Ranking the critical sections of railway networks

Ratthaphong Meesit* and John Andrews**

*Department of Civil Engineering, Faculty of Engineering, Burapha, Thailand

*, **Resilience Engineering Research Group, Faculty of Engineering, University of Nottingham, UK

*Corresponding author: ratthaphong.me@buu.ac.th

Abstract

Railway systems are now facing an increasing number of threats such as aging infrastructures and climate changes. The identification of critical network sections provides infrastructure managers with the ability to understand the impact of a disruption and creates a suitable preventive strategy to counter such threats. To this end, various vulnerability analysis methods have been proposed for railway networks. Two main types of methods, network topological analysis and network flow-based analysis, have been developed. Both approaches are constructed based on macroscopic models which take only some railway properties such as network structure, train and passenger flow into account. Thus, the results obtained are high level approximations. This study proposes a new analysis method which is developed based on the stochastic-microscopic railway network simulation model. The method can be applied to identify the critical sections of a railway network. The effect of impact levels and occurrence times of a disruption on the network section criticality are presented. An application of the proposed model is demonstrated using the Liverpool railway network in the UK.

Keywords: Vulnerability analysis, Risk-based criticality, Simulation model, Railway network.

1 Introduction

Railways are considered as the backbone of the transportation system in various countries [1]. However, disruptive events, either planned or unplanned disruptions, can lead to a reduction in system performance, such as delays and service cancellations. In the UK, these events cause approximately 640 train journey cancellations and significant delays (more than 30 minutes) daily, and the percentage of this number to the total number of planned train services tends to increase every year as can be seen in Figure 1 [2].

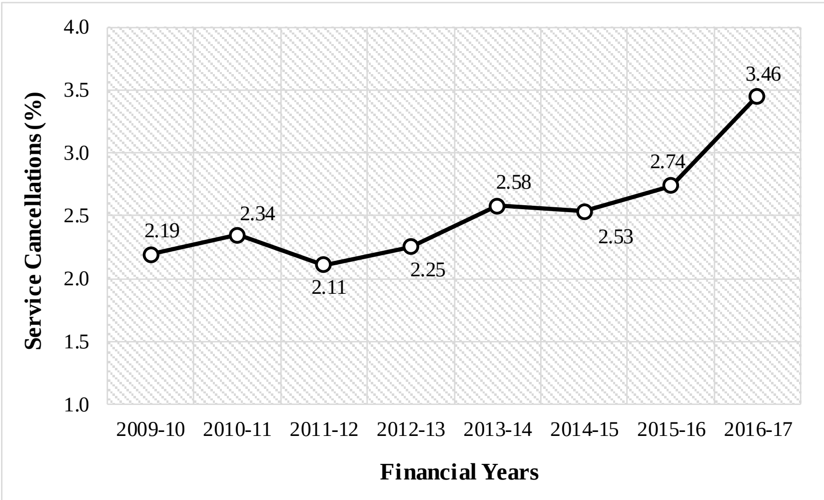


Figure 1 – The percentage of service cancellations of the last 8 financial years [2].

To mitigate this problem, improving the robustness of a railway network (RN) is a key to withstand the impact of disruptions. However, changes to the network are limited through physical barriers (e.g. restricted access) and budget constraints. Infrastructure managers need to decide on the effectiveness and priority of any improvements or added protection. This creates a need to identify the critical sections of a RN to prioritise enhancement features to be incorporated into the network.

This paper presents an analysis method for identifying the critical sections of a RN. A critical section is defined as a railway link connecting between stations, junctions, or a station and a junction, that if it fails, causes large negative impacts on the network [3]. The method is developed based on a microscopic railway network simulation model to evaluate the network vulnerability [4]. This enables the significant characteristics of a RN (e.g. train movement and passenger behaviour) to be considered and allows the effect of the different impact durations and the occurrence times of a disruption on the network section criticality to be analysed. The method also considers the type and the likelihood of disruptive events. Thus, the risk-based criticality of network sections can be evaluated.

The next section provides an overview of the relevant literature and highlights the research gap. Then, the following sections describe the modelling framework and its application using the real-world case study.

2 Railway network capability modelling

2.1 The concepts of robustness, vulnerability and criticality

Robustness is defined as the ability of a system to resist the impacts of disruptions [5]. Even when exposed to disruptions, a highly robust system can continue functioning without the need for adaptions or implementations of mitigation strategies [6]. The identification of critical components can provide system managers with significant information to improve the robustness of a system (e.g. adding redundancy and increasing system tolerance) [3].

In contrast to the robustness concept [7], vulnerability is defined as the degree of negative consequences for a system suffering from a disruption [8]. Negative consequences can be expressed in terms of structural damage (e.g. length of track failure) or functional losses (e.g. train delays) [9]. This study focuses on the functional losses to a RN. The higher the losses, the greater the vulnerability.

Finally, criticality refers to the importance of components to a system. Criticality directly relates to both the vulnerability of a system and the probability of component disruptions [10]. If the probability of a component disruption is high and its consequences are substantial, the component is critical. Nevertheless, if the occurrence probability of a component disruption is very low, the criticality can be examined with only the vulnerability of a system [11].

2.2 An overview of critical component analysis methods for railway networks

In recent years, various analysis methods for identifying the critical components (track sections and stations) of RNs have been proposed and can be classified into two main groups:

2.2.1 Network topological analysis methods

Network topological analysis methods apply the concept of complex network theory to assess the vulnerability of railway systems. The methods define a RN as a set of nodes and links. Nodes represent stations and junctions, while links represent track sections in a network. Disruptions of these components are simulated by removing them from the network. The indicators predicted are, for example, global efficiency (i.e. the ability of a network to transfer the flow), average path length (i.e. the average value of all path lengths between all pairs of nodes) and maximal connected subgraph (i.e. the maximum number of connected nodes in the network after disruptions) [12] - [13]. These indicators are suitable for a large-scale network because they can provide a quick solution from the structural point of view to robustness improvement [14]. Recent research has attempted to combine complex network theory with some characteristics of RNs. Yin et al. [15] adapted the betweenness concept, which refers to the number of shortest paths between all pairs of nodes that pass a specified node or link, to identify the importance of stations and railway lines. The study created a more realistic measure by taking the number of passengers on the shortest path into account.

2.2.2 Network flow-based methods

Network flow-based methods consider the functional vulnerability of a RN. The network is still modelled as a set of nodes and links. However, more system characteristics (e.g. the number of passengers and trains) are considered. This enables the performance of a RN during a disruption to be predicted and used to provide the vulnerability indicators. For example, Ouyang et al. [16] constructed a train flow model by considering the number of trains operating daily on each link. Two performance metrics: the number of stations that could be reached from a station and the number of services that could still operate after a station disruption, were calculated for the station criticality analysis. Hong et al. [17] used a Monte-Carlo simulation model to evaluate the impact of a flood on a RN. The model comprised three main parts: generating the flood scenario, evaluating the vulnerability of individual railway links and calculating the vulnerability of the whole network. The method proposed by Ouyang et al. [16] was applied to evaluate the performance indicator (the number of cancelled trains).

Other studies were based on passenger flow models. For instance, an application of an all-or-nothing approach was used to predict passenger demand on each railway link after a disruption [18]. Two indicators proposed were: the number of passengers that are unable to reach their destinations and the increase in passenger travel time during a disruption (link removal). Zhang et al. [19] built a gravity model to predict the movement of passengers between stations for evaluating the criticality of railway links. This model was constructed based on the number of passengers, the distance between stations and economic factors such as the gross domestic products. Sun et al. [20] investigated station criticality using the passenger Origin and Destination (O-D) ratio. Some passenger movement characteristics such as interchange times and route selection based on the shortest travel time were considered. The indicators were the total passenger flow volume after a station disruption and the passenger volume at station during a certain period.

