsilon} \frac{\beta}{(\eta + \gamma)}.$$

305 Thus, $R_{r \rightarrow u}$ gives the number of cases in urban area which are derived
 306 from an infected person in the rural area.

307 *Case study*

308 One of the main barriers for dengue eradication in Latin America is the
 309 problem of stored tires (Cantanhede and Monge, Lima, 2002). These are
 310 favorable sites for the breeding of multiple vector insects, with implications
 311 for disease transmission and human health. Used tires are one of the sites in
 312 which *A. aegypti* females deposit their eggs, becoming an important pathway
 313 for their proliferation and thus causing outbreaks of dengue in tropical and
 314 subtropical countries (Cantanhede and Monge, Lima, 2002). Conditional on
 315 successful introduction, an endemic disease state can arise in areas where the
 316 environmental conditions are suitable (Cantanhede and Monge, Lima, 2002).

317 Some Latin American countries, such as Costa Rica and Peru, have
 318 banned the import of used tires (Cantanhede and Monge, Lima, 2002). This
 319 is due to the perceived danger to public health, in addition to concerns re-

320 guarding road safety and protection of the environment. Costa Rica does not
321 possess the necessary technology to treat used tires without causing environ-
322 mental pollution (Cantanhede and Monge, Lima, 2002).

323 In Puerto Rico the accumulation of discarded tires in *gomer*as and facili-
324 ties around the island represent an environmental and health crisis (A.D.S.,
325 2014). The country has therefore implemented a tire management program.
326 According the Solid Waste Authority (ADS), around 18,000 tires are dis-
327 carded every day; this amounts to 4.7 million tires a year. Despite the tire
328 management program, it is not possible to collect all discarded tires. Among
329 the major public health risks of excessive accumulation of tires is the spread
330 of pests and diseases such as dengue.

331 A total of 6,766 confirmed cases of dengue were reported in Puerto Rico
332 in 2013 (A.D.S., 2014). For this reason the authorities have decided to reduce
333 the disposal of tires. They have introduced authorized solid waste facilities
334 in which the accumulation of tires is permitted for up to 90 days. The law
335 also allows local governments to collect used tires voluntarily and temporar-
336 ily. The collection and transport of discarded tires is carried out by official
337 vehicles (A.D.S., 2014).

338 In this section we use the data provided by ADS to estimate $\Psi(\tau_p/\tau_d)$ and
339 make an estimation of the program benefits in the reduction of dengue cases.
340 We also employ our model to analyze the implications of tire management
341 for dengue transmission in this specific geographical context. We use the
342 number of discarded tires and processed tires per *gomer*a reported by ADS

343 to calculate the parameter r representing the rate of tire transportation from
344 rural to urban areas. In order to do so we merge all urban populations into a
345 single population, and the same for rural populations, i.e. we have used the
346 homogeneous mixing hypothesis. This approximation leads to an overesti-
347 mation of disease cases because it is assumed that there are more interactions
348 between the populations than occur in reality. Thus the estimates given by
349 this analysis represent a worst case scenario, but also provide the starting
350 point for a geographically-structured model.

Parameter	Value	Units	Reference
η	0.002	1/days	Estimated
γ	1/7	1/days	Adams and Boots (2010)
α	0.67	1/days	Adams and Boots (2010)
C_a	10000 and 1000	eggs	Estimated
ϕ	10	1/days	Esteva et al. (2006)
ϵ	1/8	1/days	Adams and Boots (2010)
π	1/8	1/days	Adams and Boots (2010)
ν	0.3	proportion	Adams and Boots (2010)
ω	1/8	1/days	Adams and Boots (2010)
κ	0.5	proportion	Estimated
$\frac{r}{\theta}$	10, 20	1/days	Estimated
χ	0.1	proportion	Estimated
$\psi(\tau_s/\tau_d)$	0.64	proportion	Estimated using A.D.S. (2014)
τ_s, τ_d	90, 10	days	A.D.S. (2014), Esteva et al. (2006)

Table 2: Parameter values for simulating the dynamics of dengue transmission in Puerto Rico based on literature sources.

351 Figs 5 and 6 show the populations of infected humans in rural and urban
352 areas. In Fig. 5 the infected population in rural areas with no transportation
353 of tires (red dashed line) demonstrates that the disease is endemic. If trans-
354 portation of tires takes place, the infected population in rural areas declines

355 by 9.8% (blue solid line).red So, in this situation tire transportation acts as
 356 another mortality rate.

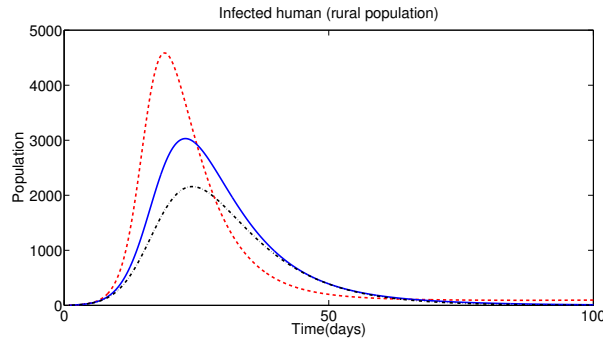


Figure 5: **Dynamics of dengue infection in rural areas under three scenarios.** Red dashed line shows the population size of infected humans when there is no transportation of tires. Blue solid line shows the infected population when tire transportation takes place ($r/\theta = 10$). Black dash-dot line shows the infected population when tire transportation rate is increased ($r/\theta = 20$).

