

# Accepted Manuscript

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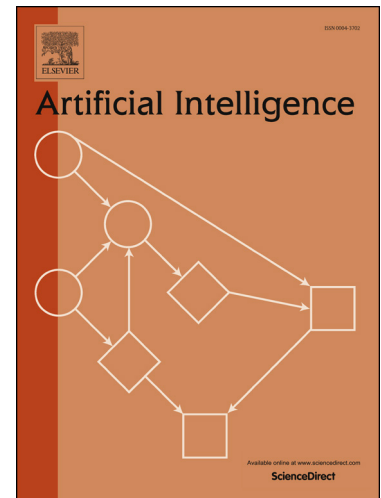
PII: S0004-3702(17)30001-2  
DOI: <http://dx.doi.org/10.1016/j.artint.2016.12.005>  
Reference: ARTINT 2990

To appear in: *Artificial Intelligence*

Received date: 26 April 2016  
Revised date: 13 December 2016  
Accepted date: 30 December 2016

Please cite this article in press as: N. Alechina et al., The virtues of idleness: A decidable fragment of resource agent logic, *Artif. Intell.* (2017), <http://dx.doi.org/10.1016/j.artint.2016.12.005>

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# The Virtues of Idleness: A Decidable Fragment of Resource Agent Logic

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## Abstract

Alternating Time Temporal Logic (ATL) is widely used for the verification of multi-agent systems. We consider Resource Agent Logic (RAL), which extends ATL to allow the verification of properties of systems where agents act under resource constraints. The model checking problem for RAL with unbounded production and consumption of resources is known to be undecidable. We review existing (un)decidability results for fragments of RAL, tighten some existing undecidability results, and identify several aspects which affect decidability of model checking. One of these aspects is the availability of a ‘do nothing’, or *idle* action, which does not produce or consume resources. Analysis of undecidability results allows us to identify a significant new fragment of RAL for which model checking is decidable.

*Keywords:* strategy logics, resource constraints, model checking

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## 1. Introduction

Many problems in AI and multi-agent systems research are most naturally formulated in terms of the abilities of a group or coalition of agents. For example, a group of agents may be able to cooperate to achieve an outcome which cannot be achieved by any agent in the group acting individually. In many cases, whether the outcome can be achieved depends critically on the resources available to the agents. Money is an obvious example, but there are many kinds of resources that may be produced or consumed by the actions of agents. For example, whether a team of agents can cooperate to extinguish a fire may depend on the amount of fuel and water they have available. Several logics for reasoning about coalitional ability under resource bounds have been proposed in the literature [1, 2, 3, 4, 5, 6, 7]. These *resource logics* allow us to express properties such as: ‘a coalition of agents  $A$  has a strategy (a choice of actions) requiring no more than  $b$  resources, such that whatever the actions by the agents outside the coalition, any evolution of the system generated by the strategy satisfies some temporal property’. Using model checking techniques we can then verify that a given coalition has a strategy requiring less than  $b$  resources to enforce an outcome, whatever the other agents in the system (or the environment) do. The ability to verify such properties can be useful when designing or developing a resource-constrained multi-agent system.

Unfortunately, the model checking problem for many resource logics where actions can produce resources is undecidable [2, 5]. Recently, however, it was shown that some resource logics where actions can produce resources have a decidable model checking problem [6, 7, 8]<sup>1</sup>. In this paper, we investigate the reasons for the decidability or undecidability of the model checking problem for resource logics. Different syntactic and semantic choices give different variants of resource logics. Some of these choices are known to affect the decidability of the model checking problem. In particular, the decidability result in [6] was proven in the presence of two major restrictions, called, in the terminology of [2], *resource flat* and *proponent restricted*. The former assumes that agents are always re-equipped with fresh resources when they reconsider their strategies; the latter assumes that only the proponents act under resource bounds (i.e., agents outside the coalition are not resource bounded). In addition to these restrictions,

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<sup>1</sup>A preliminary version of [8] is available as a technical report [9].

1 another choice in the semantics is relevant for the decidability result in [6]. This choice, which is also related to the  
 2 finitary and infinitary semantics of [2], stipulates that, in every model, agents always have a choice of doing nothing  
 3 (executing an idle action) that produces and consumes no resources. Having an *idle* action makes model checking  
 4 easier: intuitively, its availability ensures that in order to determine whether a coalition can enforce a  $\phi$ -state after  
 5 finitely many steps and within a given resource bound, we only need to find a finite strategy to enforce  $\phi$  under the  
 6 given resource bound, and after  $\phi$  is achieved, the agents can always choose the idle action forever, which does not  
 7 increase the ‘cost’ of the strategy. The presence of an idle action in the logic also guarantees some attractive formal  
 8 properties. For example, as stated in [3], it ensures coalition monotonicity: if a coalition  $A$  can ensure a property under  
 9 resource bound  $b$ , then any larger coalition can also ensure this property under the same resource bound (intuitively,  
 10 the extra agents can always perform idle).

11 In this paper, we investigate the effects of various semantic choices, such as the availability of an idle action, on  
 12 the decidability of the model checking problem for resource logics. First we show that both the resource-flat and the  
 13 proponent-restricted fragments of resource agent logic remain undecidable in the presence of idle actions. We then  
 14 identify and motivate a significant, non resource-flat fragment that has a decidable model checking property in the  
 15 presence of idle actions, and is not decidable otherwise. It follows that idle actions can make a difference for the  
 16 decidability of model checking with respect to the semantics we consider.

17 The new fragment, which we call pprRAL, allows us to express statements about the existence of nested strategies  
 18 for a coalition of agents given some *initial* allocation of resources. Unlike the resource-flat fragment considered in  
 19 [6], where for each new strategy agents are re-equipped with a fresh set of resources, pprRAL allows us to express  
 20 properties such as ‘given their initial battery charge, rescue robots  $A$  can safely get to a position from which they  
 21 can perform rescue while in visual contact with the base’. There are two nested strategies implicit in this property:  
 22 first, the robots should be able to reach some position (not necessarily maintaining visual contact with the base), and  
 23 second, from this position, the agents should be able to perform rescue while in visual contact with the base. The first  
 24 strategy (getting into position) will require certain resources (in this case battery charge), and the amount of resources  
 25 required will depend on the environment. Then, with *whatever resources are left*, the agents need a strategy to perform  
 26 the rescue. In this example, the model checking problem essentially corresponds to finding two nested conditional  
 27 resource-constrained plans, see e.g., [10]. The plans are nested because it is impossible to decouple the second plan  
 28 (for rescue) from the results of the first plan (getting into position), since we do not know the resource availability for  
 29 the initial state of the second plan; the resource availability in that state is determined by resource consumption of the  
 30 first plan. Compared to conditional planning with resources, resource logics provide an easy way to talk not just about  
 31 reachability, but also about invariants and nested goals/strategies achieved by (potentially different) coalitions.

32 This paper extends results presented in [7] in several respects, including: a more general definition of a decidable  
 33 fragment, more elaborated intuitions regarding the (un)decidability results, detailed proofs of all theorems, and tighter  
 34 undecidability results (in terms of the number of agents and resource types required for undecidability). The remainder  
 35 of the paper is organised as follows. In Section 2 we briefly survey related work. In Section 3 we introduce resource  
 36 agent logic, its models and the semantics. In Section 4, we review known decidability results for resource agent  
 37 logic, and investigate the reasons for (un)decidability. We present new undecidability results for systems with a single  
 38 resource type, and, based on these results, we motivate and introduce a new non resource flat fragment of RAL,  
 39 pprRAL. In Section 5, we present our second main technical result: a decidability result for pprRAL. We conclude in  
 40 Section 6.

## 41 2. Related Work

42 Early work on resource logics considered only the consumption of resources (i.e., no action produces resources),  
 43 and initial results on the complexity of model checking were encouraging. One of the first logics capable of expressing  
 44 resource requirements of agents was a version of Coalition Logic (CL)<sup>2</sup> called Resource-Bounded Coalition Logic  
 45 (RBCL), where actions only consume (and do not produce) resources. It was introduced in [1] with the primary  
 46 motivation of modelling systems of resource-bounded reasoners; however the framework is sufficiently general to  
 47 model any type of action. The model checking problem for RBCL was shown to be decidable in time polynomial

<sup>2</sup>CL is a fragment of ATL with only the next time  $\langle\langle A \rangle\rangle X$  modality, introduced in [11].

1 in the size of the transition system and of the property, and exponential in the number of resource types in [12]. A  
 2 resource-bounded version of ATL, RB-ATL, where again actions only consume (and do not produce) resources was  
 3 introduced in [3]. The model checking problem for this logic is also decidable in time polynomial in the size of the  
 4 transition system and of the property, and exponential in the number of resource types [3]. (For a single resource  
 5 type, e.g., energy, the model checking problem is no harder than for ATL.) Practical work on model checking many  
 6 standard computer science transition systems (not multi-agent systems) with resources also falls in the category of  
 7 consumption-only systems. For example, probabilistic model checking of systems with numerical resources as in the  
 8 PRISM model checker [13] assumes that costs increase monotonically with time.

9 However, when resource production is considered in addition to consumption, the situation changes. In a separate  
 10 strand of work, a range of different formalisms for reasoning about resources was introduced in [14, 2], including Re-  
 11 source Agent Logic RAL which is the main focus of this paper. In these formalisms, both consumption and production  
 12 of resources was considered. In [2], it was shown that the model checking problem for most variants and fragments of  
 13 RAL is undecidable. The only decidable cases considered in [2] (and the related [14]) are an extension of Computation  
 14 Tree Logic (CTL) with resources (essentially one-agent ATL), and a version where on every path only a fixed finite  
 15 amount of resources can be produced. The models satisfying this property were referred to as bounded in [2]. It was  
 16 pointed out in [2] that RBCL and RB-ATL are logics over a special kind of bounded models (where no resources are  
 17 produced at all). Other decidability results for bounded resource logics have also been reported in the literature. For  
 18 example, in [15] a decidable logic, PRB-ATL (Priced Resource-Bounded ATL) is defined, where the total amount of  
 19 resources in the system has a fixed bound. The model checking algorithm for PRB-ATL requires time polynomial  
 20 in the size of the transition system, and exponential in the number of resource types and the resource bound on the  
 21 system. In [4] an EXPTIME lower bound in the number of resource types for the PRB-ATL model checking problem  
 22 is shown. In [16], an extension of PRB-ATL to  $\mu$ -calculus is also shown to have a decidable model checking problem.

23 A general logic over systems with numerical constraints, Quantitative ATL (QATL\*), was introduced in [5], and  
 24 undecidability results for the model checking problem for QATL\* and some of its fragments were shown. For example,  
 25 QATL is undecidable even if no nestings of cooperation modalities are allowed. The main proposals for restoring  
 26 decidability to the model checking problem for QATL in [5] are removing negative payoffs (similar to removing  
 27 resource production), and introducing memoryless strategies (the latter idea is not pursued in any detail).

28 This brief survey of work suggests that the boundary between decidability and undecidability for the model check-  
 29 ing problem of resource logics is very subtle. No systematic study of the reasons for decidability and undecidability  
 30 of this problem has been undertaken to date, and with this paper we aim to address this task. We believe a better  
 31 understanding of the boundary between decidability and undecidability will be useful in developing new decidable  
 32 fragments of resource logics.

33 Of course, searching for decidable fragments is not the only way of addressing the undecidability of model check-  
 34 ing for temporal logics with infinite-state transition systems (RAL can be seen as a special case of such logics).  
 35 Another approach is to design algorithms which return definite answers where possible, and ‘unknown’ otherwise  
 36 (see, e.g., [17, 18, 19, 20, 21]). A promising direction of future research would be to explore connections between  
 37 the two approaches. Our work connects to many other areas of computer science, such as planning [10] and the  
 38 verification of autonomous systems [22, 23]. The model checking problem can essentially be seen as an approach  
 39 to computing a robust plan for a set of autonomous agents, such as robots. Techniques used in our work are related  
 40 to many subfields of theoretical computer science. In particular, techniques developed for Petri nets, vector addition  
 41 systems and model checking over pushdown systems (see, e.g., [24, 25]), are closely connected to the techniques we  
 42 use in establishing our decidability and undecidability results. The existing, deep theoretical results in these areas also  
 43 provide a starting point for establishing complexity bounds for our model checking algorithms, and ideas for restric-  
 44 tions that give fragments with good computational properties. Finally, another branch of work on reasoning about  
 45 resources is based on linear logic [26, 27], and related logics such as the logic of bunched implication [28, 29, 30, 31].  
 46 In the future, it would be interesting to explore deeper connections between the resource logics considered in this  
 47 paper, and reasoning about resources using linear logic techniques.

### 48 3. Resource Agent Logic

1 In this section we define *resource agent logic* (RAL) and *resource-bounded models* (RBMs). We essentially  
 2 follow [2], combined with aspects from [6]. We summarise the similarities and differences between RAL and the

3 resource logics considered in [2, 6] in more detail in Section 3.5.

### 4 3.1. Syntax of RAL

5 The logic is defined over a set of agents  $\mathbb{A}gt$ , a set of resources types  $Res$ , and a set of propositional symbols  $\Pi$ .  
 6 We denote the set of natural numbers by  $\mathbb{N}$ , the set of natural numbers with zero by  $\mathbb{N}_0$ , the set of natural numbers  
 7 with infinity by  $\mathbb{N}^\infty$ , and the set of natural numbers with zero and infinity by  $\mathbb{N}_0^\infty$ . An *endowment (function)*  $\eta : \mathbb{A}gt \times Res \rightarrow \mathbb{N}_0^\infty$   
 8 assigns resources to agents;  $\eta_a(r) = \eta(a, r)$  is the number of resources agent  $a$  has of resource type  $r$ .  
 9  $\text{En}$  denotes the set of all possible endowments. Resource types can represent, for example, money, fuel, battery power,  
 10 etc. Special minimal and maximal endowment functions are denoted by  $\bar{0}$  and  $\infty$ , respectively. The former expresses  
 11 that there are no resources at all, whereas the latter equips all agents with an infinite amount of each resource type. (In  
 12 what follows, for readability we will talk about amounts of some resource, rather than of some resource type.) The  
 13 logic RAL is defined according to the grammar of ATL [32]. RAL-formulae are defined by:

$$14 \quad \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle A \rangle\rangle_B^\downarrow \mathbf{X}\phi \mid \langle\langle A \rangle\rangle_B^\uparrow \mathbf{X}\phi \mid \langle\langle A \rangle\rangle_B^\downarrow \varphi \mathbf{U}\psi \mid \langle\langle A \rangle\rangle_B^\uparrow \varphi \mathbf{U}\psi \mid \langle\langle A \rangle\rangle_B^\downarrow \mathbf{G}\phi \mid \langle\langle A \rangle\rangle_B^\uparrow \mathbf{G}\phi$$

15 where  $p \in \Pi$  is a proposition,  $A, B \subseteq \mathbb{A}gt$  are sets of agents, and  $\eta$  is an endowment. We also define  $\langle\langle A \rangle\rangle^\downarrow$  and  
 16  $\langle\langle A \rangle\rangle^\uparrow$  as abbreviations for  $\langle\langle A \rangle\rangle_A^\downarrow$  and  $\langle\langle A \rangle\rangle_A^\uparrow$ , respectively. The operators  $\mathbf{X}$ ,  $\mathbf{U}$ , and  $\mathbf{G}$  denote the standard temporal  
 17 operators expressing that some property holds in the *next* point in time, *until* some other property holds, and *now and*  
 18 *always* in the future, respectively. There are two types of cooperation modalities,  $\langle\langle A \rangle\rangle_B^\downarrow$  and  $\langle\langle A \rangle\rangle_B^\uparrow$ . In both types of  
 19 cooperation modality, the actions performed by agents in  $A \cup B$  consume and produce resources (actions by agents  
 20 in  $\mathbb{A}gt \setminus (A \cup B)$  do not change their resource endowment). The reading of  $\langle\langle A \rangle\rangle_B^\uparrow \varphi$  is that *when agents  $A \cup B$  have a*  
 21 *resource endowment  $\eta$ , agents  $A$  have a strategy compatible with this endowment to enforce  $\varphi$  (whatever the agents*  
 22 *in  $\mathbb{A}gt \setminus A$  do, compatible with their resource constraints, if any). The evaluation of a modality  $\langle\langle A \rangle\rangle_B^\uparrow$  (re-)equips all*  
 23 *agents with a fresh amount of resources: the current resource endowment is overwritten by endowment  $\eta$ . The formula*  
 24  *$\langle\langle A \rangle\rangle_B^\downarrow \varphi$  reads similarly but the strategy must be compatible with the resources currently available to the agents. In*  
 25 *both cases compatible means that the strategy can be executed given the agents' resources. For both modalities it is*  
 26 *therefore necessary to keep track of resource production and consumption during the execution of a strategy.*

### 27 3.2. Semantics of RAL

28 We define the models of RAL as in [2]. Following [6] we also define a special case of these models in which all  
 29 agents have an *idle action* in their repertoire which neither consumes nor produces resources.

30 **Definition 1 (RBM, iRBM).** A *resource-bounded model (RBM)* is given by  $\mathfrak{M} = (\mathbb{A}gt, Q, \Pi, \pi, Act, d, o, Res, t)$   
 31 where

- 32 •  $\mathbb{A}gt = \{1, \dots, k\}$ , is a non-empty set of agents;
- 33 •  $Q$  is a non-empty set of states;
- 34 •  $\pi : Q \rightarrow \wp(\Pi)$  is a valuation of propositions;
- 35 •  $Act$  is a finite non-empty set of actions;
- 36 •  $d : \mathbb{A}gt \times Q \rightarrow \wp(Act) \setminus \{\emptyset\}$  indicates the actions available to agent  $a \in \mathbb{A}gt$  in state  $q \in Q$ ;
- 37 •  $o$  maps each state  $q \in Q$  and action profile  $\alpha = (\sigma_1, \dots, \sigma_k)$  such that  $\sigma_a \in d(a, q)$  for each  $a \in \{1, \dots, k\}$ , to  
 38 another state  $q' = o(q, \alpha)$ ;
- 39 •  $t : Act \times Res \rightarrow \mathbb{Z}$  models the resources consumed and produced by actions; if  $t(\sigma, r)$  is positive resource  $r$  is  
 40 produced by  $\sigma$ , if  $t(\sigma, r)$  is negative resource  $r$  is consumed by  $\sigma$ .

41 An **RBM** with idle actions, **iRBM** for short, is an **RBM**  $\mathfrak{M}$  such that for all agents  $a$  and states  $q$  in  $\mathfrak{M}$ , there is an  
 1 action  $\sigma \in d(a, q)$  with  $t(\sigma, r) = 0$  for all resource types  $r$ . We refer to this action (or to one of them if there is more  
 2 than one) as the idle action of  $a$  and denote it by *idle*.

We will write  $d_a(q)$  instead of  $d(a, q)$ , and use  $d(q)$  to denote the set  $d_1(q) \times \dots \times d_k(q)$  of action profiles in state  $q$ . Similarly,  $d_A(q)$  denotes the action tuples available to  $A \subseteq \text{Agt}$  in  $q$ . For  $\alpha = (\sigma_1, \dots, \sigma_k)$ , we use  $\alpha_A$  to denote the sub-tuple consisting of the actions of agents  $A \subseteq \text{Agt}$ ; moreover, we write  $\alpha_a$  to refer to  $\sigma_a$  for  $a \in \text{Agt}$ .  $\text{Act}^A$  is a set of tuples of actions by agents in  $A$ . We define  $\text{prod}(\sigma, r) := \max\{0, t(\sigma, r)\}$  (resp.  $\text{cons}(\sigma, r) := |\min\{0, t(\sigma, r)\}|$ ) as the amount of resource  $r$  produced (resp. consumed) by action  $\sigma$ . Note that  $\text{cons}(\sigma, r)$  is a non-negative number, and for any action  $\sigma$  and a resource type  $r$  it is not possible that both  $\text{prod}(\sigma, r)$  and  $\text{cons}(\sigma, r)$  are greater than 0.

In what follows,  $Q^\omega$  denotes the set of all infinite sequences of elements from  $Q$ , and  $Q^+$  denotes the set of all finite sequences. A *path*  $\lambda \in Q^\omega$  is an infinite sequence of states such that there is a transition between two adjacent states. A *finite path* is a finite segment of a path. We define  $\lambda[i]$  to be the  $(i+1)$ -th state of  $\lambda$ , and  $\lambda[i, \infty]$  to be the suffix  $\lambda[i]\lambda[i+1]\dots$ . We denote a finite sequence  $\lambda$  extended by  $q$  by  $\lambda q$ . A *resource-extended path*  $\lambda \in (Q \times \text{En})^\omega$  is an infinite sequence over  $Q \times \text{En}$  such that the restriction to states (the first component), denoted by  $\lambda|_Q$ , is a path in the underlying model. The projection of  $\lambda$  to the second component of each element in the sequence is denoted by  $\lambda|_{\text{En}}$ . We call any initial (finite) suffix of a resource-extended path a *finite resource extended path*.

A *strategy* for a coalition  $A \subseteq \text{Agt}$  is a function  $s_A : Q^+ \rightarrow \text{Act}^A$  such that  $s_A(\lambda q) \in d_A(q)$  for  $\lambda q \in Q^+$ . Such a strategy gives rise to a set of (resource-extended) paths. A  $(\eta, s_A, B)$ -*path* is a resource-extended path  $\lambda$  where for all  $i = 0, 1, \dots$  with  $\lambda[i] := (q_i, \eta^i)$  there is an action profile  $\alpha \in d(\lambda|_Q[i])$  such that:

1.  $\eta^0 = \eta$  ( $\eta$  describes the initial resource distribution);
2.  $s_A(\lambda|_Q[0, i]) = \alpha_A$  ( $A$  follow their strategy);
3.  $\lambda|_Q[i+1] = o(\lambda|_Q[i], \alpha)$  (transition according to  $\alpha$ );
4. for all  $a \in A \cup B$  and  $r \in \text{Res}$ :  $\eta_a^i(r) \geq \text{cons}(\alpha_a, r)$  (each agent has enough resources to perform its action);
5. for all  $a \in A \cup B$  and  $r \in \text{Res}$ :  $\eta_a^{i+1}(r) = \eta_a^i(r) + t(\alpha_a, r)$  (resources are updated);
6. for all  $a \in \text{Agt} \setminus (A \cup B)$  and  $r \in \text{Res}$ :  $\eta_a^{i+1}(r) = \eta_a^i(r)$  (the resources of agents not in  $A \cup B$  do not change).

The  $(\eta, B)$ -*outcome* of a strategy  $s_A$  in  $q$ ,  $\text{out}(q, \eta, s_A, B)$ , is defined as the set of all  $(\eta, s_A, B)$ -paths starting in  $q$ . Truth is defined over an **RBM**  $\mathfrak{M}$ , a state  $q \in Q_{\mathfrak{M}}$ , and an endowment  $\eta$ .