Another vulnerability model was introduced by Pant et al. [21]. Timetable information was considered to estimate passenger trips lost during a disruption. The interdependency between a railway system and other infrastructures, such as electricity substations and telecommunication equipment, was analysed. Disruption types considered were the failure of the different infrastructures and the flood likelihood in the UK. M'Cleod et al. [22] estimated passenger delays as an indicator. A RN was modelled using a directed graph weighted by travel time on links and waiting time and transfer time at stations. When a node disruption occurred, the number of passengers between each pair of nodes was estimated using a multinomial logit discrete choice model. Then, the criticality of a station was calculated by the product of the sum of delays and the number of passengers on all routes.

2.3 Discussion

The concepts of robustness, vulnerability and criticality are connected and are extensively applied for identifying the critical sections and stations of RNs. Network topological and network flow-based analysis methods are commonly developed to evaluate the vulnerability of RNs during a disruption. Both methods are constructed based on macroscopic models. A RN is considered as a set of nodes and links. Disruptive events are simulated using the node and link removal approach. The time of occurrence and duration of disruptions are not incorporated into the analysis. In particular, the network topological analysis studies only use complex network theory to provide indicators. Their results do not reflect the functional vulnerability of RNs. Hence, they might be too simplistic to support actual policy actions [14]. Although the network flow-based methods attempt to overcome the limitations of network topological analysis methods by considering more railway properties such as train and passenger flow, the results obtained disregard other significant characteristics of RNs such as the conditions of train movement and passenger behaviour when a disruption occurs. Therefore, this paper aims to introduce an analysis method using a microscopic railway network simulation model. This modelling approach is able to predict the functional vulnerability of RNs to reflect the consequences of a disruption as in reality and identify the critical sections of RNs in terms of risk-based criticality.

3 Critical track section analysis framework

The railway network simulation model by Meesit and Andrews [4] was applied in this study. The model was developed using a stochastic-discrete event simulation technique. The significant properties of a RN and the uncertainty of model parameters, such as the duration of disruptions and the number of passenger arrivals at stations, were considered. The model can be used to predict the performance of RNs during disruptions. Thus, this paper intends to use the capability of this model to identify the critical sections of RNs in terms of risk-based criticality. For the methodology produced, the critical sections of a RN can be investigated using four main steps: assigning a disruption to a section, assessing the vulnerability, evaluating the risk-based criticality and ranking the critical sections of a RN.

3.1 Assigning a disruption to track sections

The first step is to assign a disruptive event to each tested track section one at a time. A disruptive event can be simulated by setting the occurrence time and the impact duration of a disruption and changing the state of the tested section to '*unavailable'*. This generates a line blockage for a specific period affecting the movement conditions of trains. Then, the vulnerability of a RN due to the section disruption is calculated using the railway network simulation model. Basically, the use of a short or a large-impact disruption can be considered. The impact of the different occurrence times of a disruption on the criticality of network sections can be analysed, using the capability of the microscopic model.

3.2 Assessing the vulnerability of a railway network

3.2.1 Train service vulnerability

Train service vulnerability takes both train delay and cancellation into account. Train delay (*TD*) is calculated by the summation of the difference between the actual arrival time (*TAT*) and the expected planned arrival time (*TPT*) of all trains (***T***) at all stations (***S***), Equation (1).

$$TD = \sum_{s}^{\boldsymbol{S}} \sum_{t}^{\boldsymbol{T}} (TAT_{s,t} - TPT_{s,t}) \tag{1}$$

Train service cancellations (*TC*) are necessary when a high impact disruption happens in order to prevent the propagation of delays throughout the network. For example (Figure 2), after the system recovers, the trains at terminal stations (trains *T1* and *T3*) take the next service based on the original timetable and the missed services are counted as cancellations. Meanwhile, the trains facing a disruption (*T2*) will continue their services as soon as possible (considered as delayed trains). In this study, train service cancellations are presented as the number of departure services that needs to be cancelled at all stations along the route (*DSC*). In this example, there are 6 full-service cancellations (3 *DSCs* per 1 full service). Thus, the total *DSC* is equal to 18 services.

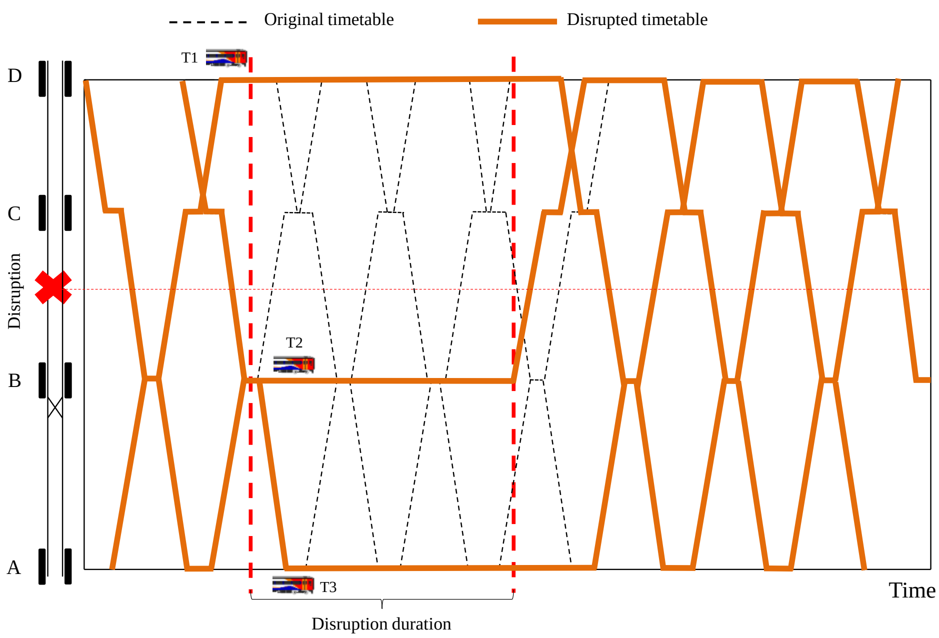


Figure 2 – Train cancellation rule set in the model.

The train service vulnerability (V_T) can then be calculated by a weighted summation of the train delay (*TD*) and the number of departure service cancellations (*DSCs*) on each route (r), Equation (2). The weighting factors in the equation, ω_r^{td} and ω_r^{dsc} represent the penalty for delay and cancellation of train services on each route. These factors can be specified based on the type of service routes (e.g. the penalty for cancelling inter-city trains should be higher than that of local train services).

$$V_T = \sum_{r}^{R} (\omega_r^{td} \cdot TD_r + \omega_r^{dsc} \cdot DSC_r) \tag{2}$$

3.2.2 Passenger vulnerability

Passenger vulnerability (V_p) describes the vulnerability of a RN from the perspective of rail users. It can be calculated by a weighted summation of the total passenger delay (*PD*) and the number of passenger journey cancellations (*PC*), see Equation (3). The weighting factors, ω_{pd} and ω_{pc} are the delay and cancellation penalty of passengers in the network respectively.

$$V_P = \omega_{pd} \cdot PD + \omega_{pc} \cdot PC \tag{3}$$

Passenger delay (*PD*) can be determined by the summation of the difference between the actual arrival time (*PAT*) and the expected arrival time (*PET*) of all passengers (***P***) at their destination stations, Equation (4). Passenger journey cancellations (*PC*) are defined as the number of passengers who cancel their journeys when the expected travel time during a disruption exceeds the defined threshold. This threshold can be set based on the experience of train operators. However, in this study, we assumed that the acceptable delay for all passengers in the network follows a Normal distribution with mean of 3,600 seconds and 300 seconds standard deviation.

$$PD = \sum_{p}^{\boldsymbol{P}} (PAT_p - PET_p) \tag{4}$$

After assessing the vulnerability of all testing sections, the highest vulnerability of a section can be normalised to 1. Then, the vulnerability of other sections can be determined as its proportion of the highest value. This method enables the vulnerability of track sections to be presented in the range of 0 to 1, which is easier to compare the criticality.

