

1 **Design of Cold-formed High Strength Steel Diamond Bird-beak Tubular**

2 **T- and X-Joints**

3 Madhup Pandey¹; and Ben Young², F.ASCE

4 **Abstract**

5 Numerical investigation and design of cold-formed high strength steel (CFHSS) diamond
6 bird-beak (DBB) T- and X-joints are presented in this paper. The 0.2% proof stress of braces
7 and chords was 960 MPa. Tests of CFHSS DBB T- and X-joints undergoing compression loads
8 were carried out by Pandey and Young (2021a and 2022). The test results were used to develop
9 accurate finite element (FE) models in this study. A comprehensive FE parametric study was
10 then performed using the verified FE models. The nominal strengths predicted from the
11 literature and European code were compared to the joint failure strengths and ultimate capacities
12 of 244 DBB T- and X-joints specimens, including 224 FE specimens investigated in this work.
13 The failure of DBB T- and X-joints specimens at chord crown locations was identified as the
14 dominant failure mode. It has been shown that the design provisions given in the literature and
15 European code are unsuitable and uneconomical for cold-formed S960 steel grade DBB T- and
16 X-joints investigated in this study. Hence, accurate, less dispersed and reliable design equations
17 are proposed in this work, using two design approaches, to predict the joint failure strengths
18 and ultimate capacities of the investigated DBB T- and X-joints.

19 **Keywords:** *Cold-formed steel; Design rules; Diamond bird-beak joints; FE analysis; High*
20 *strength steel; S960 steel, Tubular joints.*

¹ Assistant Professor, Department of Civil Engineering, University of Nottingham, Nottingham, United Kingdom. (Corresponding author) E-mail: madhup.pandey@nottingham.ac.uk

² Professor, Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong, China. E-mail: ben.young@polyu.edu.hk

21 **Introduction**

22 Bird-beak joints are one of the novel configurations of hollow section joints. The diamond
23 bird-beak (DBB) configuration is obtained by rotating the brace and chord members about their
24 respective centroidal axes. In addition to the aesthetic superiority of this configuration, it also
25 brings many other technical advantages, including (a) smooth transfer of load from brace to
26 chord members, which averted the development of bending and buckling in chord member; (b)
27 high stiffness around the brace-chord junction; (c) less hindrance for wind loads; and (d)
28 enhanced ultimate capacities of joints. The practical applications of DBB joints can be seen in
29 the convention centre in Minneapolis (Minnesota, USA), national stadium (Beijing, China) and
30 Takishita bridge (Ibaraki, Japan).

31 High strength steel (HSS) (in this study, referred to steels with steel grades higher than
32 S460) circular, square and rectangular hollow sections (CHS, SHS and RHS) members are in
33 high demand in various civil engineering projects because of their superior strength per unit
34 weight, reduced handling cost and reduced erection time. However, the lack of adequate
35 research work and design recommendations are the primary reasons hampering the widespread
36 use of HSS tubular members. Nonetheless, some studies have recently been conducted to
37 investigate the structural performance of HSS open section members (Wang et al. 2019 and
38 2020), tubular members (Li and Young, 2018 and 2019; Ma et al., 2016, 2017, 2019 and 2021),
39 built-up box section joints (Lan et al. 2019 and 2020), and cold-formed high strength steel
40 (CFHSS) tubular joints (Pandey and Young, 2020, 2021b, 2021c, 2021d and 2021e).

41 The DBB joint configuration was first introduced by Ono et al. (1991), where DBB K-
42 and T-joints made of SHS (hereafter, RHS also represents SHS) were tested to determine their
43 static strengths. Using test results, semi-empirical design equations were proposed to predict
44 the static ultimate capacities of normal strength steel (in this study, referred to steels with steel
45 grades lower than or equal to S460) DBB K- and T-joints. Numerical and analytical methods
46 were used by Davies et al. (1996) and Davies and Kelly (1995) to determine the ultimate
47 capacities of S275 steel grade DBB K-, X- and T-joints. In addition, Kelly (1998) used the
48 numerical method to study the influence of member rotation on the static strengths of S275 steel

49 grade K-, X-, and T-joints. A comparative numerical investigation between conventional (RHS
50 and CHS joints) and DBB X-joints made of S275 steel grade was carried out by Owen et al.
51 (2001). The numerical results obtained after assuming an elasto-plastic material behaviour were
52 used to propose a design equation to predict the static ultimate capacities of S275 steel grade
53 DBB X-joints. The numerical work conducted by Owen et al. (2001) was extended by Peña and
54 Chacón (2014) by investigating the effects of different steel grades (S235, S275 and S460) on
55 the ultimate capacities of DBB X-joints. Subsequently, an improved design equation was
56 proposed for the static ultimate capacities of DBB X-joints with steel grades up to S460. Chen
57 and Wang (2015) performed a detailed numerical parametric study on Q235 steel grade DBB
58 T-joints and proposed a design equation for the static ultimate capacities of the investigated
59 joints.

60 This literature review confirmed that no research is available on CFHSS DBB T- and X-
61 joints, except for the experimental investigations carried out by Pandey and Young (2021a and
62 2022). The test results were used to develop accurate finite element (FE) models of DBB T- and
63 X-joints in this study. Subsequently, a comprehensive FE parametric study, comprising 224 FE
64 analyses, was performed using the verified FE models. The nominal strengths predicted using
65 the equations in the literature (Chen and Wang, 2015; Ono et al., 1991; Peña and Chacón, 2014)
66 and EC3 (2021) were evaluated with respect to the joint failure strengths (N_f) and ultimate
67 capacities (N_{max}) of DBB T- and X-joints test and FE specimens. The existing design rules have
68 been demonstrated to be unsuitable for the range of DBB joints investigated in this work. Hence,
69 using two design approaches, new, economical and reliable design rules are proposed in this
70 work to estimate the N_f and N_{max} of cold-formed S960 steel grade DBB T- and X-joints.

71 **Summary of experimental investigations**

72 The static behaviour of cold-formed S960 steel DBB T- and X-joints were experimentally
73 studied by Pandey and Young (2021a and 2022). Axial compression loads were applied on the
74 DBB T- and X-joints test specimens via braces. The chord ends of DBB T-joint test specimens
75 were supported on rollers through specially fabricated V-shaped end blocks, while braces of
76 DBB X-joint test specimens were fixed at both ends. The braces and chords were made of S960

77 steel grade RHS members. A fully robotic metal active gas welding was employed to weld
 78 braces and chords. In total, 20 tests were conducted, including 10 DBB T-joints and 10 DBB
 79 X-joints. Moreover, chord ends were not welded to end plates and were freely deformed during
 80 the tests. Figs. 1(a) and 1(b) present various notations for DBB T- and X-joints, respectively.
 81 The static strengths of DBB T- and X-joints primarily depend on non-dimensional geometric
 82 ratios, including β' (b'_1/b'_0), 2γ (b_0/t_0) and τ (t_1/t_0). The symbols b , h , t and R stand for cross-
 83 section width, depth, thickness and external corner radius of RHS member, respectively. The
 84 subscripts 0 and 1 denote chord and brace, respectively. In the test programs, β' varied from
 85 0.40 to 0.65, 2γ varied from 25.5 to 39.0 and τ varied from 0.67 to 1.28. The lengths of braces
 86 (L_1) of DBB T- and X-joints were determined as $2\sqrt{b_1^2 + h_1^2}$ mm. On the other hand, the lengths
 87 of chords (L_0) of DBB T- and X-joints were determined as $h_1 + 3h'_0 + 180$ mm and $h_1 + 4h'_0$ mm,
 88 respectively. The symbols b'_1 and h'_1 stand for effective width and depth of brace cross-
 89 section, respectively. On the other hand, the symbols b'_0 and h'_0 stand for effective width and
 90 depth of chord cross-section, respectively. For SHS brace, b'_1 and h'_1 were equal to $\sqrt{b_1^2 + h_1^2}$
 91 $- 0.83R_1$. However, for RHS brace, b'_1 and h'_1 are equal to $2\max[b_1, h_1]\sin\omega - 0.83R_1$ and
 92 $\sqrt{b_1^2 + h_1^2} - 0.83R_1$, respectively. For chord member, b'_0 and h'_0 were equal to $\sqrt{b_0^2 + h_0^2} -$
 93 $0.83R_0$. The measured static 0.2% proof stresses of RHS members varied between 952 to 1059
 94 MPa, while the measured static 0.2% proof stress of welding filler material was 965 MPa. The
 95 chord crown failure (C) mode was identified as the dominant failure mode for all test specimens.
 96 Moreover, the N_f of all test specimens was governed by the ultimate deformation limit (i.e. 0.03
 97 b'_0) criterion. The test results were obtained in the form of N vs u curves, where N and u
 98 respectively stand for static load and chord indentation at the crown location. The typical
 99 member-rotation angles (ω) along the centroidal axis of the brace member are shown in Fig.
 100 1(c).

101 **Numerical investigation**

102 *Finite element models*

103 *General*

104 One of the popular FE software, ABAQUS (2017), was used to perform comprehensive
105 FE analyses in this study. As the induced strains in the FE model during the applied load were
106 unidirectional (i.e. no load reversal), the isotropic strain hardening law was selected for the
107 analysis. The yielding onsets of FE models in this study were based on the von-Mises yield
108 theory. In the FE analyses, the growth of the time step was kept non-linear to reduce the overall
109 computation time. Furthermore, the default Newton-Raphson method was used to find the roots
110 of non-linear equilibrium equations. In addition to the accuracy associated with the Newton-
111 Raphson method, one of the popular benefits of using this numerical technique is its quadratic
112 convergent approach, which in turn significantly increases the convergence rate of non-linear
113 problems. The material non-linearity was considered in the FE models by assigning the
114 measured values of static stress-strain curves of flat and corner regions of RHS members in the
115 plastic material definition part of the FE model. On the other hand, the geometric non-linearities
116 in FE models were considered by enabling the non-linear geometry parameter (*NLGEOM) in
117 ABAQUS (2017), which in turn allow FE models to undergo large displacement during the
118 analyses. Furthermore, various factors, including through-thickness division, contact
119 interactions, mesh seed spacing, corner region extension and element types, were also studied
120 and discussed in the following sub-sections of this paper. The labelling of parametric DBB T-
121 and X-joint FE specimens was kept identical to the label system used in the test programs
122 (Pandey and Young, 2021a and 2022).

123 *Material properties, element type and mesh size*

124 The test specimens of the experimental programs (Pandey and Young, 2021a and 2022)
125 were fabricated from tubular members that belonged to the same batch of tubes used in other
126 investigations conducted by Pandey and Young (2019a, 2019b, 2020, 2021b, 2021c, 2021d and
127 2021e). Additionally, Pandey and Young (2019b) investigated the material properties of welding
128 filler material. The details pertaining to the material properties of welding filler material and

129 tubular members can be referred to Pandey and Young (2019a and 2019b). The inclusions of
130 static stress-strain curves in FE models helped avert the effect of loading rate from FE results.
131 The true stress-strain curves of welding filler material as well as flat and corner portions of RHS
132 members were allocated to the corresponding parts of the FE specimens. In this study, the
133 influence of cold-working in RHS members was included in FE models by assigning wider
134 corner regions. Various distances for corner extension in RHS members were considered in the
135 sensitivity analyses, and finally, the corner portions were elongated by $2t$ into the neighbouring
136 flat portions, which was in agreement with other studies conducted on CFHSS tubular members
137 and joints (Pandey et al. 2021a and 2021b; Liu et al. 2021; Xu et al. 2017). Except for the welds,
138 all other parts of the FE models were developed using the C3D20 element. On the other hand,
139 the C3D10 element was used to model the weld parts due to their complicated shapes. The use
140 of solid elements helped in making realistic fusions between tubular and weld parts of DBB T-
141 and X-joints FE models.