The semantics is given by the satisfaction relation  $\models$  where the cases for propositions, negation and conjunction are standard and omitted:

$\mathfrak{M}, q, \eta \models \langle\langle A \rangle\rangle_B^\downarrow \varphi$  iff there is a strategy  $s_A$  for  $A$  such that for all  $\lambda \in \text{out}(q, \eta, s_A, B)$ ,  $\mathfrak{M}, \lambda, \eta \models \varphi$

$\mathfrak{M}, q, \eta \models \langle\langle A \rangle\rangle_B^\zeta \varphi$  iff there is a strategy  $s_A$  for  $A$  such that for all  $\lambda \in \text{out}(q, \zeta, s_A, B)$ ,  $\mathfrak{M}, \lambda, \zeta \models \varphi$

$\mathfrak{M}, \lambda, \eta \models \mathbf{X}\varphi$  iff  $\mathfrak{M}, \lambda|_Q[1], \lambda|_{\text{En}}[1] \models \varphi$

$\mathfrak{M}, \lambda, \eta \models \varphi \mathbf{U} \psi$  iff there exists  $i$  with  $i \geq 0$  and  $\mathfrak{M}, \lambda|_Q[i], \lambda|_{\text{En}}[i] \models \psi$  and for all  $j$  with  $0 \leq j < i$ ,  $\mathfrak{M}, \lambda|_Q[j], \lambda|_{\text{En}}[j] \models \varphi$

$\mathfrak{M}, \lambda, \eta \models \mathbf{G}\varphi$  iff for all  $i \geq 0$ ,  $\mathfrak{M}, \lambda|_Q[i], \lambda|_{\text{En}}[i] \models \varphi$

The *model checking problem* for RAL is stated as follows: does  $\mathfrak{M}, q, \eta \models \varphi$  hold? When the context is clear, we simply write  $q, \eta \models \varphi$ ; if  $\varphi$  is only a propositional formula, we sometimes also omit  $\eta$ .

Observe that the standard ATL modalities  $\langle\langle A \rangle\rangle$  can be defined as  $\langle\langle A \rangle\rangle_{\text{Agt}}^\infty$ , so the logic is a proper extension of ATL.

**Remark 1** (Infinitary and finitary semantics). *We refer to the semantics introduced above as infinitary semantics. In [2] the main semantics also allows for finite (maximal) paths. We refer to that semantics as finitary semantics. We note that both semantics coincide over iRBM's, as it is always possible to extend a path using idle actions.*

### 3.3. The Syntactic Fragments rfRAL, prRAL and rfprRAL

Following [2] we define three fragments of RAL. The *resource-flat fragment*, rfRAL, only allows cooperation modalities of type  $\langle\langle A \rangle\rangle_B^\eta$ : agents are always (re-)equipped with a fresh set of resources whenever they re-consider their strategies. The *proponent-restricted fragment*, prRAL, only allows cooperation modalities of types  $\langle\langle A \rangle\rangle^\downarrow$  and  $\langle\langle A \rangle\rangle^\eta$ : only the proponents are resource bounded. The fragment combining both restrictions (resource-flat and proponent-restricted) is denoted by rfprRAL.

### 3.4. Running example

We introduce a simple running example to illustrate the syntax and semantics of RAL and its fragments. The example represents interactions between two agents: a robot (agent 1) and its environment (agent 2). We consider only one resource type, energy. The robot needs to move into a position where it is capable of sending information to the base regularly and is also able to charge its battery. Both moving and the communication action require energy, and the charging action produces energy. We denote ‘being in a suitable position to send information to the base’ by  $p$ . The environment can make moving more or less difficult for the agent. We model this by giving the environment an ‘obstruct’ action which has the effect of requiring the agent to execute two move actions instead of one (and hence to spend more energy) in order to get into position; for the sake of the example, obstructing requires energy. The initial state is  $q_0$  where the agent can move and the environment can obstruct, and both also can do nothing. If the agent moves and the environment idles, then the system reaches a state  $q_1$  where  $p$  holds and the agent can loop forever between  $q_1$  and  $q_3$  (which also satisfies  $p$ ) sending data and charging. If the agent does nothing upon reaching the position, it returns to the initial state (under the influence of gravity, for example). If in  $q_0$  the environment obstructs the agent, the system reaches a state  $q_2$  where the agent can execute the move action again to reach  $q_1$ . To keep the example simple, we assume that in all states apart from  $q_0$  the environment can only idle.

Formally, we have a resource-bounded model  $\mathfrak{M} = (\mathbb{A}gt, Q, \Pi, \pi, Act, d, o, Res, t)$  where

- $\mathbb{A}gt = \{1, 2\}$
- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Pi = \{p\}$
- $\pi(q_0) = \pi(q_2) = \emptyset, \pi(q_1) = \pi(q_3) = \{p\}$
- $Act = \{idle, move, send, charge, obstruct\}$
- Actions available to the robot:  $d(1, q_0) = \{idle, move\}, d(1, q_1) = \{idle, send\}, d(1, q_2) = \{idle, move\}, d(1, q_3) = \{idle, charge\}$ . Actions available to the environment:  $d(2, q_0) = \{idle, obstruct\}, d(2, q_1) = d(2, q_2) = d(2, q_3) = \{idle\}$ .
- The transition function is as follows:

Transitions from $q_0$	Transitions from $q_1$	Transitions from $q_2$	Transitions from $q_3$
$o(q_0, (idle, idle)) = q_0$	$o(q_1, (idle, idle)) = q_0$	$o(q_2, (idle, idle)) = q_2$	$o(q_3, (idle, idle)) = q_0$
$o(q_0, (move, idle)) = q_1$	$o(q_1, (send, idle)) = q_3$	$o(q_2, (move, idle)) = q_1$	$o(q_3, (charge, idle)) = q_1$
$o(q_0, (move, obstruct)) = q_2$			
$o(q_0, (idle, obstruct)) = q_0$			

- $Res = \{energy\}$
- $t(idle, energy) = 0, t(move, energy) = -2, t(send, energy) = -1, t(charge, energy) = 1, t(obstruct, energy) = -1$ .

The model is shown in Figure 1.

Here are some example RAL properties which hold in the model:

- If both agents are resource-bounded, and the initial allocation of resources is 3 units of energy for the robot and 0 units for the environment, then the robot has a strategy to reach a state from where with the remaining resources it can maintain the invariant  $p$ . We represent an endowment  $\eta$  that assigns 3 units of energy to agent 1 and 0 units to agent 2 by 1:3, 2:0:

$$\langle\langle 1 \rangle\rangle_{(1,2)}^{1:3,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle_{(1,2)}^\downarrow \mathbf{G} p$$

In fact, with environment unable to obstruct, the robot is guaranteed to reach  $q_1$  in one step:

$$\langle\langle 1 \rangle\rangle_{(1,2)}^{1:3,2:0} \mathbf{X} \langle\langle 1 \rangle\rangle_{(1,2)}^\downarrow \mathbf{G} p$$

This property belongs to full RAL: it is not in rRAL since it uses  $\downarrow$  in  $\langle\langle 1 \rangle\rangle_{(1,2)}^\downarrow \mathbf{G}$ , nor in prRAL since it restricts the resources of the opponent agent 2.

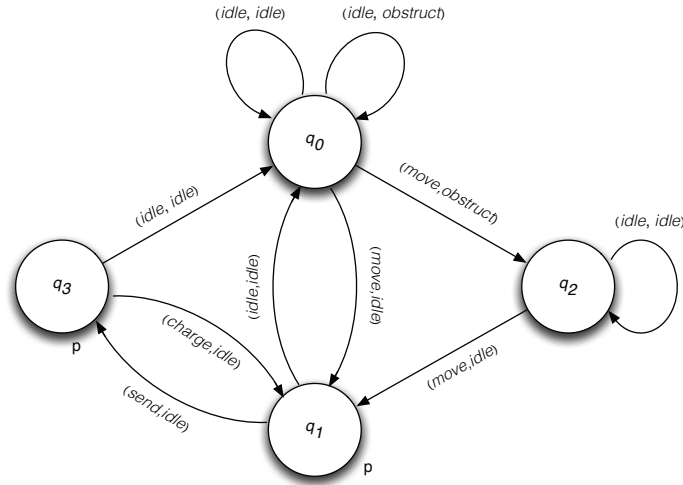


Figure 1: State transition system.

- When both agents are resource bounded, and the environment is restricted to 0 units of energy, then with 2 units of energy, the agent can reach the state where it can maintain the invariant with 1 unit of energy:

$$\langle\langle 1 \rangle\rangle_{(1,2)}^{1:2,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle_{(1,2)}^{1:1,2:0} \mathbf{G} p$$

33 This property belongs to the resource-flat fragment, since the second strategy for the invariant uses a fresh  
1 resource allocation.

- If only the robot is resource-bounded, and the initial allocation of resources is 5 units of energy for the robot and 0 units for the environment, then the robot has a strategy to reach a state from where with the remaining resources it can maintain the invariant  $p$ . The strategy is to execute the move action until the state  $q_1$  is reached; in the worst case this would require 4 units of energy (since the environment is not resource-bounded, its initial allocation does not matter and it can perform the obstruct action). Then with at least one unit of energy remaining, the agent can enter the loop between  $q_1$  and  $q_3$ :

$$\langle\langle 1 \rangle\rangle_{(1)}^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle_{(1)}^{\downarrow} \mathbf{G} p$$

This property does not belong to rfRAL but it does belong to prRAL. It can be written without the argument for the set of resource-bounded agents:

$$\langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$$

2 In fact, this property belongs to the fragment with a decidable model checking problem, pprRAL (positive  
3 fragment of prRAL).

- If only the robot is resource-bounded, then with initial allocation of 4 units of energy, it can reach a state where with one unit of energy it can maintain the invariant:

$$\langle\langle 1 \rangle\rangle^{1:4,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{1:1,2:0} \mathbf{G} p$$

4 This property belongs to rfprRAL.



### 5 3.5. Similarities and Differences

1 We conclude this section with a discussion of similarities and differences between the variant of RAL presented  
2 here and the original resource agent logics of [2] and the logic of [6]. In the interests of readability, we refer to the  
3 setting of [2] by  $S_1$ , and to that of [6] by  $S_2$ .

4 The language of RAL as given above is almost identical to the setting of  $S_1$ , except that we do not allow the release  
5 operator.<sup>3</sup> Setting  $S_2$  essentially corresponds to the resource-flat and proponent-restricted fragment of RAL. **RBM**s  
6 serve as models of  $S_1$ , where  $S_2$  uses **iRBM**s. There are also differences in how the production and consumption  
7 of resources are handled. In  $S_2$  the resources of a coalition of agents are combined before the resource requirements  
8 of actions are evaluated. A shortage of resources of one agent can thus be balanced by surplus resources of another  
9 agent in the coalition. The implicit assumption is that agents in the proponent coalition share their resources. It is  
10 not necessary to decide how to divide any resources produced, as the coalition sticks together throughout the relevant  
11 part of the evaluation of the current formula. When a new cooperation modality is encountered, all agents are re-  
12 equipped with a new endowment. This is a property of the resource-flat and proponent restricted fragment of the  
13 logic. This approach cannot be used if the restriction of resource-flatness or proponent restrictiveness is dropped.  
14 First, a coalition may split-up in a nested modality in which agents are not re-equipped with new resources. In this  
15 case it is important to know how many resources each individual agent has. A similar difficulty arises if an agent in  
16 the proponent coalition becomes an opponent in a nested cooperation modality. If the logic is not proponent restricted  
17 it is necessary to know how many resources this agent possesses. In  $S_1$  this issue is addressed by introducing *shares*.  
18 A share models how many resources an individual agents contributes to the pool of resources needed to execute the  
19 joint action, and also the amount of resources each agent receives when resources are produced. This can be seen as  
20 a binding agreement about the resource distribution. Again, the underlying assumption is that agents in the proponent  
21 and opponent coalitions share their resources within the coalition.

22 As we consider a non resource-flat variant of RAL here, the approach of  $S_2$  is not sufficient, whereas the approach  
23 of  $S_1$  complicates the presentation. We therefore adopt a less involved formalisation for ease of readability: resources  
24 *cannot* be shared within a coalition and each agent is entirely responsible for its own resource balance. Thus, at  
25 each moment agents have a clearly defined resource endowment. Finally, most results of  $S_1$  are given in terms of the  
26 finitary semantics whereas we require that paths are always infinite (cf. Remark 1).

## 27 4. The Quest for Decidability

28 If unbounded production of resources is allowed, the model checking problem for many resource logics is unde-  
29 cidable. In particular, most fragments of the resource agent logic considered in [2, 5] are undecidable. The case of the  
30 resource-flat, proponent-restricted fragment remained open in [2], but was shown to be decidable in [6, 8] (see also  
31 [9]):

32 **Observation 1.** *rfprRAL is decidable over iRBM.s.*

33 A natural question arises: *can we extend decidability to more expressive fragments?* Which restrictions are essen-  
34 tial for decidability, and which can be relaxed?

35 The result above relies on three restrictions on RAL: (1) the availability of an idle action; (2) resource flatness, that  
36 is, each nested quantifier has a fresh endowment; and (3) proponent restriction, that is, there are no resource bounds  
37 on the opponents. It turns out that all three restrictions are essential for the decidability of rfprRAL, as we explain  
38 below.

39 It follows from [2] that the availability of an idle action is essential for the decidability of rfprRAL:

40 **Observation 2.** *rfprRAL is undecidable over RBM.s.*

41 However, the availability of an idle action on its own is not sufficient for decidability. Replacing **RBMs** with  
42 **iRBMs** does not always make the model checking problem decidable.

<sup>3</sup>In  $S_1$  the release operator is used to show the undecidability of model checking resource-bounded agents using memoryless strategies. As we focus on perfect-recall strategies in this paper, we do not need the extra expressivity provided by the release operator.

43 In this section we present the main idea underlying the undecidability proofs of model checking RAL from [2]  
 1 and investigate the reasons for the (un)decidability. We show that the model checking problem for RAL, i.e., the logic  
 2 without any additional restrictions, remains undecidable over **iRBMs** (see Theorem 1 below). This result also holds  
 3 for both the proponent-restricted fragment and the resource-flat fragment. For these fragments we also investigate  
 4 the effect of the number of agents and resource types on the undecidability. The results of [2, 7] depend on the  
 5 availability of two resource types. Here we show that undecidability holds even if each agent has only a *single*  
 6 resource type available. In this case, however, additional agents are required for undecidability; more precisely, one  
 7 or two additional agents are required depending on the setting. We also show that in the case of prRAL over **iRBMs**,  
 8 although the model checking problem remains undecidable, the formula expressing an undecidable problem in the  
 9 logic is more complex than the formula required for rfRAL. This suggests the idea of a syntactic restriction of prRAL  
 10 which does not allow expression of the undecidable property. We end this section by motivating a new fragment of  
 11 RAL, the positive proponent restricted fragment, pprRAL. This fragment is more expressive than that introduced in  
 12 [7], in that the formula  $\varphi_1$  on the left-hand-side of  $\varphi_1 \mathbf{U} \varphi_2$  is not constrained to be purely propositional. In Section 5  
 13 we show that the model checking problem for pprRAL is decidable over **iRBMs**.

14 For all the results below, the undecidability of the model checking problem is shown by a reduction of the halting  
 15 problem for two-counter machines (also called Minsky machines, see [33] for details). A *two-counter machine* (TCM)  
 16 is essentially a pushdown automaton with two stacks. The stacks are represented as two counters over natural numbers.  
 17 Each of the counters (1 and 2) can be incremented, decremented (if non-zero), and tested for zero. In [33] it is shown  
 18 that these machines are expressively equivalent to Turing machines. As a consequence the halting problem of two-  
 19 counter machines is undecidable as well. For this paper we only need to consider TCMs with empty inputs; therefore,  
 20 we only introduce this special type of TCMs.

21 **Definition 2** (Empty-band two-counter machine (cf. [33]), empty-band). *An empty band TCM  $\mathcal{A}$  is given by  $(S, s_{init}, S_f,$   
 22  $\Delta)$  where  $S$  is a finite set of states,  $s_{init} \in S$  is the initial state,  $S_f \subseteq S$  is a set of final states, and  $\Delta \subseteq (S \times \{0, 1\}^2) \times$   
 23  $(S \times \{-1, 0, 1\}^2)$  is the transition relation such that if  $((s, E_1, E_2), (s', C_1, C_2)) \in \Delta$  and  $E_i = 0$  then  $C_i \neq -1$  for  
 24  $i = 1, 2$  (to ensure that an empty counter is not decremented). In the following we sometimes use infix notation and  
 25 write  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  instead of  $((s, E_1, E_2), (s', C_1, C_2)) \in \Delta$ . We call  $((s, E_1, E_2), (s', C_1, C_2))$  a transition if  
 26  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  and denote a typical transition by  $\tau$ .*

27 As we focus on empty-band TCMs, we often simply say *automaton* or *machine* to refer to such a TCM. A TCM  
 28 can be considered as a transition system equipped with two counters that influence the transitions. Each transition  
 29 step of the automaton depends on whether the counters are zero or non-zero, and in each step the counters can be  
 30 incremented or decremented. It is important to emphasise that a TCM cannot access the specific value of the counters.  
 31 In the following let  $\tau = ((s, E_1, E_2), (s', C_1, C_2))$  be a transition. Here,  $E_i = 1$  (resp.  $= 0$ ) represents that counter  
 32  $i$  is non-zero (resp. zero), and  $C_k = 1$  (resp.  $= -1$ ) denotes that counter  $i$  is incremented (resp. decremented) by 1.  
 33 A value  $C_k = 0$  indicates that counter  $k$  is left unchanged. The transition encodes that in state  $s$  the automaton can  
 34 change its state to  $s'$  provided that the first (resp. second) counter meets condition  $E_1$  (resp.  $E_2$ ). The value of counter  
 35  $k$  changes according to  $C_k$  for  $k = 1, 2$ . For example, the transition  $((s, 1, 0), (s', -1, 1))$  is enabled if the current state  
 36 is  $s$ , counter 1 is non-zero, and counter 2 is zero. If the transition is enabled and taken, the state changes to  $s'$ , counter  
 37 1 is decremented and counter 2 is incremented by 1.

38 The general mode of operation is as for pushdown automata. In particular, a *configuration* is a triple  $(s, v_1, v_2) \in$   
 39  $S \times \mathbb{N}_0^2$  describing the current state ( $s$ ), the value of counter 1 ( $v_1$ ) and of counter 2 ( $v_2$ ). An  $\mathcal{A}$ -*computation*  $\rho$  (or simply  
 40 *computation* if the two-counter machine is clear from context) is a sequence of subsequent configurations resulting  
 41 from transitions according to  $\Delta$ , such that the first state is  $s_{init}$ . An *accepting* computation is a finite computation  
 42  $\rho = (s^i, v_1^i, v_2^i)_{i=1, \dots, l}$  where the last state  $s^l \in S_f$  is a final state. We use  $\rho_i = ((s_i, E_1^i, E_2^i), (s_{i+1}, C_1^i, C_2^i))$  to denote the  
 43 transition that leads from the  $i$ th configuration  $(s_i, v_1^i, v_2^i)$  to the  $(i+1)$ th configuration  $(s_{i+1}, v_1^{i+1}, v_2^{i+1})$  for  $i < l$ . Note  
 44 that we have that  $v_k^{i+1} = v_k^i + C_k^i$  for  $k = 1, 2$ .

45 Finally, we say that a transition  $\tau = ((s, E_1, E_2), (s', C_1, C_2))$  is *enabled* in a configuration  $(s, v_1, v_2)$  if the value  $v_k$   
 46 of counter  $k$  satisfies condition  $E_k \in \{0, 1\}$  for  $k \in \{1, 2\}$  (with the obvious meaning of being zero or non-zero), i.e.  
 47  $v_k > 0$  iff  $E_k = 1$ . If an enabled transition  $\tau$  is taken the automaton changes its control state from  $s$  to  $s'$ , and counter  $i$   
 48 is updated by adding  $C_i \in \{-1, 0, +1\}$ . The automaton halts on empty input iff there is an *accepting computation*.<sup>4</sup>

<sup>4</sup>We only require that there is an accepting computation; in particular, there could be other (infinite) non-accepting computations of the automa-

#### 4.1. Undecidability of rRAL and prRAL with Two Resources Types over iRBMs

In this section we essentially extend the undecidability results of [2] to **iRBMs**. We first give a generic construction of an **iRBM**  $\mathfrak{M}_1^{\mathcal{A}}$  for a two-counter machine  $\mathcal{A}$  which is used to show that model checking rRAL and prRAL are undecidable over **iRBMs** (Theorems 1 and 2). The key is provided by the Simulation Lemma 1.

##### 4.1.1. Encoding of Two-Counter Machines

In [2] it was shown that model checking formulae of the form  $\langle\langle 1 \rangle\rangle_{\text{Agf}}^0 \mathbf{F} \text{halt}$  is undecidable over **RBMs**, where  $\text{halt}$  is an arbitrary proposition encoding that the TCM halts. In the following we show that this result carries over to **iRBMs**. Before we give the formal definitions and proof we present the basic idea underlying the reductions of [2] regarding **RBMs**.<sup>5</sup> The key idea is to encode the transition table of the automaton as an **RBM**, where the two counters are simulated by two resource types  $R_1$  and  $R_2$ . We give a reduction for both one and two agents. First, we describe the variant with two agents. In this variant, agent 1 is the *simulator* and agent 2 is the *spoiler*. Essentially, the role of agent 1 is to select transitions  $\tau$  of the automaton, while the role of agent 2 is to ensure that only enabled transitions are selected by agent 1. As an illustration, let us consider a single transition  $\tau = ((s, E_1, E_2), (s', C_1, C_2))$ . The model has different types of states, including *automaton states*  $S$ . In states  $s \in S$  the simulator agent 1 can execute an action  $E_1E_2$  followed by an action  $s_{E_1E_2}^{C_1C_2}$ . Both actions together simulate the selection of  $\tau$ . The first action,  $E_1E_2$ , is used to select and to (partially) check whether a transition of the TCM is enabled. That is, if  $E_i = 1$ , agent 1 must have a resource of type  $R_i$  to execute the action. After executing  $E_1E_2$  the model enters a *test state*  $sE_1E_2$ . The purpose of the test state is to check whether a transition with  $E_i = 0$  was selected by agent 1 only if counter  $i$  was indeed zero. That is, the test state ensures that the simulation is sound. Note that, in general, nothing prevents agent 1 from executing such an action if it has resources available. The problem is that it is not possible to *test for zero* directly in the model.<sup>6</sup> The workaround proposed in [2] is to use the spoiler agent 2 to encode the “zero test”. In a test state  $sE_1E_2$ , agent 2 must not be able to reach the *fail state*  $q_f$ . Reaching the fail state is only possible if resources are available in cases where there should not be any. This is encoded in the model by test actions  $test_i$  with  $i \in \{1, 2\}$ . For example, if counter 1 should be empty,  $E_1 = 0$ , the action  $test_1$  can only be executed if resources of type 1 are available. That is, the executability of the action indicates a flawed simulation. To work correctly, this requires that agent 2 correctly mirrors agent 1’s resource balance, i.e. agent 2 also simulates the counter values. This is achieved by essentially making the model turn-based, in the sense that agent 2 frequently has no alternatives: once agent 1 has executed an action  $s_{E_1E_2}^{C_1C_2}$  to update the counter values, an *intermediate state*  $s'C_1C_2$  is introduced in which agent 2 has a single choice with the same effect on its resource balance as agent 1’s previous action.<sup>7</sup> Based on this idea, it is shown in [2] (using the finitary semantics) that the TCM  $\mathcal{A}$  halts on the empty input if, and only if, the formula  $\langle\langle 1 \rangle\rangle_{\{1,2\}}^0 \mathbf{F} \text{halt}$  holds in the corresponding model. The state corresponding to the automaton’s accepting state is labelled **halt**.

In extending the reduction to **iRBMs**, the main difficulty is correctly mirroring agent 1’s resources by agent 2 in the presence of idle actions. It is no longer possible to give agent 2 only a single action to execute; an action with no costs must also be available. We extend the construction outlined above accordingly. The key idea is that executing the idle action does not help agent 2 spoil the execution. The next definition formalises an appropriate encoding to work over **iRBMs**.