3.3 Evaluating risk-based criticality

Risk-based criticality (*RC*) is determined by the product of the vulnerability (V_T *or* V_P) and the frequency (F_D^S) or the probability (P_D^S) of section disruption, Equation (5). The vulnerability can be evaluated as described previously, while the frequency/probability of section disruption can be obtained from historical data. This data can be specific to events of interest such as flooding or landslides, if the focus is on a particular disruption type, or it can be the overall frequency/probability of all disruptive events on the track section.

$$RC = V_T \text{ or } V_P \times \ F_D^S \text{ or } P_D^S \tag{5}$$

In Table 1, there are three main causes that can potentially lead to a disruption of a railway section: technical failures, natural disasters and man-made disasters. Technical failures refer to the failure of sub-systems or the absence of essential support systems. The failure of sub-systems (e.g. control systems and railway infrastructures) is mainly due to aging components. The absence of essential support systems relates to the failure or inoperability of systems such as electrical powers or telecommunications. This type of disruption might not only affect a particular railway link but tends to impact a widespread area on a RN. The second cause of a disruption is natural disasters. Basically, natural disasters are considered as rare events. However, when they happen, their impacts are substantial. Three common events in the UK are taken into account: floods, landslides and strong winds. Finally, man-made disasters describe the disruptions caused by human actions. Three issues are considered: accidents, trespasses/suicides and terrorist attacks. Accidents, such as derailments and bridge strikes, are mostly unintentional events, and the others generally are intentional events that have a low chance of occurrence but can be considered in the list.

Table 1 – The causes of a railway section disruption.

Consequence	**Main causes**	**Sub-causes**	**Descriptions**
Railway section disruption	Technical failures (TF)	Control system failure (CF)	A: Signal components fail (e.g. signals, axle counters, track circuits and other lineside equipment).
		Electrification system failure (EF)	B: Electrification components fail (e.g. overhead line or third rail equipment faults).
		Infrastructure failure (IF)	C: Track failures (e.g. rails, fastening components, sleepers and foundations).
			D: Point failures (switches and crossings).
			E: Level crossing failures.
			F: Bridge collapses.
			G: Tunnel collapses.
		Dependant system failure (DF)	H: Power supply shortage.
			I: Telecommunications failures.
			J: Other related systems such as water networks.
	Natural disasters (ND)	Landslides (LS)	K: Relate to earthworks and embankment failures.
		Flood events (FE)	L: E.g. flood risk from the sea and rivers or surface water.
		Strong winds (SW)	M: E.g. leaves on the track or trees fall.
		Other events such as huge wave and snow (OE)	N: Might be common in some parts of the network.
	Man-made disruptions (MD)	Accidents (AC)	O: E.g. derailments, bridge strikes, and collisions.
		Terrorist attacks (TA)	P: Rare but can create substantial impacts on a railway network.
		Trespasses and suicides (TS)	Q: Unpredictable, but their impacts are significant.

3.4 Prioritising the critical sections

Risk-based criticality is applied to prioritise the criticality of railway network sections. Sections with a higher predicted *RC* are used to prioritise the robustness improvement actions. The detail of the improvements, such as enhancement features and cost, can be obtained by considering the causes and the frequency of the disruptions on each critical section. For example, if the cause of the disruptions is vandalisms and trespasses, the strategies for both preventive, such as creating safety campaigns to make local people aware of the dangers of trespass, and corrective enhancements such as installing fences can be established to increase the robustness of the RN.

3.5 Simulation procedure

The simulation procedure of the critical track section analysis model is presented in Figure 3. The process begins with the data loading step. Three new data sets, apart from the data for the RN performance model, are needed. These data sets include: a test disruption, a list of test track sections and the frequency or the probability of the section disruption as described in Section 3.3. At the next step, the section counter (*SN*) is initialised to "*1*", and the first section on the list is considered by generating a disruption on the section and evaluating the vulnerability of the RN. After the vulnerability assessment step is completed, the RN performance prediction model is initialised, and the section counter is compared to the total number of test sections (*TSN*) on the list. If $SN < TSN$, *SN* is increased by one and the process is repeated by considering the next section on the list. However, if the condition is false, the vulnerability of all sections is normalised and used in the risk-based criticality calculation. Then, the results obtained will be prioritised and transformed to the robustness improvement policy.

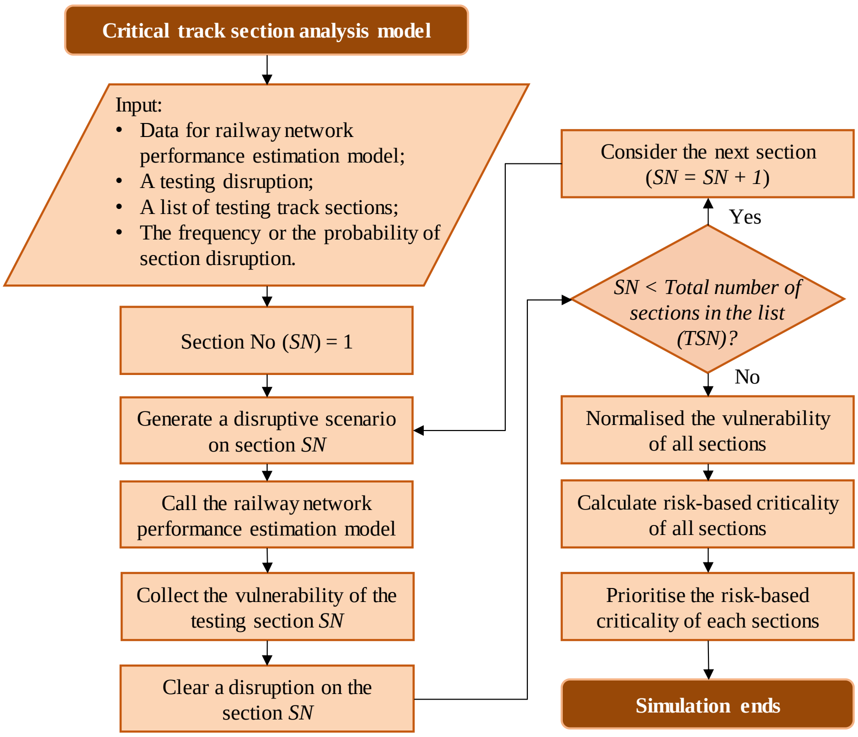


Figure 3 – Simulation procedure of the critical track section analysis model.

4 Application of the Proposed Model

4.1 Case study

The Liverpool railway network was selected as a case study (Figure 4). This network is an electrified system (third rail, 750V DC), serving approximately 110,000 passengers daily. The network has 67 stations, 72 links (ID0 to ID71) and 4 main junctions (J0 to J3). The total length of this network is approximately 120 km. On this network, two railway lines: the Northern line and the Wirral line, are operated daily from 6:00 to 24:00. The Northern line is represented by a thick line. This line offers three service routes: Southport to Hunts Cross (R0), Ormskirk to Liverpool Central (R1) and Kirkby to Liverpool Central (R2). The first route is operated with 4 trains per hour throughout the whole day. The other two routes are operated with 4 trains per hours until evening (19:00), when the frequency reduces to 2 trains per hour and remains at this point until the end of operation. The Wirral line (thin line) extends the network to four terminus stations: Ellesmere Port (R3), Chester (R4), West Kirkby (R5) and New Brighton (R6). The train services from these terminus stations run to the Liverpool Central station and return to their terminus stations using the single-track underground loop tunnel. Route R3 is operated with 2 trains per hour, and the service patterns of the other routes are the same as Routes 1 and 2 on the Northern line. All trains on the network are the British rail class 507/508 (3 coaches), and they stop at every intermediate station along their routes. The timetable of each service route was obtained from the Merseyrail timetable [23].