357 Puerto Rico has instituted a program to recover used tires. Fig. 6 shows
 358 the comparison between number of infected people depending on whether the
 359 tires are handled appropriately. The blue line depicts the infected population
 360 in the absence of a recycling program ($\Psi(\tau_s/\tau_d) = 1$). The recycling program
 361 recovers and processes 36% of tires in the three temporary storage centers
 362 ($\Psi(\tau_s/\tau_d) = 0.64$). Our model suggests that the tire recycling program may
 363 have reduced the number of dengue cases considerably. Compared to the
 364 case with no recycling program, the program reduced the number of infected
 365 people in urban areas by 13.0% (red line).

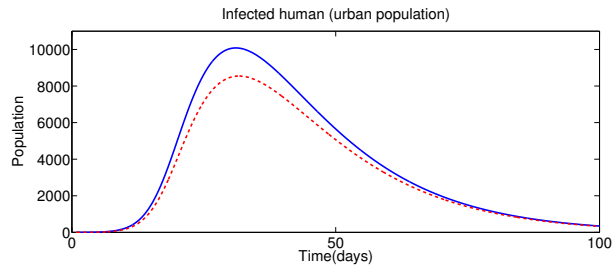


Figure 6: **Dynamics of dengue infection in urban areas, indicating the impact of the tire recycling program.** Blue solid line shows the outbreak dynamics with unregulated transport of tires, red dashed line shows the predicted urban outbreak given the existence of this program.

366 **Discussion**

367 The trade in second-hand tires, if unregulated, can under certain cir-
 368 cumstances be an important factor in the generation of dengue outbreaks.
 369 Tire movement can both trigger epidemics and sustain disease states through
 370 continuous reintroduction. Regulation can help to avoid these impacts and
 371 reduce the availability of mosquitoes hatcheries. Thus tires are an impor-
 372 tant component of disease dynamics whenever environmental conditions are
 373 conducive to a dengue epidemic.

374 Discarded tires are believed to be one of the most productive hatcheries
 375 of the *Aedes* mosquitoes which transmit dengue fever (Alves Honório et al.,
 376 2006), with many eggs transported whilst in a diapause state. As a result
 377 of vertical transmission across generations, infected mosquitoes can pass the
 378 virus to their offspring, and therefore a new generation of infective vectors
 379 emerge, able to transmit the disease without first feeding on an infected indi-
 380 vidual. Evidence also exists of vertical transmission of Zika and Chikungunya

381 viruses by *A. aegypti* and *A. albopictus* (Thangamani et al., 2016; Ferreira-
382 de-Brito et al., 2016; Niyas et al., 2010; Agarwal et al., 2014), extending
383 the implications of our model to other vector-borne diseases. Our model
384 therefore has important consequence for the spread of a range of emerging
385 diseases. Even in hostile environmental conditions, the resistance of *Aedes*
386 eggs to desiccation, combined with vertical transmission of the virus, is likely
387 to facilitate the persistence and spread of disease.

388 Our model demonstrates that the movement of tires containing mosquito
389 eggs has the potential to transfer both vector and virus from rural to urban
390 regions, and with a sufficient rate of input, can induce a persistent dengue
391 state in the urban area even if environmental conditions or control measures
392 such as fumigation and hatchery elimination would otherwise cause it to be
393 eradicated. Management of the tire trade to reduce their storage time is a
394 potential strategy for reducing spread of the disease, and we demonstrate
395 using an empirical case study from Puerto Rico that even a modest program
396 of tire collection can lead to major declines in the disease burden experienced
397 in urban areas. Tires left in the open are productive *A. aegypti* hatcheries
398 (Higa et al., 2010; Yee, 2008; Alves Honório et al., 2006), increasing the risk
399 of dengue transmission. There is also evidence that tire transportation has
400 led to the introduction of dengue in areas previously free of disease (Belli
401 et al., 2015; Medlock et al., 2012; Kourí et al., 1998).

402 The model explicitly takes into account the movement and storage of
403 second-hand tires, typically from rural to urban areas for processing, a com-

404 mon feature of their trade. Thus, this study can help to guide tire-trade
405 policy. The particular policy can be strengthened or relaxed depending on
406 local conditions with respect to the scenarios analyzed above.

407 Our model can be used as the basis for evidence-based policy-making in
408 a range of contexts with appropriate parametrization. Practical issues such
409 as the frequency of fumigation campaigns, limits to the batch sizes of tires,
410 and regulations concerning the storage time and conditions of tires, can all
411 have quantifiable impacts on disease dynamics. Incorporating these actions
412 as model extensions would inform effective investment of limited resources.
413 Most importantly, we show that control of dengue transmission in urban
414 areas provides insufficient protection of public health when tire movement
415 continues as a source of constant reintroduction.

416 The homogeneous mixing assumption for the rural area and the urban
417 area is likely an overestimation of cases in our model. In order to improve
418 accuracy of the predictions it would be necessary to build a spatially ex-
419 plicit model where different rural areas with its interactions are taken into
420 account. In addition to this, the model analyses the impact of tire movement
421 alone. Extending the model in order to take into account human migration
422 can guide more complex policies of simultaneous interventions on tire trade
423 and human mobility.