**Definition 3** ( $\mathfrak{M}_1^{\mathcal{A}}$ ). Let  $\mathcal{A} = (S, s_{\text{init}}, S_f, \Delta)$  be an empty-band TCM. From  $\mathcal{A}$  we construct the **iRBM**  $\mathfrak{M}_1^{\mathcal{A}} = (\{1, 2\}, Q, \Pi, \pi, \text{Act}, d, o, \{R_1, R_2\}, t)$  with

1.  $Q = S \cup Q_1 \cup Q_2 \cup \{q_f, q_h, q_\circ\}$  where  $Q_1 = \{sE_1E_2 \mid s \in S, E_1, E_2 \in \{0, 1\}\}$  and  $Q_2 = \{sC_1C_2 \mid s \in S, C_1, C_2 \in \{-1, 0, 1\}\}$ . State  $q_f$  (resp.  $q_h$  and  $q_\circ$ ) is called a *fail state* (resp. *auxiliary halting state* and *loop state*).
2. The set *Act* of actions is defined as follows. For each transition  $((s, E_1, E_2), (s', C_1, C_2))$  of  $\mathcal{A}$  the set contains actions  $E_1E_2$ ,  $s_{E_1E_2}^{C_1C_2}$ , and  $s'C_1C_2$ . Additionally, there are the idle action *idle* and test actions  $test_i$  for  $i \in \{1, 2\}$ .

ton due to non-determinism.

<sup>5</sup>No complete, formal proofs are given in [2], only proof sketches.

<sup>6</sup>Testing for zero is a delicate property, the satisfaction of which seems crucial for the undecidability of other formalisms, such as Petri nets [34].

<sup>7</sup>We note that we need the underscore in order to make the two types of states syntactically different. Otherwise, in case  $C_1 = E_1, C_2 = E_2$  and  $s = s'$ , we could not have two ‘copies’ of a state.

3. The action availability is defined according to  $\Delta$ . For agent 1 we have:

$$\begin{aligned} E_1E_2 &\in d_1(s) \text{ iff there is a transition } ((s, E_1, E_2), (s', C_1, C_2)) \in \Delta \\ \text{idle} &\in d_1(q) \text{ for all } q \in Q \\ s'_{E_1E_2}{}^{C_1C_2} &\in d_1(sE_1E_2) \text{ iff } ((s, E_1, E_2), (s', C_1, C_2)) \in \Delta \end{aligned}$$

and for agent 2:

$$\begin{aligned} \text{idle} &\in d_2(q) \text{ for all } q \in Q \\ s'C_1C_2 &\in d_2(s'C_1C_2) \text{ for all } s'C_1C_2 \in Q_2 \\ \text{test}_i &\in d_2(sE_1E_2) \text{ iff } E_i = 0 \text{ with } i \in \{1, 2\} \end{aligned}$$

- 43 4. The set of propositions is defined by  $\Pi = \{\text{halt}, \text{fail}\}$ . All states in  $\{q_h\} \cup S_f$  are labelled with **halt** and  $q_f$  is  
1 labelled with **fail**.
- 2 5. The transition function is defined as follows:

$$\begin{aligned} o(s, (E_1E_2, \text{idle})) &= sE_1E_2 \\ o(s, (\text{idle}, \text{idle})) &= o(sE_1E_2, (\text{idle}, \text{idle})) = q_\circ \\ o(q_\circ, (\text{idle}, \text{idle})) &= q_\circ \\ o(sE_1E_2, (s'_{E_1E_2}{}^{C_1C_2}, \text{idle})) &= s'C_1C_2 \\ o(sE_1E_2, (\star, \text{test}_i)) &= o(q_f, (\text{idle}, \text{idle})) = q_f \\ o(s'C_1C_2, (\text{idle}, s'C_1C_2)) &= s' \\ o(s'C_1C_2, (\text{idle}, \text{idle})) &= o(q_h, (\text{idle}, \text{idle})) = q_h \end{aligned}$$

3 where  $\star$  represents any action available to the respective agent in that state.

- 4 6. The actions' resource consumption/production is defined by function  $t$  where  $i, r \in \{1, 2\}$ :

$$\begin{aligned} t(E_1E_2, R_r) &= -E_r \\ t(\text{idle}, R_r) &= 0 \\ t(s'_{E_1E_2}{}^{C_1C_2}, R_r) &= C_r + E_r \\ t(sC_1C_2, R_r) &= C_r \\ t(\text{test}_i, R_r) &= \begin{cases} -1 & \text{if } i = r \\ 0 & \text{else} \end{cases} \end{aligned}$$

5 Let us consider a TCM  $\mathcal{A} = (S, s_{\text{init}}, S_f, \Delta)$ . The construction of the model  $\mathfrak{M}_1^{\mathcal{A}}$  is sketched in Figure 2 (left), and  
6 the encoding of a single transition  $\tau = ((s, E_1, E_2), (s', C_1, C_2))$  is illustrated in Figure 2 (right). As explained above,  
7 the action  $E_1E_2$  consumes  $-E_r$  resources of  $R_r$  for  $r = 1, 2$ . This simulates that only enabled transitions  $\tau$  can be  
8 taken. If  $E_r = 1$  then the action  $E_1E_2$  can only be taken if resources  $R_r \geq 1$ . Actions of type  $s'_{E_1E_2}{}^{C_1C_2}$  consume/produce  
9  $C_r + E_r$  units of resource  $R_r$ ,  $r = 1, 2$ . The component  $C_r$  simulates the decrement and increment of counter  $r$  where  $E_r$   
10 corrects the possible (temporary) subtraction from the previous action  $E_1E_2$ . The necessary information to select the  
11 correct action is stored in the state  $sE_1E_2$ . Clearly, actions can only be performed if sufficient resources are available.  
12 The difficulty is to ensure that actions  $E_1E_2$  with some  $E_r = 0$  are only performed if the counter  $r$  is actually 0; that  
13 is, if *no* resources of type  $R_r$  are available. For this purpose, *test actions*  $\text{test}_r$  that cost  $-1$  units of resource  $R_r$  for  
14  $r \in \{1, 2\}$ , are introduced. Such an action  $\text{test}_r$  can only be performed in states  $sE_1E_2$  if  $E_r = 0$ , and it always leads to  
15 the fail state  $q_f$ . Now, in a state  $sE_1E_2$  with some element equal to 0, say  $E_1 = 0, E_2 = 1$ , (representing that counter  
16 1 should be zero and 2 should be non-zero) the action  $\text{test}_1$  can be used to verify whether the currently available  
17 resources model the counter correctly: if  $q_f$  is reachable, resources of type  $R_1$  are available, although this should not  
18 be the case according to  $E_1$ .

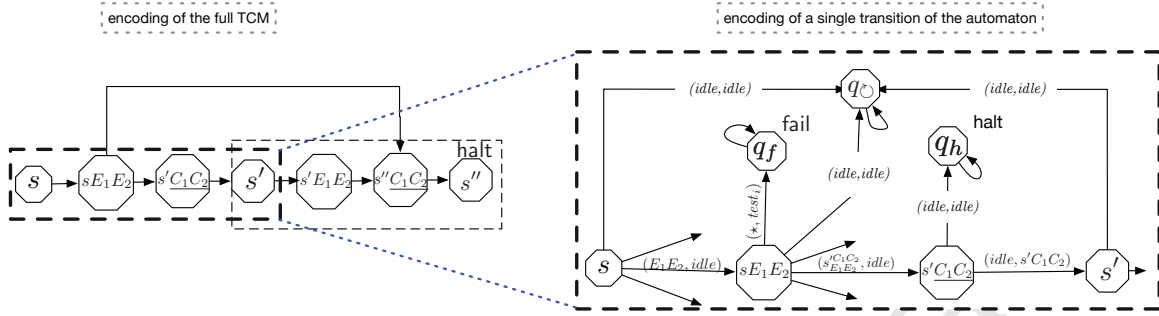


Figure 2: The left figure schematically shows an abstraction of the encoding of the automaton  $\mathfrak{M}_1^{\mathcal{A}}$  for  $\Delta = \{(s, E_1, E_2), (s', C_1, C_2), ((s, E_1, E_2), (s'', C_1, C_2)), ((s', E_1, E_2), (s'', C_1, C_2))\}$ . The right figure shows an excerpt of the encoding of a transition  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  using two resource types and two agents.

#### 4.1.2. Properties of the Encoding: The Simulation Lemma

In the following, we state properties of the encoding, and prove a simulation lemma which relates runs of the two-counter machine with paths in the model. First, we make a straightforward observation:

**Observation 3.** *The model  $\mathfrak{M}_1^{\mathcal{A}}$  is an iRBM.*

We define the concept of a *computation pre-encoding*. This is a finite path in the model which will later be shown to encode a partial computation of the automaton.

**Definition 4** (Computation pre-encoding of  $\mathfrak{M}_1^{\mathcal{A}}$ ). *Let  $\mathcal{A}$  be an empty-band TCM. A finite resource-extended path  $\lambda \in (Q \times \text{En})^+$  in  $\mathfrak{M}_1^{\mathcal{A}}$  is called an  $\mathcal{A}$ -computation pre-encoding of  $\mathfrak{M}_1^{\mathcal{A}}$  if it satisfies the following properties:*

1.  $\eta(1, R_r) = \eta(2, R_r)$  where  $\lambda[0] = (q, \eta)$  for  $r \in \{1, 2\}$ ; and
2.  $\lambda|_Q = (s^i(s^i E_1^i E_2^i)(s^{i+1} C_1^i C_2^i))_{i=1, \dots, k} s^{k+1}$  or  $\lambda|_Q = s^1$ .

An  $\mathcal{A}$ -computation pre-encoding of  $\mathfrak{M}_1^{\mathcal{A}}$  is called *accepting* if its final state  $s^{k+1}$  is an accepting state of the TCM,  $s^{k+1} \in S_f$ , or if  $\lambda|_Q = s^1 \in S_f$ . In the following we shall often omit “of  $\mathfrak{M}_1^{\mathcal{A}}$ ”.

The first requirement states that the endowments of both agents must be the same in the initial state. The second requirement expresses that a fail, auxiliary halting, or loop state must never be visited, and the path ends in a state that is also a state of  $\mathcal{A}$ ; moreover, it specifies the order in which states in the model are visited. The latter is inherent in the construction of the model. The next proposition states that, on a computation pre-encoding, agent 2 correctly mirrors the resources of agent 1 whenever a state in the model, representing a state of the TCM, is visited.

**Proposition 1.** *Let  $\mathcal{A}$  be an empty-band TCM and  $\lambda = (q^i, \eta^i)_{i=1, \dots, 3(k-1)+1}$  be an  $\mathcal{A}$ -computation pre-encoding.*

- (a) *We have that  $\eta^{3(k-1)+1}(1, R_r) = \eta^{3(k-1)+1}(2, R_r)$  for  $r = 1, 2$ .*
- (b) *If  $\lambda \circ (sE_1E_2, \eta)(s'C_1C_2, \eta')$  is a partial resource-extended path, then  $\lambda \circ (sE_1E_2, \eta)(s'C_1C_2, \eta')(s', \eta'')$  is a partial resource-extended path with a uniquely defined endowment  $\eta''$ .*
- (c) *If  $\lambda[i] = (q^i, \eta^i) = (sE_1E_2, \eta^i)$  with  $E_r = 0$ , then  $(\eta^i(1, R_r) = 0$  if, and only if,  $\eta^i(2, R_r) = 0)$ , where  $r \in \{1, 2\}$ .*

The proof is given in Appendix A.1. Part (a) expresses that the resource endowments of agents 1 and 2 are identical whenever a state  $q$  corresponding to an automaton state  $s$  is reached. Part (b) says that agent 2 always has enough resources in states of type  $s'C_1C_2$  to mirror the action executed by agent 1 one step before. Finally, in (c) the crucial observation is made that in the test states  $sE_1E_2$  with  $E_r = 0$  for  $r \in \{1, 2\}$ , both agents both have either no resources of  $R_r$  available, or they both have resources of  $R_r$  available. This ensures that the test actions are correctly executed.

The next definition relates a computation pre-encoding to the computations of the automaton it simulates. This means that essentially the same automaton states are visited, and the resources of agent 1 correctly simulate the counter values.

**Definition 5** (Simulation,  $\mathcal{A}$ -computation encoding). We say that the  $\mathcal{A}$ -computation pre-encoding  $\lambda = (q^i, \eta^i)_{i=1, \dots, 3(k-1)+1}$ ,  $k \in \mathbb{N}$ , simulates the computation  $\rho$  iff the following holds:

1.  $\rho$  has length  $k$ ;
2. for every  $i \in \{1, \dots, k\}$  if  $\rho[i] = (s, v_1, v_2)$  then  $\lambda[3(i-1)+1] = (s, \eta)$  with  $\eta(1, R_r) = v_r$  for  $r \in \{1, 2\}$ ; and
3. for any configuration  $\lambda[i] = (sE_1E_2, \eta)$  on  $\lambda$  with  $E_r = 0$ ,  $r \in \{1, 2\}$ , it holds that  $\eta(1, R_r) = 0$ .

An  $\mathcal{A}$ -computation pre-encoding is called an  $\mathcal{A}$ -computation encoding in  $\mathfrak{M}_1^{\mathcal{A}}$  if it simulates some computation of  $\mathcal{A}$ . Consequently, an  $\mathcal{A}$ -computation encoding in  $\mathfrak{M}_1^{\mathcal{A}}$  is called accepting if it simulates an accepting computation of  $\mathcal{A}$ .

The following lemma is the key step in our reduction. It specifies that the computation pre-encodings do exactly characterise the computations of the automaton. In other words, the behaviour of the automaton is exactly captured by the computation pre-encoding in the constructed **iRBM**. This lemma concludes the construction of  $\mathfrak{M}_1^{\mathcal{A}}$  and the analysis of its structural properties.

**Lemma 1** (Simulation Lemma for  $\mathfrak{M}_1^{\mathcal{A}}$ ). There is a bijection  $f^{\mathcal{A}}$  between computations of  $\mathcal{A}$  and  $\mathcal{A}$ -computation encodings of  $\mathfrak{M}_1^{\mathcal{A}}$  such that  $f^{\mathcal{A}}(\rho)$  simulates the computation  $\rho$ . In particular, if  $\rho$  is an accepting computation then  $f^{\mathcal{A}}(\rho)$  is also accepting.

The proof is given in Appendix A.1.

#### 4.1.3. Resource-Flat Fragment with Two Resource Types

We first show that proponent restrictedness is essential for decidability over **iRBMs**, by showing that resource flatness is not sufficient, and **rFRAL** is undecidable over **iRBMs**. In [2] it was shown that model checking formulae of type  $\langle\langle 1 \rangle\rangle_{\text{Agt}}^0 \mathbf{F} \text{halt}$  is undecidable over **RBM**s. The decidability of this fragment was open over **iRBM**s. In Theorem 1, we show that undecidability continues to hold. The proof adapts the approach of [2] to work over **iRBM**s.

As in [2] we show that the empty-band TCM  $\mathcal{A}$  halts iff  $\mathfrak{M}_1^{\mathcal{A}}, q_{\text{init}}, \bar{0} \models \langle\langle 1 \rangle\rangle_{(1,2)}^0 \mathbf{F} \text{halt}$ . By Lemma 1 this is equivalent to showing that there is an accepting  $\mathcal{A}$ -computation encoding iff  $\mathfrak{M}_1^{\mathcal{A}}, q_{\text{init}}, \bar{0} \models \langle\langle 1 \rangle\rangle_{(1,2)}^0 \mathbf{F} \text{halt}$ .

Consider the encoding of  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  shown in Figure 2. First, we observe that executing an idle action is not helpful for agent 1 in states  $s$  and  $sE_1E_2$ ; neither is it helpful for agent 2 to idle in a state  $s'C_1C_2$ . If agent 1 executes idle in states  $s$  or  $sE_1E_2$ , this would yield the state  $q_f$  or  $q_{\odot}$  which cannot help to make the formula true; on the contrary, if the formula is not already true, these states make it false. Similarly, if agent 2 executes idle in  $s'C_1C_2$ , the formula would be true when state  $q_h$  is reached. As we are looking for a winning strategy for agent 1 against all strategies of agent 2, we can neglect the cases where either agent executes an idle action in the aforementioned states. As a result, we only need to consider paths that have the structure of  $\mathcal{A}$ -computation pre-encodings. The second agent is needed to ensure that agent 1 chooses actions that yield an  $\mathcal{A}$ -computation encoding, i.e. that the selection of actions simulates a possible behaviour of the automaton. By construction, agent 2 only has a choice in states  $sE_1E_2$  and  $s'C_1C_2$ . In the former state the agent can execute a *test action* if sufficient resources are available. In the latter, it could idle—as discussed above, an action the agent should not execute. As a consequence, in states of type  $s'C_1C_2$ , agent 2 should essentially only perform the action  $s'C_1C_2$  ensuring that the agent's resources mirror agent 1's resources (cf. Proposition 1). This essentially ensures condition 3 of Definition 5 (simulation). Formally, we have:

**Lemma 2.** The empty-band TCM  $\mathcal{A}$  halts iff  $\mathfrak{M}_1^{\mathcal{A}}, s_{\text{init}}, \bar{0} \models \langle\langle 1 \rangle\rangle_{(1,2)}^0 \mathbf{F} \text{halt}$ .

We briefly sketch the main idea of the proof below; the full proof is given in Appendix A.1:

- Suppose that  $\mathcal{A}$  halts. Then, agent 1 simulates the transitions of the machine's accepting run. Due to the simulation, agent 2 will never be able to enforce the fail state  $q_f$ . Moreover, either agent 2's resources correctly mirror agent 1's resources, or agent 2 performs the idle action. In both cases, either an accepting state or the auxiliary halting state  $q_h$ , both labelled halt, are reached. The formula is true.
- Let the formula be true. Agent 1 must have a strategy that guarantees reaching a state labelled halt against all strategies of agent 2, including agent 2's strategy in which agent 2 never performs the idle action. This strategy of agent 2 correctly mirrors agent 1's resources and ensures that agent 1's strategy only selects enabled transitions. Thus, the strategy of agent 1 yields a  $\mathcal{A}$ -computation encoding.

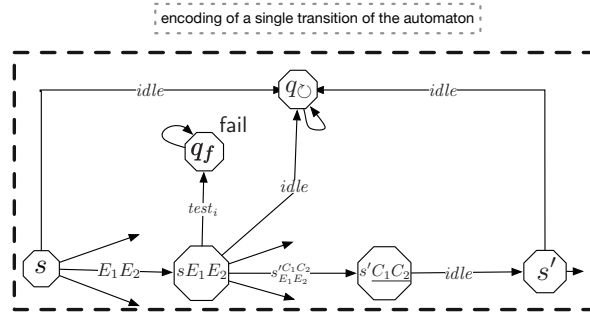


Figure 3: Excerpt of model  $\widehat{\mathfrak{M}}_1^{\mathcal{A}}$ : encoding of a transition  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  using two resource types and one agent.

46 The previous lemma immediately yields the following theorem which concludes our study of rRAL with two  
1 resource types.

2 **Theorem 1.** *Model checking rRAL over the class of iRBMs is undecidable, even for two agents and two resource*  
3 *types.*

#### 4 4.1.4. The Proponent Restricted Fragment with Two Resource Types

5 Theorem 1 shows that the restriction of resource-flatness is not enough to obtain a decidable model checking  
6 property. In this section, we consider the proponent-restricted fragment. We show that proponent-restrictedness  
7 on its own is also not sufficient for decidability, and prRAL is undecidable over iRBMs. We do this by adapting  
8 the undecidability proof of [2] for prRAL to work over iRBMs. This is a negative result. However, in contrast  
9 to Theorem 1, the formula used in the reduction is more complex. This leaves room for restrictions on the temporal  
10 structure of prRAL. Indeed, this is the motivation for the decidable fragment of prRAL that we introduce in Section 4.3.

11 The proof of the undecidability result for prRAL over RBMs of [2] essentially follows an encoding similar to  
12 the one shown in Figure 3. However, in contrast to the previous encoding, the second agent is removed, and agent  
13 1 itself is used to perform the zero test. This requires a slightly more sophisticated formula. First, we show a  
14 reduction with respect to our original, two-player model  $\mathfrak{M}_1^{\mathcal{A}}$ : the automaton  $\mathcal{A}$  halts if, and only if,  $\mathfrak{M}_1^{\mathcal{A}}, q_{init}, \bar{0} \models$   
15  $\langle\langle 1, 2 \rangle\rangle^0 ((-\langle\langle 1, 2 \rangle\rangle^{\downarrow} \mathbf{X} \text{fail}) \mathbf{U} \text{halt})$ . The main idea is that in test state  $sE_1E_2$ , agents  $\{1, 2\}$  must not be able to reach the  
16 fail state  $q_f$ , which is expressed by  $\neg\langle\langle 1, 2 \rangle\rangle^{\downarrow} \mathbf{X} \text{fail}$ . The next lemma follows essentially as a Corollary of [2], by  
17 adding idle loops to the construction. However, for uniformity, we base the proof on the model  $\mathfrak{M}_1^{\mathcal{A}}$ .

18 **Lemma 3.** *The empty-band TCM  $\mathcal{A}$  halts iff  $\mathfrak{M}_1^{\mathcal{A}}, q_{init}, \bar{0} \models \langle\langle 1, 2 \rangle\rangle^0 ((-\langle\langle 1, 2 \rangle\rangle^{\downarrow} \mathbf{X} \text{fail}) \mathbf{U} \text{halt})$ .*

19 *Proof.* The proof is analogous to the one given for Lemma 2. For the direction " $\Rightarrow$ " it is sufficient to observe that  
20 the only states from which the fail state  $q_f$  can be reached within one step are of the form  $sE_1E_2$  with  $E_r = 0$  for  
21  $r \in \{1, 2\}$ . Thus, the strategy profile  $(s_1, s_2)$ , where  $s_1$  is the strategy of agent 1 as defined in Lemma 2 and  $s_2$   
22 is an arbitrary strategy for agent 2, witnesses the truth of the formula. The other direction is done analogously to  
23 Lemma 2.  $\square$

24 In the formula used in the reduction of Lemma 3,  $\langle\langle 1, 2 \rangle\rangle^0 ((-\langle\langle 1, 2 \rangle\rangle^{\downarrow} \mathbf{X} \text{fail}) \mathbf{U} \text{halt})$ , the two agents 1 and 2 always  
25 act as a team; there is no opponent. Thus, the two agents can be merged into a single agent. The next result makes  
26 this observation precise.