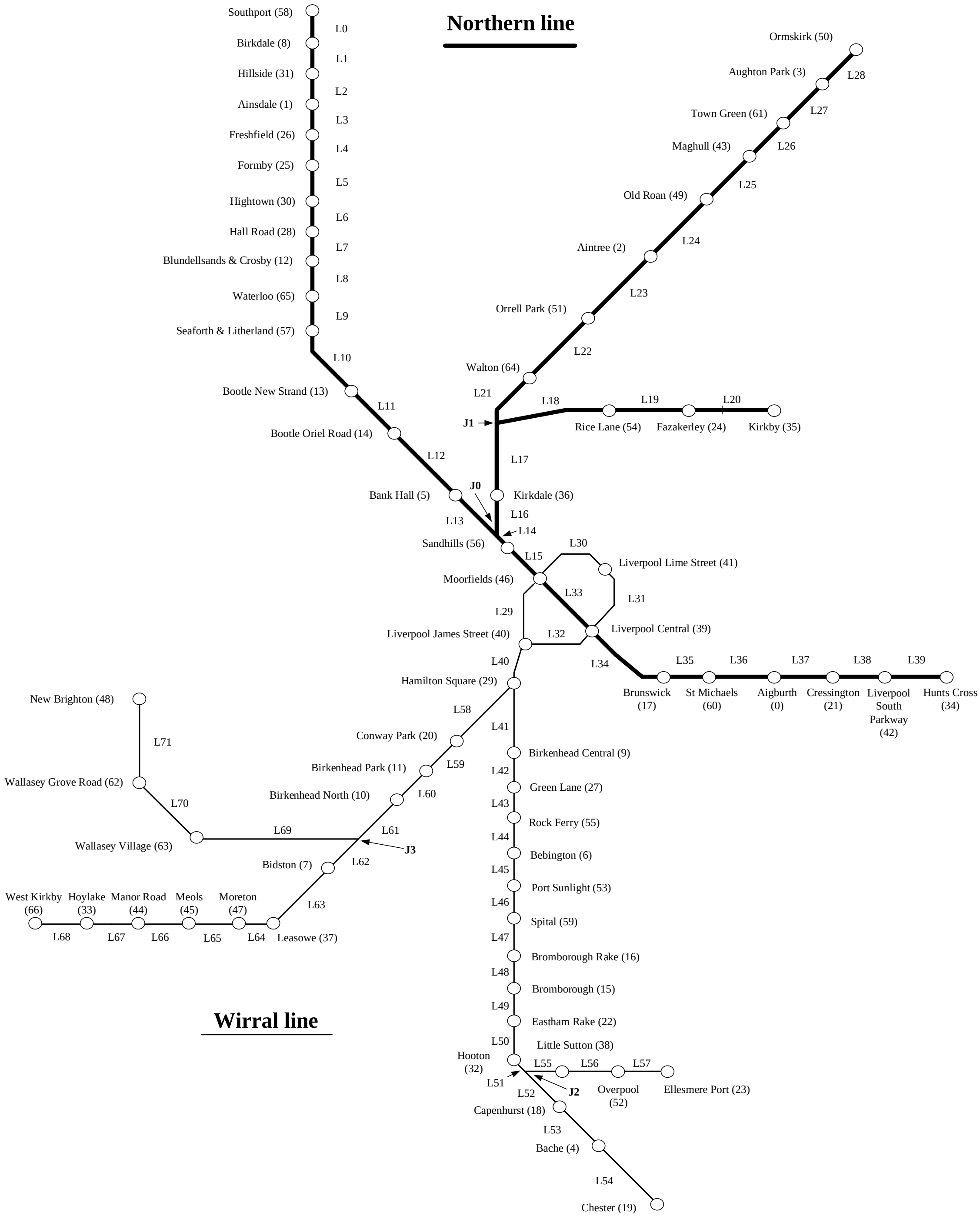
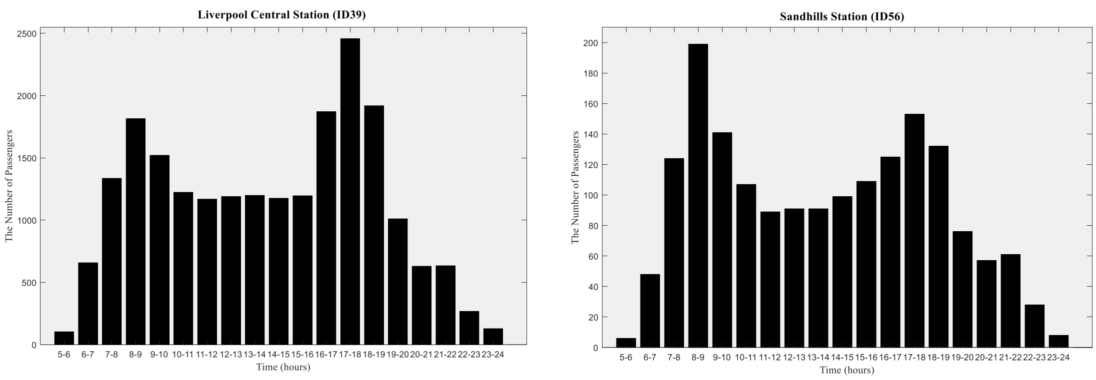


Figure 4 – The Liverpool railway network.

4.2 Passenger data

Since real passenger data was not available for the reason of commercial sensitivity, estimated passenger data was based on the Office of Rail and Road's station usage dataset [24]. The passenger arrival rate at each station was determined by solving the proportion of the daily number of passengers at a station regarding to the times of the day. The percentage of passenger arrival rates at peak hours (7:00-10:00 and 16:00-19:00) was set to be higher than that of at off-peak hours by approximately 50 to 60 percent. Examples of the number of passenger arrivals at Liverpool central and Sandhills station from a simulation are depicted in Figures 5*(a)* and *(b)*.



a) Liverpool central station (ID39) *b) Sandhills station (ID56)*

Figure 5 – The number of passenger arrivals at Liverpool central (a) and Sandhills station (b) results from a simulation.

For the passenger Origin-Destination (O-D) matrix, a gravity model was applied to predict the passenger flow between stations ($N_{i,j}$), Equation (6). The daily number of passengers using a station, both entries and exits, was considered as the number of trips produced and attracted by a station. For some terminus stations, such as Liverpool Lime Street, which have a connection with other networks, the number of entries and exits cannot be directly used in the model. This is because passengers who enter, exit or change a train at these stations might not travel to or from the stations in the network. Thus, the trip production (P) and trip attraction (A) of these stations were obtained by assuming the proportion of the number of entries (*EN*), exits (*EX*) and interchanges made (*IC*) at the stations as present in Equations (9) and (10).

Friction factors ($F_{i,j}$) indicates the impact of travel time on the trips made between stations. Passengers are more likely to travel to a station when the friction factor between origin and destination station is high. In this study, the friction factor was assumed based on the study of Hartholt [25] to demonstrate the model. In Figure 6, the friction factor varies directly with the travel time until the point where most passengers are willing to travel on the network (set to 20 minutes). After that it decreases exponentially as the travel time increases.

After calculating the passenger flow between each pair of stations, the total trip production and trip attraction of each station was calculated using Equations (7) and (8). The results obtained were compared with the actual trip production and trip attraction of each station. If the computed numbers did not match with the actual numbers - the difference was more than 1%, factoring was applied to adjust the values in the matrix. This was performed by determining the error ratio of each station (i.e. the actual number divided by the computed number of trip production/attraction) and multiplying it by all trips in each row (in the case of trip production) or each column (in the case of trip attraction) in the matrix. This process was repeated until a converged solution was achieved. Finally, the number of trips between each pair of stations was transformed into a percentage and used in the simulation to distribute passengers in the network.

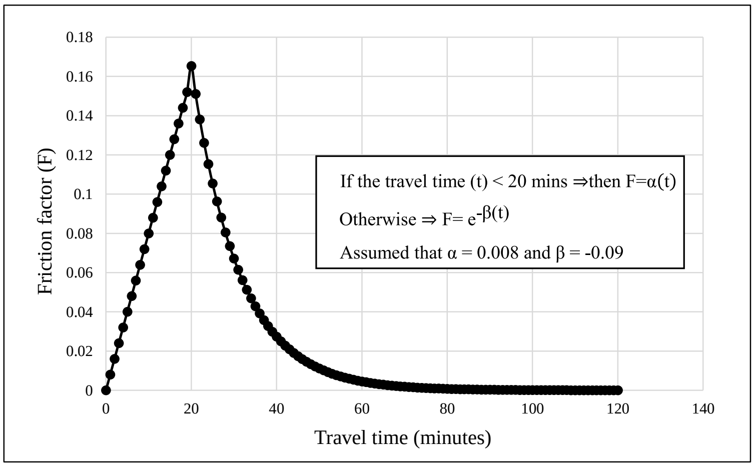


Figure 6 – The relationship between friction factor and travel time used in the model.

$$N_{i,j} = P_i \frac{A_j F_{i,j}}{\sum_j A_j F_{i,j}} \tag{6}$$

$$P_i = \sum_j N_{i,j} \tag{7}$$

$$A_j = \sum_i N_{i,j} \tag{8}$$

$$P_i = EN_i(a_i) + IC_i(b_i) \tag{9}$$

$$A_i = EX_i(c_i) + IC_i(d_i) \tag{10}$$

where: a_i and c_i are the proportional number of passengers who enter and exit the network at station i respectively. Meanwhile, b_i and d_i are the proportional number of passengers who make an interchange at station i to travel to the other stations inside and outside the network, respectively. These parameters were reasonably assumed for the stations that have a connection to other networks as shown in Table 2.