142 Convergence studies were conducted using different mesh sizes, and finally, chord and
143 brace members were seeded at 4 mm and 7 mm intervals, respectively, along both longitudinal
144 and transverse directions. Moreover, the seeding intervals of weld parts reciprocated the seeding
145 spacings of their respective brace parts. In order to ensure the smooth transfer of stresses
146 between the flat portions of the RHS cross-section, the corner portions of the RHS cross-section
147 were split into ten elements. FE analyses were also conducted to examine the influence of
148 divisions along the wall thickness (t) of RHS members. The results of these FE analyses
149 demonstrated the trivial influence of wall thickness divisions on the load vs chord indentation
150 curves of the investigated DBB T- and X-joints. The use of the C3D20 element having one built-
151 in node along the thickness direction as well as the small wall thickness of test specimens (i.e.
152 $t \leq 6$ mm) led to such observations. The presence of a built-in node naturally provides one
153 division along the wall thickness of tubular members (i.e. two layers). It is worth noting that a
154 similar observation was also noticed in other studies (Crockett, 1994; Pandey et al., 2021a and
155 2021b). Thus, for the validations of DBB T- and X-joints FE models, the wall thicknesses of
156 tubular members were kept undivided.

157 ***Modelling of welds and contact interactions***

158 The dihedral angle between brace and chord members of a DBB joint is 120° , as shown
159 in Fig. 2. According to the weld design recommendations given in EC3 (2005), CIDECT (2009)
160 and AWS D1.1M (2020), the fillet weld can be used up to the maximum dihedral angle equal
161 to 120° , therefore, fillet weld was modelled for all FE specimens investigated in this study. The
162 welds were modelled using the average values of measured fillet weld leg sizes. The inclusions
163 of weld geometries and weld material properties appreciably improved the overall accuracies
164 of DBB T- and X-joints FE models. Furthermore, weld component modelling aided in achieving
165 realistic load transfer between brace and chord members. The selection of the C3D10 element
166 maintained optimum stiffness around the joint perimeter due to its ability of taking complicated
167 shapes. A total of two types of contact interactions was defined in DBB T- and X-joints FE
168 models. First, contact interaction between brace and chord members of DBB T- and X-joints
169 FE models. Second, contact interaction between chord members and V-shaped end blocks of
170 DBB T-joint FE models. In addition, a tie constraint was also established between weld and
171 tubular members of DBB T- and X-joints FE models. Both contact interactions were established
172 using the built-in surface-to-surface contact definition. The contact interaction(s) between brace
173 and chord members of DBB T- and X-joints FE models was kept frictionless, while a frictional
174 penalty of 0.3 was imposed on the contact interaction between chord member and V-shaped end
175 blocks of DBB T-joint FE models. Along the normal direction of these two contact interactions,
176 a ‘hard’ contact pressure overclosure was used. In addition, finite sliding was permitted between
177 the interaction surfaces. This technique of fusion between various parts of FE models has been
178 successfully used in several other investigations (Pandey et al., 2021a and 2021b; Hu et al.,
179 2021; Li and Young, 2022a and 2022b).

180 ***Boundary conditions***

181 The boundary conditions in DBB T- and X-joints FE models were assigned by creating
182 reference points. Three reference points were created for the DBB T-joint FE model, including
183 one top reference point (TRP) and two bottom reference points (BRP-1 and BRP-2). The TRP
184 replicated the fixed boundary condition of the top brace end, while BRP-1 and BRP-2 replicated

185 the boundary conditions of the roller positioned at each chord end. As shown in Fig. 3, the TRP
186 was created at the cross-section centre of the top brace end and BRP-1 and BRP-2 were created
187 at 20 mm below the centre of the bottom surfaces of V-shaped end blocks. The TRP, BRP-1 and
188 BRP-2 were then coupled to their corresponding surfaces using a built-in kinematic coupling
189 type.

190 In order to exactly replicate the boundary conditions of the DBB T-joint test setup, all
191 degrees of freedom (DOF) of TRP were restrained. On the other hand, for BRP-1 and BRP-2,
192 except for the translations along the vertical and longitudinal directions of the DBB T-joint FE
193 specimen as well as the rotation about the chord width direction, all other DOF of BRP-1 and
194 BRP-2 were also restrained. In addition, all DOF of other nodes of DBB T-joint FE specimens
195 were kept unrestrained for both rotation and translation. On the other hand, in DBB X-joint FE
196 model, the top and bottom reference points (TRP and BRP) were created at the cross-section
197 centres of braces, as shown in Fig. 4. Subsequently, TRP and BRP were coupled to their
198 respective brace end cross-section surfaces using kinematic coupling type. In order to exactly
199 replicate the boundary conditions of the DBB X-joint test setup, all DOF of TRP were restrained.
200 However, except for the translation along the vertical direction of the DBB X-joint specimen,
201 all other DOF of BRP were also restrained. Moreover, all DOF of other nodes of the DBB X-
202 joint FE specimen were kept unrestrained for both rotation and translation. Using the
203 displacement control method, compression load was then applied at the bottom reference points
204 of the DBB T- and X-joints FE models. Following this approach, the boundary conditions and
205 load application in FE analyses were identical to the test programs (Pandey and Young 2021a
206 and 2022).

207 ***Weld heat affected region (WHAR)***

208 The heat transferred to parent tubular members during the welding process has a
209 considerable impact on the overall behaviour of hollow section joints (Pandey et al. 2021a).
210 The design rules in international standards (AISC 360, 2016; ISO 14346, 2013; IIW, 2012;
211 CIDECT, 2009; EC3, 2005) are identical for HSS produced from different methods, namely by
212 adding alloying elements and by various heat treatment techniques. However, it has been

213 reported in some recent studies (Pandey and Young, 2021c; Stroetmann et al., 2018; Javidan et
214 al., 2016; Amraei et al., 2019 and 2020) that HSS produced by different methods exhibited
215 different extents of softening around the welds. Investigations carried out by Stroetmann et al.
216 (2018), Javidan et al. (2016) and Amraei et al. (2019 and 2020) reported 16% to 32% reductions
217 in the ultimate strengths of S960 steel grade parent materials around the welds.

218 Pandey and Young (2021c) examined the material properties of the weld heat affected
219 region (WHAR) of RHS members with 0.2% proof stress of 960 MPa and wall thickness
220 varying between 3-6 mm. A 14% to 32% reduction in the ultimate strengths of the parent metals
221 was reported in the first 6 mm distance of the WHAR. The definition of WHAR for tubular
222 joints was proposed by Pandey et al. (2021a), as shown in Fig. 5. For DBB T- and X-joints FE
223 models, the spreads of WHAR are shown in Figs. 3 and 4, respectively. In addition, a simplified
224 strength reduction (S_{rl}) model was proposed by Pandey et al. (2021a) for S900 and S960 steel
225 grades tubular joints to integrate the material properties of WHAR in FE models, as illustrated
226 in Fig. 6. The proposed strength reduction model was successfully used to perform the
227 numerical investigation and design of CFHSS T- and TF-joints (Pandey et al. 2021a and 2021b).
228 Therefore, it was also included in this investigation, and accordingly, material properties were
229 assigned to the WHAR of DBB T- and X-joints FE models. The adoption of WHAR appreciably
230 improved the accuracies of FE models and, thus, the numerical results.

231 ***Validations of DBB T- and X-joints FE models***

232 The DBB T- and X-joints FE models were developed using the modelling approaches
233 described in the preceding sections of this paper. The test results of DBB T- and X-joints
234 reported in Pandey and Young (2021a and 2022) were used to validate their corresponding FE
235 models. The validations were performed by comparing the N_f , N_{max} , load-chord indentation
236 histories and failure modes of test and FE specimens. The measured dimensions of tubular
237 members and welds were used to develop all DBB T- and X-joints FE models. In addition,
238 measured material properties of tubular members, welds and WHAR were also included. The
239 N_f and N_{max} of DBB T- and X-joints test specimens were compared to those predicted from their
240 corresponding FE models ($N_{f,FE}$ and $N_{max,FE}$). Referring to Table 1, when the joint failure

241 strengths of DBB T-joint ($N_{f,T}$) test specimens were compared to the strengths predicted from
242 DBB T-joint FE models, the mean (P_m) and coefficients of variation (COV) (V_p) of the overall
243 comparisons were 0.98 and 0.022, respectively. However, when the ultimate capacities of DBB
244 T-joint ($N_{max,T}$) test specimens were compared to the FE strengths, the mean (P_m) and COV (V_p)
245 of the overall comparisons were 1.01 and 0.007, respectively.

246 When the joint failure strengths of DBB X-joint ($N_{f,X}$) test specimens were compared to
247 the strengths predicted from DBB X-joint FE models, the mean (P_m) and COV (V_p) of the
248 overall comparisons were 1.01 and 0.020, respectively. However, when the ultimate capacities
249 of DBB X-joint ($N_{max,X}$) test specimens were compared to the FE strengths, the mean (P_m) and
250 COV (V_p) of the overall comparisons were 1.01 and 0.016, respectively. Likewise in the
251 experimental investigation, the N_f of DBB T- and X-joints was determined by jointly
252 considering the ultimate capacity and ultimate deformation limit (i.e. $0.03 b'_0$) loads, whichever
253 occurred earlier in the load vs chord indentation curves. In addition, the comparisons of N vs u
254 curves between typical DBB T- and X-joints test and FE specimens are shown in Figs. 7 and 8,
255 respectively. Moreover, Figs. 9 and 10 present the comparisons of failure modes between typical
256 DBB T- and X-joints test and FE specimens, respectively. Therefore, from Table 1 and Figs. 7-
257 10, it can be concluded that the validated FE models precisely replicated the overall static
258 behaviour of DBB T- and X-joints.

259 ***Parametric study***

260 ***Introduction***

261 The test results reported in Pandey and Young (2021a and 2022) were not sufficient to
262 develop a broad understanding of various governing factors affecting the static strengths of
263 CFHSS DBB T- and X-joints. Therefore, the data pool was widened by performing a
264 comprehensive numerical parametric study using the validated DBB T- and X-joints FE models.
265 In total, 224 parametric FE analyses were performed in this study, including 112 DBB T-joints
266 and 112 DBB X-joints. Table 2 presents the overall ranges of various critical parameters
267 considered in the numerical parametric study. It is important to mention that, in this study, the

268 $0.03 b_0'$ deformation limit criterion governed the N_f of all DBB FE specimens. In the parametric
269 study, four member-rotation angles (ω) were included for braces ($\omega = 15^\circ, 25^\circ, 40^\circ$ and 45°),
270 while for all chords, ω was equal to 45° . All FE modelling techniques used in the validations
271 of DBB T- and X-joints were also employed in the parametric study.