27 **Theorem 2.** *Model checking prRAL over the class of iRBMs is undecidable, even in the case of a single agent and*  
28 *two resource types.*

29 *Proof.* We modify the model  $\mathfrak{M}_1^{\mathcal{A}} = (\{1, 2\}, Q, \Pi, \pi, Act, d, o, \{R_1, R_2\}, t)$  to a model  $\widehat{\mathfrak{M}}_1^{\mathcal{A}} = (\{1\}, Q \setminus \{q_h\}, \Pi, \widehat{\pi}, \widehat{Act}, \widehat{d}, \widehat{o},$   
30  $\{R_1, R_2\}, \widehat{t})$ . We remove the auxiliary halting state as it must not be reached by the proponent agent 1. Essentially, we  
31 merge the two agents into one. The encoding of a single automaton transition is shown in Figure 3. We define  $\widehat{d}_1(q)$  as

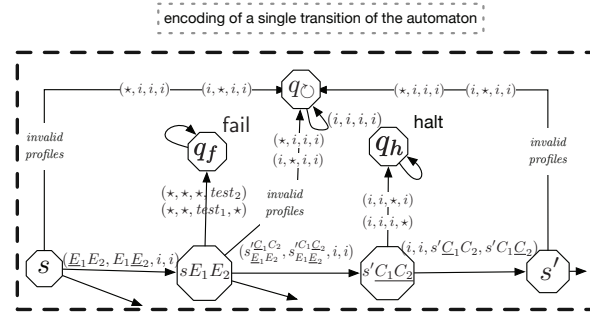


Figure 4: Excerpt of model  $\mathfrak{M}_2^A$ . Encoding of a transition  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  using only one resource type and four agents. In the figure, we use  $i$  to refer to the idle action idle.

32  $d_1(q)$  where we additionally require that  $test_i \in \widehat{d}_1(sE_1E_2)$  iff  $E_i = 0$ ; agent 1 can now make all decisions. The action  
 1 set  $Act$  equals  $Act$  but all actions of type  $sC_1C_2$  are removed. The transition function  $\widehat{o}$  is obtained from  $o$ :

$$\begin{aligned} \widehat{o}(s, E_1E_2) &= sE_1E_2 \\ \widehat{o}(s, idle) = \widehat{o}(sE_1E_2, idle) &= q_\circ \\ \widehat{o}(q_\circ, idle) &= q_\circ \\ \widehat{o}(sE_1E_2, sC_1C_2) &= sC_1C_2 \\ \widehat{o}(sE_1E_2, test_i) = \widehat{o}(q_f, idle) &= q_f \\ \widehat{o}(sC_1C_2, idle) &= s' \end{aligned}$$

2 The cost function  $\widehat{t}$  and the labelling function  $\widehat{\pi}$  are defined as before restricted to the new action set.

3 Now, it is easy to see that each resource-extended path  $\lambda = (q^i, \eta^i)_{i \in \mathbb{N}}$  in  $\mathfrak{M}_1^A$  that does not visit state  $q_h$  cor-  
 4 responds to a path  $\lambda = (q^i, \widehat{\eta}^i)_{i \in \mathbb{N}}$  in  $\widehat{\mathfrak{M}}_1^A$  with  $\widehat{\eta}^i(1, R_r) = \eta^i(1, R_r)$ . By Proposition 1(c), the zero-test in the test  
 5 states can equivalently be defined with respect to agent 1's resource endowment. It is immediate that  $\mathfrak{M}_1^A, q_{init}, \bar{0} \models$   
 6  $\langle\langle 1, 2 \rangle\rangle^0 ((-\langle\langle 1, 2 \rangle\rangle^\downarrow \mathbf{X} \text{ fail}) \mathbf{U} \text{ halt})$  if, and only if,  $\widehat{\mathfrak{M}}_1^A, q_{init}, \bar{0} \models \langle\langle 1 \rangle\rangle^0 ((-\langle\langle 1 \rangle\rangle^\downarrow \mathbf{X} \text{ fail}) \mathbf{U} \text{ halt})$ .  $\square$

7 **Remark 2.** We note that we can further simplify model  $\widehat{\mathfrak{M}}_1^A$ . For example, states of type  $sC_1C_2$  are not needed.  
 8 Keeping them, however, allows us to reuse the previous notation and thus simplifies the presentation.

#### 9 4.2. Undecidability of prRAL and rRAL with One Resource Type over iRBMs

10 The reductions presented in the previous section use two resource types to simulate the two counters of the two-  
 11 counter machine. In this section we show that the model checking problem for prRAL and rRAL remain undecidable  
 12 even if each agent has only one resource type available. This shows that proponent restriction and resource flatness  
 13 are essential even with one resource type. Settings restricted to a single resource type are an important special case,  
 14 as having only one resource type available might be expected to make model checking less complex. For example,  
 15 with one resource type, the complexity of  $RB \pm \text{ATL}$  goes from EXPSPACE-hard to PSPACE-complete [8, 9]. The  
 16 reductions in the case of one resource type are more complex. Specifically, the number of agents doubles: instead of  
 17 one agent in the proponent-restricted setting, we need two; and instead of two agents in the resource-flat setting, we  
 18 need four.

##### 19 4.2.1. Encoding Two-Counter Machines

20 The encoding of the TCM is very similar to the encoding presented in Section 4.1.1. The key difference is that  
 21 we can no longer use a single action to update both resource types at the same time. The actions must be split into  
 22 two. Instead of an action  $E_1E_2$ , we introduce two actions  $\underline{E}_1E_2$  and  $E_1\underline{E}_2$ , where the first and second action are  
 23 controlled by the first and second agent, respectively, and change the agent's single resource type according to  $E_1$  and



24  $E_2$ , respectively. Similarly, actions of type  $s_{C_1C_2}^{E_1E_2}$  are split into  $s_{\underline{C}_1C_2}^{E_1E_2}$  and  $s_{C_1\underline{C}_2}^{E_1E_2}$ . The underscore indicates which parts  
1 of the action should be used to update the resources of the agent executing the action.

2 The technical presentation requires a little more notation. The decomposition ensures that the agents coordinate  
3 their actions so that the action tuples consisting of the actions of the first and second agent have a counterpart in the  
4 TCM. This is best illustrated by an example. Suppose the automaton contains the transitions  $((s, 0, 1), (s', 1, 1))$  and  
5  $((s, 1, 0), (s', 1, 1))$ , and that these are the only two transitions which should be enabled in state  $s$ . In the new encoding,  
6 agent 1 would have the actions  $\underline{01}$ ,  $\underline{10}$ , and idle in its repertoire at state  $s$ . Similarly, agent 2 has actions  $\underline{01}$ ,  $\underline{10}$ , and  
7 idle available at state  $s$ . As both agents are autonomous decision makers they are free to choose actions independently.  
8 Hence, the action profile  $(\underline{01}, \underline{10})$  may result from their action selection. Clearly, this is an undesirable action tuple,  
9 as it does not correspond to any transition of the automaton. We need to ensure that such action profiles never yield a  
10 behaviour which encodes an accepting run of the automaton. Therefore, the model is constructed in such a way that  
11 *invalid* action profiles result in the loop state  $q_\circ$ .

12 With this change, the other parts of the previous encoding can be used with only minor modifications. Agents  
13  $\{1, 2\}$  are used to simulate the behaviour of the automaton, and agents  $\{3, 4\}$  play the role of the spoiler agents who  
14 ensure that the simulation is sound. Thus, the coalition  $\{3, 4\}$  is used to encode the zero-test. This requires that agent  
15 3 mirrors the resources of agent 1, and agent 4 the resources of agent 2. The new encoding of a transition is illustrated  
16 in Figure 4, and the formal definition of the model is given in Appendix A.2 (Definition 7).

#### 17 4.2.2. Properties of the Encoding: The Simulation Lemma

18 Analogously to Section 4.1.2, we present a simulation lemma. We also need to introduce computation pre-  
19 encodings etc. with respect to  $\mathfrak{M}_2^{\mathcal{A}}$ . For the sake of readability, we mostly refrain from giving formal definitions,  
20 and focus on the key modifications. An  $\mathcal{A}$ -computation pre-encoding of  $\mathfrak{M}_2^{\mathcal{A}}$  is defined as for  $\mathfrak{M}_1^{\mathcal{A}}$ , but the initial  
21 condition is changed to  $\eta(a, R) = \eta(b, R)$  for agents  $(a, b) \in \{(1, 3), (2, 4)\}$  where  $R$  refers to the single resource type.  
22 (In the following we use  $a$  and  $b$  to denote the agents, where  $b$  simulates the resources of  $a$ .) That is, the initial  
23 endowment for agents 1 and 3, as well as for agents 2 and 4 must be identical. With this notion we can also prove  
24 basic properties analogously to Proposition 1. The new version of Proposition 1 reads as follows:

- 25 (a) We have that  $\eta^{3(k-1)+1}(a, R) = \eta^{3(k-1)+1}(b, R)$  for  $(a, b) \in \{(1, 3), (2, 4)\}$ .
- 26 (b) If  $\lambda \circ (sE_1E_2, \eta)(s'\underline{C}_1C_2, \eta')$  is a resource-extended path, then  $\lambda \circ (sE_1E_2, \eta)(s'\underline{C}_1C_2, \eta')(s', \eta')$  is a resource-  
27 extended path with a uniquely defined endowment  $\eta''$ .
- 28 (c) If  $\lambda[i] = (q^i, \eta^i) = (sE_1E_2, \eta^i)$  with  $E_a = 0$ , then  $(\eta^i(a, R) = 0$  if and only if  $\eta^i(b, R) = 0$ ), where  $(a, b) \in$   
29  $\{(1, 3), (2, 4)\}$

30 (b) remains unchanged, and expresses that agents 3 and 4 correctly mirror the resources of agents 1 and 2, respectively.  
31 We introduce the revised notion of simulation. We say that the  $\mathcal{A}$ -computation pre-encoding  $\lambda = (q^i, \eta^i)_{i \in \{1, \dots, 3(k-1)+1\}}$   
32 of  $\mathfrak{M}_2^{\mathcal{A}}$ ,  $k \in \mathbb{N}$ , *simulates the  $\mathcal{A}$ -computation  $\rho$*  (wrt.  $\mathfrak{M}_2^{\mathcal{A}}$ ) if the following holds:

- 33 1.  $\rho$  has length  $k$ ;
- 34 2. for every  $i \in \{1, \dots, k\}$  if  $\rho[i] = (s, v_1, v_2)$  then  $\lambda[3(i-1)+1] = (s, \eta)$  with  $\eta(1, R) = v_1$  and  $\eta(2, R) = v_2$ ; and  
35 3. for any configuration  $\lambda[i] = (sE_1E_2, \eta)$  on  $\lambda$  with  $E_r = 0$ ,  $r \in \{1, 2\}$ , it holds that  $\eta(r, R) = 0$ .

36 Note that counters now refer to the unique resource type of different agents, rather than to different resource types of  
37 a single agent.

38 Analogously to Lemma 1 we can prove the following adapted simulation lemma.

39 **Lemma 4** (Simulation Lemma for  $\mathfrak{M}_2^{\mathcal{A}}$ ). *There is a bijection  $f^{\mathcal{A}}$  between computations of  $\mathcal{A}$  and  $\mathcal{A}$ -computation*  
40 *encodings of  $\mathfrak{M}_2^{\mathcal{A}}$  such that  $f^{\mathcal{A}}(\rho)$  simulates the computation  $\rho$ . In particular, if  $\rho$  is an accepting computation then*  
41  *$f^{\mathcal{A}}(\rho)$  is also accepting.*

#### 4.2.3. Resource-Flat Fragment with One Resource Type

In this section we prove undecidability of rRAL with only one resource type. We proceed analogously to Section 4.1.3. We show that the empty-band TCM  $\mathcal{A}$  halts if, and only if,  $\mathfrak{M}_2^{\mathcal{A}}, s_{\text{init}}, \bar{0} \models \langle\langle 1, 2 \rangle\rangle_{(1,2,3,4)}^{\bar{0}} \mathbf{F} \text{halt}$ . Again, we observe that for any encoding of a transition  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  shown in Figure 4, agents 1 and 2 (resp. 3 and 4) have no incentive to idle in states  $s$  and  $sE_1E_2$  (resp. in state  $s'C_1C_2$ ). Instead of looking for a winning strategy for  $\{1\}$  we look for a winning strategy for  $\{1, 2\}$ . The key idea is that the coalition behaves in such a way that their combined action corresponds to the action selection of  $\{1\}$  in Section 4.1.3. Given the reformulation of properties above, we can make the following observations:

- If  $\mathcal{A}$  halts, then coalition  $\{1, 2\}$  simulates the transitions of the machine's accepting run. Due to the simulation, coalition  $\{3, 4\}$  will never be able to enforce the fail state  $q_f$ . Moreover, either agent  $b$ 's resources correctly mirror agent  $a$ 's resources, or agent  $b$  performs the idle action for  $(a, b) \in \{(1, 3), (2, 4)\}$ . In both cases, either an accepting state or the auxiliary halting state  $q_h$ , both labelled halt, are reached. The formula is true.
- Let the formula be true. Coalition  $\{1, 2\}$  must have a strategy that guarantees reaching a state labelled halt against all strategies of  $\{3, 4\}$ , including the collective strategy of  $\{3, 4\}$  in which an idle action is never performed in states of type  $s'C_1C_2$ . This strategy of agent  $b$  correctly mirrors agent  $a$ 's resources and ensures that agent  $a$ 's strategy only selects enabled transitions,  $(a, b) \in \{(1, 3), (2, 4)\}$ . Thus, the strategy of  $\{1, 2\}$  yields an  $\mathcal{A}$ -computation encoding. By Simulation Lemma 4 the TCM has an accepting run and halts.

Formally, we capture the previous discussion in the following Lemma and Theorem.

**Lemma 5.** *The empty-band TCM  $\mathcal{A}$  halts iff  $\mathfrak{M}_2^{\mathcal{A}}, s_{\text{init}}, \bar{0} \models \langle\langle 1, 2 \rangle\rangle_{(1,2,3,4)}^{\bar{0}} \mathbf{F} \text{halt}$ .*

**Theorem 3.** *Model checking rRAL over the class of iRBMs is undecidable, even in the case of a single resource type and four agents.*

The proofs of Lemma 5 and Theorem 3 are analogous to those of Lemma 2 and Theorem 1, respectively.

#### 4.2.4. The Proponent Restricted Fragment with One Resource Type

Finally, we consider the proponent-restricted fragment. Again, the line of argument is analogous to the one followed in Section 4.1.4, but there is one caveat when it comes to merging the agents. We first state the analogue of Lemma 3.

**Lemma 6.** *The empty-band TCM  $\mathcal{A}$  halts iff  $\mathfrak{M}_2^{\mathcal{A}}, q_{\text{init}}, \bar{0} \models \langle\langle 1, 2, 3, 4 \rangle\rangle^{\bar{0}} ((-\langle\langle 1, 2, 3, 4 \rangle\rangle^{\downarrow} \mathbf{X} \text{fail}) \mathbf{U} \text{halt})$ .*

Next, we observe that the agents  $\{1, 2, 3, 4\}$  act as a team. Thus, we can consider their decision making as the decision making of a single “merged” agent. However, in contrast to the setting of Section 4.1.4, we cannot explicitly model this by a single agent as there is only one resource type. Thus, we merge agents  $\{1, 3\}$  and agents  $\{2, 4\}$  into two distinct agents. The resulting model  $\widehat{\mathfrak{M}}_2^{\mathcal{A}}$  is illustrated in Figure 5.

**Theorem 4.** *Model checking prRAL over the class of iRBMs is undecidable, even in the case of a single resource type and two agents.*

The proof is given in Appendix A.2.

### 4.3. The Positive Proponent-Restricted Fragment of RAL

Following the observation made in the previous sections about the proponent-restricted variants of RAL (cf. Theorems 2 and 4) we define a proponent-restricted but not resource-flat fragment of RAL, pprRAL, that has a decidable model checking property over iRBMs. We first define the *positive fragment* of RAL as the set of all RAL-formulae where no cooperation modality is under the scope of a negation symbol.

**Definition 6** (The fragment pprRAL). *The logic pprRAL is defined as the proponent-restricted and positive fragment of RAL.*<sup>8</sup>

<sup>8</sup>A more restricted version of pprRAL was introduced in [7], where in addition the formula  $\varphi_1$  on the left-hand-side of  $\varphi_1 \mathbf{U} \varphi_2$  is constrained to be propositional.

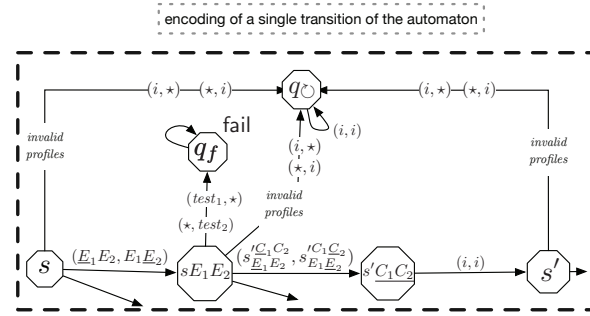


Figure 5: Excerpt of model  $\mathbb{M}_2^{\mathcal{A}}$ . Encoding of a transition  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  using only one resource type and two agents. Again, we denote the idle action idle by  $i$ .

41 As noted in the introduction, the pprRAL-fragment allows us to express properties of coalitions of agents which  
 1 re-consider their strategies *without* being re-equipped with fresh resources. For example, we can formalise the prop-  
 2 erty “given their initial battery charge, rescue robots  $A$  can safely get to a position from which they can perform  
 3 rescue while in visual contact with the base” as:  $\langle\langle A \rangle\rangle^{\text{init}}(\text{safe } \mathbf{U}(\langle\langle A \rangle\rangle^{\downarrow}(\text{visual } \mathbf{U} \text{ rescue})))$ . Intuitively, this reflects the  
 4 constraint that the robots cannot recharge their batteries after reaching the position where they can perform rescue  
 5 while in visual contact with the base. This is expressible in pprRAL but not in rfRAL. Another example is the formula  
 6  $\langle\langle 1, 2 \rangle\rangle^{\text{init}} \mathbf{F}(\text{rob} \wedge \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{F} \text{ escape})$ , expressing that the coalition  $\{1, 2\}$  can cooperate to eventually rob a bank, following  
 7 which agent 1 has a strategy to escape on its own using only its remaining resources.

8 Before we show the decidability of pprRAL over iRBMs in the next section, we make the following observation  
 9 which follows from [2, Theorem 6]:

10 **Observation 4.** *Model checking pprRAL over RBMs is undecidable.*

11 To give a flavour of the basic idea of the undecidability proof, let us consider Figure 2. We have to modify the  
 12 construction in such a way that, in test states  $sE_1E_2$ , the opponent can always execute a test action corresponding to a  
 13 counter that should be zero, after which a new state is reached in which the proponent has to execute a specific action.  
 14 In a sense the opponent can challenge the proponent to execute this specific action. Now, there are two options. First,  
 15 the proponent has not sufficient resources to execute the action, which means that the counter is simulated correctly.  
 16 In that case the history leading to the current state is disregarded as it cannot be extended to a resource-extended path  
 17 (recall that such paths have to be infinite). Second, the proponent can execute the action. This would result in a new  
 18 fail state labelled with a specific proposition, say *error*, indicating that the reduction is flawed. Then, the simulation  
 19 is continued by connecting the fail state with the state  $s'C_1C_2$  which would have been reached if the opponent had  
 20 not executed the test action. Given this modification, we can show formally that  $\mathbb{M}, q_{\text{init}}, \bar{0} \models \langle\langle 1 \rangle\rangle^{\bar{0}}(-\text{error})\mathbf{U} \text{ halt}$  iff  
 21 the automaton  $\mathcal{A}$  halts on the empty input, where  $\mathbb{M}$  is a modified version of  $\mathbb{M}_2^{\mathcal{A}}$  essentially along the lines sketched  
 22 above (in particular, all idle actions are removed).

23 It is important to note that in the presence of idle actions, this reduction no longer works, as the proponent always  
 24 has a choice. Even in the case where the proponent has no resources left, the computation of the system can always be  
 25 extended to be infinite, either by visiting the fail state, or by looping in some state. As a consequence, a halting state  
 26 may never be reached. This implies that the opponent has too much power, and can always spoil the simulation by  
 27 performing a test action in cases where the simulation is sound and no resources are available. That there is no way  
 28 to save the reduction is shown by the decidability result we present in the next section.

## 29 5. Model-Checking pprRAL over iRBMs

30 In this section we prove that the model checking problem for the fragment pprRAL over iRBMs is decidable. We  
 31 first present the model checking algorithm for pprRAL over iRBMs, and then prove termination and correctness of  
 32 the algorithm in Lemmas 7 and 8, respectively.

### 5.1. Model Checking Algorithm for pprRAL

The model checking algorithm for pprRAL over iRBMs takes as input a model  $\mathfrak{M}$ , formula  $\phi$ , and initial endowment  $\eta$ , and labels the set of states  $[\phi]_{\mathfrak{M}}^{\eta}$ , where  $[\phi]_{\mathfrak{M}}^{\eta} = \{q \mid \mathfrak{M}, q, \eta \models \phi\}$  is the set of states satisfying  $\phi$  (see Algorithm 1).<sup>9</sup>

---

#### Algorithm 1 Labelling $\phi$

---

```

1: procedure LABEL( $\mathfrak{M}, \phi, \eta$ )
2:   for  $\phi' \in \text{Sub}(\phi)$  do
3:      $[\phi']_{\mathfrak{M}}^{\eta} \leftarrow \text{ATL-LABEL}(\mathfrak{M}, \phi')$ 
4:    $[\phi]_{\mathfrak{M}}^{\eta} \leftarrow \{q \mid q \in Q \wedge \text{STRATEGY}(\text{node}_0(q, \eta, \text{prop}(\phi)), \phi)\}$ 

```

---

Given  $\phi$ , we produce the set of subformulae of  $\phi$ ,  $\text{Sub}(\phi)$ , in the usual way, except that  $\langle\langle A \rangle\rangle^{\downarrow}$  and  $\langle\langle A \rangle\rangle^{\zeta}$  modalities are replaced by standard ATL modalities  $\langle\langle A \rangle\rangle$ .  $\text{Sub}(\phi)$  is ordered in increasing order of complexity. For a formula  $\phi' \in \text{Sub}(\phi)$ , we will write  $s \models \phi'$  to indicate that state  $s$  has been labelled by  $\phi'$ . Note that if a state  $s$  is not annotated with the standard ATL modality  $\langle\langle A \rangle\rangle$ , then it cannot satisfy  $\langle\langle A \rangle\rangle^{\downarrow}$  or  $\langle\langle A \rangle\rangle^{\zeta}$ . Algorithm 1 simply labels states with the subformulae of  $\phi$  using the standard ATL labelling algorithm [32] (lines 2–3). It then calls the function STRATEGY to label states with  $\phi$  (line 4). *prop* is a function that returns either the proponents  $A \subseteq \mathbb{A}_{\text{gt}}$  if  $\phi$  is of the form  $\langle\langle A \rangle\rangle^* \mathbf{X}\psi$ ,  $\langle\langle A \rangle\rangle^* \psi_1 \mathbf{U}\psi_2$ ,  $\langle\langle A \rangle\rangle^* \mathbf{G}\psi$  where  $*$  is either  $\downarrow$  or an endowment, or  $\mathbb{A}_{\text{gt}}$  otherwise. The function  $\text{node}_0$  initialises the root node for the function STRATEGY and is explained below.<sup>10</sup>

---

#### Algorithm 2 Strategy

---

```

1: function STRATEGY( $n, \phi$ )
2:   case  $\phi = p \in \Pi$ 
3:     return  $s(n) \models \phi$ 
4:   case  $\phi = \neg p$  where  $p \in \Pi$ 
5:     return  $s(n) \not\models \phi$ 
6:   case  $\phi = \psi_1 \wedge \psi_2$ 
7:     return  $\text{STRATEGY}(\text{node}_0(s(n), e(n), v(n), c(n)), \psi_1) \wedge \text{STRATEGY}(\text{node}_0(s(n), e(n), v(n), c(n)), \psi_2)$ 
8:   case  $\phi = \psi_1 \vee \psi_2$ 
9:     return  $\text{STRATEGY}(\text{node}_0(s(n), e(n), v(n), c(n)), \psi_1) \vee \text{STRATEGY}(\text{node}_0(s(n), e(n), v(n), c(n)), \psi_2)$ 
10:  case  $\phi = \langle\langle A \rangle\rangle^{\downarrow} \mathbf{X}\psi$ 
11:    return  $\mathbf{X}\text{-STRATEGY}(\text{node}_0(s(n), e(n), v(n), A), \phi)$ 
12:  case  $\phi = \langle\langle A \rangle\rangle^{\zeta} \mathbf{X}\psi$ 
13:    return  $\mathbf{X}\text{-STRATEGY}(\text{node}_0(s(n), \zeta, \zeta, A), \phi)$ 
14:  case  $\phi = \langle\langle A \rangle\rangle^{\downarrow} \psi_1 \mathbf{U}\psi_2$ 
15:    return  $\mathbf{U}\text{-STRATEGY}(\text{node}_0(s(n), e(n), v(n), A), \phi)$ 
16:  case  $\phi = \langle\langle A \rangle\rangle^{\zeta} \psi_1 \mathbf{U}\psi_2$ 
17:    return  $\mathbf{U}\text{-STRATEGY}(\text{node}_0(s(n), \zeta, \zeta, A), \phi)$ 
18:  case  $\phi = \langle\langle A \rangle\rangle^{\downarrow} \mathbf{G}\psi$ 
19:    return  $\mathbf{G}\text{-STRATEGY}(\text{node}_0(s(n), e(n), v(n), A), \phi)$ 
20:  case  $\phi = \langle\langle A \rangle\rangle^{\zeta} \mathbf{G}\psi$ 
21:    return  $\mathbf{G}\text{-STRATEGY}(\text{node}_0(s(n), \zeta, \zeta, A), \phi)$ 

```

---

The function STRATEGY is shown in Algorithm 2 and proceeds by depth-first and-or search. That is, we examine each path in the search space in turn, as in standard depth-first search, but treat nodes corresponding to a particular

<sup>9</sup>The model checking algorithm for pprRAL is a slightly modified version of the algorithm given in [7]. In particular, Algorithm 4 incorporates an extra call to STRATEGY to allow arbitrary positive formulae on the left of  $\mathbf{U}$ . Other changes simply clarify the presentation and/or correct minor bugs in the algorithm given in [7].