Table 2 – Parameter assumptions for calculating trip production and trip attraction.

Stations	**Parameter assumptions**			
	a	*b*	*c*	*d*
Bidston (ID7)	0.7	0.5	0.7	0.5
Chester (ID19)	0.2	0.2	0.2	0.2
Ellesmere Port (ID23)	0.5	0.5	0.5	0.5
Hunts Cross (ID34)	0.5	0.5	0.5	0.5
Kirkby (ID35)	0.6	0.5	0.6	0.5
Liverpool Lime Street (ID41)	0.2	0.2	0.2	0.2
Liverpool south parkway (ID42)	0.4	0.2	0.4	0.2
Ormskirk (ID50)	0.6	0.5	0.6	0.5
Southport (ID58)	0.5	0.5	0.5	0.5

4.3 Results

The examples of railway link criticality analysis are presented. The computational experiments were conducted using a computer with Intel i7 processor, CPU at 2.60 GHz and 16 GB of RAM running on Window 10. Regarding the stochastic behaviour of the model, the vulnerability of each railway link was expressed in terms of average vulnerability from the results of 1,000 simulations, after which the statistics have converged.

4.3.1 Testing with a single railway link

As described in Section 3.1, the test disruption needs to be assigned to each link on the network one at a time to identify the criticality of railway links. However, due to the long distance of railway links, the location of a disruption on a railway link is important. Different locations might lead to different levels of railway link criticality. Thus, this issue needs to be addressed before testing all the links of the network. In this study, the railway link between Ainsdale (ID1) and Freshfield station (ID26) was selected as an example. This link comprises nine pairs of track sections (S0-S8). The length of each section is approximately 500 meters (Figure 7).

In this experiment, four different impact durations were used, including the recovery times of 15, 30, 60 and 120 minutes (assumed to follow a Uniform distribution on the interval of -10% and +10%). Six different occurrence times at both peak and off-peak hours of the disruptions were considered. These consist of 8:00, 8:05, 8:10, 12:00, 12:05 and 12:10. Then, the disruptions defined were assigned to each pair of track sections to create a blockage on the railway link. After that, the impacts on train services and passengers were estimated using Equations (2) and (3). The weighting factors set in the test were 1, 100, 1 and 60 for ω_r^{td}, ω_r^{dsc}, ω_{pd} and ω_{pc}, respectively. The results obtained from the simulation are presented in Tables 3 and 4.

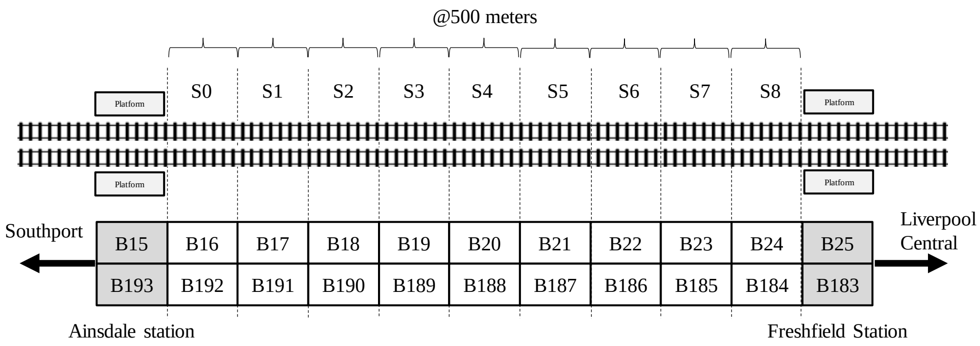


Figure 7 – An example of a railway link and its track sections used in the test.

It is obvious that the vulnerability of each track section varies directly with the duration of the disruptions. The larger impact duration, the greater vulnerability. Moreover, the occurrence time of the disruption has also a significant impact on the vulnerability of track sections, especially in the different periods of the day. In this case, the occurrence of a disruption at the peak hours causes higher consequences, especially for passenger vulnerability, compared to that of at the off-peak hours wherever the disruption was on the link. However, for the train service vulnerability, the consequences predicted seem to be the same at both peak and off-peak. This might be because there is no significant difference in the timetable of for these periods in the network.

In detail, both train and passenger indicators provide the similar trend of the results that the criticality of the example railway link needs to be calculated from the vulnerability of all sections. This is because the vulnerability of each section of the link fluctuated when the occurrence time and the location of the disruption were changed. A disruption that occurs at a specific times and locations created different effects to train services depending on the timetable. This phenomenon will happen with every railway link in the network due to the schedule-based nature of the railway operation. Therefore, to represent the vulnerability of the railway link, the average vulnerability of all sections of the link tested by different disruption occurrence times can be applied to perform the railway link criticality analysis. This hypothesis will be clarified in the next section.

Table 3 – Train service vulnerability of the tested single railway link.

Periods of the day	**Occurrence times**	**Disruption durations**	**Train service vulnerability (weighting factor, ω_r^{td} = 1 and ω_r^{dsc} = 100)**								
			Section IDs								
			0	1	2	3	4	5	6	7	8
Peak hours	8:00	15	4500	4500	4500	4500	4500	4500	4500	4500	4500
		30	9300	9300	9400	9400	9500	9300	9500	9400	9300
		60	19000	19100	19000	19000	19000	19300	19000	19100	19000
		120	38800	38600	38700	39000	38600	38900	38800	38900	39000
	8:05	15	5800	5500	5900	5800	5500	5900	5500	5800	5600
		30	11000	10900	11200	10700	11100	11200	11200	11000	10900
		60	20700	20900	20800	20700	21000	20700	20500	20900	20500
		120	40500	40200	40400	40500	40500	40000	40000	40500	40400
	8:10	15	4600	4500	4500	4500	4800	4800	4800	4800	4700
		30	9900	9700	9800	9700	10300	10400	10200	10100	10400
		60	18800	19000	18800	19000	20200	20000	20000	19800	20000
		120	37800	38000	38000	38000	40600	40000	40100	39800	40700
Off-peak hours	12:00	15	4500	4500	4500	4500	4500	4500	4500	4500	4500
		30	9400	9300	9300	9700	9300	9400	9300	9500	9300
		60	19100	19100	19300	19100	19100	19000	19100	19100	19100
		120	39000	39300	39300	38800	39100	38900	38800	39000	38900
	12:05	15	5600	5500	5700	5600	6100	5700	5600	5900	5500
		30	11100	11200	11000	11400	10900	11100	10900	10800	11100
		60	20400	20700	20600	20700	20200	20900	20200	20400	21200
		120	40200	40600	40400	40900	40500	40200	40700	41000	40600
	12:10	15	4700	4800	4500	4500	4800	4800	4800	4800	4900
		30	10000	9800	9700	9600	10200	10500	10200	10200	10400
		60	18800	18900	19100	18900	19800	20100	19800	20200	20100
		120	38200	38500	37900	38300	40100	40500	40400	39900	40700

*The values in the table were rounded to the nearest hundred.

Table 4 – Passenger vulnerability of the tested single railway link.