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Appendix A. Urban net reproductive rate

In the following we determine the urban net reproductive rate, for this we consider the equations for mosquitoes in urban area, that is \dot{M}_{SU} , \dot{M}_{IU} , \dot{E}_{SU} and \dot{E}_{IU} . Also $\dot{M} = \dot{M}_{SU} + \dot{M}_{IU}$ and $\dot{E} = \dot{E}_{SU} + \dot{E}_{IU}$, then we have the following system:

$$\begin{aligned}\dot{M} &= \kappa\omega E - \epsilon M, \\ \dot{E} &= \phi M - (\pi + \omega)E + \frac{r\chi}{\theta}\psi\left(\frac{\tau_s}{\tau_d}\right) E_R.\end{aligned}\tag{A.1}$$

We calculate the mosquito urban net reproduction rate using the method of (Diekmann et al., 1990). The system(A.1) can be defined as $\dot{\mathfrak{X}} = \mathfrak{F} - \mathfrak{W}$:

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{M} \\ \dot{E} \end{pmatrix}, \quad \mathfrak{F} = \begin{pmatrix} \kappa\omega E \\ 0 \end{pmatrix}, \quad \mathfrak{V} = \begin{pmatrix} \epsilon M \\ (\pi + \omega)E - \phi M \left(1 - \frac{E}{C}\right) - \frac{r_X}{\theta} \psi \left(\frac{\tau_s}{\tau_d}\right) E_R \end{pmatrix}.$$

The Jacobian matrices F and V , associated with \mathfrak{F} and \mathfrak{V} respectively, at the vector free equilibrium $M^* = 0$, $E^* = 0$ are:

$$F = \begin{pmatrix} 0 & \kappa\omega \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \epsilon & 0 \\ -\phi & (\pi + \omega) \end{pmatrix}, \quad V^{-1} = \begin{pmatrix} \frac{1}{\epsilon} & 0 \\ \frac{\phi}{\epsilon(\pi + \omega)} & \frac{1}{\pi + \omega} \end{pmatrix},$$

$$K = FV^{-1} = \begin{pmatrix} \frac{\kappa\omega\phi}{\epsilon(\pi + \omega)} & \frac{\kappa\omega}{\pi + \omega} \\ 0 & 0 \end{pmatrix}.$$

The eigenvalues of K are 0 and $\frac{\kappa\omega\phi}{\epsilon(\pi + \omega)}$, so the mosquito urban net reproductive rate is given by:

$$R_M^u = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega)}$$

The meaning of the parameter R_M^u is the average number of mosquitoes produced by a single mosquito during her lifetime in the urban area.

Appendix B. Rural net reproductive rate

In the following we determine the vector demography, we start with rural net reproductive rate, for this we consider the equations for mosquitoes in rural area, that is \dot{M}_{SR} , \dot{M}_{IR} , \dot{E}_{SR} and \dot{E}_{IR} . Also $\dot{M} = \dot{M}_{SR} + \dot{M}_{IR}$ and $\dot{E} = \dot{E}_{SR} + \dot{E}_{IR}$, then we have the following system:

$$\begin{aligned}\dot{M} &= \kappa\omega E - \epsilon M, \\ \dot{E} &= \phi M - \left(\pi + \omega + \frac{r}{\theta}\right)E.\end{aligned}\tag{B.1}$$

We calculate the net reproductive rate using the method of (Diekmann et al., 1990). We write the system(B.1) as $\dot{\mathfrak{X}} = \mathfrak{F} - \mathfrak{V}$:

$$\dot{\mathfrak{X}} = \begin{pmatrix} \dot{M} \\ \dot{E} \end{pmatrix}, \quad \mathfrak{F} = \begin{pmatrix} \kappa\omega E \\ 0 \end{pmatrix}, \quad \mathfrak{V} = \begin{pmatrix} \epsilon M \\ \left(\pi + \omega + \frac{r}{\theta}\right)E - \phi M \left(1 - \frac{E}{C}\right) \end{pmatrix}.$$

The Jacobian matrices F and V , associated with \mathfrak{F} and \mathfrak{V} respectively, at the vector free equilibrium $M^* = 0$, $E^* = 0$ are:

$$F = \begin{pmatrix} 0 & \kappa\omega \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \epsilon & 0 \\ -\phi & (\pi + \omega + \frac{r}{\theta}) \end{pmatrix}, \quad V^{-1} = \begin{pmatrix} \frac{1}{\epsilon} & 0 \\ \frac{\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})} & \frac{1}{\pi + \omega + \frac{r}{\theta}} \end{pmatrix},$$

$$K = FV^{-1} = \begin{pmatrix} \frac{\kappa\omega\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})} & \frac{\kappa\omega}{\pi + \omega + \frac{r}{\theta}} \\ 0 & 0 \end{pmatrix}.$$

The eigenvalues of K are 0 and $\frac{\kappa\omega\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})}$, so the rural net reproductive rate is given by:

$$R_M^r = \frac{\kappa\omega\phi}{\epsilon(\pi + \omega + \frac{r}{\theta})}.$$

Where R_M^r is the average number of mosquitoes produced by a single mosquito during her lifetime in rural area. On the other hand, the system (B.1) has two stationary states $E^* = M^* = 0$ and $M^* = \frac{C\kappa\omega}{\epsilon} \left(\frac{R_M^r - 1}{R_M^r} \right)$, $E^* = C \left(\frac{R_M^r - 1}{R_M^r} \right)$.