<sup>10</sup>Note that we do not label states with subformulae of  $\phi$  involving  $\langle\langle A \rangle\rangle^{\downarrow}$  or  $\langle\langle A \rangle\rangle^{\zeta}$  modalities as in [6].

14 choice of action by  $A$  as and-nodes, i.e., all branches corresponding to this choice must return true for the choice  
 1 to be part of a successful strategy. The function STRATEGY processes each coalition modality in turn, starting from  
 2 the outermost modality. The logical connectives are standard, and simply call STRATEGY on the subformulae. Each  
 3 temporal operator is handled by a separate function: X-STRATEGY for  $\mathbf{X}\psi$ , U-STRATEGY for  $\psi_1\mathbf{U}\psi_2$ , and G-STRATEGY for  
 4  $\mathbf{G}\psi$ , and are explained below. We record information about the state of the search in a search tree of nodes. A *node*  
 5 is a structure which consists of a state of  $\mathfrak{M}$ , the resources available to all the agents in that state, and a finite path  
 6 of nodes leading to this node from the root node. Edges in the search tree correspond to joint actions by all agents.  
 7 Note that the resources available to the agents in a state on a path constrain the edges from the corresponding node  
 8 to be those action profiles  $\alpha_A$  where for all proponent agents  $a$ ,  $(\text{cons}(\alpha_a, r))_{r \in \text{Res}}$  is less than or equal to the available  
 9 resources of agent  $a$ . We compare vectors of resources in the usual way; for example,  $\zeta_a \geq (\text{cons}(\alpha_a, r))_{r \in \text{Res}}$  stands  
 10 for  $\zeta_a(r) \geq \text{cons}(\alpha_a, r)$  for all resources  $r$ . For an action profile  $\alpha_A$  of  $A \subseteq \text{Agt}$ , we write  $\text{cons}(\alpha)$  to refer to the tuple  
 11  $((\text{cons}(\alpha_a))_{r \in \text{Res}})_{a \in A}$ . For each node  $n$  in the tree, we have a function  $s(n)$  which returns its state,  $p(n)$  which returns  
 12 the sequence of nodes on the path to  $n$ ,  $e(n)$  which returns an endowment specifying the resource availability for all  
 13 agents as a result of following  $p(n)$ ,  $v(n)$  which returns the resources potentially available to the agents as a result  
 14 of traversing cycles on  $p(n)$  additional times,<sup>11</sup> and  $c(n)$  which returns the current set of proponents. The function  
 15  $\text{node}_0(s, \eta, \eta', A)$  returns the root node, i.e., a node  $n_0$  such that  $s(n_0) = s$ ,  $p(n_0) = []$  (empty list),  $e(n_0) = \eta$ ,  $v(n_0) = \eta'$ ,  
 16 and  $c(n_0) = A \subseteq \text{Agt}$  is the current set of proponents. The function  $\text{node}(n, s', \alpha, A)$  returns a node  $n'$  where  $s(n') = s'$ ,  
 17  $p(n') = [p(n) \cdot n]$ ,  $c(n') = A = c(n)$ ,

$$e_a(n') = \begin{cases} e_a(n) & \text{if } a \notin A \\ e_a(n) + \text{prod}(\alpha) - \text{cons}(\alpha) & \text{if } a \in A \end{cases}$$

18 and

$$v_a(n') = \begin{cases} v_a(n) & \text{if } a \notin A \\ v_a(n) + \text{prod}(\alpha) - \text{cons}(\alpha) & \text{if } a \in A \end{cases}$$

19 where vectors are added and subtracted as usual unless their components are not integers. For technical reasons, we  
 20 introduce an extra value for an agent's resource endowment,  $arb$ , which denotes an arbitrary finite value; for any  
 21  $m \in \mathbb{Z}$ ,  $m < arb$ .  $arb$  cannot be incremented or decremented:  $arb + m = arb$  and  $arb - m = arb$ . Above,  $e(n)(a, r)$  is  
 22 used to keep track of the 'real' cost of the loops and does not contain  $arb$  values, while  $v(n)(a, r) = arb$  indicates that  
 23 the path  $p(n)$  contains a *productive loop*, which can be traversed multiple times to generate an arbitrary finite amount  
 24 of resource  $r$  for agent  $a$ . Intuitively,  $arb$  represents an arbitrary finite number; hence, having  $arb$  resources allows the  
 25 agent to execute any action as well as any finite number of loop traversals, but does not allow the agent to traverse a  
 26 loop infinitely many times.

---

### Algorithm 3 X-strategy (both types of modalities)

---

```

1: function X-STRATEGY( $n, \langle\langle A \rangle\rangle^* \mathbf{X}\psi$ )
2:   if  $s(n) \not\models \text{atl}(\langle\langle A \rangle\rangle^* \mathbf{X}\psi)$  then
3:     return false
4:    $ActA \leftarrow \{\alpha' \in d_A(s(n)) \mid \text{cons}(\alpha') \leq v_A(n)\}$ 
5:   for  $\alpha' \in ActA$  do
6:      $ActAgt \leftarrow \{\alpha \in d(s(n)) \mid \alpha_A = \alpha'\}$ 
7:      $strat \leftarrow true$ 
8:     for  $\alpha \in ActAgt$  do
9:        $s' \leftarrow o(s(n), \alpha)$ 
10:       $strat \leftarrow strat \wedge \text{STRATEGY}(\text{node}(n, s', \alpha, A), \psi)$ 
11:    if  $strat$  then
12:      return true
13:  return false

```

---

<sup>11</sup>A cycle on  $p(n)$  is a subsequence of  $p(n)$  with start and end nodes sharing the same state.

27 The function  $x$ -STRATEGY for formulae of types  $\langle\langle A \rangle\rangle^{\downarrow} \mathbf{X}\psi$  and  $\langle\langle A \rangle\rangle^{\zeta} \mathbf{X}\psi$  is shown in Algorithm 3 and is straightfor-  
 1 ward. After checking if the search should be terminated with false because the ATL version of the formula is false  
 2 (lines 2–3),<sup>12</sup> we simply check if there is an action of  $A$  that is possible given the current endowment (line 4), and  
 3 where in all outcome states  $A$  has a strategy to enforce  $\psi$  (lines 5–12).  $atl(\phi)$  is a function that returns the formula  
 4 where each  $\langle\langle A \rangle\rangle^{\downarrow}$  and  $\langle\langle A \rangle\rangle^{\zeta}$  in  $\phi$  is replaced by  $\langle\langle A \rangle\rangle$ .

5 The function  $u$ -STRATEGY for formulae of types  $\langle\langle A \rangle\rangle^{\downarrow} \psi_1 \mathbf{U} \psi_2$  and  $\langle\langle A \rangle\rangle^{\zeta} \psi_1 \mathbf{U} \psi_2$  is shown in Algorithm 4. First  $u$ -  
 6 STRATEGY checks whether the search should be terminated with false because either the ATL version of the formula is  
 7 false (lines 2–3), or the current path ends in an unproductive loop (lines 4–5). We then check the path for a productive  
 8 loop, and update  $v(n)$  if we find one (lines 6–7). If the ATL version of  $\psi_2$  is true, we try to find a strategy to enforce  
 9  $\psi_2$  from  $s(n)$ , and, if we are successful,  $u$ -STRATEGY returns true (lines 8–9). We then check if the endowment in  $n$   
 10 is insufficient to enforce  $\psi_1$ , and terminate the search with false if it is not (lines 10–11). (This check is required as  
 11 only the ATL version of the formula is checked at lines 2–3.) Otherwise the search continues, as the node where  
 12 STRATEGY( $n, \psi_2$ ) returns true may be found later on the path. Each action available at  $s(n)$  is considered in turn (lines  
 13 12–20). For each action  $\alpha' \in ActA$ , we check whether a recursive call of the algorithm returns true in all outcome  
 14 states  $s'$  of  $\alpha'$  (i.e.,  $\alpha'$  is part of a successful strategy). If such an  $\alpha'$  is found, the algorithm returns true. Otherwise  
 15 the algorithm returns false. Note that we never traverse a productive loop more than twice: if an arbitrary amount of  
 16 the resource(s) produced by the loop is insufficient to enforce  $\psi_2$  (and hence return true), at the beginning of the third  
 17 traversal the search will be terminated with false at the test for an unproductive loop (since the second traversal of the  
 18 loop did not result in a change in the endowment).

19 The function  $g$ -STRATEGY for formulae of types  $\langle\langle A \rangle\rangle^{\downarrow} \mathbf{G}\phi$  and  $\langle\langle A \rangle\rangle^{\zeta} \mathbf{G}\phi$  is shown in Algorithm 5. Again we check  
 20 if the search should be terminated with false, either because the standard ATL modality does not hold (lines 2–3), or  
 21 because the current path terminates in a resource consuming cycle (lines 4–7). The first check is for cycles where  
 22 at least one resource is consumed and no resources are produced (lines 4–5). The second check is for cycles which  
 23 both produce and consume resources (so the previous test does not apply), and where we have already shown we  
 24 can produce an arbitrary amount of the resource being consumed (lines 6–7). As any arbitrary amount of resource  
 25 is insufficient to maintain such a loop indefinitely, we terminate the search with false. We then check the path for a  
 26 productive loop, and update  $v(n)$  if we find one (lines 8–9). Note that, to enforce an invariant, only a path ending in a  
 27 nondecreasing loop (as opposed to a productive loop) is required. However we must correctly update the endowment  
 28 available in  $n$  in order to evaluate  $\psi$  in  $\langle\langle A \rangle\rangle^{\downarrow} \mathbf{G}\psi$ . We then check if the endowment in  $n$  is insufficient to enforce  $\psi$  from  
 29  $s(n)$ , and terminate the search with false if it is not (lines 10–11). If the current path terminates in a nondecreasing  
 30 loop, we return true (lines 12–13):  $\psi$  is enforceable from each of the states on the path, and the loop can be traversed  
 31 indefinitely. Otherwise we continue the search for a nondecreasing loop (lines 14–22).

To illustrate the execution of the algorithm, we revisit the running example from Section 3.4 and consider the property

$$\langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G}p$$

We skip the ATL labelling step and consider the initial call to

$$u\text{-STRATEGY}(node_0(q_0, (1:5, 2:0), (1:5, 2:0), \{1\}), \langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G}p)$$

32 where  $n_0 = node_0(q_0, (1:5, 2:0), (1:5, 2:0), \{1\})$ . For  $n_0$ , no cases of Algorithm 4 are applicable until line 12.  $ActA$   
 33 (actions of agent 1 for which the resource consumption is less than  $v_1(n_0)$ , i.e., less than 5) consists of *idle* and *move* ac-  
 34 tions. Let us trace the algorithm calls for *idle* first. There are two joint actions we need to consider (line 14): (*idle, idle*)  
 35 and (*idle, obstruct*). In both cases the result will be the same, the next call to  $u$ -STRATEGY( $n, \langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G}p$ ),  
 36 where  $n$  is the successor node by the joint action, will return false on line 4. This is because  $n$  will have the same state  
 37 ( $q_0$ , since  $o(q_0, (idle, idle)) = o(q_0, (idle, obstruct)) = q_0$ ) and  $v_1(n) = v_1(n_0)$  (resources available to agent 1 have not  
 38 changed since *idle* costs nothing). Let us consider the choice of  $\sigma = move$  on line 13. There are two joint actions on  
 39 line 14,  $\{(move, idle), (move, obstruct)\}$ .

40 First let us consider (*move, idle*). On line 17,  $s'$  is  $q_1$  and  $node(n_0, q_1, (move, idle), \{1\})$  is  $n_1$  where  $s(n_1) = q_1$ ,  
 41  $p(n_1) = [n_0]$ ,  $e_1(n_1) = v_1(n_1) = 3$  (since move actions cost 2 units of energy, agent 1's resources are decremented)

<sup>12</sup>Note that the checks for the ATL versions of the formula in  $x$ -STRATEGY,  $u$ -STRATEGY and  $g$ -STRATEGY are only for efficiency, and are not required for the correctness of the algorithms.

**Algorithm 4** U-strategy (both types of modalities)

---

```

1: function U-STRATEGY( $n, \langle\langle A \rangle\rangle^* \psi_1 \mathbf{U} \psi_2$ )
2:   if  $s(n) \not\models \text{atl}(\langle\langle A \rangle\rangle^* \psi_1 \mathbf{U} \psi_2)$  then
3:     return false
4:   if  $\exists n' \in p(n) : s(n') = s(n) \wedge v_A(n') \geq v_A(n)$  then
5:     return false
6:   for  $(a, r) \in \{(a, r) \in A \times \text{Res} \mid \exists n' \in p(n) : s(n') = s(n) \wedge v_A(n') \leq v_A(n) \wedge v(n')(a, r) < v(n)(a, r)\}$  do
7:      $v(n)(a, r) \leftarrow \text{arb}$ 
8:   if  $s(n) \models \text{atl}(\psi_2) \wedge \text{STRATEGY}(n, \psi_2)$  then
9:     return true
10:  if  $\neg \text{STRATEGY}(n, \psi_1)$  then
11:    return false
12:   $\text{ActA} \leftarrow \{\alpha' \in d_A(s(n)) \mid \text{cons}(\alpha') \leq v_A(n)\}$ 
13:  for  $\alpha' \in \text{ActA}$  do
14:     $\text{ActAgt} \leftarrow \{\alpha \in d(s(n)) \mid \alpha_A = \alpha'\}$ 
15:     $\text{strat} \leftarrow \text{true}$ 
16:    for  $\alpha \in \text{ActAgt}$  do
17:       $s' \leftarrow o(s(n), \alpha)$ 
18:       $\text{strat} \leftarrow \text{strat} \wedge \text{U-STRATEGY}(\text{node}(n, s', \alpha, A), \langle\langle A \rangle\rangle^* \psi_1 \mathbf{U} \psi_2)$ 
19:    if  $\text{strat}$  then
20:      return true
21:  return false

```

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42 and  $e_2(n_1) = v_2(n_1) = 0$  (agent 2's resources do not change since it is not in the proponent coalition). When we call  
1 U-STRATEGY( $n_1, \langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ), the first applicable case is on line 8. The ATL version of  $\langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$  is true, and  
2 in fact G-STRATEGY( $n_1, \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ) returns true. (We will show this after we finish tracing the calls to U-STRATEGY.) So on  
3 line 8 the call to U-STRATEGY( $n_1, \langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ) returns true. Let us consider (*move, obstruct*). On line 17,  $s'$  is  
4  $q_2$  and  $\text{node}(n_0, q_2, (\text{move}, \text{obstruct}), \{1\})$  is  $n_2$  where  $s(n_2) = q_2$ ,  $p(n_2) = [n_0]$ ,  $e_1(n_2) = v_1(n_2) = 3$  (since move actions  
5 cost 2 units of energy, agent 1's resources are decremented) and  $e_2(n_2) = v_2(n_2) = 0$  (agent 2's resources do not change  
6 since it is not in the proponent coalition). When we call U-STRATEGY( $n_2, \langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ), the **if** statement on  
7 line 8 is not applicable since the invariant formula is not true in  $q_2$ . We continue to line 12 and collect all actions by  
8 agent 1 with resource consumptions of at most  $v_1(n_2)$ . Such actions are *idle* and *move*. We skip the case of *idle*, as  
9 it is identical to choosing *idle* in  $n_0$ . Let us consider *move*. The only joint action possible if agent 1 choses *move* is  
10 (*move, idle*), since in  $q_2$ , agent 2 has only the *idle* action. On line 17,  $s'$  is  $q_1$  and  $\text{node}(n_2, q_1, (\text{move}, \text{idle}), \{1\})$  is  $n_3$   
11 where  $s(n_3) = q_1$ ,  $p(n_3) = [n_0, n_2]$ ,  $e_1(n_3) = v_1(n_3) = 1$  (since move actions costs 2 units of energy, agent 1's resources  
12 are decremented) and  $e_2(n_3) = v_2(n_3) = 0$  (agent 2's resources do not change since it is not in the proponent coalition).  
13 When we call U-STRATEGY( $n_3, \langle\langle 1 \rangle\rangle^{1:5,2:0} \top \mathbf{U} \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ), the call to G-STRATEGY( $n_3, \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ) returns true (which will be  
14 shown next), and hence all calls to U-STRATEGY from  $n_0$  for the joint actions extending *move* return true.

15 Now we show that G-STRATEGY( $n_3, \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ) returns true (the case of G-STRATEGY( $n_1, \langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ ) is similar but easier,  
16 since agent 1 in  $n_1$  has a greater resource availability). None of the cases in Algorithm 5 before line 14 are applicable.  
17 Here, the available actions are *idle* and *send* (the agent still has one unit of energy left). If we try *idle*, then the next  
18 call to the algorithm will return false on line 2 since *idle* will bring us back to  $q_0$  which does not satisfy the ATL  
19 version of the formula  $\langle\langle 1 \rangle\rangle^{\downarrow} \mathbf{G} p$ . If we select *send*, then in the resulting node  $n_4$  the applicable actions are *idle* and  
20 *charge*; the choice of *idle* will again lead to failure, but by selecting *charge* we reach  $n_5$  where the state is  $q_1$  again  
21 (so  $s(n_5) = s(n_3)$ ), and  $e_1(n_5) = e_1(n_3)$ , so the algorithm returns true on line 13.

22 **5.2. Correctness of the Model Checking Algorithm**

23 In this section we show that the algorithm always terminates (Lemma 7) and that it gives the correct answer  
24 (Lemma 8). Together, the two lemmas give the proof of the main result:

**Algorithm 5** G-strategy (both types of modalities)

---

```

1: function G-STRATEGY( $n, \langle\langle A \rangle\rangle^* \mathbf{G}\psi$ )
2:   if  $s(n) \not\models \text{atl}(\langle\langle A \rangle\rangle^* \mathbf{G}\psi)$  then
3:     return false
4:   if  $\exists n' \in p(n) : s(n') = s(n) \wedge e_A(n') \geq e_A(n) \wedge$ 
       $(\exists a \in A, r \in \text{Res} : e(n')(a, r) > e(n)(a, r))$  then
5:     return false
6:   if  $\exists n' \in p(n) : s(n') = s(n) \wedge$ 
       $\forall a \in A \forall r \in \text{Res} : (v(n')(a, r) = v(n)(a, r) = \text{arb} \vee e(n')(a, r) = e(n)(a, r)) \wedge$ 
       $\exists a \in A \exists r \in \text{Res} : e(n')(a, r) > e(n)(a, r)$  then
7:     return false
8:   for  $(a, r) \in \{(a, r) \in A \times \text{Res} \mid \exists n' \in p(n) : s(n') = s(n) \wedge v_A(n') \leq v_A(n) \wedge v(n')(a, r) < v(n)(a, r)\}$  do
9:      $v(n)(a, r) \leftarrow \text{arb}$ 
10:  if  $\neg \text{STRATEGY}(n, \psi)$  then
11:    return false
12:  if  $\exists n' \in p(n) : s(n') = s(n) \wedge e_A(n') \leq e_A(n)$  then
13:    return true
14:   $\text{Act}A \leftarrow \{\alpha' \in d_A(s(n)) \mid \text{cons}(\alpha') \leq v_A(n)\}$ 
15:  for  $\alpha' \in \text{Act}A$  do
16:     $\text{Act}Agt \leftarrow \{\alpha \in d(s(n)) \mid \alpha_A = \alpha'\}$ 
17:     $\text{strat} \leftarrow \text{true}$ 
18:    for  $\alpha \in \text{Act}Agt$  do
19:       $s' \leftarrow o(s(n), \alpha)$ 
20:       $\text{strat} \leftarrow \text{strat} \wedge \text{G-STRATEGY}(\text{node}(n, s', \alpha, A), \langle\langle A \rangle\rangle^* \mathbf{G}\psi)$ 
21:    if  $\text{strat}$  then
22:      return true
23:  return false

```

---



25 **Theorem 5.** *The model checking problem for pprRAL over iRBMs is decidable.*

1 *Proof.* Follows from Lemma 7 and Lemma 8 below. □

2 **Lemma 7.** *Algorithm 2 terminates.*

3 *Proof.* The proof is by induction on the length of the formula. Calls for propositional formulae clearly terminate.  
4 For the inductive step, we need to show that a call for any connective terminates provided calls for lower complexity  
5 formulae terminate. Conjunction and disjunction are obvious. X-STRATEGY makes a recursive call to determine if there  
6 is a strategy for a smaller complexity formula after one step. The only non-trivial cases are U-STRATEGY and G-STRATEGY.