Periods of the day	**Occurrence times**	**Disruption durations**	**Passenger vulnerability (weighting factor, ω_{pd} = 1 and ω_{pc} = 60)**								
			Section IDs								
			0	1	2	3	4	5	6	7	8
Peak hours	8:00	15	12300	12700	13100	12800	13000	13100	13500	13500	13600
		30	43400	42400	43100	45000	44900	43900	45300	43900	44400
		60	149500	152100	152000	150800	150200	152600	150400	152500	150900
		120	312500	310800	312500	313400	311900	312600	311300	311700	313000
	8:05	15	18700	17300	19400	19100	17900	19200	17800	19300	18300
		30	44400	59000	58800	61200	56400	59800	60500	60400	58400
		60	168000	167200	166600	169100	166500	165300	167500	166900	151800
		120	329400	328500	329600	330600	329500	325200	324400	329500	328000
	8:10	15	15400	14900	15100	14900	14800	15100	14700	14800	14200
		30	53100	52300	53600	52500	53800	54500	53400	52800	54300
		60	151800	152900	152400	152500	160400	159600	160200	159900	162900
		120	313100	314900	314900	314300	331800	326600	327800	326400	333200
Off-peak hours	12:00	15	7300	7500	7500	7500	7800	7800	7800	7900	7900
		30	23800	23900	24000	25900	24300	24500	24500	24800	24200
		60	88200	88300	89500	88000	87400	88100	87600	87200	86700
		120	222700	225800	225800	221700	223000	222900	219900	222000	221200
	12:05	15	10500	10500	11400	11100	12400	11400	11000	12000	10800
		30	32600	33100	32100	33700	31900	32400	32100	31000	32600
		60	97300	99100	98500	98600	96400	100100	96900	97200	101400
		120	233000	237000	234200	238000	235000	232100	234500	237500	236200
	12:10	15	9200	9500	8900	8700	9200	9100	9100	9000	9300
		30	31800	31200	31000	30400	29600	30300	29400	29200	30300
		60	96000	97100	97900	96600	97400	98500	96200	97700	97500
		120	234500	237200	234200	236200	235300	237700	237100	234600	238600

*The values in the table were rounded to the nearest hundred.

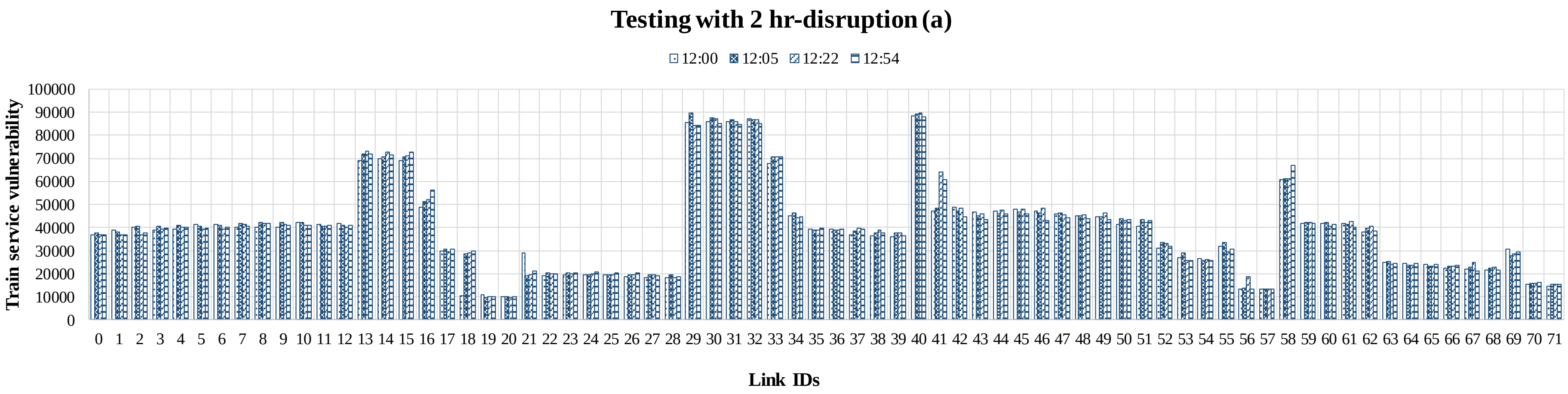
4.3.2 Vulnerability evaluation of the railway links

According to the previous experiment, this experiment takes the effect of disruption durations, disruption occurrence times and disruption locations on the railway links into account. Since the study is more interested in the large impact disruptions, two large impact disruptions with the approximated recovery times of 2 and 3 hours on the interval of -10% and +10% (Uniform distribution) were considered as examples. For the occurrence times of these disruptions, since the test network operated with a cyclic timetable, a certain pattern of train services repeats itself every hour. Thus, the effect of occurrence times can be analysed by considering times within a service hour. In this experiment, four different occurrence times during off-peak hours were randomly analysed, which are: 12:00PM, 12:05PM, 12:22PM and 12:54PM. These disruptions were assigned to each pair of block sections on each railway link on the Liverpool network one at a time. Then, the average vulnerability of the network when the disruption happened was estimated as the criticality indicator of the railway link.

Figures 8 and 9 present the results obtained from the simulations. It seems that the trend of the results predicted from both impact durations is more likely to be the same. Although the vulnerability prediction of each railway link was slightly different when the occurrence time was changed, there was no effect on the rank of the railway link criticality. For the indicators, the results can be explained by considering the links that have similar criticality in the same group. The train service vulnerability and the passenger vulnerability tend to provide the different results for the critical link prediction. For example, the first indicator illustrates that links 29, 30, 31, 32 and 40 were the most critical links in the network. However, the second indicator shows these links were only in the second group. Links 13, 14, 15 and 33 were in the most critical group for this indicator.

One reason why the criticality results from these two indicators were different is due to the nature of trains and passengers. As described, large impact disruptions lead to a blockage of a railway link for a long period of time. This directly affects the train services operating through the disrupted locations. If no mitigation strategy, such as rail replacement bus services, is implemented to keep providing services at some parts of the route, the train services need to be delayed and cancelled to avoid the propagation of delays to non-affected routes. The delay of the train is mainly based on the duration of the disruption and the number of service cancellations is dependent on the frequency of train on the route. For this network, most of the routes are operated with 4 trains per hour. Consequently, the criticality of many railway links tends to be the same as shown in Figures 8*(a)* and *(b)*. The second indicator depends on the number of passengers travelling on each route and inherent passenger behaviour that they have flexibility to travel on the network. Passengers might still be able to travel to their destinations during a severe disruption. However, their journey times will significantly increase due to the limited availability of train services on the network.

Although both indicators give the different results of the railway link criticality, they are still useful to create robustness improvement strategies for RNs. Train service vulnerability could be applied in the case of freight operation networks, while passenger vulnerability might be suitable for passenger-railway networks. Hence, there will be a focus on the passenger indicator in the next section.