Linearization around the trivial stationary solutions requires calculation of the Jacobian matrix around the equilibrium point $(0, 0)$:

$$\mathbf{DF}(\mathbf{0}, \mathbf{0}) = \begin{pmatrix} -\epsilon & \kappa\omega \\ \phi & -(\pi + \omega + \frac{r}{\theta}) \end{pmatrix}.$$

This obtains the following characteristic polynomial:

$$\lambda^2 + \left(\epsilon + \pi + \omega + \frac{r}{\theta}\right)\lambda + \epsilon\left(\pi + \omega + \frac{r}{\theta}\right)(1 - R_M) = 0$$

whose roots are of the shape

$$\lambda_{\pm} = \frac{1}{2}(\gamma \pm \sqrt{\xi})$$

where $\gamma = -\left(\epsilon + \pi + \omega + \frac{r}{\theta}\right)$, $\xi = \gamma^2 - 4\Xi$ and $\Xi = \epsilon\left(\pi + \omega + \frac{r}{\theta}\right)(1 - R_M)$.

Appendix C. Basic reproduction number without vertical

In the following we calculate the basic reproduction number in urban area. The infected classes in the urban model are I_U , E_{IU} and M_{IU} , so the matrices \mathfrak{F} and \mathfrak{V} take the following shape:

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_U}{N_U} M_{IU} \\ 0 \\ \alpha \frac{I_U}{N_U} M_{SU} \end{pmatrix} \quad \mathfrak{V} = \begin{pmatrix} (\eta + \gamma) I_{IU} \\ (\pi + \omega) E_{IU} - \frac{r}{\theta} \chi \psi(\tau_s/\tau_d) E_{IR} \\ \epsilon M_{IU} - \kappa \omega E_{IU} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & 0 & \alpha \frac{S_U}{N_U} \\ 0 & 0 & 0 \\ \alpha \frac{M_{SU}}{N_U} & 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 \\ 0 & (\pi + \omega) & 0 \\ 0 & -\kappa\omega & \epsilon \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta + \gamma)} & 0 & 0 \\ 0 & \frac{1}{\pi + \omega} & 0 \\ 0 & \frac{\kappa\omega}{\epsilon(\pi + \omega)} & \frac{1}{\epsilon} \end{pmatrix}$$

We evaluated the Jacobian matrices at the disease free equilibrium $S_U = N_U, M_{SU} = M_U, E_{SU} = E_U, I_U = M_{IU} = E_{IU} = 0$. The eigenvalues of $K = FV^{-1}$ since the basic reproduction number is the spectral radius. The maximum of the eigenvalues of K will be the basic reproduction number.

$$K = \begin{pmatrix} 0 & \frac{\alpha\kappa\omega}{\epsilon(\pi + \omega)} & \frac{\alpha}{\epsilon} \\ 0 & 0 & 0 \\ \frac{\alpha N}{(\eta + \gamma)M^*} & 0 & 0 \end{pmatrix}$$

There are three eigenvalues, one of them is zero, the other is smaller, so the maximum is

$$R_0^u = \sqrt{\frac{\alpha N}{(\mu + \gamma)M^*} \frac{\alpha}{\epsilon}}.$$

The basic reproduction number without vertical transmission in urban area R_0^u is the expected number of secondary cases produced by a single infection in a completely susceptible urban population

Appendix D. Basic reproduction number in rural area

We calculated the basic reproduction number in rural area using the next generation matrix method $\dot{\mathfrak{X}} = \mathfrak{F} - \mathfrak{V}$ (Diekmann et al., 1990). The infected classes in the rural model are: I_R , E_{IR} and M_{IR} . The information is separated into two matrices, the first one corresponding to new infection and the second corresponding to disease progression, that is:

$$\dot{\mathfrak{X}} = \begin{pmatrix} \dot{I} \\ \dot{E}_{IR} \\ \dot{M}_{IR} \end{pmatrix}$$

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_R}{N_R} M_{IR} \\ \nu \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) \\ \alpha \frac{I_R}{N_R} M_{SR} \end{pmatrix} \quad \mathfrak{V} = \begin{pmatrix} (\eta + \gamma) I_{IR} \\ (\pi + \omega + \frac{r}{\theta}) E_{IR} \\ \epsilon M_{IR} - \kappa \omega E_{IR} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & 0 & \alpha \frac{S_R}{N_R} \\ 0 & -\frac{\nu\phi M_{IR}}{C_r} & \nu\phi \left(1 - \frac{E_R}{C_r}\right) \\ \alpha \frac{M_{SR}}{N_R} & 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 \\ 0 & (\pi + \omega) + \frac{r}{\theta} & 0 \\ 0 & -\kappa\omega & \epsilon \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta + \gamma)} & 0 & 0 \\ 0 & \frac{\theta}{\theta(\pi + \omega) + r} & 0 \\ 0 & \frac{\theta\kappa\omega}{\epsilon(\theta(\pi + \omega) + r)} & \frac{1}{\epsilon} \end{pmatrix}$$