272 ***FE modelling details***

273 In the numerical investigation, the dimensions of tubular members included practical sizes.
274 Overall, the values of cross-section width and depth of braces and chords of parametric FE
275 specimens varied between 40 mm to 200 mm, while wall thickness of braces and chords varied
276 between 2.5 mm to 12 mm. The exterior corner radii of RHS brace and chord members (R_l and
277 R_o) conformed to the commercially produced HSS members (SSAB, 2017a and 2017b). In this
278 study, R_l and R_o were kept as $2t$ for $t \leq 6$ mm, $2.5t$ for $6 < t \leq 10$ mm and $3t$ for $t > 10$ mm,
279 which in turn also meet the limits detailed in EN 10219-2 (2019). The brace and chord lengths
280 of DBB FE specimens were identical to those adopted in the experimental programs (Pandey
281 and Young, 2021a and 2022). For meshing along the longitudinal and transverse directions of
282 RHS members, seedings were approximately spaced at the minimum of $[b/30, h/30]$, where b
283 and h stand for cross-section width and depth of RHS member. Overall, the adopted mesh sizes
284 of parametric FE specimens varied between 3 mm to 10 mm. On the other hand, the seeding
285 interval of weld parts of parametric FE specimens reciprocated the seeding interval of their
286 corresponding brace parts. For RHS members with $t \leq 6$ mm, no divisions were made along the
287 wall thickness of braces and chords. However, for RHS members with $t > 6$ mm, the wall
288 thickness of braces and chords was divided using a node. The use of the C3D20 element and
289 one division along the wall thickness of FE specimens with $6 < t \leq 12$ provided four layers
290 along the thickness direction. Further wall thickness divisions made the element assembly quite
291 complex and led to unconverged results.

292 Following the prequalified tubular joint details given in AWS D1.1M (2020), the leg size
293 (w) of FW of DBB T- and X-joints FE specimens was designed as 1.5 times the minimum of t_l
294 and t_o . The weld designs of both DBB T- and X-joints FE specimens were consistent with the
295 experimental programs (Pandey and Young, 2021a and 2022). In the parametric study, the

296 material properties of flat and corner portions of RHS 150×150×6 were assigned to the flat and
297 corner portions of braces and chords of FE specimens. Besides, weld parts of all DBB T- and
298 X-joints parametric FE specimens were given the measured material properties of welding filler
299 material. Table 3 presents the measured material properties of RHS 150×150×6 and welding
300 filling material adopted in the parametric study, which include Young's modulus (E), 0.2% proof
301 stress and strain ($\sigma_{0.2}$ and $\varepsilon_{0.2}$), ultimate stress and strain (σ_u and ε_u), fracture strain (ε_f) and
302 Ramberg-Osgood parameter (n). On the other hand, the material properties and spread of
303 WHAR were in accordance with the recommendations proposed by Pandey et al. (2021a). In
304 this study, the ignorance of WHAR in FE analyses of DBB T-joints over-estimated the N_f and
305 N_{max} in the range of 8.1% to 29.9% and 6.7% to 31.0%, respectively. On the other hand, for
306 DBB X-joints, the ignorance of WHAR in FE analyses over-estimated the N_f and N_{max} in the
307 range of 7.1% to 63.3% and 3.5% to 19.1%, respectively.

308 ***Influence of governing geometric parameters and failure mode***

309 With the increase of β' ratio, the brace member(s) extends up to a greater depth on the
310 chord connecting regions. As a result, the chord member of DBB T- and X-joints suffered more
311 local plastic deformation, which in turn enhanced both N_f and N_{max} . On the other hand, initial
312 stiffness, N_f and N_{max} of DBB T- and X-joints decreased as 2γ ratio increased. The increase of
313 2γ ratio reduced the out-of-plane bending stiffness of chord connecting flanges, which in turn
314 reduced the load bearing capacity of DBB joints. With regard to the effect of τ ratio, the stiffness
315 and strength of DBB T- and X-joints first increased with the increase of τ ratio up to $\tau=1.0$ and
316 then decreased. Generally, for small values of τ ratio ($\tau < 0.50$), the joint could fail by the local
317 buckling of brace member(s). However, for large values of τ ratio ($\tau > 2.0$), punching failure at
318 chord crown and saddle locations could take place. For the design of tubular joints, generally,
319 brace and chord members of identical nominal thicknesses (i.e. $\tau = 1.0$) are selected. For DBB
320 joints, the value of ω of chord member is fixed and equal to 45°. However, generally, the N_f
321 and N_{max} increased with the increase of the value of ω of brace member.

322 All DBB T- and X-joints test (Pandey and Young, 2021a and 2022) and FE specimens
323 were failed by chord crown failure mode, which was denoted by the letter 'C'. In the chord

324 crown failure (C) mode, the test and FE specimens were failed by predominant convex
325 deformation at the crown locations of the chords. It is important to note that this failure mode
326 was defined corresponding to the N_f of DBB T- and X-joints, which in turn was computed by
327 combinedly considering the ultimate capacity and deformation limit loads, whichever occurred
328 earlier in the N vs u curve. It is noteworthy to mention that the convex deformation at the chord
329 crown location of all DBB T- and X-joint test and FE specimens was always larger than the
330 corresponding concave deformation at the chord saddle location. The predominance of
331 deformation at crown location remained valid for both the N_f and N_{max} of DBB T- and X-joints
332 test and FE specimens. The deformation capacities of the DBB T- and X-joints test and FE
333 specimens were significantly large. The attainment of the N_{max} of test and FE specimens was
334 accompanied by large deformation at the crown and saddle regions of the chords. Generally, the
335 N vs u curves of the DBB test and FE specimens entered a stagnant phase near the N_{max} , followed
336 by a very gradual load drop in the post-ultimate regions. It should be stressed that for all DBB
337 T- and X-joints test and FE specimens, the $0.03 b'_0$ loads occurred quite earlier than their
338 corresponding ultimate capacities. In this investigation, the test and parametric FE specimens
339 were failed by the C mode for $0.20 \leq \beta' \leq 0.84$. Moreover, none of the test and FE specimens
340 were failed by the global buckling of braces.

341 It should be noted that, in the experimental investigations (Pandey and Young, 2021a and
342 2022) and in this study, DBB T- and X-joints were purposely designed for failure in the chord
343 member. However, DBB T- and X-joints undergoing axial load (compression or tensile) through
344 brace members could also fail by other failure modes, including local buckling of braces,
345 combination of local buckling of braces and chord failure, and weld rupture failure. The DBB
346 T- and X-joints undergoing brace axial compression load could fail by local buckling of braces
347 for small values of τ ratio ($\tau < 0.50$). In addition, DBB T- and X-joints with small values of τ
348 ratio and large values of 2γ ratio could fail by the combination of local buckling of braces and
349 chord failure. Moreover, DBB T- and X-joints with inadequate weld design and subjected to
350 tensile loads could fail by the rupture of welds, i.e. weld rupture failure.

351 Existing design rules

352 Currently, DBB joint configuration is not included in any international code of practice.
353 The overall static behaviour of tubular T- and X-joints when subjected to axial compression
354 loads via braces are nearly similar. Therefore, in this investigation, the N_f and N_{max} of test and
355 parametric FE specimens were evaluated against the nominal strengths of DBB T- and X-joints
356 design rules given in the literature (Chen and Wang, 2015; Ono et al., 1991; Peña and Chacón,
357 2014). Moreover, owing to the rotated brace and chord members of DBB T- and X-joints, the
358 DBB joint configuration resembles to that of the CHS-to-CHS configuration. Thus, the N_f and
359 N_{max} of test and parametric FE specimens were also evaluated against the nominal strengths of
360 CHS-to-CHS T- and X-joint design rules given in EC3 (2021). The nominal strengths were
361 determined using the measured dimensions and mechanical properties. Under axial
362 compression load, the chord members of DBB T-joints were subjected to chord-in-plane
363 bending. In this investigation, the effect of normal stresses developed due to chord-in-plane
364 bending on the static strengths of DBB T-joints was considered through chord stress functions
365 ($k_n, f(n')$ and Q_f). On the other hand, in this study, no preload was applied to the chord members
366 of DBB X-joints. Therefore, the values of $k_n, f(n')$ and Q_f for DBB X-joints were set to unity in
367 Eqs. (1) to (5).

368 *Ono et al. (1991)*

369 Ono et al. (1991) experimentally studied the static behaviour of DBB T-joints with yield
370 strengths varied between 365-415 MPa. The chord ends of test specimens were supported by
371 pins, while axial compression was applied via braces. The following design equation was
372 proposed to determine the static ultimate capacity of the investigated DBB T-joints:

$$N_{Ono} = f_{y0} t_0^2 \left[\frac{1}{0.211 - 0.147 (b_1/b_0)} + \frac{b_0/t_0}{1.794 - 0.942 (b_1/b_0)} \right] f(n') \quad (1)$$

373 In order to assess the suitability of Eq. (1) for CFHSS DBB joints studied in this work, a
374 material factor (C_f) equal to 0.80 was multiplied to Eq. (1). The revised nominal strength was
375 symbolised by N_{Ono}^{\wedge} . The function $f(n')$ is equal to $1 + 0.3n' - 0.3(n')^2$. The chord stress
376 parameter (n') is equal to $0.25N_f(L_0-h'_1)$.

377 **Peña and Chacón (2014)**

378 Peña and Chacón (2014) numerically investigated the static behaviour of DBB X-joints.
379 The numerical investigation was based on elasto-plastic material curves, where the yield
380 strengths of braces and chords were assumed as 235, 275 and 460 MPa. In the numerical
381 investigation, both compression and tensile loads were applied to the FE specimens via braces.
382 Based on the parametric FE results, the following design equation was proposed to determine
383 the static ultimate resistances of the investigated joints:

$$N_{PC} = \left(\frac{1}{1.05} \right) f_{y0} \left[\frac{(6.06 - 5.6\beta + 11.4\beta^2)(0.6 + 1.97\sqrt{\beta})t_0^2}{(6.06 - 5.6\beta + 11.4\beta^2)\frac{t_0}{b_0} + \frac{(0.6 + 1.97\sqrt{\beta})}{3}} \right] \quad (2)$$

384 The appropriateness of Eq. (2) for CFHSS DBB joints was examined by multiplying the
385 Eq. (2) with $C_f=0.80$. After multiplying the C_f factor to Eq. (2), the revised nominal strength
386 was symbolised by N_{PC}^{\wedge} .

387 **Chen and Wang (2015)**

388 Chen and Wang (2015) proposed a design equation (Eq. (3)) to predict the ultimate
389 resistances of DBB T-joints with nominal 0.2% proof stress of 235 MPa. The chord ends of test
390 specimens were supported by pins, while axial compression was applied via braces.

$$N_{CW} = 1.814\beta^{\frac{1}{2}}\gamma^{\frac{1}{2}}\tau^{\frac{1}{6}}\left(\frac{1-\beta}{k_n}\right)\left[Q_f\frac{f_{y0}t_0^2}{\sin\theta_1}\left(\frac{2\beta}{(1-\beta)\sin\theta_1} + \frac{4}{\sqrt{1-\beta}}\right)\right]^{\gamma_{M5}} \quad (3)$$

391 In order to prolong the suitability of Eq. (3) for CFHSS DBB joints studied in this work,
392 $C_f=0.80$ was multiplied to Eq. (3). After multiplying the C_f factor to Eq. (3), the revised nominal
393 strength was symbolised by N_{CW}^{\wedge} . In Eq. (3), f_{y0} is the yield stress of the chord member, γ_{M5} is
394 the partial safety factor of tubular joints as per EC3 (2021) and θ_1 represents angle between
395 brace and chord members (in degrees).