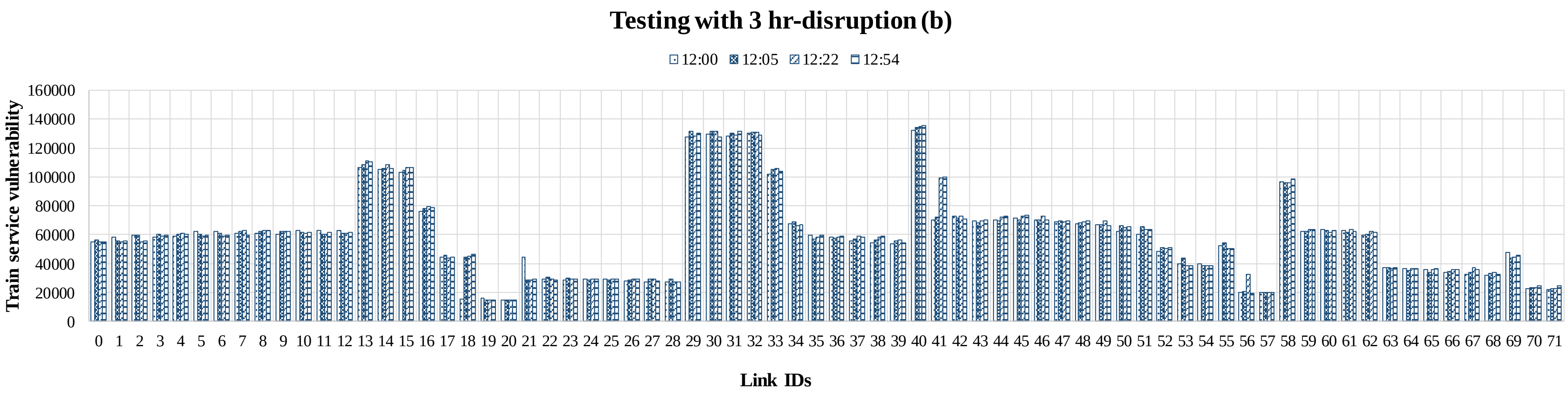
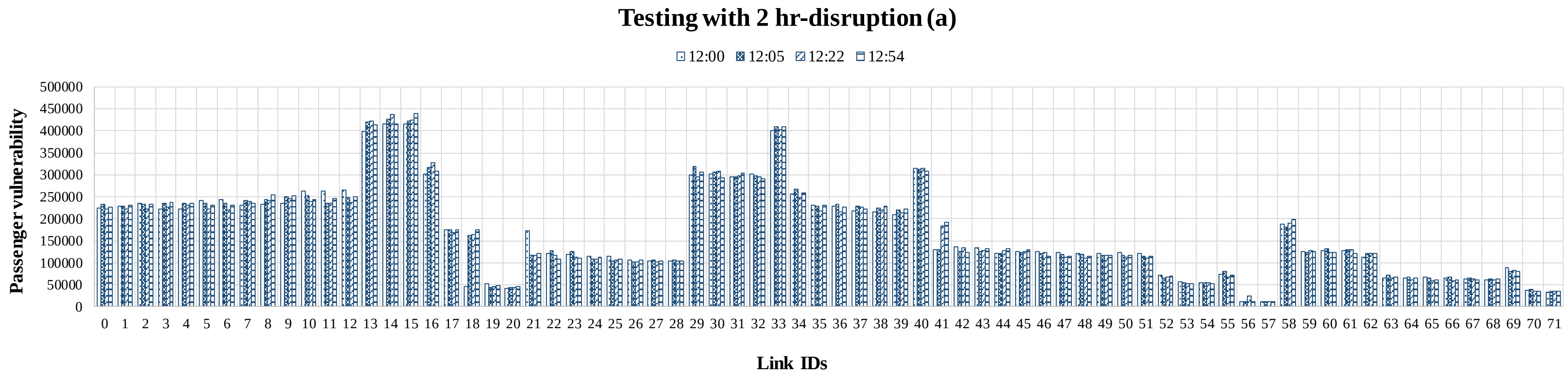


Figure 8 – Train service vulnerability of each railway link testing with 2 hr (a) and 3 hr-disruption (b).



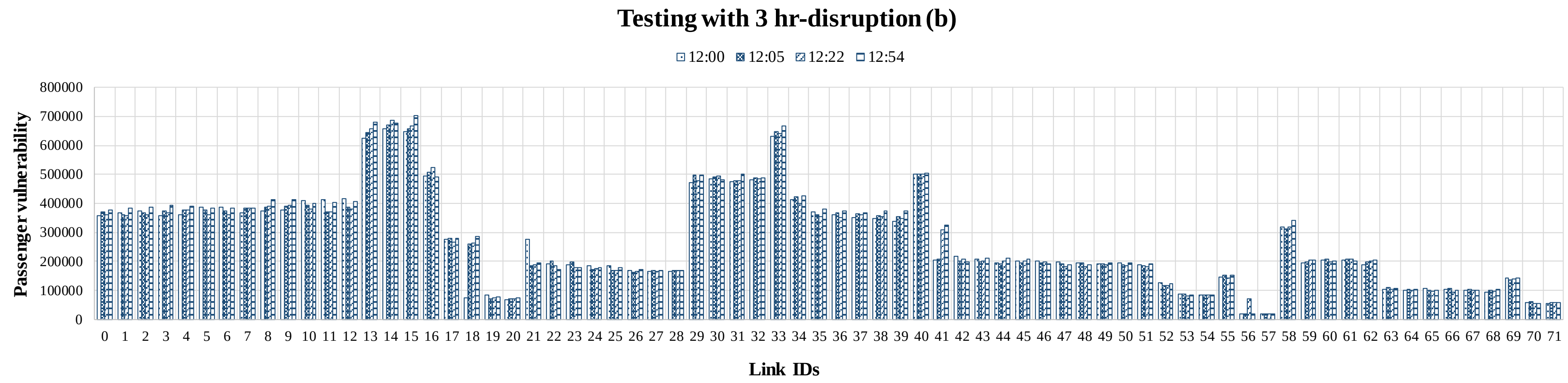


Figure 9 – Passenger vulnerability of each railway link testing with 2 hr (a) and 3 hr-disruption (b).

4.3.3 *Risk-based criticality analysis*

The risk-based criticality of each railway link was evaluated using the product of the normalised vulnerability and the likelihood of the railway link disruptions. The first factor was calculated from the average of the normalised vulnerability of the passenger indicator (both testing with 2 and 3 hour-disruptions, Figure 10). Then, the second factor was separated into two examples: the flood risk and the overall frequency of disruptions.

4.3.3.1 *Flood risk-based criticality*

The flood likelihood from rivers and the sea of the Liverpool network was considered. This data was obtained from the flood risk map provided by the Environment Agency [26]. The map illustrates the likelihood of the railway links exposed to flood by considering any flood defences in the area. The likelihood results (i.e. a chance of flooding in each year) are shown in four categories: high (greater than 3.3%), medium (between 1.0% and 3.3%), low (between 0.1% and 1.0%) and very low (less than 0.1%). In the analysis, these likelihoods were considered as constant values. The first and the last category were assumed to be 3.3% (high) and 0.1 % (very low), and the rest were based on the median of the range which are: 2.15% (medium), 0.55% (low), respectively.

The results obtained are presented in Figure 11. It was found that links 25 and 26, which connect the Old Roan, the Maghull and the Town Green station, were the highest critical links in the network. Their risk exposures were 8.462×10^{-3} and 8.080×10^{-3}, which were approximately 63% higher than the next critical link which is link 63. The least critical links in this network were links 56 and 57, which connect the Little Sutton, the Overpool and the Ellesmere Port station. These railway links have a risk exposure of only 4.0×10^{-5} and 2.6×10^{-5}, respectively. From these results, the limited budget available for enhancing the robustness of the network, such as flood barriers, raised tracks and lineside equipment protection and drainage clearance, would be most effectively directed towards links 25 and 26, and the others could be ranged based on their risk results.

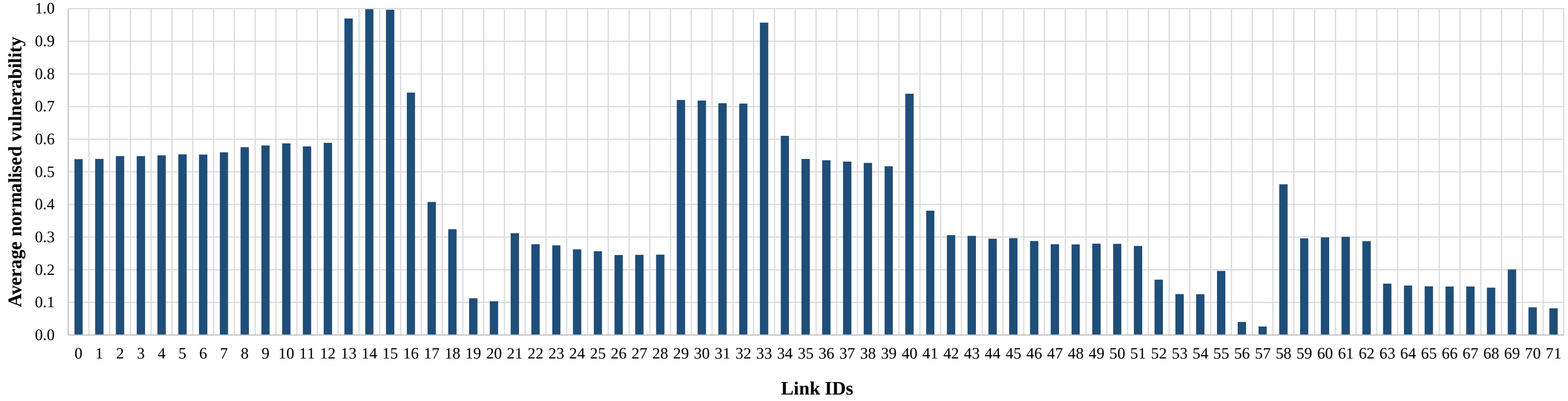


Figure 10 – Average normalised vulnerability from the passenger delays indicator.