We evaluated the Jacobian matrices at the disease free equilibrium $S_R = N_R$, $M_{SR} = M_R$, $E_{SR} = E_R$, $I_R = M_{IR} = E_{IR} = 0$. Then we found the eigenvalues of $K = FV^{-1}$; from this we need the maximum of the eigenvalues of K , which is the basic reproductive number.

$$K = \begin{pmatrix} 0 & \frac{\alpha\theta\kappa\omega}{\epsilon(\theta(\pi + \omega) + r)} & \frac{\alpha}{\epsilon} \\ 0 & \frac{\nu\phi\theta\kappa\omega}{\epsilon(\theta(\pi + \omega) + r)} \left(1 - \frac{E}{C_r}\right) & \frac{\nu\phi}{\epsilon} \left(1 - \frac{E}{C_r}\right) \\ \frac{\alpha N}{M^*\eta + \gamma} & 0 & 0 \end{pmatrix}$$

There are three eigenvalues, one of them is zero, the other is smaller, so the maximum is

$$R_0^r = \frac{1}{2} \frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi + \omega) + r)} \left(1 - \frac{E_R^*}{C_r}\right) + \frac{1}{2} \sqrt{\left(\frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi + \omega) + r)} \left(1 - \frac{E_R^*}{C_r}\right)\right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta + \gamma)M^*}}.$$

The basic reproduction number in rural area R_0^r is the expected number of secondary cases produced by a single infection in a completely susceptible rural population

Appendix E. Basic reproduction number in urban area

To calculate the basic reproduction number in urban area we consider the infected classes in the urban model, that is I_U , E_{IU} and M_{IU} , so the matrices \mathfrak{F} and \mathfrak{V} take the following form:

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_U}{N_U} M_{IU} \\ \nu \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) \\ \alpha \frac{I_U}{N_U} M_{SU} \end{pmatrix} \quad \mathfrak{V} = \begin{pmatrix} (\eta + \gamma) I_{IU} \\ (\pi + \omega) E_{IU} - \frac{\tau}{\theta} \chi \psi(\tau_s/\tau_d) E_{IR} \\ \epsilon M_{IU} - \kappa \omega E_{IU} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & 0 & \alpha \frac{S_U}{N_U} \\ 0 & -\frac{\nu \phi M_{IU}}{C_u} & \nu \phi \left(1 - \frac{E_U}{C_u}\right) \\ \alpha \frac{M_{SU}}{N_U} & 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 \\ 0 & (\pi + \omega) & 0 \\ 0 & -\kappa \omega & \epsilon \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta+\gamma)} & 0 & 0 \\ 0 & \frac{1}{\pi+\omega} & 0 \\ 0 & \frac{\kappa\omega}{\epsilon(\pi+\omega)} & \frac{1}{\epsilon} \end{pmatrix}$$

We evaluated the Jacobian matrices at the disease free equilibrium $S_U = N_U, M_{SU} = M_U, E_{SU} = E_U, I_U = M_{IU} = E_{IU} = 0$. We found the eigenvalues of $K = FV^{-1}$; the maximum of the eigenvalues of K is the basic reproduction number.

$$K = \begin{pmatrix} 0 & \frac{\alpha\kappa\omega}{\epsilon(\pi+\omega)} & \frac{\alpha}{\epsilon} \\ 0 & \frac{\nu\phi\kappa\omega}{\epsilon(\pi+\omega)} \left(1 - \frac{E_U}{C_u}\right) & \frac{\nu\phi}{\epsilon} \left(1 - \frac{E_U}{C_u}\right) \\ \frac{\alpha N}{(\eta+\gamma)M^*} & 0 & 0 \end{pmatrix}$$

There are three eigenvalues, one of them is zero, the other is smaller, so the maximum is

$$R_0^u = \frac{1}{2} \frac{\nu\phi\kappa\omega}{\epsilon(\pi+\omega)} \left(1 - \frac{E_U^*}{C_u}\right) + \frac{1}{2} \sqrt{\left(\frac{\nu\phi\kappa\omega}{\epsilon(\pi+\omega)} \left(1 - \frac{E_U^*}{C_u}\right)\right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta+\gamma)M^*}}$$

The basic reproduction number in urban area R_0^r is the expected number of secondary cases produced by a single infection in a completely susceptible urban population.

The basic reproduction number of the complete model is the maximum

of the two numbers, basic reproduction number in rural area and basic reproductive number in urban area.

$$R_0 = \max \{R_0^r, R_0^u\}$$

Appendix F. Number of transmissions from rural to urban area

To evaluate transmission from rural to urban dengue cases, we utilized the basic reproduction number of rural and urban area. The desired parameter is the number of infections an individual in the rural population would generate in the urban population through the movement of infected tires. For this we assume that dengue is endemic in the rural population. The infected classes in the full model are: $I_R, E_{IR}, M_{IR}, I_U, E_{IU}, M_{IU}$. The information is separated into two matrices, the first one corresponds to new infections \mathfrak{F} and the second to disease progression \mathfrak{V} , that is:

$$\mathfrak{F} = \begin{pmatrix} \alpha \frac{S_U}{N_U} M_{IU} \\ \nu \phi M_{IU} \left(1 - \frac{E_U}{C_u}\right) \\ \alpha \frac{I_U}{N_U} M_{SU} \\ \alpha \frac{S_R}{M_R} M_{IR} \\ \nu \phi M_{IR} \left(1 - \frac{E_R}{C_r}\right) \\ \alpha \frac{I_R}{N_R} M_{SR} \end{pmatrix} \quad \mathfrak{Y} = \begin{pmatrix} (\eta + \gamma) I_{IU} \\ (\pi + \omega) E_{IU} - \frac{r}{\theta} \chi \psi (\tau_s / \tau_d) E_{IR} \\ \epsilon M_{IU} - \kappa \omega E_{IU} \\ (\eta + \gamma) I_{IR} \\ (\pi + \omega) E_{IR} + \frac{r}{\theta} E_{IR} \\ \epsilon M_{IR} - \kappa \omega E_{IR} \end{pmatrix}$$