396 **EC3 (2021)**

397 The design equations given in EC3 (2021) are applicable for tubular joints with steel

398 grades up to S700. However, a material factor (C_f) is required to be multiplied with the design
 399 rules when the steel grade exceeds S355. When steel grade ranged between 550-700 MPa, the
 400 value of material factor (C_f) is equal to 0.80. Furthermore, EC3 (2021) explicitly recommended
 401 the value of partial safety factor for tubular joints (γ_{M5}) equal to 1.0. The nominal strengths of
 402 joints failed by chord failure mode can be determined as follows:

403 For CHS-to-CHS T-joint:

$$N_{EC3,T}^{\wedge} = \frac{C_f}{\gamma_{M5}} \left[Q_f \frac{f_{y0} t_0^2}{\sin \theta_1} (2.6 + 17.7 (\beta')^2) \gamma^{0.2} \right] \quad (4)$$

404 For CHS-to-CHS X-joint:

$$N_{EC3,X}^{\wedge} = \frac{C_f}{\gamma_{M5}} \left[Q_f \frac{f_{y0} t_0^2}{\sin \theta_1} \left(\frac{2.6 + 2.6 \beta'}{1 - 0.7 (\beta')} \right) \gamma^{0.15} \right] \quad (5)$$

405 Reliability analysis

406 In order to examine the reliability of existing and proposed design equations, a reliability
 407 study was performed as per AISI S100 (2016). The Eq. (6) was used to calculate the reliability
 408 index (β_0). In this investigation, a lower bound value of 2.50 was taken as the target β_0 .
 409 Therefore, when $\beta_0 \geq 2.50$, the design equation was treated as reliable in this study.

$$\beta_0 = \frac{\ln(C_\phi M_m F_m P_m / \phi)}{\sqrt{V_M^2 + V_F^2 + C_P V_P^2 + V_Q^2}} \quad (6)$$

410 A dead load (DL)-to-live load (LL) ratio of 0.20 was used to compute the calibration
 411 coefficient (C_ϕ) in Eq. (6). For the material factor, the mean value and COV are respectively
 412 denoted by M_m and V_M . For the fabrication factor, the mean value and COV are respectively
 413 denoted by F_m and V_F . Referring to AISI S100 (2016), the M_m and V_M were adopted as 1.10 and
 414 0.10, respectively. Additionally, F_m and V_F were adopted as 1.00 and 0.10, respectively. The
 415 resistance factor required to convert the nominal strength to design strength is denoted by ϕ .
 416 The mean value of ratios of test and FE strengths-to-nominal strengths predicted from literature
 417 and code was denoted by P_m , while the corresponding COV was denoted by V_P . The correction
 418 factor (C_P) proposed by AISI S100 (2016) was also used in Eq. (6) to incorporate the effect of

419 the number of data under consideration. Besides, V_Q denoted the COV of load effects. To
420 evaluate the reliability levels of EC3 (2021) design provisions, the DL and LL were combined
421 as $1.35DL + 1.5LL$ as per EN (2002), and thus, the calculated value of C_ϕ was 1.463. Further,
422 to examine the reliability levels of design equations given in the literature (Chen and Wang,
423 2015; Ono et al., 1991; Peña and Chacón, 2014) as well as for the proposed design rules, the
424 DL and LL were combined as $1.2DL + 1.6LL$ as per ASCE 7 (2016), and the calculated value
425 of C_ϕ was 1.521. For extreme cases, where the values of P_m were very small, the calculated
426 values of β_0 were less than zero. Therefore, such values of β_0 are not reported in this paper.

427 ***Comparisons between test and FE strengths with nominal strengths***

428 Table 4 presents the summary of overall comparisons of $N_{f,T}$ and $N_{max,T}$ of DBB T-joint
429 test and parametric FE specimens with nominal strengths predicted from Chen and Wang (2015),
430 Ono et al. (1991), Peña and Chacón (2014) and EC3 (2021). The comparisons of the $N_{f,T}$ of
431 DBB T-joints with nominal strengths revealed that the predictions from design rule given in
432 Ono et al. (1991) were very unconservative, quite dispersed and unreliable. The predictions of
433 design rule proposed by Chen and Wang (2015) were slightly unconservative and unreliable for
434 the $N_{f,T}$ of DBB T-joints. However, the predictions from design rule given in Peña and Chacón
435 (2014) were found to be satisfactory but unreliable for the $N_{f,T}$ of DBB T-joints. On the contrary,
436 the comparisons of predictions from CHS-to-CHS T-joint design rule given in EC3 (2021) with
437 the $N_{f,T}$ of DBB T-joints were found to be very conservative but quite dispersed and unreliable.
438 From the comparisons of the $N_{max,T}$ of DBB T-joint test and parametric FE specimens with
439 nominal strengths, it can be noticed that the predictions from the design rule given in Ono et al.
440 (1991) were unconservative, quite dispersed and unreliable. On the other hand, the predictions
441 from design rules given in Chen and Wang (2015) and Peña and Chacón (2014) were quite
442 conservative but dispersed for the $N_{max,T}$ of DBB T-joints. In addition, for the $N_{max,T}$ of DBB T-
443 joints, the CHS-to-CHS T-joint design rule of EC3 (2021) was found to be significantly
444 conservative but quite dispersed. Figs. 11(a) and 11(b) graphically present the comparisons of
445 $N_{f,T}$ and $N_{max,T}$ of DBB T-joint test and parametric FE specimens with nominal strengths
446 predicted from design equations given in Chen and Wang (2015) and Ono et al. (1991),

447 respectively.

448 The summary of overall comparisons of $N_{f,X}$ and $N_{max,X}$ of DBB X-joint test and
449 parametric FE specimens with nominal strengths predicted from Chen and Wang (2015), Ono
450 et al. (1991), Peña and Chacón (2014) and EC3 (2021) are presented in Table 5. The predictions
451 from design rules given in Chen and Wang (2015), Ono et al. (1991) and Peña and Chacón
452 (2014) were found to be very unconservative, quite dispersed and unreliable for the $N_{f,X}$ of DBB
453 X-joints. On the contrary, the comparisons of predictions of CHS-to-CHS X-joint design rule
454 of EC3 (2021) with the $N_{f,X}$ of DBB X-joints were found to be conservative but dispersed and
455 unreliable. With regard to the comparisons with the $N_{max,X}$ of DBB X-joints, the predictions
456 from design rule given in Ono et al. (1991) were found to be very unconservative, quite
457 dispersed and unreliable. The predictions from design rule given in Chen and Wang (2015) were
458 slightly unconservative and unreliable for the $N_{max,X}$ of DBB X-joints. However, design rule
459 given in Peña and Chacón (2014) satisfactorily predicted the $N_{max,X}$ of DBB X-joints, however,
460 the design equation was found to be unreliable. On the contrary, the comparisons of predictions
461 of CHS-to-CHS X-joint design rule of EC3 (2021) with the $N_{max,X}$ of DBB X-joints were found
462 to be very conservative and uneconomical. Figs. 12(a) and 12(b) graphically present the
463 comparisons of $N_{f,X}$ and $N_{max,X}$ of DBB X-joint test and parametric FE specimens with nominal
464 strengths predicted from design equations given in Peña and Chacón (2014) and Ono et al.
465 (1991), respectively.

466 ***Discussion of comparison results***

467 This section of the paper presents the possible reasons behind the inaccuracies of existing
468 design rules for the static strength predictions of CFHSS DBB T- and X-joints. Ono et al. (1991)
469 carried out tests on DBB T-joints made of normal strength steel. A total of twenty-five DBB T-
470 joints was tested, and the obtained test results were used to propose the semi-empirical design
471 rule given by Eq. (1). The simplified theoretical ring model, originally used to formulate the
472 design rule for conventional CHS-to-CHS joints, was employed to develop the Eq. (1). The
473 analytical model was derived using the strain distribution in the chord member as well as
474 assuming that the chord deformation only depends on β . However, strain distribution in the

475 chord of conventional CHS-to-CHS T-joint is quite different to those of DBB T-joint.
476 Furthermore, it has also been reported by Mang (1978) and Kurobane (1981) that the joint
477 strength appreciably decreased as the ratio of yield stress-to-ultimate stress increased, which is
478 one of the characteristics of HSS. In order to calibrate the theoretical ring model for DBB T-
479 joints, numerical parameters in Eq. (1) were derived by curve fitting the test data. Owing to
480 these possible reasons, the design equation given in Ono et al. (1991) yielded very
481 unconservative predictions for the investigated CFHSS DBB joints.

482 The design rule proposed by Peña and Chacón (2014) for DBB X-joint was derived using
483 the design equation given in Owen et al. (2001) for S275 steel grade DBB X-joints. However,
484 using a reduction factor, Peña and Chacón (2014) numerically extended the validity of the
485 design equation proposed by Owen et al. (2021) up to S460 steel grade. Nonetheless, the revised
486 design equation was found to be inadequate for CFHSS DBB T- and X-joints investigated in
487 this work. More importantly, one of the critical geometric parameters, 2γ (b_0/t_0), affecting the
488 behaviour of DBB joints was left out from the design rule given in Peña and Chacón (2014).

489 Chen and Wang (2015) proposed design rule for DBB T-joints by applying correction
490 factors to the conventional RHS T-joint design equation given in CIDECT (2009). However, it
491 is worth mentioning that the structural behaviour of DBB T- and X-joints is very different
492 compared to conventional RHS T-joints. Therefore, the extension of the RHS T-joint design rule
493 for DBB T-joint by merely applying correction factors on the former could lead to inaccurate
494 joint strengths. Further, it is essential to note that the design rule given in Chen and Wang (2015)
495 was only valid for Q235 steel grade tubular members. The COV of the proposed design equation
496 (Eq. (3)) was 0.323 (Chen and Wang, 2015), which in turn revealed that the predictions of Eq.
497 (3) were highly dispersed even for the investigated Q235 steel DBB T-joints. Owing to the $(1-$
498 $\beta)$ factor in Eq. (3), the strength of the DBB T-joint decreased as the value of β increased, which
499 was contrary to the general behaviour of DBB joints. Moreover, the influence of chord-in-plane
500 bending was considered using functions present in both the numerator and denominator of Eq.
501 (3), which eventually eliminated the total chord-in-plane bending influence from the joint
502 strength. The points mentioned above could be the possible reasons behind the inaccuracies of
503 the design rule given in Chen and Wang (2015) for the investigated CFHSS DBB joints.

504 In this study, the comparisons of N_f and N_{max} of DBB T- and X-joints with nominal
505 strengths predicted from CHS-to-CHS T- and X-joints design rules given in EC3 (2021) are
506 presented only for illustrative purposes. The design rules for DBB joints are not given in EC3
507 (2021). The CHS-to-CHS T- and X-joints design rules in previous and latest versions of
508 Eurocode 3 (part-8) are semi-empirical in nature. These design equations (refer to Eqs. (4) and
509 (5)) were developed by calibrating the simplified analytical ring model primarily against the
510 test results of CHS-to-CHS T- and X-joints made of mild steel grades (i.e. steel grades lower
511 than and equal to S355). Although the overall configuration of DBB T- and X-joints looks
512 similar to those of CHS-to-CHS T- and X-joints, however, the interlocking of corner regions of
513 brace and chord members remarkably enhanced the stiffness and strength of DBB T- and X-
514 joints. As a result, current CHS-to-CHS T- and X-joints design rules given in EC3 (2021)
515 provided very conservative predictions for the range of DBB T- and X-joints investigated in
516 this study.