7 Let us consider termination of U-STRATEGY first. We need to show that there cannot be an infinite sequence of  
8 recursive calls to U-STRATEGY( $node(n, s', \alpha, A), \langle\langle A \rangle\rangle^* \psi_1 \cup \psi_2$ ) (see line 18 of Algorithm 4). Such an infinite sequence  
9 would imply that the search is stuck in an infinite loop and hence encounters the same state infinitely often. There are  
10 three types of loops to consider: (1) a consuming or neutral loop (where for all proponent agents and all resources,  
11 the amount of each resource stays the same or decreases); (2) a productive loop (where for all proponent agents and  
12 all resources, the amount of each resource stays at least the same and increases for at least one agent and resource  
13 type) and (3) mixed (for some agents and resource types, resource availability increases and for some it goes down).  
14 Clearly the search will terminate in case (1) because of the loop check on line 4. Note that we compare  $v$  rather than  $e$   
15 endowments because we do not want to keep looping after discovering a way to earn  $arb$  resources. If the agents are  
16 in a productive loop (case (2)), eventually all increasing resources are set to  $arb$  in line 7, and  $v$  stops changing, hence  
17 we fall back to case (1) and terminate due to the check on line 4. Finally, consider a case of a mixed loop (case (3)).  
18 Here we have two sub-cases: (3a) when for at least one agent and resource pair,  $(a, r)$ , the loop decreases the value of  
19  $e(n)(a, r)$ ,  $v(n)(a, r) \neq arb$ ; and (3b) when for all such pairs where resources are consumed,  $v(n)(a, r) = arb$ . In case  
20 (3a) termination is trivial since the actions which constitute the loop will not be possible after the required resources  
21 are consumed. In case (3b) the ‘decreasing’ resources are all  $arb$ , so the same actions are still available. However, we  
22 assumed that in (3b) for all other pairs  $(a, r)$  resources grow (so  $v(n)(a, r)$  is eventually set to  $arb$ ) or stay the same.  
23 After all growing resources are set to  $arb$ , there is no further change and the search will terminate in the loop check  
24 on line 4. This concludes the proof that U-STRATEGY terminates.

25 Let us consider G-STRATEGY. Again we need to show that there cannot be an infinite sequence of recursive calls to  
26 G-STRATEGY( $node(n, s', \alpha, A), \langle\langle A \rangle\rangle^* G\psi$ ) (line 20 of Algorithm 5). Again such a sequence would need to involve a loop  
27 and there are three cases to consider: (1) an increasing or neutral loop (for all agents and resource types, endowments  
28  $e$  increase or remain the same); (2) a decreasing loop (also for endowments  $e$ ); and (3) a mixed loop. In case (1) we  
29 terminate on lines 12-13; in case (2) we terminate on lines 4-5. Case (3) again has two subcases: (3a) and (3b). The  
30 reasoning is the same as in the case for U-STRATEGY; case (3a) is straightforward, case (3b) is covered by the check on  
31 lines 6-7. □

32 Given  $v : \mathbb{A}gt \times Res \rightarrow \mathbb{N}_0^\infty \cup \{arb\}$ , the set of endowments compatible with  $v$  is defined as  $compatible(v) = \{e :$   
33  $\mathbb{A}gt \times Res \rightarrow \mathbb{N}_0^\infty \mid \forall a \in \mathbb{A}gt, r \in Res : v(a, r) = arb \vee e(a, r) = v(a, r)\}$ , i.e., all individual endowments of  $e$  for each  
34 agent and resource agree with  $v$  whenever  $v(a, r) \neq arb$ .

35 **Lemma 8.** *Algorithm 2 is correct, that is, STRATEGY( $n, \phi$ ) returns true iff  $\exists e' \in compatible(v(n)) : s(n), e' \models \phi$ .*

36 The proof is given in Appendix B.

## 37 6. Discussion

38 Over the last few years, logics for reasoning about strategic, resource-bounded agents and models have become a  
39 popular research topic, e.g. [1, 3, 6, 2, 14, 15, 4, 5], and, given current trends in the development of intelligent systems  
40 (e.g., driverless cars, unmanned vehicles, autonomous robots), the formal verification of resource-bounded systems  
41 will become even more important in the near future. Unfortunately, formal, logic-based techniques for the verification  
42 of resource-bounded systems are often intractable or even undecidable.

43 In this paper we investigated the boundary of (un)decidable logics for verifying resource-bounded systems. We  
44 identified a significant fragment of Resource Agent Logic (RAL) with a decidable model checking property, and  
45 proved two new undecidability results. We have shown that a rather natural property of models — that agents can

46 always decide to do nothing — can make model checking decidable. In particular, the positive proponent-restricted  
 1 fragment of RAL that we present, pprRAL, is decidable in the presence of idle actions and undecidable without them.  
 2 However, the availability of idle actions is not sufficient on its own to make the model checking of RAL, or even of the  
 3 proponent-restricted fragment prRAL, decidable. We show that considering opponents acting under resource bounds  
 4 makes model checking undecidable, as does allowing coalition modalities in the scope of negations. The summary of  
 5 known decidability and undecidability results is presented in Table 1.

Logic	Models	
	<b>RBM</b>	<b>iRBM</b>
RAL	Undecidable [2]	Undecidable (Corollary of [2])
rfRAL	Undecidable [2]	Undecidable for two agents and two resource types (Theorem 1) Undecidable for four agents and one resource type (Theorem 3)
prRAL	Undecidable [2]	Undecidable (Corollary of [2]) Undecidable for one agent and two resource types (Theorem 2) Undecidable for two agents and one resource type (Theorem 4)
rfprRAL	Undecidable [2]	Decidable (Corollary of [6])
pprRAL	Undecidable (Corollary of [2])	Decidable (Theorem 5)

Table 1: Summary of known decidability and undecidability results

6 Note that **iRBMs** are very similar to **RBMs** with finite semantics of [2] (see [8] and [9] for a formal statement  
 7 of correspondence). The result presented here, together with those of [8, 9] implies that pprRAL is decidable over  
 8 **RBMs** under finite semantics.

9 Finally, we did not discuss the complexity of the model checking problem for pprRAL over **iRBMs** in this paper.  
 10 We conjecture that it is the same as the model checking problem for  $\text{RB}\pm\text{ATL}$ , which was recently shown in [35] to  
 11 be  $2\text{EXPTIME}$ -complete.

## 12 Acknowledgements

13 We thank the reviewers for their detailed and helpful reviews. This work was supported by the Engineering and  
 14 Physical Sciences Research Council [grant EP/K033905/1].

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52 **A. Encodings and Proofs from Section 4**1 *A.1. Encodings and Proofs from Section 4.1*2 **Proposition 1.** *Let  $\mathcal{A}$  be an empty-band TCM and  $\lambda = (q^i, \eta^i)_{i=1, \dots, 3(k-1)+1}$  be an  $\mathcal{A}$ -computation pre-encoding.*3 (a) *We have that  $\eta^{3(k-1)+1}(1, R_r) = \eta^{3(k-1)+1}(2, R_r)$  for  $r = 1, 2$ .*4 (b) *If  $\lambda \circ (sE_1E_2, \eta)(s'C_1C_2, \eta')$  is a finite resource-extended path, then so is  $\lambda \circ (sE_1E_2, \eta)(s'C_1C_2, \eta')(s', \eta')$  for a*  
5 *uniquely defined endowment  $\eta'$ .*6 (c) *If  $\lambda[i] = (q^i, \eta^i) = (sE_1E_2, \eta^i)$  with  $E_r = 0$ , then  $\eta^i(1, R_r) = 0$  if, and only if,  $\eta^i(2, R_r) = 0$ , for  $r \in \{1, 2\}$ .*7 *Proof.* Let  $\lambda = (q^i, \eta^i)_{i=1, \dots, 3(k-1)+1}$ . (a) We show this by induction on  $k$ . For  $k = 1$  the claim follows by definition.  
8 Now, suppose the claim is true for all computation pre-encodings up to (and excluding)  $k \geq 2$ . That is, it is true for  
9  $\lambda = (q^i, \eta^i)_{i=1, \dots, 3(k-1)+1}$ , which is also an  $\mathcal{A}$ -computation pre-encoding. That is,  $\eta^{3(k-1)+1}(1, R_r) = \eta^{3(k-1)+1}(2, R_r)$  for  
10  $r = 1, 2$ . Consider the  $\mathcal{A}$ -computation pre-encoding (where we simply consider how  $\lambda$  has to be extended given the  
11 construction of the model)

$$\begin{aligned} \lambda' &= (q^i, \eta^i)_{i=1, \dots, 3k+1} \\ &= \lambda \circ (q^{3(k-1)+2}, \eta^{3(k-1)+2})(q^{3(k-1)+3}, \eta^{3(k-1)+3})(q^{3k+1}, \eta^{3k+1}) \\ &= \lambda \circ (s^k E_1^k E_2^k, \eta^{3(k-1)+2})(s^{k+1} C_1^k C_2^k, \eta^{3(k-1)+3})(s^{k+1}, \eta^{3k+1}) \end{aligned}$$

12 Now, a simple computation, taking into consideration the structure of the model, gives:  $\eta^{3k+1}(1, R_r) = \eta^{3(k-1)+3}(1, R_r) =$   
13  $\eta^{3(k-1)+2}(1, R_r) + C_r^k + E_r^k = \eta^{3(k-1)+1}(1, R_r) + C_r^k \stackrel{IH}{=} \eta^{3(k-1)+1}(2, R_r) + C_r^k = \eta^{3(k-1)+2}(2, R_r) + C_r^k = \eta^{3(k-1)+3}(2, R_r) + C_r^k =$   
14  $\eta^{3k+1}(2, R_r)$ .15 (b) We have to show that  $\eta''(1, R_r) \geq 0$  and  $\eta''(2, R_r) \geq 0$  for  $r \in \{1, 2\}$ . The former is clear because  $\eta'(1, R_r) \geq 0$   
16 and the endowment for player 1 does not change in the transition from  $s'C_1C_2$  to  $s'$ . For the latter, we make the  
17 following observation:  $\eta''(2, R_r) = \eta'(2, R_r) + C_r = \eta^{3(k-1)+1}(2, R_r) + C_r$  where the latter equality holds as the resources  
18 of agent 2 do not change between the state  $q^{3(k-1)+1}$  and  $s'C_1C_2$ . Now, by (a), we have that  $\eta^{3(k-1)+1}(2, R_r) + C_r =$   
19  $\eta^{3(k-1)+1}(1, R_r) + C_r$ . Finally, we can compute that  $\eta^{3(k-1)+1}(1, R_r) + C_r = \eta(1, R_r) + E_r + C_r = \eta'(1, R_r) \geq 0$  as  
20  $\lambda \circ (sE_1E_2, \eta)(s'C_1C_2, \eta')$  is a finite resource-extended path by assumption. Finally,  $\eta''$  is uniquely defined as there is  
21 only a unique action from  $s'C_1C_2$  to  $s'$ .22 (c) Suppose a configuration  $(sE_1E_2, \eta^i)$  with  $E_r = 0$  is reached. Then, action  $E_1E_2$  (resp. idle) was performed in  
23  $\lambda[i-1] = (s, \eta^{i-1})$  by agent 1 (resp. 2). Note that none of the actions changes the resource balance of  $R_r$ . Suppose now  
24 that  $\eta^i(x, R_r) = 0$  for  $x \in \{1, 2\}$ . Then, also  $\eta^{i-1}(x, R_r) = 0$  and by (a)  $\eta^{i-1}(3-x, R_r) = 0$ . With the above observation  
25 we can conclude that also  $\eta^i(3-x, R_r) = 0$ .  $\square$ 26 **Lemma 1** (Simulation Lemma for  $\mathfrak{M}_1^{\mathcal{A}}$ ). *There is a bijection  $f^{\mathcal{A}}$  between computations of  $\mathcal{A}$  and  $\mathcal{A}$ -computation*  
27 *encodings of  $\mathfrak{M}_1^{\mathcal{A}}$  in such a way that  $f^{\mathcal{A}}(\rho)$  simulates the computation  $\rho$ . In particular, if  $\rho$  is an accepting computation*  
28 *then  $f^{\mathcal{A}}(\rho)$  is also accepting.*29 *Proof.* Let  $\rho = (s^i, v_1^i, v_2^i)_{i=1, \dots, k}$  be a finite  $\mathcal{A}$ -computation. From  $\rho$  we inductively construct the following finite  
30 resource-extended path  $\lambda = \lambda^\rho = (q^j, \eta^j)_{j=1, \dots, 3(k-1)+1}$ .31 (a) First, we consider the first configuration  $i = 1$ . Assume that  $k > 1$ . Let  $\rho_1 = ((s^1, E_1, E_2), (s^2, C_1, C_2))$ . We  
32 define (i)  $q^1 = s^1$ ,  $\eta^1(1, R_r) = \eta^1(2, R_r) = v_r^1$  for  $r \in \{1, 2\}$ ; (ii)  $q^2 = s^1 E_1 E_2$ ,  $\eta^2(1, R_r) = \eta^1(1, R_r) - E_r$  for  
33  $r \in \{1, 2\}$ ; and (iii)  $q^3 = s^2 C_1 C_2$ ,  $\eta^3(1, R_r) = \eta^2(1, R_r) + E_r + C_r$  for  $r \in \{1, 2\}$ . In the case that  $k = 1$ , we define  
34  $q^1 = s^1$ ,  $\eta^1(1, R_r) = \eta^1(2, R_r) = v_r^1$  for  $r \in \{1, 2\}$ .

(b) Inductively, we assume that the path has been constructed up to position  $i - 1$  of computation  $\rho$ , that is up to  $3(i - 1) + 3$  on  $\lambda$ . Suppose  $i < k$  and let  $\rho_i = ((s^i, E_1, E_2), (s^{i+1}, C_1, C_2))$ . Again, we define (i)  $q^{3i+1} = s^i$ ,  $\eta^{3i+1}(1, R_r) = v_r^i$  for  $r \in \{1, 2\}$ ; (ii)  $q^{3i+2} = s^i E_1 E_2$ ,  $\eta^{3i+2}(1, R_r) = \eta^{3i+1}(1, R_r) - E_r$  for  $r \in \{1, 2\}$ ; and (iii)  $q^{3i+3} = s^{i+1} C_1 C_2$ ,  $\eta^{3i+3}(1, R_r) = \eta^{3i+2}(1, R_r) + E_r + C_r$  for  $r \in \{1, 2\}$ . In the case that  $k = i$ , we define  $q^{3i+1} = s^i$ ,  $\eta^{3i+1}(1, R_r) = v_r^i$  for  $r \in \{1, 2\}$ .

First, we prove the following claim which is essential for the rest of the proof.

**Claim.** For every  $i = 1, \dots, k$  we have that (i)  $s^i = q^{3(i-1)+1}$ , (ii)  $v_r^i = \eta^{3(i-1)+1}(1, R_r)$  for  $r \in \{1, 2\}$ , and if  $i < k$ , then (iii)  $q^{3(i-1)+2} = s^i E_1 E_2$  with  $E_r^i = 0$  for  $r \in \{1, 2\}$  implies  $\eta^{3(i-1)+2}(1, R_r) = 0$ , and also (iv)  $v_r^{i+1} = \eta^{3(i-1)+3}(1, R_r)$  for  $r \in \{1, 2\}$ .

*Proof of claim.* We prove the claim by induction on  $i \leq k$ .

*Induction base:* Let  $i = 1 \leq k$ . We have that (i)  $s^1 = q^1$  and (ii)  $v_r^1 = \eta_j^1(1, R_r)$  by construction.

*Induction step:* Suppose the claim is true up to  $i < k$ . We show the case for  $i + 1$ . First, suppose that  $i + 1 = k$ . By induction, we have that  $\eta^{3(i-1)+1}(1, R_r) = v_r^i$ ,  $\eta^{3(i-1)+3}(1, R_r) = v_r^{i+1}$ , and by Proposition 1(a)  $\eta^{3(i-1)+1}(1, R_r) = \eta^{3(i-1)+1}(2, R_r)$ . Thus, by Proposition 1(b) configuration  $q^{3i+1} = (s^{i+1}, \eta^{3i+1})$  can be reached. Moreover, the resources for agent 1 do not change in the step transition from  $q^{3(i-1)+3}$  to  $q^{3i+1}$ , which shows that  $\eta^{3i+1}(1, R_r) = v_r^{i+1}$ .

Now, we consider the case  $i + 1 < k$ . Cases (i) and (ii) follow completely analogously. By construction, if  $q^{3i+2} = s^{i+1} E_1^{i+1} E_2^{i+1}$  with  $E_r^{i+1} = 0$  then  $\rho_{i+1} = ((s^{i+1}, E_1, E_2), (s^{i+2}, C_1, C_2))$  with  $E_r = 0$ . This transition can only be taken by the automaton if  $v_r^{i+1} = 0$ . Then, by (ii)  $\eta^{3i+1}(1, R_r) = 0$ . Finally, action  $E_1^{i+1} E_2^{i+1}$  can only consume resources by construction of the automaton. This shows that also  $\eta^{3i+2}(1, R_r) = 0$ . For (iv) we observe that  $\eta^{3i+2}(1, R_r) = \eta^{3i+1}(1, R_r) - E_r^{i+1}$  and thus  $\eta^{3i+3}(1, R_r) = \eta^{3i+2}(1, R_r) + C_r + E_r^{i+1} = \eta^{3i+1}(1, R_r) + C_r \stackrel{(ii)}{=} v_r^{i+1} + C_r = v_r^{i+2}$ .  $\square$

Using the claim we now have to show that the thus constructed sequence is indeed an  $\mathcal{A}$ -computation pre-encoding (Definition 4) and that it is a simulation (Definition 5). The two conditions of Definition 4 follow immediately. Also, by the claim it follows that  $\lambda$  is indeed a path in  $\mathfrak{M}_1^{\mathcal{A}}$ .

For the first condition of Definition 5, we consider two cases. If  $\rho = (s, v_1, v_2)$  consists of a single configuration, then by (a)  $\lambda = (s, \eta)$ . Hence,  $\lambda$  has length  $3 \cdot 0 + 1$ . For the second case,  $|\rho| = k > 1$ , we observe that we added for each  $i < k$ , three states to  $\lambda$  ( $s, sE_1E_2$ , and  $s'C_1C_2$ ), and an additional state for  $i = k$ . Thus,  $\lambda$  has length  $3(k - 1) + 1$ . The other conditions (2) and (3) of the Definition follow from (i-iii) of the claim.

We refer to the thus constructed path as  $f^{\mathcal{A}}(\rho)$ . Different  $\rho$ 's yield different  $f^{\mathcal{A}}(\rho)$ 's. It remains to show that  $f^{\mathcal{A}}$  is surjective. For an arbitrary  $\mathcal{A}$ -computation encoding  $\lambda$ , each triple of states  $(s)(sE_1E_2)(s'C_1C_2)$  on  $\lambda$  defines a unique transition  $\tau_i = ((s, E_1, E_2), (s', C_1, C_2))$ . Given the initial configuration  $(q^1, \eta^1)$  with  $\eta^1(1, R_r) = \eta^1(2, R_r)$  for  $r \in \{1, 2\}$  and the the sequence of the  $\tau_i$ 's, we can compute the computation  $\rho$ . It is immediate that  $f^{\mathcal{A}}(\rho) = \lambda$ .  $\square$

**Lemma 2.** The empty-band TCM  $\mathcal{A}$  halts iff  $\mathfrak{M}_1^{\mathcal{A}}, s_{\text{init}}, \bar{0} \models \langle \langle 1 \rangle_{(1,2)}^{\bar{0}} \mathbf{F} \text{ halt}$ .

*Proof.* Let the empty-band TCM  $\mathcal{A} = (S, s_{\text{init}}, S_f, \Delta)$  be given and  $\mathfrak{M}_1^{\mathcal{A}}$  be the iRBM constructed according to Definition 3.

“ $\Rightarrow$ ” Assume that  $\mathcal{A}$  halts and let  $\rho = (s^i, v_1^i, v_2^i)_{i=1, \dots, k}$  be a minimal length accepting run of  $\mathcal{A}$ . By Lemma 1 (Simulation Lemma),  $\lambda = f^{\mathcal{A}}(\rho) = (q^j, \eta^j)_{j=1, \dots, 3(k-1)+1}$  is an accepting  $\mathcal{A}$ -computation encoding that simulates  $\rho$ . Each subsequent configurations  $(q^i, \eta^i)$  and  $(q^{i+1}, \eta^{i+1})$ , for  $i = 1, \dots, 3(k - 1) + 1$  on  $\lambda$  define a unique action of agent 1. Let  $s_1$  be the strategy which assigns to each history  $q^1 \dots q^i$  this unique action of agent 1 leading to  $q^{i+1}$ . The strategy assigns idle to all other histories, including  $\lambda$  itself. We assume that player 1 follows strategy  $s_1$ . As  $\lambda$  simulates  $\rho$ , agent 2 can never perform a test action in states of type  $sE_1E_2$  according to Definition 5.3 and Proposition 1(c). Thus, agent 2 can only choose between actions in states of type  $sC_1C_2$ , otherwise it can only perform a unique action (the idle action). As a consequence, the outcome set wrt.  $s_1$  contains the following

43 paths:

$$\begin{aligned} out(q^1, \bar{0}, s_1, \{1, 2\}) &= \{(q^1, \eta^1) \dots (q^k, \eta^k)(q_\circ, \eta^{k+1}) \dots\} \\ &\cup \{(q^{3(i-1)+1}, \eta^{3(i-1)+1}) \\ &\quad (q^{3(i-1)+2}, \eta^{3(i-1)+2}) \\ &\quad (q^{3(i-1)+3}, \eta^{3(i-1)+3})_{i=1, \dots, j} (q_h, \eta^{3j+1}) \dots \mid 1 \leq j < k\}. \end{aligned}$$

1 All these paths contain a state labelled halt. This shows that  $\mathfrak{M}_1^{\mathcal{A}}, s_{\text{init}}, \bar{0} \models \langle\langle 1 \rangle\rangle_{(1,2)}^{\bar{0}} \mathbf{F} \text{halt}$ .

2 “ $\Leftarrow$ ” Assume that  $\mathfrak{M}_1^{\mathcal{A}}, s_{\text{init}}, \bar{0} \models \langle\langle 1 \rangle\rangle_{(1,2)}^{\bar{0}} \mathbf{F} \text{halt}$  holds. Then, there is a witnessing strategy  $s_1$  of agent 1 such that for  
3 all  $\lambda = (q^i, \eta^i)_{i \in \mathbb{N}} \in out(s^1, \bar{0}, s_1, \{1, 2\})$  there is a minimal index  $k$  such that  $\pi(q^k) = \text{halt}$ . In particular, the  
4 set contains a path in which the state  $q_h$  is never visited. This follows from Proposition 1(b). We denote the  
5 prefix of this path that is cut directly after the state labelled halt by  $\lambda'$ . On  $\lambda'$  it can never be the case that in  
6 a state  $q^i = sE_1E_2$  with  $E_r = 0$  we have that  $\eta^i(2, E_r) > 0$ ; otherwise, the outcome would also contain a path  
7 which loops in  $q_f$  because agent 2 could perform the test action. But this would contradict  $s_1$  being a witnessing  
8 winning strategy. Thus, we have  $\eta^i(2, E_r) = 0$  and by Proposition 1(c) also  $\eta^i(1, E_r) = 0$ , for  $r \in \{1, 2\}$ . Thus,  
9 by the Simulation Lemma, there is  $\rho$  with  $f^{\mathcal{A}}(\rho) = \lambda'$  that is an accepting run of the automaton. The automaton  
10 halts.

11

□

## 12 A.2. Encodings and Proof from Section 4.2

13 The formal definition of the encoding of a two counter automaton as a resource-bounded model with idle actions  
14 and a single resource type is given next.