The Jacobian matrices are:

$$F = \begin{pmatrix} 0 & \alpha \frac{S_U}{N_U} & 0 & 0 & 0 & 0 \\ \alpha \frac{M_{SU}}{N_U} & 0 & 0 & 0 & 0 & 0 \\ 0 & \nu \phi \left(1 - \frac{E_U}{C_u}\right) & -\frac{\nu \phi M_{IU}}{C_u} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha \frac{S_R}{N_R} & 0 \\ 0 & 0 & 0 & \alpha \frac{M_{SR}}{N_R} & 0 & 0 \\ 0 & 0 & 0 & 0 & \nu \phi \left(1 - \frac{E_R}{C_r}\right) & -\frac{\nu \phi M_{IR}}{C_r} \end{pmatrix}$$

$$V = \begin{pmatrix} (\eta + \gamma) & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon & -\kappa\omega & 0 & 0 & 0 \\ 0 & 0 & (\pi + \omega) & 0 & 0 & -\frac{r}{\theta}\chi\psi(\tau_s/\tau_d) \\ 0 & 0 & 0 & (\eta + \gamma) & 0 & 0 \\ 0 & 0 & 0 & 0 & \epsilon & -\kappa\omega \\ 0 & 0 & 0 & 0 & 0 & (\pi + \omega + \frac{r}{\theta}) \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(\eta+\gamma)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\epsilon} & \frac{\kappa\omega}{\epsilon(\pi+\omega)} & 0 & 0 & \frac{\kappa\omega r\chi}{\epsilon(\pi+\omega)(\theta(\pi+\omega)+r)}\psi\left(\frac{\tau_s}{\tau_d}\right) \\ 0 & 0 & \frac{1}{(\pi+\omega)} & 0 & 0 & \frac{r\chi}{(\pi+\omega)(\theta(\pi+\omega)+r)}\psi\left(\frac{\tau_s}{\tau_d}\right) \\ 0 & 0 & 0 & \frac{1}{(\eta+\gamma)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\epsilon} & \frac{\kappa\omega\theta}{\epsilon(\theta(\pi+\omega)+r)} \\ 0 & 0 & 0 & 0 & 0 & \frac{\theta}{\theta(\pi+\omega)+r} \end{pmatrix}$$

We evaluated the Jacobian matrices at the disease free equilibrium, then found $K = FV^{-1}$. To get the number of disease transmissions from the rural to urban area we obtained K^3 with this matrix. In the column for the infectious rural population the following is obtained:

$$R_{r \rightarrow u} = \frac{\alpha \kappa \omega r \chi}{\epsilon(\omega + \pi)(\theta(\omega + \pi) + r)} \psi \left(\frac{\tau_s}{\tau_d} \right) \frac{\nu \phi}{\epsilon} \frac{\alpha}{(\eta + \gamma)}$$

Appendix G. Appendix G: Summary of reproduction number

- Urban net reproductive rate.

$$R_M^u = \frac{\kappa \omega \phi}{\epsilon(\pi + \omega)}$$

- Rural net reproductive rate.

$$R_M^r = \frac{\kappa \omega \phi}{\epsilon(\pi + \omega + \frac{r}{\theta})}$$

- Basic reproduction number without vertical transmission in urban area.

$$R_0^u = \sqrt{\frac{\alpha N}{(\mu + \gamma) M^*} \frac{\alpha}{\epsilon}}$$

- Basic reproduction number in urban area.

$$R_0^u = \frac{1}{2} \frac{\nu \phi \kappa \omega}{\epsilon(\pi + \omega)} \left(1 - \frac{E_U^*}{C_u} \right) + \frac{1}{2} \sqrt{\left(\frac{\nu \phi \kappa \omega}{\epsilon(\pi + \omega)} \left(1 - \frac{E_U^*}{C_u} \right) \right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta + \gamma) M^*}}$$

- Basic reproduction number in rural area.

$$R_0^r = \frac{1}{2} \frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi + \omega) + r)} \left(1 - \frac{E_R^*}{C_r}\right) + \frac{1}{2} \sqrt{\left(\frac{\nu\phi\kappa\omega\theta}{\epsilon(\theta(\pi + \omega) + r)} \left(1 - \frac{E_R^*}{C_r}\right)\right)^2 + \frac{4\alpha}{\epsilon} \frac{\alpha N}{(\eta + \gamma)M^*}}$$