517 **Proposed design rules**

518 In this study, two types of design rules are proposed, under proposal-1 and -2, to predict
519 the N_f and N_{max} of cold-formed S960 steel grade DBB T- and X-joints. Under proposal-1, new
520 design equations are proposed to predict the N_f and N_{max} of DBB T- and X-joints by taking into
521 consideration the effect of important geometric factors as well as P_m and V_p of the overall
522 comparison. However, under proposal-2, the N_f and N_{max} of CFHSS DBB T- and X-joints were
523 predicted by applying correction factor(s) on the current CHS-to-CHS joint design rules (Eqs.
524 (4) and (5)) given in EC3 (2021). Furthermore, as welds were modelled in all parametric FE
525 specimens, the effects of weld and associated WHAR were implicitly included in the proposed
526 design equations. In order to calculate design strengths (N_d), the proposed nominal strengths
527 (N_{pn1} and N_{pn2}) shall be multiplied by their correspondingly recommended resistance factors (ϕ),
528 i.e. $N_d = \phi (N_{pn1} \text{ or } N_{pn2})$. All design rules proposed in this study are valid for $0.20 \leq \beta \leq 0.80$,
529 $0.20 \leq \beta' \leq 0.84$, $16.6 \leq 2\gamma \leq 40$, $0.50 \leq \tau \leq 1.28$ and $15^\circ \leq \omega$ (brace) $\leq 63^\circ$. Compared to the
530 existing design rules, the proposed design rules are more accurate, less dispersed and reliable
531 for the investigated CFHSS DBB joints.

532 ***DBB T-joints failed by chord crown failure (C) mode***

533 ***For joint failure strength***

534 Proposal-1:

$$N_{pn1} = \frac{f_{y0}t_0^2 (0.5\beta' + 1)(0.1\tau + 1)}{[0.16 - 0.001(2\gamma)]} \quad (7)$$

535 Proposal-2:

$$N_{pn2} = 0.6(\beta')^{-0.8} [N_{EC3,T}^{\wedge}] \quad (8)$$

536 The term $N_{EC3,T}^{\wedge}$ in Eq. (8) can be obtained from Eq. (4). The summary of overall
537 comparison results of proposal-1 and -2 are shown in Table 4. The comparisons of $N_{f,T}$ of test
538 and FE specimens with nominal strengths predicted from Chen and Wang (2015), Ono et al.
539 (1991) and proposal-1 are graphically presented in Fig. 11(a). In addition, the distributions of
540 the ratios of $N_{f,T}$ of test and FE specimens-to-nominal strengths predicted from existing and
541 proposal-1 design rules are shown in Fig. 13.

542 ***For joint ultimate capacity***

543 Proposal-1:

$$N_{pn1} = \frac{f_{y0}t_0^2 (0.4\beta' + 0.75)(0.12\tau + 0.94)}{[0.09 - 0.0007(2\gamma)]} \quad (9)$$

544 Proposal-2:

$$N_{pn2} = 0.75(\beta')^{-0.9} [N_{EC3,T}^{\wedge}] \quad (10)$$

545 The term $N_{EC3,T}^{\wedge}$ in Eq. (10) can be obtained from Eq. (4). The summary of overall
546 comparison results of proposal-1 and -2 are shown in Table 4. The comparisons of $N_{max,T}$ of test
547 and FE specimens with nominal strengths predicted from Chen and Wang (2015), Ono et al.
548 (1991) and proposal-1 are graphically presented in Fig. 11(b). In addition, the distributions of
549 the ratios of $N_{max,T}$ of test and FE specimens-to-nominal strengths predicted from existing and
550 proposal-1 design rules are shown in Fig. 14.

551 ***DBB X-joints failed by chord crown failure (C) mode***

552 ***For joint failure strength***

553 Proposal-1:

$$N_{pn1} = \frac{f_{y0} t_0^2 (1.5\beta' + 0.6)(0.1\tau + 1)}{[0.1 + 0.003(2\gamma)]} \quad (11)$$

554 Proposal-2:

$$N_{pn2} = (\beta')^{-0.25} [1.5 - 0.02(2\gamma)] [N_{EC3,X}^{\wedge}] \quad (12)$$

555 The term $N_{EC3,X}^{\wedge}$ in Eq. (12) can be obtained from Eq. (5). The summary of overall
556 comparison results of proposal-1 and -2 are shown in Table 5. The comparisons of $N_{f,X}$ of test
557 and FE specimens with nominal strengths predicted from Peña and Chacón (2014), Ono et al.
558 (1991) and proposal-1 are graphically presented in Fig. 12(a). In addition, the distributions of
559 the ratios of $N_{f,X}$ of test and FE specimens-to-nominal strengths predicted from existing and
560 proposal-1 design rules are shown in Fig. 15.

561 ***For joint ultimate capacity***

562 Proposal-1:

$$N_{pn1} = \frac{f_{y0} t_0^2 (1.4\beta' + 0.5)(0.1\tau + 1)}{[0.12 - 0.0002(2\gamma)]} \quad (13)$$

563 Proposal-2:

$$N_{pn2} = 0.6(\beta')^{-0.35} [2.3 - 0.013(2\gamma)] [N_{EC3,X}^{\wedge}] \quad (14)$$

564 The term $N_{EC3,X}^{\wedge}$ in Eq. (14) can be obtained from Eq. (5). The summary of overall
565 comparison results of proposal-1 and -2 are shown in Table 5. The comparisons of $N_{max,X}$ of test
566 and FE specimens with nominal strengths predicted from Peña and Chacón (2014), Ono et al.
567 (1991) and proposal-1 are graphically presented in Fig. 12(b). In addition, the distributions of
568 the ratios of $N_{max,X}$ of test and FE specimens-to-nominal strengths predicted from existing and
569 proposal-1 design rules are shown in Fig. 16.

570 ***Unified design equation***

571 As the formats of the proposed design equations under proposal-1 (Eqs. (7), (9), (11) and
572 (13)) are identical. Therefore, an attempt has been made to propose a unified design equation
573 to predict the N_f and N_{max} of cold-formed S960 steel grade DBB T- and X-joints that failed by
574 the C mode. The proposed unified design equation, shown in Eq. (15), is valid for $0.20 \leq \beta' \leq$
575 0.84 , $16.6 \leq 2\gamma \leq 40$, $0.50 \leq \tau \leq 1.28$. The values of coefficients (A to F) are given in Table 6.

$$N_{pn1} = f_{y0} t_0^2 \frac{(A\beta' + B)(C\tau + D)}{[E + F(2\gamma)]} \quad (15)$$

576 **Conclusions**

577 The detailed numerical investigation performed in this study on cold-formed S960 steel
578 grade diamond bird-beak (DBB) T- and X-joints led to the following main conclusions:

- 579 • The modelling of welds and inclusion of weld heat affected regions substantially increased
580 the accuracies of predictions from the developed DBB T- and X-joint finite element (FE)
581 models.
- 582 • The joint failure strengths (N_f) of all DBB T- and X-joints were governed by 3% ultimate
583 deformation limit criterion.
- 584 • The chord crown failure (C) mode was identified as the dominant failure mode for all DBB
585 T- and X-joints investigated in this work. This failure mode was characterised by a visible
586 convex deformation at the chord crown locations. In the load vs chord indentation curves,
587 generally, a stagnant phase was noticed near the peak strengths of DBB T- and X-joints,
588 followed by a gradual reduction of load in their post-ultimate regions.
- 589 • The design provisions given in the literature (Chen and Wang, 2015; Ono et al., 1991; Peña
590 and Chacón, 2014) and EC3 (2021) are generally found to be unsuitable and uneconomical
591 for the investigated DBB T- and X-joints.
- 592 • Accurate, less dispersed, user-friendly and reliable design equations are proposed, by two
593 approaches, to predict the joint failure strengths and ultimate capacities of cold-formed
594 S960 steel grade DBB T- and X-joints that failed by the chord crown failure (C) mode.
595 Moreover, a new unified design equation is also proposed to predict the static joint failure
596 strengths and ultimate capacities of the investigated DBB T- and X-joints.

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600 **Data Availability Statement**

601 Some or all data, models, or code that support the findings of this study are available from
602 the corresponding author upon reasonable request.

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726

Table 1. Summary of test vs FE strength comparisons for DBB T- and X-joints.

Parameters	DBB T-joints		DBB X-joints	
	$\frac{N_{f,T}}{N_{f,FE}}$	$\frac{N_{max,T}}{N_{max,FE}}$	$\frac{N_{f,X}}{N_{f,FE}}$	$\frac{N_{max,X}}{N_{max,FE}}$
	No. of Data (n)	10	10	10
Mean (P_m)	0.98	1.01	1.01	1.01
COV (V_p)	0.022	0.007	0.020	0.016

Table 2. Overall ranges of critical parameters used in numerical study.

Parameters	Validity Ranges
$\beta (b_1/b_0)$	[0.20 to 0.80]
$\beta' (b'_1/b'_0)$	[0.20 to 0.84]
$2\gamma (b_0/t_0)$	[16.6 to 40]
$\tau (t_1/t_0)$	[0.50 to 1.28]
ω (brace)	[15° to 63°]
ω (chord)	45°

Table 3. Material properties of RHS member and weld used in parametric FE analyses.

Materials	Measured Material Properties							
	E (GPa)	$\sigma_{0.2}$ (MPa)	$\epsilon_{0.2}$ (%)	σ_u (MPa)	$0.8\sigma_u$ (MPa)	ϵ_u (%)	ϵ_f (%)	n
RHS (150×150×6)*	208.5	1059.1	0.71	1145.7	916.6	1.48	9.37 [#]	5.31
Weld Material@	202.7	965.2	0.68	1023.4	818.7	5.41	17.15 ^{\$}	8.13

Note: * Pandey and Young (2019a); @Pandey and Young (2019b); [#]fracture strain based on 50 mm gauge length; ^{\$}fracture strain based on 25 mm gauge length.

Table 4. Comparisons between test and FE strengths with existing and proposed nominal strengths for DBB T-joints failed by chord crown failure (C) mode.

Parameters	Comparisons for Joint Failure Strengths						Comparisons for Joint Ultimate Capacities					
	$\frac{N_{f,T}}{N_{CW}^{\wedge}}$	$\frac{N_{f,T}}{N_{Ono}^{\wedge}}$	$\frac{N_{f,T}}{N_{PC}^{\wedge}}$	$\frac{N_{f,T}}{N_{EC3,T}^{\wedge}}$	$\frac{N_{f,T}}{N_{pn1}}$	$\frac{N_{f,T}}{N_{pn2}}$	$\frac{N_{max,T}}{N_{CW}^{\wedge}}$	$\frac{N_{max,T}}{N_{Ono}^{\wedge}}$	$\frac{N_{max,T}}{N_{PC}^{\wedge}}$	$\frac{N_{max,T}}{N_{EC3,T}^{\wedge}}$	$\frac{N_{max,T}}{N_{pn1}}$	$\frac{N_{max,T}}{N_{pn2}}$
	Mean (P_m)	0.91	0.58	1.00	1.51	1.00	1.01	1.23	0.86	1.37	2.31	1.00
Maximum	1.24	0.95	1.19	3.07	1.28	1.18	1.63	1.41	1.67	5.35	1.17	1.19
Minimum	0.56	0.32	0.74	0.61	0.80	0.80	0.77	0.40	0.90	0.75	0.74	0.76
COV (V_p)	0.161	0.276	0.095	0.341	0.077	0.078	0.143	0.293	0.119	0.395	0.086	0.090
Resistance factor (ϕ)	1.00	1.00	1.00	1.00	0.85	0.85	1.00	1.00	1.00	1.00	0.85	0.85
Reliability index (β_0)	1.38	-	1.90	2.08	2.55	2.60	2.48	0.95	2.95	2.77	2.53	2.57

Note: “ - ” denotes not applicable as $\beta_0 < 0$.