15 **Definition 7** ( $\mathfrak{M}_2^{\mathcal{A}}$ ). Let  $(S, s_{\text{init}}, S_f, \Delta)$  be an empty-band TCM. From  $\mathcal{A}$  we construct the **iRBM**  $\mathfrak{M}_2^{\mathcal{A}} = (\{1, 2, 3, 4\}, Q, \Pi, \pi,$   
16  $Act, d, o, \{R\}, t)$  where:

- 17 1. The sets of states  $Q = S \cup Q_1 \cup Q_2 \cup \{q_f, q_h, q_\circ\}$  and of propositions and their valuations are defined as in  
18 Definition 3.
- 19 2. The set  $Act$  is defined as follows. For each transition  $(s, E_1, E_2)\Delta(s', C_1, C_2)$  of  $\mathcal{A}$  the set contains actions  $\underline{E}_1E_2,$   
20  $E_1\underline{E}_2, s'_{\underline{E}_1E_2}C_1C_2, s'_{E_1\underline{E}_2}C_1C_2, s'\underline{C}_1C_2,$  and  $s'C_1\underline{C}_2$ . Additionally, there is an action idle and test actions  $test_i$  for  $i \in \{1, 2\}$ .
- 21 3. The action availability is defined according to  $\Delta$ . For agent 1 we have:

$$\begin{aligned} \underline{E}_1E_2 &\in d_1(s) \text{ iff } (s, E_1, E_2)\Delta(s', C_1, C_2) \text{ for some } (s', C_1, C_2) \\ \text{idle} &\in d_1(q) \text{ for all } q \in Q \\ s'_{\underline{E}_1E_2}C_1C_2 &\in d_1(sE_1E_2) \text{ iff } (s, E_1, E_2)\Delta(s', C_1, C_2) \end{aligned}$$

22 and analogously for agent 2 but  $E_2$  and  $C_2$  are underlined instead of  $E_1$  and  $C_1$ , respectively. For agent 3 we  
23 have:

$$\begin{aligned} \text{idle} &\in d_3(q) \text{ for all } q \in Q \\ s'\underline{C}_1C_2 &\in d_3(s'\underline{C}_1C_2) \text{ for all } s'\underline{C}_1C_2 \in Q_2 \\ \text{test}_1 &\in d_3(sE_1E_2) \text{ iff } E_1 = 0 \end{aligned}$$

24 and again analogously for agent 4 but  $E_2$  and  $C_2$  are underlined instead of  $E_1$  and  $C_1$ , respectively. Moreover,  
25  $test_2 \in d_4(sE_1E_2)$  iff  $E_2 = 0$ , that is, the test action is only available if the counter which the agent is supposed  
26 to simulate is empty.

- 27 4. We abstain from giving the transition function  $o$  and refer to Figure 4 where we call an action profile  $(\underline{E}_1 E_2, E'_1 \underline{E}'_2, \text{idle}, \text{idle})$   
 1 invalid if (i)  $E_i \neq E'_i$  for some  $i \in \{1, 2\}$  or (ii) if there is no transition  $((s, E_1, E_2), (s', C_1, C_2)) \in \Delta$ . Similarly, we  
 2 call an action profile  $(s_{\underline{E}_1 E_2}^{C_1 C_2}, s_{E'_1 \underline{E}'_2}^{C'_1 C'_2}, \text{idle}, \text{idle})$  invalid in state  $sE_1 E_2$  if (i)  $C_i \neq C'_i$  or  $E_i \neq E'_i$  for some  $i \in \{1, 2\}$   
 3 or (ii) if there is no transition  $((s, E_1, E_2), (s', C_1, C_2)) \in \Delta$ .  
 4 5. For  $i \in \{1, 2, 3, 4\}$ , the actions' resource consumption/production is defined by the function  $t$ :

$$\begin{aligned}
 t(\underline{E}_1 E_2, R) &= -E_1 \\
 t(E_1 \underline{E}_2, R) &= -E_2 \\
 t(\text{idle}, R) &= 0 \\
 t(s_{\underline{E}_1 E_2}^{C_1 C_2}, R) &= C_1 + E_1 \\
 t(s_{E_1 \underline{E}_2}^{C_1 C_2}, R) &= C_2 + E_2 \\
 t(s_{\underline{C}_1 C_2}, R) &= C_1 \\
 t(s_{C_1 \underline{C}_2}, R) &= C_2 \\
 t(\text{test}_1, R) &= -1 \\
 t(\text{test}_2, R) &= -1
 \end{aligned}$$

5 **Theorem 4.** Model checking prRAL over the class of iRBMs is undecidable even in the case of single resource and  
 6 two agents.

7 *Proof.* We modify the model  $\mathfrak{M}_2^{\mathcal{A}} = (\{1, 2, 3, 4\}, Q, \Pi, \pi, \text{Act}, d, o, \{R\}, t)$  to a model  $\widehat{\mathfrak{M}}_2^{\mathcal{A}} = (\{1, 2\}, Q \setminus \{q_h\}, \Pi, \widehat{\pi}, \widehat{\text{Act}}, \widehat{d}, \widehat{o}, \{R\}, \widehat{t})$ .  
 8 Essentially, we merge agents  $\{1, 3\}$  and agents  $\{2, 4\}$  into agent 1 and 2 in the new model, respectively. The main en-  
 9 coding is shown in Figure 5. We define  $\widehat{d}_i(q)$  as  $d_i(q)$  for  $i \in \{1, 2\}$  where it is additionally required that  $\text{test}_1 \in$   
 10  $\widehat{d}_1(sE_1 E_2)$  iff  $E_1 = 0$ , and  $\text{test}_2 \in \widehat{d}_2(sE_1 E_2)$  iff  $E_2 = 0$ ; coalition  $\{1, 2\}$  can now make all decisions. The action set  $\widehat{\text{Act}}$   
 11 equals  $\text{Act}$  but all actions of type  $s_{\underline{C}_1 C_2}$  and  $s_{C_1 \underline{C}_2}$  are removed. The transition function  $\widehat{o}$  is obtained from  $o$ :

$$\begin{aligned}
 \widehat{o}(s, (\underline{E}_1 E_2, E_1 \underline{E}_2)) &= sE_1 E_2 \\
 \widehat{o}(s, (\text{idle}, \star)) &= q_{\circlearrowleft} \\
 \widehat{o}(s, (\star, \text{idle})) &= q_{\circlearrowleft} \\
 o(sE_1 E_2, (\text{idle}, \star)) &= q_{\circlearrowleft} \\
 o(sE_1 E_2, (\star, \text{idle})) &= q_{\circlearrowleft} \\
 \widehat{o}(q_{\circlearrowleft}, (\text{idle}, \text{idle})) &= q_{\circlearrowleft} \\
 \widehat{o}(sE_1 E_2, (s_{\underline{E}_1 E_2}^{C_1 C_2}, s_{E_1 \underline{E}_2}^{C'_1 C'_2})) &= s_{\underline{C}_1 C_2} \\
 \widehat{o}(sE_1 E_2, (\text{test}_1, \star)) &= q_f \\
 \widehat{o}(sE_1 E_2, (\star, \text{test}_2)) &= q_f \\
 \widehat{o}(q_f, (\text{idle}, \text{idle})) &= q_f \\
 \widehat{o}(s' \underline{C}_1 C_2, (\text{idle}, \text{idle})) &= s'
 \end{aligned}$$

12 Moreover, invalid action profiles executed in states  $s$  and  $sE_1 E_2$  also result in the loop state  $q_{\circlearrowleft}$ . Here, we call an action  
 13 profile  $(\underline{E}_1 E_2, E'_1 \underline{E}'_2)$  *invalid* if (i)  $E_i \neq E'_i$  for some  $i \in \{1, 2\}$  or (ii) if there is no transition  $((s, E_1, E_2), (s', C_1, C_2)) \in \Delta$ .  
 14 Similarly, we call an action profile  $(s_{\underline{E}_1 E_2}^{C_1 C_2}, s_{E'_1 \underline{E}'_2}^{C'_1 C'_2})$  *invalid* in state  $sE_1 E_2$  if (i)  $C_i \neq C'_i$  or  $E_i \neq E'_i$  for some  $i \in \{1, 2\}$  or  
 15 (ii) if there is no transition  $((s, E_1, E_2), (s', C_1, C_2)) \in \Delta$ .

16 The cost function  $\widehat{t}$  is defined as before restricted to the new action set. (States of type  $s_{\underline{C}_1 C_2}$  are only kept due  
 17 to compatibility reasons.) As in the related case, it is easy to see that each resource-extended path  $\lambda = (q^i, \eta^i)_{i \in \mathbb{N}}$  in  
 18  $\mathfrak{M}_2^{\mathcal{A}}$  that does not contain state  $q_h$  corresponds to a path  $\lambda = (q^i, \widehat{\eta}^i)_{i \in \mathbb{N}}$  in  $\widehat{\mathfrak{M}}_2^{\mathcal{A}}$  with  $\widehat{\eta}^i(1, R) = \eta^i(1, R)$  and  $\widehat{\eta}^i(2, R) =$   
 19  $\eta^i(2, R)$  for all  $i$ . By the analogue of Proposition 1(c) given in Section 4.2.2, the zero-test simulated in the test states

20 can equivalently be defined with respect to agent 1's and 2's resource endowments. It holds that  $\mathfrak{M}_2^{\mathcal{A}}, q_{\text{init}}, \bar{0} \models$   
 1  $\langle\langle\{1, 2, 3, 4\}\rangle\rangle^{\bar{0}}((-\langle\{1, 2, 3, 4\}\rangle)^{\dagger} \mathbf{X} \text{fail}) \mathbf{U} \text{halt}$  if, and only if,  $\mathfrak{M}_2^{\mathcal{A}}, q_{\text{init}}, \bar{0} \models \langle\langle\{1, 2\}\rangle\rangle^{\bar{0}}((-\langle\{1, 2\}\rangle)^{\dagger} \mathbf{X} \text{fail}) \mathbf{U} \text{halt}$ .  $\square$

## 2 B. Proof from Section 5

3 **Lemma 8.** *Algorithm 2 is correct, that is,  $\text{STRATEGY}(n, \phi)$  returns true iff  $\exists e' \in \text{compatible}(v(n)) : s(n), e' \models \phi$ .*

4 *Proof.* The proof is by induction on the structure of the formulae.

5 **Base case:**

6 *Case  $\phi = p$  for some  $p \in \Pi$ :*  $\text{STRATEGY}(n, p)$  returns true iff  $s(n) \models p$  (by lines 2-3 in Algorithm 2) iff  $s(n), e' \models p$  for  
 7 any  $e' \in \text{compatible}(v(n))$  (by the semantics of RAL). Obviously,  $\text{compatible}(v(n)) \neq \emptyset$ .

8 *Case  $\phi = \neg p$  for some  $p \in \Pi$ :* Similarly,  $\text{STRATEGY}(n, \neg p)$  returns true iff  $s(n) \not\models p$  (by lines 4-5 in Algorithm 2) iff  
 9  $s(n), e' \not\models p$  for any  $e' \in \text{compatible}(v(n))$ . Again,  $\text{compatible}(v(n)) \neq \emptyset$ .

10 **Induction step:** The proof is done for each case of pprRAL formulae.

11 *Case  $\phi = \phi_1 \wedge \phi_2$ :*  $\text{STRATEGY}(n, \phi)$  returns true iff  $\text{STRATEGY}(n, \phi_1)$  and  $\text{STRATEGY}(n, \phi_2)$  return true (by lines 6-7), iff  
 12  $\exists e'_1 \in \text{compatible}(v(n)) : s(n), e'_1 \models \phi_1$  and  $\exists e'_2 \in \text{compatible}(v(n)) : s(n), e'_2 \models \phi_2$  (by induction hypothesis), iff  
 13  $s(n), e' \models \phi_1 \wedge \phi_2$  (by the semantics of RAL) where  $e' = \overline{\max}\{e'_1, e'_2\} \in \text{compatible}(v(n))$  where  $\overline{\max}V = v$  denotes the  
 14 pointwise maximum from a finite set  $V$  of endowments, i.e.,  $v(i, r) = \max\{e'(i, r) \mid e' \in V\}$  for all  $i \in \text{Agt}$  and  $r \in \text{Res}$   
 15 (note that this is straightforward by induction on the structure of formulae in pprRAL that  $s, v \models \varphi$  implies  $s, e' \models \varphi$   
 16 for all states  $s$  and  $e' \geq v$  since pprRAL contains only positive formulae).

17 *Case  $\phi = \phi_1 \vee \phi_2$ :*  $\text{STRATEGY}(n, \phi)$  returns true iff  $\text{STRATEGY}(n, \phi_1)$  or  $\text{STRATEGY}(n, \phi_2)$  return true (by lines 8-9), iff  $\exists e'_1 \in$   
 18  $\text{compatible}(v(n)) : s(n), e'_1 \models \phi_1$  or  $\exists e'_2 \in \text{compatible}(v(n)) : s(n), e'_2 \models \phi_2$  (by induction hypothesis) iff  $s(n), e' \models$   
 19  $\phi_1 \vee \phi_2$  (by the semantics of RAL) for some  $e' \in \{v_1, v_2\} \subseteq \text{compatible}(v(n))$ .

20 *Case  $\phi = \langle\langle A \rangle\rangle^{\dagger} \mathbf{X}\psi$ :*  $\text{STRATEGY}(n, \phi)$  returns true iff  $\text{x-STRATEGY}(n', \phi)$  returns true (by lines 10-11) where  $n' = \text{node}_0(s(n),$   
 21  $e(n), v(n), A)$ , iff there exists  $\alpha' \in \text{Act}A$  (according to lines 4-5 of Algorithm 3) such that for every  $\alpha \in \text{ActAgt}$   
 22 (line 8 of Algorithm 3)  $\text{STRATEGY}(n'', \psi)$  returns true where  $n'' = \text{node}(n', s_{\alpha}, \alpha, A)$  and  $s_{\alpha} = o(s(n'), \alpha)$  (recall  
 23 that  $v_a(n'') = v_a(n) - \text{cons}(\alpha_a) + \text{prod}(\alpha_a)$  for  $a \in A$ ,  $v_a(n'') = v_a(n)$  for  $a \notin A$  and  $s(n'') = s_{\alpha}$ ), iff for every  
 24  $\alpha \in \text{ActAgt}$ , there exists  $v_{\alpha} \in \text{compatible}(v(n) - \text{cons}(\alpha) + \text{prod}(\alpha))$  such that  $s_{\alpha}, v_{\alpha} \models \psi$  (by induction hypothe-  
 25 sis), iff there exists  $e' \in \text{compatible}(v(n) - \text{cons}(\alpha') + \text{prod}(\alpha'))$  such that for every  $\alpha \in \text{ActAgt}$   $s_{\alpha}, e' \models \psi$ , where  
 26  $e' = \overline{\max}\{v_{\alpha} \mid \alpha \in \text{ActAgt}\} \in \text{compatible}(v(n) - \text{cons}(\alpha') + \text{prod}(\alpha'))$ , iff  $s, e' + \text{cons}(\alpha') - \text{prod}(\alpha') \models \langle\langle A \rangle\rangle^{\dagger} \mathbf{X}\psi$  where  
 27  $e' + \text{cons}(\alpha') - \text{prod}(\alpha') \in \text{compatible}(v(n))$ .

28 The case for  $\langle\langle A \rangle\rangle^{\leq} \mathbf{X}\psi$  is similar to the above case, hence omitted here.

29 *Case  $\phi = \langle\langle A \rangle\rangle^{\dagger} \psi_1 \mathbf{U} \psi_2$ :*  $\text{STRATEGY}(n, \phi)$  returns true iff  $\text{u-STRATEGY}(n_0, \phi)$  returns true (by lines 14-15) where  $n_0 =$   
 30  $\text{node}_0(s(n), e(n), v(n), A)$ .

31 ( $\Rightarrow$ ): Let  $T$  denote the search tree rooted at  $n_0$  when  $\text{u-STRATEGY}(n_0, \phi)$  returns true. For the purpose of this proof, we  
 32 assume that each interior node  $n$  in  $T$  has an additional function  $a(n)$  which returns the action tuple of the proponent  
 33 at  $s(n)$ ; and the function  $\text{node}(n, s, \alpha, A)$  also assigns  $a(n) = \alpha$ .

34 For every leaf  $n$  of  $T$ , we have that  $\text{STRATEGY}(n, \psi_2)$  returns true according to lines 8-9 of Algorithm 4. By the  
 35 induction hypothesis, we also have  $s(n), v_n \models \psi_2$  for some  $v_n \in \text{compatible}(v(n))$ . Similarly, for every interior node  
 36  $n$  of  $T$ ,  $\text{STRATEGY}(n, \psi_1)$  returns true according to lines 10-11 of Algorithm 4 and, hence,  $s(n), v_n \models \psi_1$  for some  
 37  $v_n \in \text{compatible}(v(n))$ . We first update the value of  $e(n)$  for every node in  $T$  so that resource availability is enough to  
 38 satisfy  $\psi_1$  at every interior node and  $\psi_2$  at every leaf node. The update is carried out from the leaves to the root of  $T$   
 39 as follows:

- 40 • For a leaf  $n$ ,  $e(n) := \overline{\max}\{v_n, e(n)\}$ ;
- 41 • For an interior node  $n$  with  $k$  children  $n_1, \dots, n_k$ , if  $e(n_i)$  has been updated for all  $i \in \{1, \dots, k\}$ , then  $e(n) :=$   
 42  $\overline{\max}\{v_n, e(n), e(n_1) + \text{cons}(a(n)), \dots, e(n_k) + \text{cons}(a(n))\}$ .

43 Let  $s_T$  denote the strategy for  $A$  where for each node  $n \in T$ , with  $p(n) = n_0 \dots n_k$ ,  $s_T(s(n_0) \dots s(n_k)s(n)) = (a(n))_A$ .  
 44 However, this strategy may not be executable from  $n_0$  if, as a result of the initial resource availability  $e(n_0)$ , there is  
 45 some node  $n$  in  $T$  such that  $e(n)(i, r) < 0$  for some resource  $r$  and agent  $i$ . Note that whenever  $e(n)(i, r) < 0$ , we have



46  $v(n)(i, r) = arb$  according to lines 6, 7 and 12 of Algorithm 4. If  $v(n_0)(i, r) = arb$ , we can simply increase the value of  
 1  $e(n_0)(i, r)$  to compensate for the lack of resources above. In particular, we increase  $e(n_0)(i, r)$  to  $e(n_0)(i, r) - e(n)(i, r)$ ,  
 2 then recalculate the value of  $e(n')(i, r)$  for every node  $n'$  in  $T$ . Then,  $e(n)(i, r)$  becomes 0. Obviously, this step only  
 3 removes negative values and can be repeated until no further negative values can be removed. This means for any  
 4  $e(n)(i, r) < 0$  we have  $v(n_0)(i, r) \neq arb$ . Then, there must be a loop within the path  $p(n)$  (according to lines 6-7  
 5 of Algorithm 4) that strictly increases resource  $r$  for agent  $i$ . To increase  $e(n)(i, r)$  to a positive value, we need to  
 6 determine the number of times the loop should be performed. In particular, there must be a node  $n_1 \in p(n)$  such that  
 7  $v(n_1)(i, r)$  is assigned  $arb$  by the statement in lines 6-7 of Algorithm 4. Let  $n_2$  be the node  $n'$  in line 6. We denote  $n_1$   
 8 by  $stop_r(n)$  and  $n_2$  by  $start_r(n)$ . Let  $T(\hat{n})$  denote the subtree of  $T$  rooted in  $\hat{n}$  for every  $\hat{n}$  in  $T$ . For a resource  $r$ , if there  
 9 is a node  $n$  with  $e(n)(i, r) < 0$  for some  $i$  we extend  $T$  until  $e(n)(i, r) \geq 0$  by repeating the branch between  $start_r(n)$   
 10 and  $end_r(n)$  finitely many times. There are two sub-cases to consider:

- 11 • Sub-case 1:  $n$  is the only node in  $T(start_r(n))$  with  $e(n)(i, r) < 0$  as depicted by Figure B.6(a).

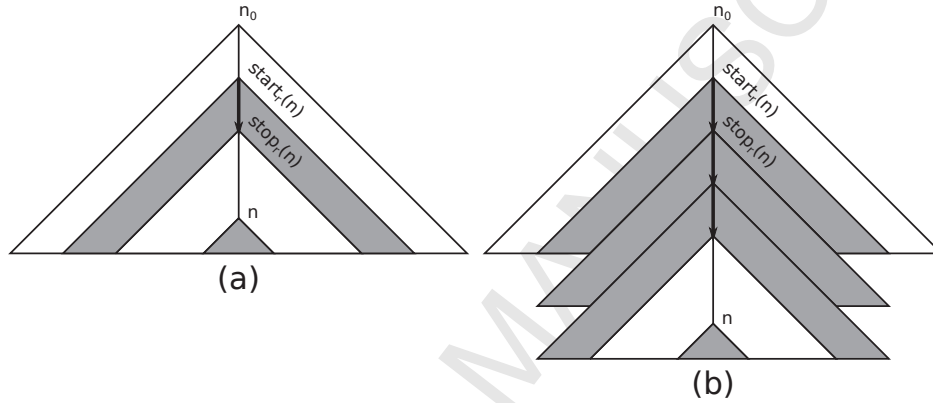


Figure B.6:  $n$  is the only node with  $e(n)(i, r) < 0$  in  $T(start_r(n))$

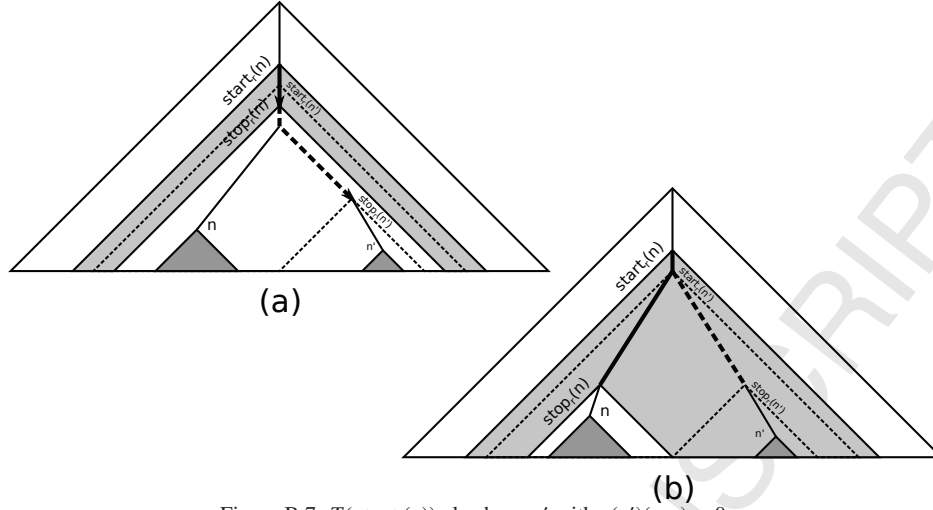
12 As the path from  $start_r(n)$  to  $stop_r(n)$  increases  $r$  for agent  $i$  by an amount of  $e(stop_r(n))(i, r) - e(start_r(n))(i, r)$ ,  
 13 it is necessary to repeat this path  $\lceil \frac{|e(n)(i, r)|}{e(stop_r(n))(i, r) - e(start_r(n))(i, r)} \rceil$  times, as depicted in Figure B.6(b).

- 14 • Sub-case 2:  $n$  is not the only node in  $T(start_r(n))$  with  $e(n)(i, r) < 0$ . Other nodes  $n'$  are either in the subtree  
 15  $T(stop_r(n))$  (as depicted by Figures B.7(a)) or in the subtree of  $T(start_r(n))$  but not  $T(stop_r(n))$  (as depicted by  
 16 Figures B.7(b)). Without loss of generality, we assume that  $start_r(n)$  is the ancestor of  $start_r(n')$  for any of such  
 17  $n'$ .