- Number of transmissions from rural to urban area

$$R_{r \rightarrow u} = \frac{\alpha\kappa\omega r\chi}{\epsilon(\omega + \pi)(\theta(\omega + \pi) + r)} \psi\left(\frac{\tau_s}{\tau_d}\right) \frac{\nu\phi}{\epsilon} \frac{\alpha}{(\eta + \gamma)}$$

Appendix H. Appendix H

The stationary state of infected mosquitoes in the urban area M_{IU}^* is found from the solution of the following quadratic equation

$$\begin{aligned} & (R_o^u)^2(\eta + \gamma)\left(\frac{\eta + \gamma}{\beta} + N_u\right)(M_{IU}^*)^2 \\ & - (\eta + \gamma) \left((R_o^u)^2 \left(\frac{\kappa\omega}{\beta} E_{IU}^* + N_U M_U\right) - \eta N_U \right) M_{IU}^* \\ & - \frac{\kappa\omega}{\epsilon} \eta N_U E_{IU}^* = 0 \end{aligned} \quad (\text{H.1})$$

and E_{IU}^* is given by

$$E_{IU}^* = \frac{r\chi\psi\left(\frac{\tau_s}{\tau_d}\right)}{\theta(\omega + \pi)} E_{IR}^*$$

References

- Adams, B., Boots, M., 2010. How important is vertical transmission in mosquitoes for the persistence of dengue? Insights from a mathematical model. *Epidemics* 2 (1).
- A.D.S., 2014. Autoridad de Desperdicios Sólidos. Manejo de Neumáticos en Puerto Rico. <http://www.ads.pr.gov/programas/neumaticos/>.
- Agarwal, A., Dash, P., Singh, A., Sharma, S., Gopalan, N., Rao, P., Parida, M., Reiter, P., 2014. Evidence of experimental vertical transmission of emerging novel ECSA genotype of chikungunya virus in aedes aegypti. *Plos Neglect Trop D* 8 (7), e2990.
- Alves Honório, N., Cabello, P. H., et.al., 2006. Preliminary data on the performance of aedes aegypti and aedes albopictus immatures developing in water-filled tires in rio de janeiro. *Memórias do Instituto Oswaldo Cruz* 101 (2), 225–228.
- Belli, A., Arostegui, J., Garcia, J., Aguilar, C., Lugo, E., Lopez, D., Valle, S., Lopez, M., Harris, E., Coloma, J., 2015. Introduction and establishment of aedes albopictus(diptera: Culicidae) in managua, nicaragua. *Journal of Medical Entomology* 52 (4).
- Benedict, M., Levine, R., Hawley, W., Lounibos, L., 2007. Spread of the tiger: Global risk of invasion by the mosquito aedes albopictus. *Vector borne and zoonotic diseases* 7 (1), 76–85.

- Bengis, R. G., Leighton, F. A., Fischer, J. R., Artois, M., Morner, T., Tate, C. M., 2004. The role of wildlife in emerging and re-emerging zoonoses. *Revue scientifique et technique-office international des epizooties* 23 (2), 497–512.
- Bonizzoni, M., Gasperi, G., Chen, X., James, A. A., 2013. The invasive mosquito species *aedes albopictus*: current knowledge and future perspectives. *Trends Parasitology* 9 (29), 460–468.
- Brauer, F., Van den Driessche, P., Wu, J., Allen, L. J. S. (Eds.), 2008. *Mathematical epidemiology*. No. 1945 in *Mathematical biosciences subseries*. Springer, Berlin.
- Briseño-García, B., Gómez-Dantés, H., Argott-Ramírez, E., Montesano, R., Vázquez-Martínez, A. L., Ibáñez-Bernal, S., Madrigal-Ayala, G., Ruíz-Matus, C., Flisser, A., Tapia-Conyer, R., 1996. Potential risk for dengue hemorrhagic fever: the isolation of serotype dengue-3 in Mexico. *Emerging Infectious Diseases* 2 (2).
- Cantanhede, A., Monge, G., Lima, 2002. Estado del arte del manejo de llantas usadas en las Américas. Organización Panamericana de la Salud.
- Christophers, S. S. R., 1960. *Aedes Aegypti (L.): The Yellow Fever Mosquito, Its Life History, Bionomics and Structure*, Chapter XXIII. C. U. P.
- Cook, G. C., Zumla, A. I., 2008. *Tropical Disease*. Twenty second edition, Saunder Elsevier.

- Diekmann, O., Heesterbeek, J., Metz, J., 1990. On the definition and the computation of the basic reproduction ratio r_0 in models for infectious diseases in the heterogeneous population. *Mathematical Biology* 28 (4), 365–382.
- Eritja, R., Escosa, R., Lucientes, J., Marqués, E., Molina, R., Roiz, D., Ruiz, S., 2005. Worldwide invasion of vector mosquitoes: present european distribution and challenges for spain. *Biological Invasions* 7 (1), 87–97.
- Esteva, L., Rivas, G., Yang, H., 2006. Modelling parasitism and predation of mosquitoes by water mites. *Journal of Mathematical Biology* 53 (4), 540–555.
- Esteva, L., Vargas, C., 2000. Influence of vertical and mechanical transmission on the dynamics of dengue disease. *Mathematical Biosciences* 167 (1), 51–64.
- Esteva, L., Vargas, C., 2003. Coexistence of different serotypes of dengue virus. *Journal of Mathematical Biology* 46 (1), 31–47.
- Ferreira-de-Brito, A., Ribeiro, I. P., de Miranda, R., Fernandes, R., Campos, S., da Silva, K., de Castro, M., Bonaldo, M. C., Brasil, P., Ricardo, L., 2016. First detection of natural infection of *aedes aegypti* with zika virus in brazil and throughout south america. *Memórias Instituto Oswaldo Cruz* 111 (10), 655–658.