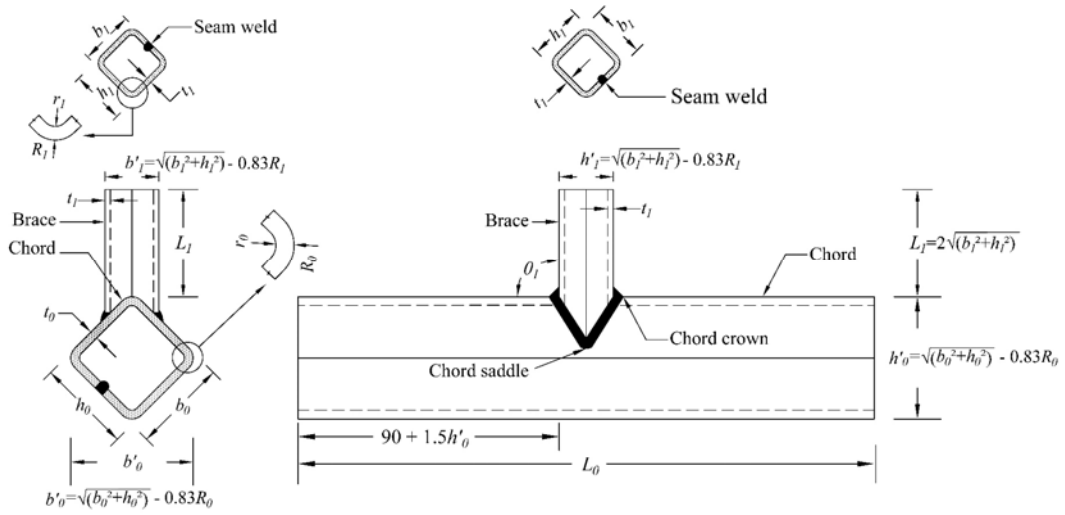
Table 5. Comparisons between test and FE strengths with existing and proposed nominal strengths for DBB X-joints failed by chord crown failure (C) mode.

Parameters	Comparisons for Joint Failure Strengths						Comparisons for Joint Ultimate Capacities					
	$\frac{N_{f,X}}{N_{CW}^{\wedge}}$	$\frac{N_{f,X}}{N_{Ono}^{\wedge}}$	$\frac{N_{f,X}}{N_{PC}^{\wedge}}$	$\frac{N_{f,X}}{N_{EC3,X}^{\wedge}}$	$\frac{N_{f,X}}{N_{pn1}}$	$\frac{N_{f,X}}{N_{pn2}}$	$\frac{N_{max,X}}{N_{CW}^{\wedge}}$	$\frac{N_{max,X}}{N_{Ono}^{\wedge}}$	$\frac{N_{max,X}}{N_{PC}^{\wedge}}$	$\frac{N_{max,X}}{N_{EC3,X}^{\wedge}}$	$\frac{N_{max,X}}{N_{pn1}}$	$\frac{N_{max,X}}{N_{pn2}}$
	Mean (P_m)	0.72	0.39	0.78	1.17	1.00	0.98	0.96	0.52	1.05	1.59	1.00
Maximum	1.31	0.66	1.17	1.96	1.29	1.20	1.47	0.74	1.39	2.25	1.24	1.29
Minimum	0.38	0.18	0.47	0.75	0.81	0.80	0.58	0.32	0.84	1.10	0.83	0.83
COV (V_p)	0.328	0.360	0.233	0.232	0.088	0.069	0.198	0.219	0.104	0.142	0.087	0.078
Resistance factor (ϕ)	1.00	1.00	1.00	1.00	0.85	0.85	1.00	1.00	1.00	1.00	0.85	0.85
Reliability index (β_0)	0.44	-	0.77	1.83	2.53	2.51	1.47	-	2.07	3.24	2.53	2.57

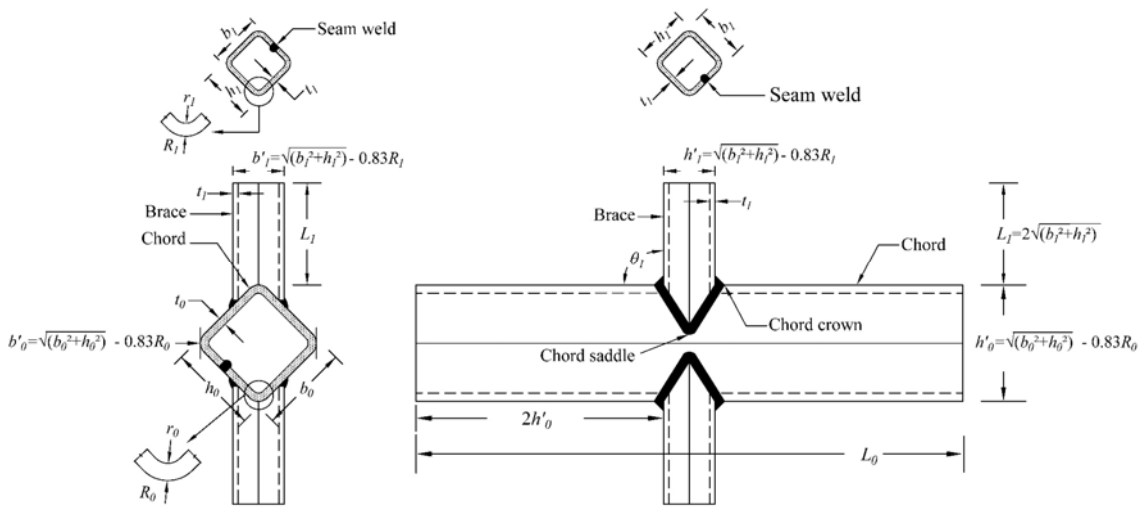
Note: “ - ” denotes not applicable as $\beta_0 < 0$.

Table 6. Values of coefficients for DBB T- and X-joints unified design rule.

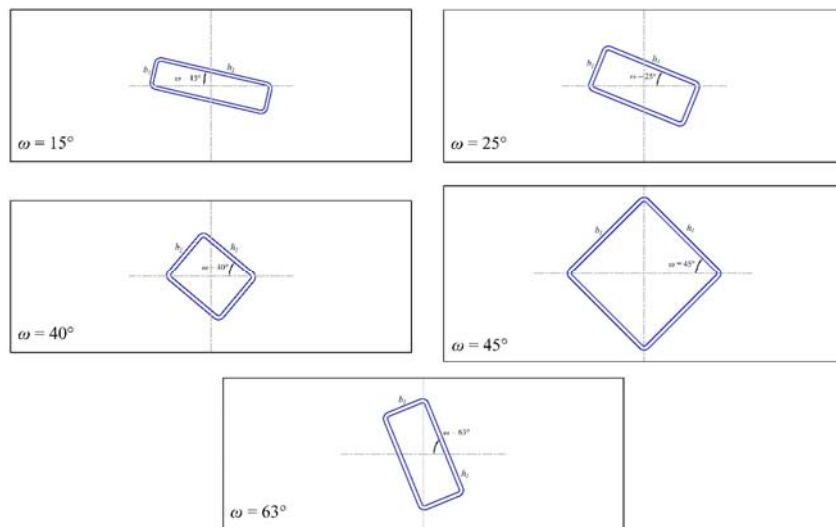
Joint Types	Joint Resistance	Coefficients					
		A	B	C	D	E	F
DBB T-joint	Joint failure strength	0.5	1	0.1	1	0.16	-0.001
	Ultimate capacity	0.4	0.75	0.12	0.94	0.09	-0.0007
DBB X-joint	Joint failure strength	1.5	0.6	0.1	1	0.1	0.003
	Ultimate capacity	1.4	0.5	0.1	1	0.12	-0.0002



(a) Definitions of notations for DBB T-joint.



(b) Definitions of notations for DBB X-joint.

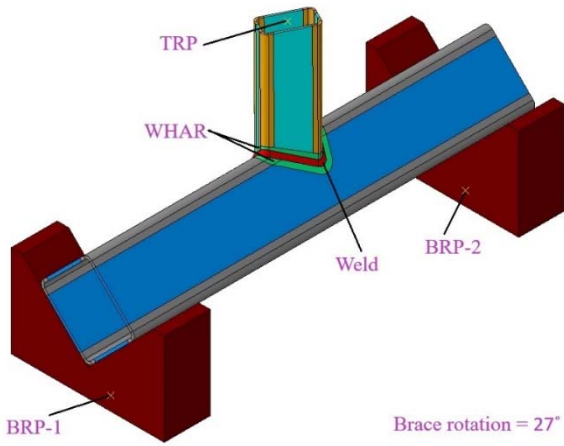


(c) Typical orientations of brace members of DBB T- and X-joints.

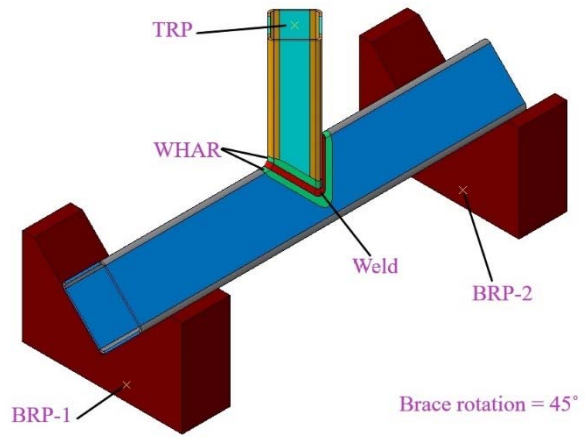
Figure 1. Notations definitions and member orientations for DBB T- and X-joints.



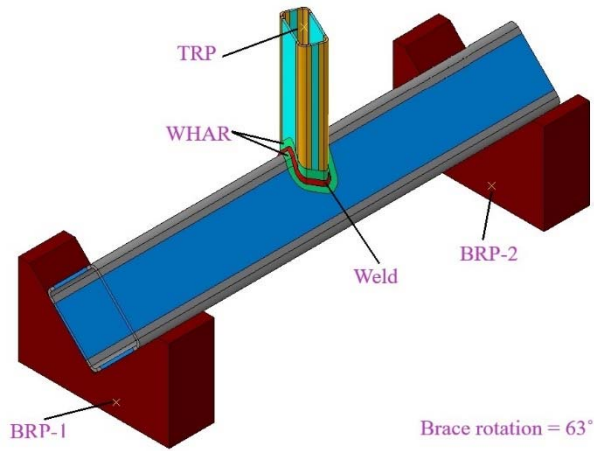
Figure 2. Dihedral angle ($\psi=120^\circ$) for DBB X-joint (also valid for DBB T-joint).



(a) Typical FE model of DBB T-joint with brace rotation of 27° .