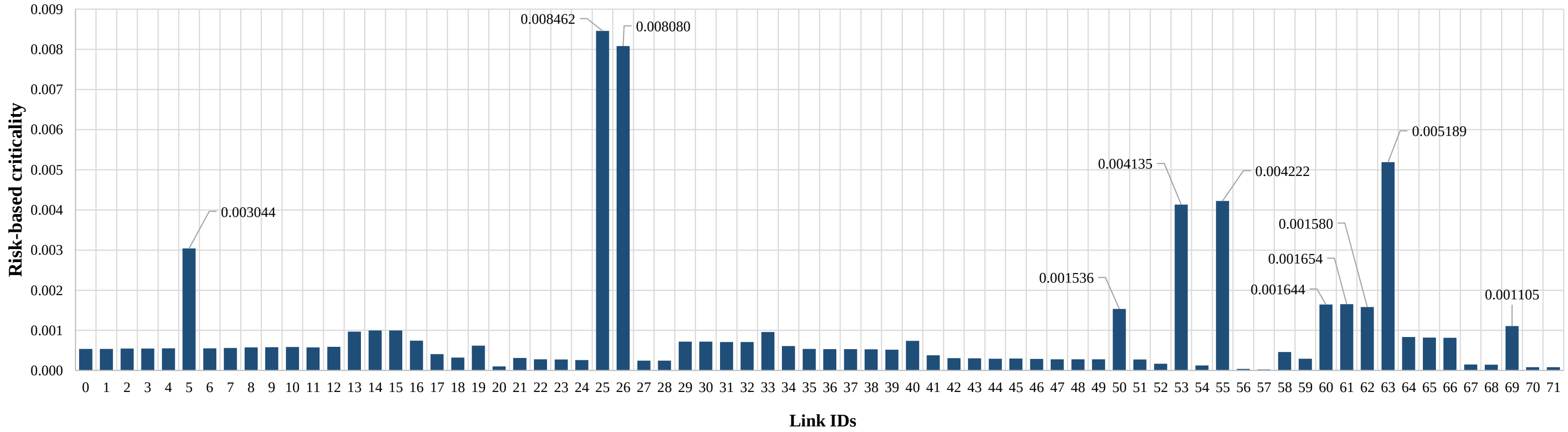


Figure 11 – Flood risk-based criticality of the railway links in the Liverpool railway network.

4.3.3.2 *Overall risk-based criticality*

The criticality of the railway links was predicted using the overall frequency of the railway link disruptions from the past five years (assumed due to the limited availability of the real data). The assumption was made based on the main causes of the disruptions explained in Section 3.3. The existence of the assets on each railway link, such as level crossings, bridges and tunnels, rivers, were taken into account based on the Railway track diagrams: Midlands and North West [27] and the Google map [28]. Only the disruptions affected the network more than 1 hour were considered in the analysis.

The disruption frequency and the risk exposures of the top 10 critical links are presented in Table 5. Link 25, which is between the Old Roan and the Maghull station, is the link with the highest frequency of disruptions in the list (21 times in 5 years). The disruptions of this rail link were found due to all defined causes: technical failures (e.g. signalling failures), natural disasters (e.g. flood and landslide), and man-made disasters (e.g. accident and trespass). The other links on the list have the disruption frequency in the range of 6-13 times in 5 years, and the failures of network components, such as signalling equipment (A), third rail equipment (B), tracks (C) and points (D), seem to be the main cause of the disruptions on these railway links.

In terms of the risk, the order of the critical links in the network is different from the previous analysis. Link 40, which is the tunnel railway link, became the most critical link in the network. Its risk exposure was 9.610, which is approximately 38% higher than the second critical link (link 15). Links 13, 14 (between the junction and the Sandhills station) and 33 (between the Moorfields and the Liverpool Central station) were in the high ranking when only the vulnerability was considered (see Figure 11). However, only link 33 was found in the top 10 (5^{th} place) from this analysis. The least critical link in the list was link 2, which is between Hillside (ID31) and Ainsdale station (ID1). This link has a risk exposure of 3.835.

Infrastructure managers can apply this information to create a plan for the network robustness improvement. The detail of the improvements, such as priority, enhancement features, and cost, can be obtained by considering the causes and the frequency of the disruptions on each critical railway link. For instance, if the cause of the disruptions is the failures of network components such as third rail equipment (links 40, 30 and 33), the strategies for both preventive (e.g. increasing inspection frequency) and corrective enhancements (e.g. replacing aging components) can be established in order to sustainably reduce the vulnerability of the RN.

Table 5 – Top 10 critical links in the network based on the assumption of the disruption frequency.

Consequence	**Main causes**	**Sub-causes**	**Description**	**Railway link ID (Top 10 critical links)**									
				40^{1st}	15^{2nd}	16^{3rd}	30^{4th}	33^{5th}	25^{6th}	0^{7th}	38^{8th}	61^{9th}	2^{10th}
Railway link disruption	TF	CF	A	4	3	3	3	3	8	1	1	3	2
		EF	B	3	1	0	2	2	1	0	1	0	0
		IF	C	3	1	1	1	1	5	2	1	3	2
			D	2	2	3	2	0	0	1	3	6	0
			E	0	0	0	0	0	2	1	0	0	1
			F	0	0	0	0	0	0	0	0	0	0
			G	0	0	0	0	0	0	0	0	0	0
		DF	H	0	0	0	0	0	0	0	1	0	0
			I	1	0	0	0	0	0	1	1	0	1
			J	0	0	0	0	0	0	0	0	0	0
	ND	LS	K	0	0	0	0	0	1	0	0	1	0
		FE	L	0	0	0	0	0	0	0	0	0	0
		SW	M	0	0	1	0	0	1	1	0	0	0
		OE	N	0	0	0	0	0	0	0	0	0	0
	MD	AC	O	0	0	0	0	0	2	1	0	0	0
		TA	P	0	0	0	0	0	0	0	0	0	0
		TS	Q	0	0	0	0	0	1	0	0	0	1
Total disruption frequency				13	7	8	8	6	21	8	8	13	7
Normalised vulnerability				0.739	0.997	0.742	0.718	0.957	0.256	0.538	0.527	0.300	0.548
Risk-based criticality				9.610	6.977	5.941	5.745	5.742	5.385	4.311	4.219	3.910	3.835

*Remark: the codes of the main causes, sub-causes and their descriptions can be refereed to Table 1.

5 Conclusion

A new analysis method for identifying the critical sections of a railway network is introduced. The method is constructed based on a stochastic-microscopic railway network simulation model. The framework of the method consists of four main parts: assigning a disruption, predicting network vulnerability, evaluating the risk-based criticality and prioritising the critical sections of a network.

For the application of this proposed method, the identification of railway link criticality in the Liverpool railway network was presented. The impact of different durations and occurrence times of disruptions were analysed, and the results obtained are useful for the future research in the field of the vulnerability analysis. Moreover, the prediction of the risk-based criticality of the railway links was also performed in this paper. The examples of both flood events and all disruptive events were given along with the interpretation of the results for supporting the establishment of robustness improvement strategies for the railway network. Although the information, such as passenger data and disruption data, is assumed due to the commercial sensitivity, the users of this model are expected to be infrastructure managers. Therefore, the data will be available for them to use in the model.

In the future, the application of the proposed method to investigate the criticality of other network components, such as stations and points, will be considered. The analysis of multiple simultaneous failures of network components will be analysed in order to identify the critical sets of components in railway networks.

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