- García-Rivera, E. J., Rigau-Pérez, J. G., 2006. Dengue virus, Chapter 17. Humana Press.
- Gotelli, N. J., others, 1995. A primer of ecology. A primer of ecology.
- Gubler, D. J., 1986. The arbovirus: Epidemiology and ecology. T.P. Monath (Ed). CRC, Boca Raton, FL 2, 213.
- Gubler, D. J., 2002. Epidemic dengue/dengue hemorrhagic fever as a public health, social and economic problem in the 21st century. Trends in Microbiology 10 (2).
- Gúnther, J., Martínez-Muñoz, J., Pérez-Ishiwara, D., Salas-Benito, J., 2007. Evidence of vertical transmission of dengue virus in two endemic localities in the state of Oaxaca, Mexico. Intervirology 50 (5), 347–352.
- Higa, Y., Thi Yen, N., Kawada, H., Hai Son, T., Thuy Hoa, N., Takagi, M., 2010. Geographic Distribution of *Aedes aegypti* and *Aedes albopictus* Collected from Used Tires in Vietnam 26.
- Juliano, S. A., 2007. Population dynamics. Journal of the American Mosquito Control Association 23 (sp2), 265–275.
- Kourí, G., Guzmán, M. G., Valdés, L., Carbonel, I., del Rosario, D., Vazquez, S., Laferté, J., Delgado, J., Cabrera, M. V., 1998. Reemergence of dengue in Cuba: a 1997 epidemic in Santiago de Cuba. Emerging Infectious Diseases 4 (1).

- Levin, S. A., Carpenter, S. R., Godfray, H. C. J., Kinzig, A. P., Loreau, M., Losos, J. B., Walker, B., Wilcove, D. S., 2009. The Princeton guide to ecology. Princeton University Press.
- Levins, R., 1969. Some demographic and genetic consequences of environmental heterogeneity for biological control. *American Entomologist* 15 (3), 237–240.
- Loreau, M., 2010. From populations to ecosystems: theoretical foundations for a new ecological synthesis. *Monographs in population biology*. Princeton University Press, Princeton.
- MacArthur, R. H., Wilson, E. O., 2016. *The Theory of Island Biogeography*. Princeton University Press, Princeton, oCLC: 936620333.
- Martins, V. E. P., Alencar, C. H., Kamimura, M. T., de Carvalho Araújo, F. M., De Simone, S. G., Dutra, R. F., Guedes, M. I. F., 2012. Occurrence of Natural Vertical Transmission of Dengue-2 and Dengue-3 Viruses in *Aedes aegypti* and *Aedes albopictus* in Fortaleza, Ceará, Brazil 7.
- Medlock, J. M., Hansford, K. M., Schaffner, F., Versteirt, V., Hendrickx, G., Zeller, H., Bortel, W. V., 2012. A Review of the Invasive Mosquitoes in Europe: Ecology, Public Health Risks, and Control Options 12.
- Murillo, D., Holechek, S. A., Murillo, A. L., Sanchez, F., Castillo-Chavez, C., 2014. Vertical Transmission in a Two-Strain Model of Dengue Fever 1.

- Niyas, K. P., Abraham, R., Unnikrishnan, R., Mathew, T., Nair, S., Manakkadan, A., Issac, A., Sreekumar, E., 2010. Molecular characterization of chikungunya virus isolates from clinical samples and adult aedes albopictus mosquitoes emerged from larvae from kerala, south india. *Virol J* 7 (1), 1–8.
- Pan American Health Organization, World Health Organization, 2014. State of the art in the prevention and control of Dengue in the Americas. Pan American Health Organization and World Health Organization.
- Rezza, G., 2012. Aedes albopictus and the reemergence of dengue. *BMC., Public health* 12 (72).
- Rochlin, I., Ninivaggi, D. V., Hutchinson, M. L., Farajollahi, A., 2013. Climate change and range expansion of the asian tiger mosquito (aedes albopictus) in northeastern usa: Implications for public health practitioners. *PLOS ONE* 8 (4), 1–9.
- Thangamani, S., Huang, J., Hart, C., Guzman, H., Tesh, R., 2016. Vertical transmission of zika virus in aedes aegypti mosquitoes. *Am J Trop Med Hyg.*
- Thomas, S. M., Obermayr, U., et.al, 2012. Low-temperature threshold for egg survival of a post-diapause and non-diapause european aedine strain, aedes albopictus (diptera: Culicidae). *Parasites and vectors* 5 (100).

Thompson, R. C. A., Nov. 2000. Giardiasis as a re-emerging infectious disease and its zoonotic potential. *International Journal for Parasitology* 30 (12–13), 1259–1267.

World Health Organization, 2009. *Dengue: Guidelines for diagnosis, treatment, prevention and control*. For research on disease of poverty.

Yee, D. A., 2008. Tires as habitats for mosquitoes: A review of studies within the eastern united states. *Journal of Medical Entomology* 45 (4), 581–593.