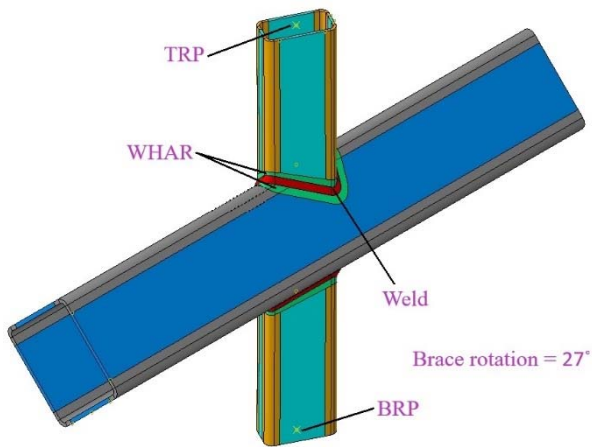


(b) Typical FE model of DBB T-joint with brace rotation of 45° .

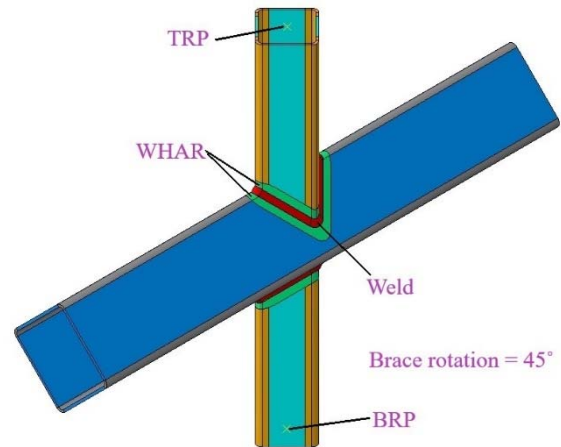


(c) Typical FE model of DBB T-joint with brace rotation of 63° .

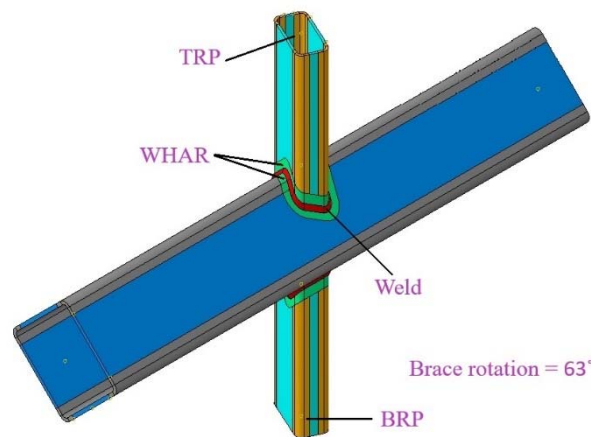
Figure 3. Typical FE models of DBB T-joints.



(a) Typical FE model of DBB X-joint with brace rotation of 27°.



(b) Typical FE model of DBB X-joint with brace rotation of 45°.



(c) Typical FE model of DBB X-joint with brace rotation of 63°.

Figure 4. Typical FE models of DBB X-joints.

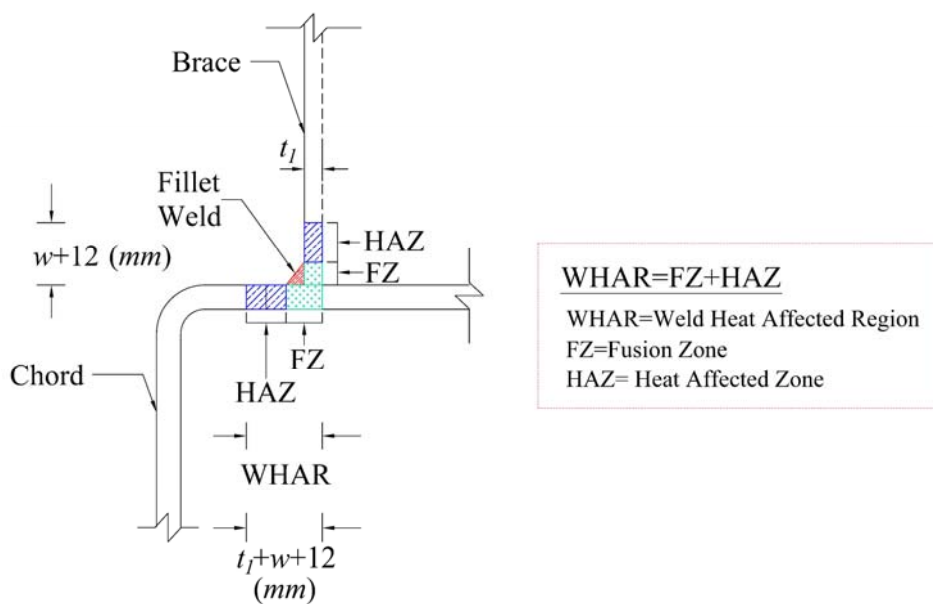


Figure 5. Definition of weld heat affected region (Pandey et al. 2021a).

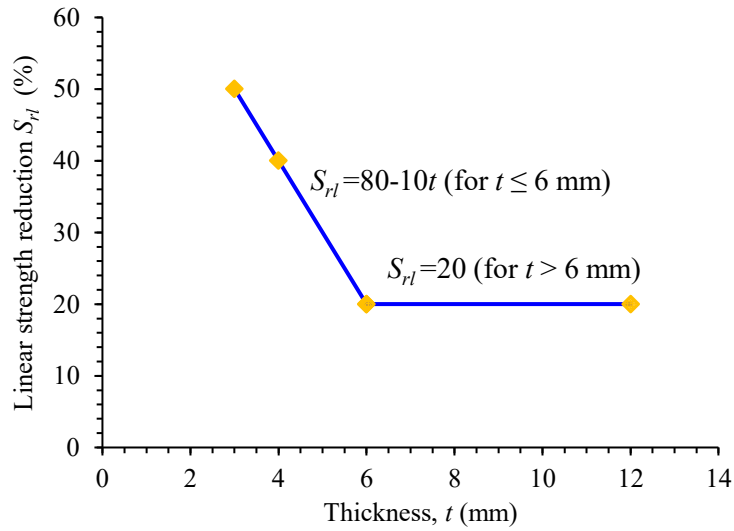


Figure 6. Linear strength reduction model for WHAR of S900 and S960 steel grades tubular joints (Pandey et al. 2021a).

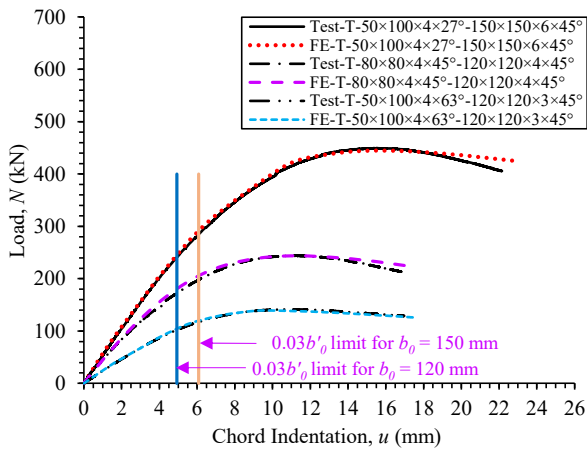


Figure 7. Test vs FE load vs chord indentation (N vs u) curves for DBB T-joints.

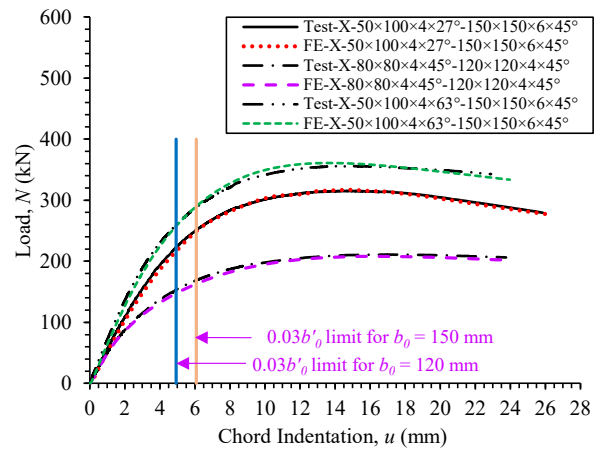
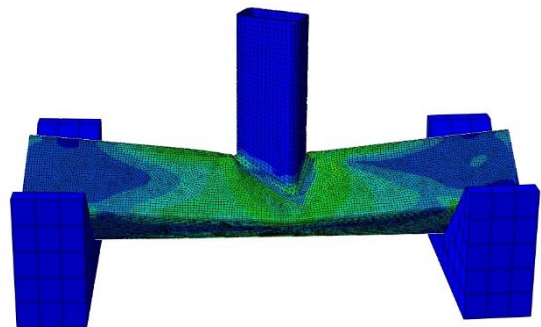
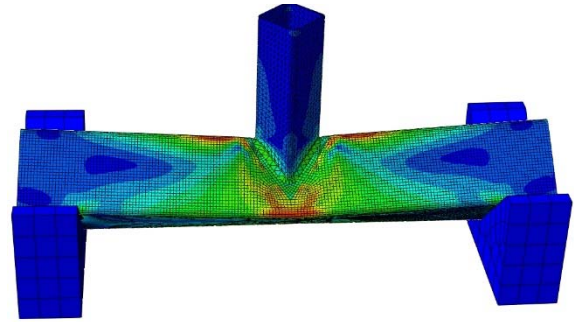


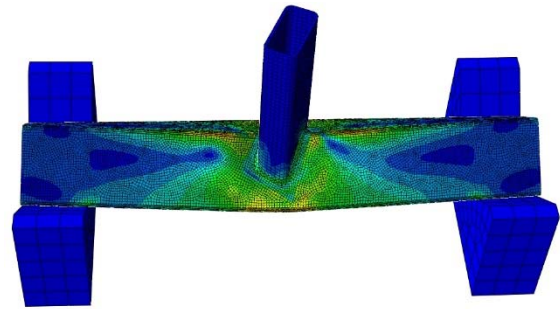
Figure 8. Test vs FE load vs chord indentation (N vs u) curves for DBB X-joints.



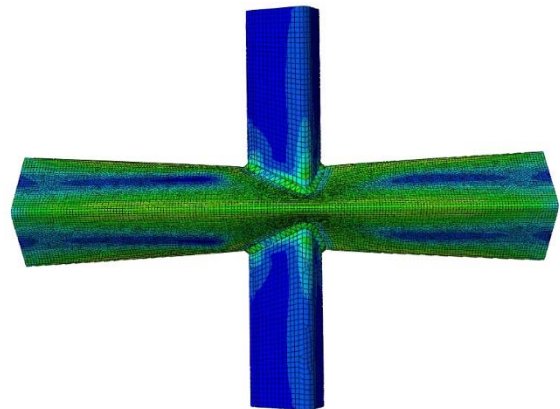
(a) Comparison of test and FE DBB T-joint ($\omega=27^\circ$) failed by chord crown failure (C) mode.



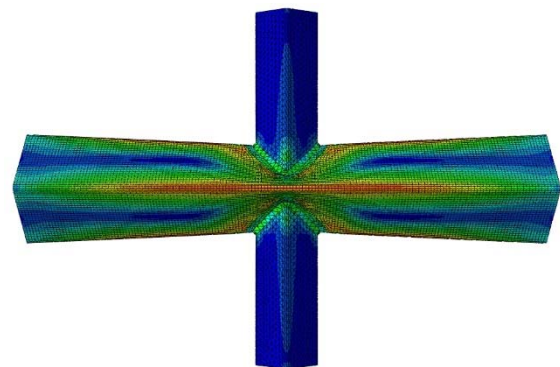
(b) Comparison of test and FE DBB T-joint ($\omega=45^\circ$) failed by chord crown failure (C) mode.