18 Again, as the path from  $start_r(n)$  to  $end_r(n)$  increases  $r$  for agent  $i$  by an amount of  $e(end_r(n))(i, r) - e(start_r(n))(i, r)$ ,  
 19 it is necessary to repeat this path  $k = \lceil \frac{|e(n)(i, r)|}{e(end_r(n))(i, r) - e(start_r(n))(i, r)} \rceil$  times, as depicted in Figure B.8(a). Note that  
 20 this repetition will also repeat nodes  $n'$  which are in the subtree of  $T(start_r(n))$  but not in  $T(stop_r(n))$  and have  
 21  $e(n')(i, r) < 0$  as  $n'_1, \dots, n'_k$  depicted in Figure B.8(b).

22 Let  $T_1$  be the obtained tree. Then, the number of nodes  $n''$  in  $T_1(n_{k-1})$  with  $e(n'')(i, r) < 0$  is strictly less than  
 23 that in  $T(start_r(n))$ . Therefore, we can reapply the above construction to obtain a tree  $T_2$  where all nodes  $n''$  in  
 24  $T_2(n_{k-1})$  have  $e(n'')(i, r) \geq 0$ . These include node  $n'$  as depicted in Figure B.8a and  $n'_k$  as depicted in Figure B.8b.  
 25 Then, we further apply step by step the above construction for nodes  $n'_{k-1}, \dots, n'_1$  and  $n'$  in Figure B.8b. Finally,  
 26 we obtain a tree  $T_3$  where all nodes  $n''$  have  $e(n'')(i, r) \geq 0$ . This construction can be repeated for other resources  
 27  $r' \neq r$  and agents  $i' \neq i$ . Finally, we obtain a tree  $T_4$  where for all nodes  $n$  in  $T_4$ ,  $e(n)(i, r) \geq 0$  for all  $r$  and  $i$  and  
 28  $s_{T_4}$  is a strategy satisfying  $\phi$  at  $s(n_0)$  where  $n_0$  is the root of  $T_4$ . In other words, we have  $s(n_0), e(n_0) \models \phi$  where it is  
 29 obvious that  $e(n_0) \in compatible(v(n_0))$ . Since  $n_0 = node_0(s(n), e(n), v(n), A)$ ,  $s(n) = s(n_0)$  and  $v(n) = v(n_0)$ ; therefore  
 30  $s(n), e(n_0) \models \phi$  where  $e(n_0) \in compatible(v(n))$ .

31 ( $\Leftarrow$ ): Assume that  $s(n), e' \models \phi$  for some  $e' \in compatible(v(n))$ , then there exists a strategy  $s_A$  such that for all  
 32  $\lambda \in out(q, e', s_A, A) : \exists i_\lambda \geq 0 : \lambda|_Q[i_\lambda], \lambda|_{En}[i_\lambda] \models \psi_2$  and  $\forall 0 \leq j < i_\lambda : \lambda|_Q[j], \lambda|_{En}[j] \models \psi_1$ . We shall now prove  
 33 that STRATEGY( $n, \phi$ ) returns true, i.e., equivalently U-STRATEGY( $node_0(s(n), e(n), v(n), A), \phi$ ) returns true. Let  $T = (V, E)$

Figure B.7:  $T(start, (n))$  also have  $n'$  with  $e(n')(a, r) < 0$ .

34 be the tree induced by all runs  $\lambda[0, i_i]$  for  $\lambda \in out(q, e', s_A, A)$ , i.e.,  $V = \{\lambda[0, i] \mid \lambda \in out(q, e', s_A, A), i \leq i_i\}$  and  
 1  $E = \{(\lambda[0, i], \lambda[0, i + 1]) \mid \lambda \in out(s, e', s_A, A), i < i_i\}$ . We first attach to each node  $\lambda[0, i]$  of  $T$  an element  $v(\lambda[0, i])$   
 2 where

$$v(\lambda[0, i])(a, r) = \begin{cases} \lambda|_{E_n}[i](a, r) & \text{if } v(n)(a, r) \neq arb \\ arb & \text{otherwise} \end{cases}$$

3 In the following, we show how to convert  $T$  into a search tree which shows that  $U\text{-STRATEGY}(node_0(s(n), e(n), v(n), A), \phi)$   
 4 returns true. Note that  $T$  must be finite and each edge in  $E$  corresponds to a join action of all agents.

5 Initially, let  $T_0 = T$ , then  $T_{l+1}$  is constructed from  $T_l$  as follows.

6 **Case 1a** If there is a node  $\lambda[0, k]$  in  $T_l$  such that  $\exists j < k : \lambda|_Q[j] = \lambda|_Q[k] \wedge v(\lambda[0, j]) \geq v(\lambda[0, k])$ , then  $T_{l+1}$  is  
 7 constructed from  $T_l$  by replacing the subtree  $T_l(\lambda[j])$  by  $T_l(\lambda[k])$ , updating the values for  $\lambda[k']|_{E_n}$  and  $v(\lambda[0, k'])$   
 8 for all  $k' \geq k$  in the subtree  $T_l(\lambda[k])$  according to the values  $\lambda[j]|_{E_n}$  and  $v(\lambda[0, j])$  and the costs of actions in  
 9  $T_l(\lambda[k])$ .

10 **Case 1b** If there is a node  $\lambda[0, k]$  in  $T_l$  such that  $\exists j < k : \lambda|_Q[j] = \lambda|_Q[k] \wedge v(\lambda[0, j]) \leq v(\lambda[0, k]) \wedge v(\lambda[0, j]) \neq$   
 11  $v(\lambda[0, k])$ , then  $T_{l+1}$  is constructed from  $T_l$  by replacing the value  $v(\lambda[0, k'])$  for all nodes  $\lambda[0, k']$  in the subtree  
 12  $T_l(\lambda[k])$  as follows

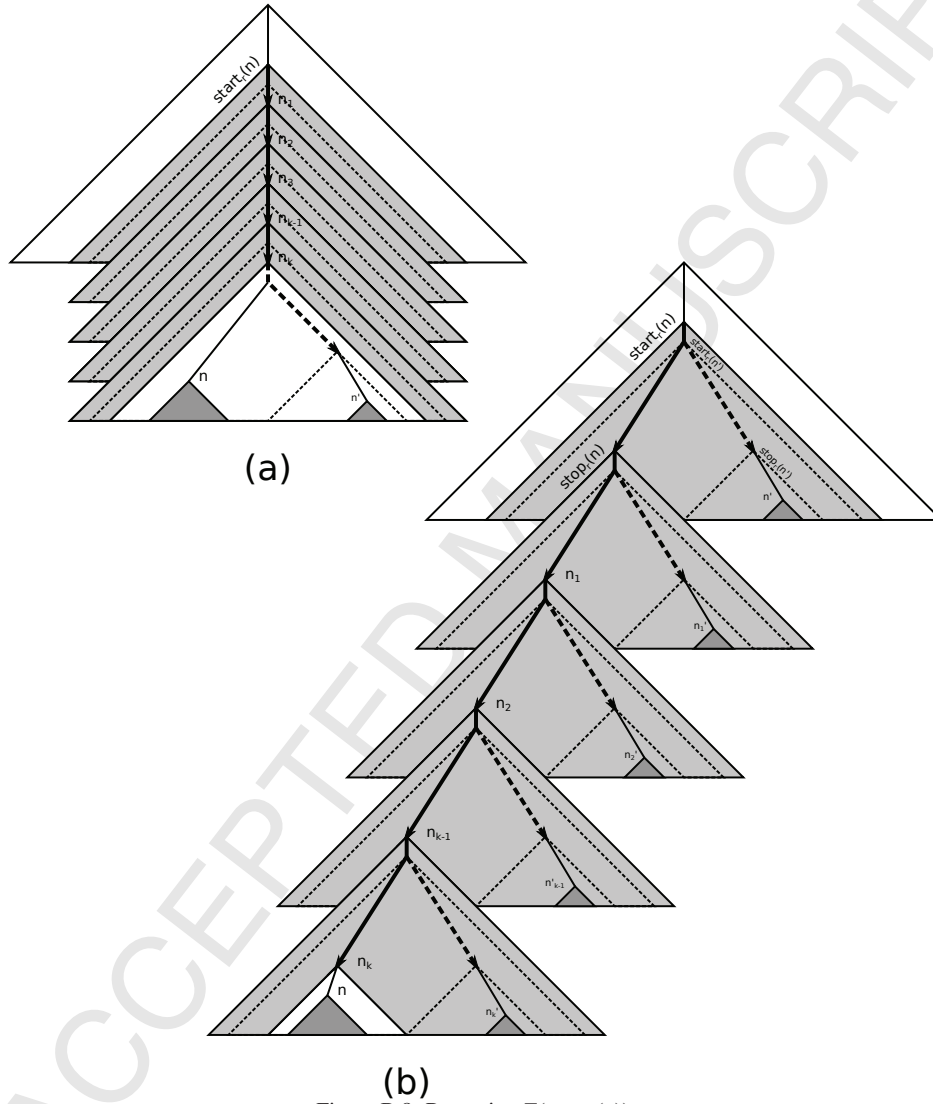
$$v(\lambda[0, k'])(a, r) = \begin{cases} v(\lambda[0, k'])(a, r) & \text{if } v(\lambda[0, j])(a, r) = v(\lambda[0, k])(a, r) \\ arb & \text{if } v(\lambda[0, j])(a, r) < v(\lambda[0, k])(a, r) \end{cases}$$

13 **Case 2** Otherwise,  $T_{l+1} = T_l$ , i.e., no more change.

14 The construction stops when  $T_{l+1} = T_l$ . Let the resulting tree  $T_{l+1} = T'$ . For each node  $\lambda[0, i]$  in  $T'$ , we define a  
 15 node  $n_{\lambda[0, i]}$  where:  $s(n_{\lambda[0, i]}) = \lambda|_Q[i]$ ,  $p(n_{\lambda[0, i]}) = [\lambda|_Q[0] \cdot \dots \cdot \lambda|_Q[i - 1]]$ ,  $c(n_{\lambda[0, i]}) = A$  and  $e(n_{\lambda[0, i]}) = \lambda|_{E_n}[i]$  and  
 16  $v(n_{\lambda[0, i]}) = v(\lambda[0, i])$ . In the following, we show by induction on the length of  $T'(\lambda[0, i])$  that  $U\text{-STRATEGY}(n_{\lambda[0, i]}, \phi)$   
 17 returns true.

18 **Base case:** Assume that  $\lambda[0, i]$  is a leaf of  $T'$ , then it is a leaf from  $T$ . Then, the condition of the **if** statement in lines  
 19 8-9 of Algorithm 4 is true, therefore,  $U\text{-STRATEGY}(n_{\lambda[0, i]}, \phi)$  returns true.

20 **Induction step:** Assume that  $\lambda[0, i]$  is not a leaf of  $T'$ . Then the condition of the **if** statement (lines 2-3) is false, since  
 21  $s(n) \models alt(\phi)$  (where  $n$  is from the input of this Lemma). The condition of the next **if** statement (lines 4-5) is also  
 22 false, since otherwise  $T'$  can be cut further by (Case 1a). The condition of the third or fourth **if** statement (lines 8-11)  
 23 is false since otherwise  $\lambda[0, i]$  must have been a leaf of  $T'$ . Therefore, the algorithm must enter the second **for** loop.

Figure B.8: Repeating  $T(\text{start}, (n))$ .

24 For  $\alpha = s_A(\lambda|_Q[0, i]) \in d_A(\lambda[i])$  we have that, for every  $s' \in out(\lambda|_Q[i], \alpha)$  with  $n' = node(n_{\lambda[0, i]}, s', \alpha, A)$ , there must  
 1 be  $\lambda'[0, i+1]$  in  $T'$  such that  $n' = n_{\lambda'[0, i+1]}$ . By the induction hypothesis,  $U\text{-STRATEGY}(n_{\lambda'[0, i+1]}, \phi)$  returns true. Thus,  
 2  $U\text{-STRATEGY}(n_{\lambda[0, i]}, \phi)$  also returns true.

3 Obviously,  $U\text{-STRATEGY}(node_0(s(n), e(n), v(n), A), \phi)$  returns true since  $n_{\lambda[0]} = node_0(s(n), e(n), v(n), A)$ .

4 The above proof can be adapted to the case  $\phi = \langle\langle A \rangle\rangle^{\zeta} \psi_1 U \psi_2$  by exchanging the role of  $v(n)$  and  $\zeta$ .

5 Case  $\phi = \langle\langle A \rangle\rangle^{\downarrow} G \psi$ :  $\text{STRATEGY}(n, \phi)$  returns true iff  $G\text{-STRATEGY}(n_0, \phi)$  returns true (by lines 18-19) where  $n_0 = node_0(s(n),$   
 6  $e(n), v(n), A)$ .

7 ( $\Rightarrow$ ): Let  $T$  denote the search tree rooted at  $n_0$  when  $G\text{-STRATEGY}(n_0, \phi)$  returns true.

8 For every node  $n$  of  $T$ , we have that  $\text{STRATEGY}(n, \psi)$  returns true according to lines 10-11 of Algorithm 5. By  
 9 induction hypothesis, we also have  $s(n), v_n \models \psi$  for some  $v_n \in compatible(v(n))$ . We first update the value of  $e(n)$  for  
 10 every node in  $T$  so that resource availability is enough to satisfy  $\psi$  at every node. The update is carried out from the  
 11 leaves to the root of  $T$  as follows:

12 • For a leaf  $n$ ,  $e(n) := \overline{\max}\{v_n, e(n)\}$ ;

13 • For an interior node  $n$  with  $k$  children  $n_1, \dots, n_k$ , if  $e(n_i)$  has been updated for all  $i \in \{1, \dots, k\}$ , then  $e(n) :=$   
 14  $\overline{\max}\{v_n, e(n), e(n_1) + \text{cons}(a(n)), \dots, e(n_k) + \text{cons}(a(n))\}$ .

15 Let  $s_T$  denote the strategy for  $A$  where for each node  $n \in T$ , with  $p(n) = n_0 \dots n_k$ ,  $s_T(s(n_0) \dots s(n_k)s(n)) = (a(n))_A$ .  
 16 However, this strategy may not be executable from  $n_0$  if there is some node  $n$  in  $T$  such that  $e(n)(i, r) < 0$  for some  
 17 resource  $r$  and agent  $i$ . We can repeat the tree expansions in the previous case to eliminate all such nodes. Let  
 18 the obtained tree be  $T_1$ . By lines 12-13, for all leaves  $n'$  of  $T_1$ , we have that there exists  $n'' \in p(n')$  such that  
 19  $e_A(n'') \leq e_A(n')$ . Then, we construct an infinite tree from  $T_1$  as follows:

20 • Given  $T_i$ , we construct  $T_{i+1}$  by replacing all leaves  $n'$  of  $T_i$  by the tree  $T_1(n'')$ .

21 •  $T' = \lim_{i \rightarrow \infty} T_i$ .

22 Then, the strategy  $s_{T'}$  from  $T'$  obviously satisfies  $\phi$ . In other words, we have  $s(n_0), e(n_0) \models \phi$  where it is obvious  
 23 that  $e(n_0) \in compatible(v(n))$ . Again, since  $n_0 = node_0(s(n), e(n), v(n), A)$ ,  $s(n) = s(n_0)$  and  $v(n) = v(n_0)$ ; therefore  
 24  $s(n), e(n_0) \models \phi$  where  $e(n_0) \in compatible(v(n))$ .

25 ( $\Leftarrow$ ): Assume that  $s(n), e' \not\models \phi$  for some  $e' \in compatible(v(n))$ , then there exists a strategy  $s_A$  such that for all  $\lambda \in$   
 26  $out(q, e', s_A, A)$  and  $i \geq 0$  :  $\lambda|_Q[i], \lambda|_{E_n}[i] \models \psi$ . We shall now prove that  $\text{STRATEGY}(n, \phi)$  returns true, i.e., equivalently  
 27  $G\text{-STRATEGY}(node_0(s(n), e(n), v(n), A), \phi)$  returns true. Let  $T = (V, E)$  be the infinite tree induced by all runs  $\lambda \in$   
 28  $out(q, e', s_A, A)$ , i.e.,  $V = \{\lambda[0, i] \mid \lambda \in out(q, e', s_A, A), i \geq 0\}$  and  $E = \{(\lambda[0, i], \lambda[0, i+1]) \mid \lambda \in out(q, e', s_A, A)\}$ . We  
 29 also attach to each node  $\lambda[0, i]$  of  $T$  an element  $v(\lambda[0, i])$  where

$$v(\lambda[0, i])(a, r) = \begin{cases} \lambda|_{E_n}[i](a, r) & \text{if } v(n)(a, r) \neq arb \\ arb & \text{otherwise} \end{cases}$$

30 Then, in the following, we cut  $T$  into a finite search tree which shows that  $G\text{-STRATEGY}(node_0(s(n), e(n), v(n), A), \phi)$   
 31 returns true. Note that each edge in  $E$  corresponds to a join action of all agents. First, we repeatedly apply the  
 32 following rule to prune the tree

33 **Case 3** If there is a node  $\lambda[0, k]$  in  $T_l$  such that  $\exists 0 \leq j < k$  :  $\lambda|_Q[j] = \lambda|_Q[k] \wedge \lambda|_{E_n}[j] \leq \lambda|_{E_n}[k]$ , then  $T_{l+1}$  is  
 34 constructed from  $T_l$  by replacing in the subtree  $T_l(\lambda[k])$  by the node  $\lambda[k]$ .

35 This step must yield a finite tree  $T'$  since for every infinite path in the original tree, there must be a state appearing  
 36 infinitely often and the corresponding endowments must be non-decreasing. These paths then are always cut by (Case  
 37 3).

38 Then, let  $T_0 = T'$ , then  $T_{l+1}$  is constructed from  $T_l$  as follows.

39 **Case 4** If there is a node  $\lambda[0, k]$  in  $T_l$  such that  $\exists j < k$  :  $\lambda|_Q[j] = \lambda|_Q[k] \wedge \lambda|_{E_n}[j] \geq \lambda|_{E_n}[k]$  and  $\lambda|_{E_n}[j] \neq \lambda|_{E_n}[k]$ ,  
 40 then  $T_{l+1}$  is constructed from  $T_l$  by replacing the subtree  $T_l(\lambda[j])$  by  $T_l(\lambda[k])$ , updating the values for  $\lambda[k']|_{E_n}$   
 41 and  $v(\lambda[0, k'])$  for all  $k' \geq k$  in the subtree  $T_l(\lambda[k])$  according to the values  $\lambda[j]|_{E_n}$  and  $v(\lambda[0, j])$  and the costs of  
 42 actions in  $T_l(\lambda[k])$ .

43 **Case 5** If there is a node  $\lambda[0, k]$  in  $T_l$  such that  $\exists j < k : \lambda|_Q[j] = \lambda|_Q[k]$  and  $(v(\lambda[0, j])(a, r) = v(\lambda[0, k])(a, r) = arb$   
 1 or  $\lambda|_{E_n}[j](a, r) = \lambda|_{E_n}[k](a, r)$  for all  $a \in A, r \in Res$ ) and  $\lambda|_{E_n}[j](a, r) > \lambda|_{E_n}[k](a, r)$  for some  $a \in A, r \in Res$ ,  
 2 then  $T_{l+1}$  is constructed from  $T_l$  by replacing the subtree  $T_l(\lambda[j])$  by  $T_l(\lambda[k])$ , updating the values for  $\lambda[k']|_{E_n}$   
 3 and  $v(\lambda[0, k'])$  for all  $k' \geq k$  in the subtree  $T_l(\lambda[k])$  according to the values  $\lambda[j]|_{E_n}$  and  $v(\lambda[0, j])$  and the costs of  
 4 actions in  $T_l(\lambda[k])$ .

5 **Case 6** If there is a node  $\lambda[0, k]$  in  $T_l$  such that  $\exists j < k : \lambda|_Q[j] = \lambda|_Q[k] \wedge v(\lambda[0, j]) \leq v(\lambda[0, k]) \wedge v(\lambda[0, j]) \neq v(\lambda[0, k])$ ,  
 6 then  $T_{l+1}$  is constructed from  $T_l$  by replacing the value  $v(\lambda[0, k'])$  for all nodes  $\lambda[0, k']$  in the subtree  $T_l(\lambda[k])$  as  
 7 follows

$$v(\lambda[0, k'])(a, r) = \begin{cases} v(\lambda[0, k'])(a, r) & \text{if } v(\lambda[0, j])(a, r) = v(\lambda[0, k])(a, r) \\ arb & \text{if } v(\lambda[0, j])(a, r) < v(\lambda[0, k])(a, r) \end{cases}$$

8 **Case 7** Otherwise,  $T_{l+1} = T_l$ , i.e., no more change.

9 The construction stops when  $T_{l+1} = T_l$ . Let the resulting tree  $T_{l+1} = T''$ .

10 For each node  $\lambda[0, i]$  in  $T''$ , we define a node  $n_{\lambda[0, i]}$  where:  $s(n_{\lambda[0, i]}) = \lambda|_Q[i]$ ,  $p(n_{\lambda[0, i]}) = [\lambda|_Q[0] \cdot \dots \cdot \lambda|_Q[i-1]]$ ,  
 11  $c(n_{\lambda[0, i]}) = A$  and  $e(n_{\lambda[0, i]}) = \lambda|_{E_n}[i]$  and  $v(n_{\lambda[0, i]}) = v(\lambda[0, i])$ . In the following, we show by induction on the height of  
 12  $T''(\lambda[0, i])$  that  $\text{G-STRATEGY}(n_{\lambda[0, i]}, \phi)$  returns true.

13 **Base case:** Assume that  $\lambda[0, i]$  is a leaf of  $T''$ , then it is a leaf from  $T'$  and is the result of applying Case 3. Then, the  
 14 condition of the **if** statement in lines 12-13 of Algorithm 5 is true, therefore,  $\text{G-STRATEGY}(n_{\lambda[0, i]}, \phi)$  returns true.

15 **Induction step:** Assume that  $\lambda[0, i]$  is not a leaf of  $T''$ . Then the condition of the **if** statement (lines 2-3) is false,  
 16 since  $s(n) \models alt(\phi)$  (where  $n$  is from the input of this Lemma). The condition of the next **if** statement (lines 4-5) is  
 17 also false, since otherwise  $T''$  can be cut further by (Case 4). The condition of the third or fourth **if** statement (lines  
 18 6-7) is false since otherwise  $T''$  must can be cut further by (Case 5). Therefore, the algorithm must enter the second  
 19 **for** loop. For  $\alpha = s_A(\lambda|_Q[0, i]) \in d_A(\lambda[i])$  we have that, for every  $s' \in out(\lambda|_Q[i], \alpha)$  with  $n' = node(n_{\lambda[0, i]}, s', \alpha, A)$ ,  
 20 there must be  $\lambda'[0, i+1]$  in  $T'$  such that  $n' = n_{\lambda'[0, i+1]}$ . By the induction hypothesis,  $\text{G-STRATEGY}(n_{\lambda'[0, i+1]}, \phi)$  returns  
 21 true. Thus,  $\text{G-STRATEGY}(n_{\lambda[0, i]}, \phi)$  also returns true.

22 Obviously,  $\text{G-STRATEGY}(node_0(s(n), e(n), v(n), A), \phi)$  returns true since  $n_{\lambda[0]} = node_0(s(n), e(n), v(n), A)$ .

23 The above proof can be adapted to the case  $\phi = \langle\langle A \rangle\rangle^{\zeta} \mathbf{G}\psi$  by exchanging the role of  $v(n)$  and  $\zeta$ . □