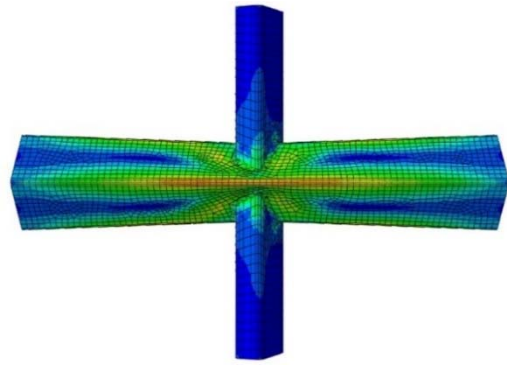
(c) Comparison of test and FE DBB T-joint ($\omega=63^\circ$) failed by chord crown failure (C) mode.
 Figure 9. Failure mode comparisons between test and FE specimens of DBB T-joints.



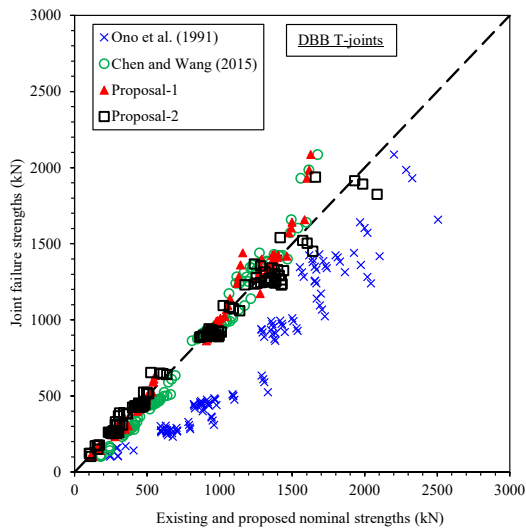
(a) Comparison of test and FE DBB X-joint ($\omega=27^\circ$) failed by chord crown failure (C) mode.



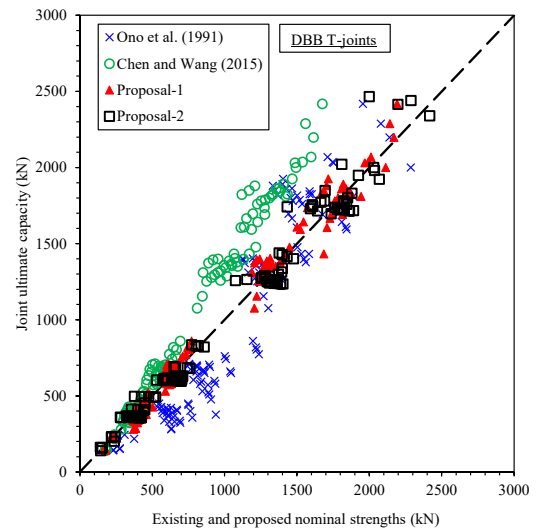
(b) Comparison of test and FE DBB X-joint ($\omega=45^\circ$) failed by chord crown failure (C) mode.



(c) Comparison of test and FE DBB X-joint ($\omega=63^\circ$) failed by chord crown failure (C) mode. Figure 10. Failure mode comparisons between test and FE specimens of DBB X-joints.

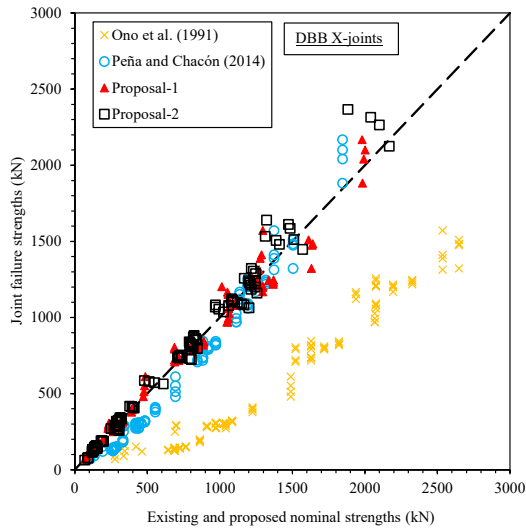


(a) Joint failure strengths ($N_{f,T}$) comparisons for DBB T-joints.

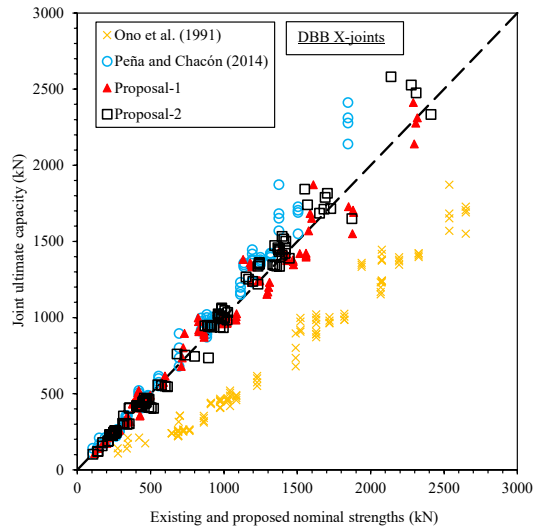


(b) Joint ultimate capacities ($N_{max,T}$) comparisons for DBB T-joints.

Figure 11. Comparisons of test and FE strengths with current and proposed strengths for DBB T-joints.



(a) Joint failure strengths ($N_{f,X}$) comparisons for DBB X-joints.



(b) Joint ultimate capacities ($N_{max,X}$) comparisons for DBB X-joints.

Figure 12. Comparisons of test and FE strengths with existing and proposed strengths for DBB X-joints.

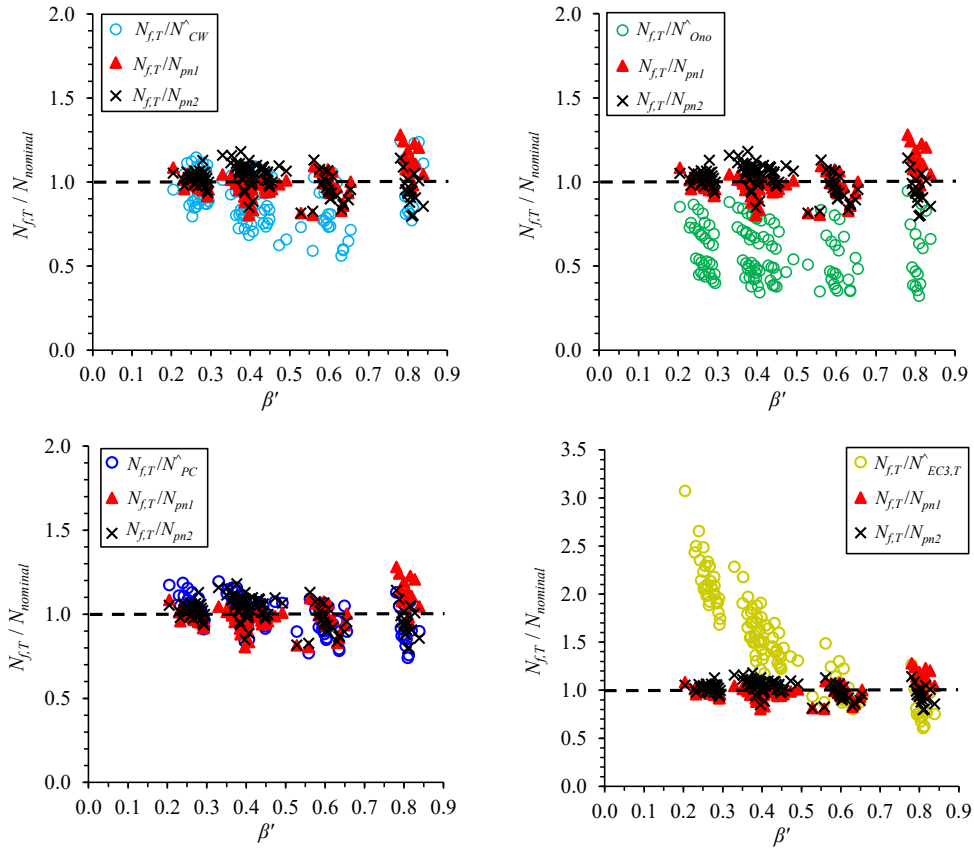


Figure 13. Distributions of joint failure strength comparisons ratios for DBB T-joints.

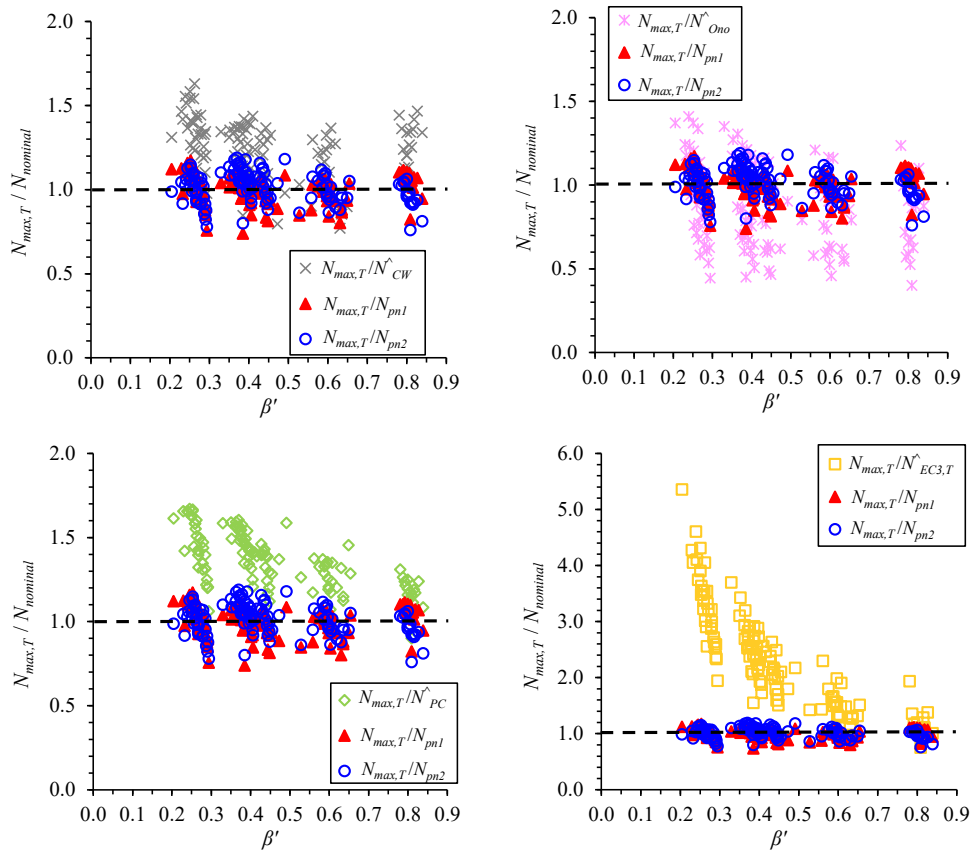


Figure 14. Distributions of joint ultimate capacity comparisons ratios for DBB T-joints.

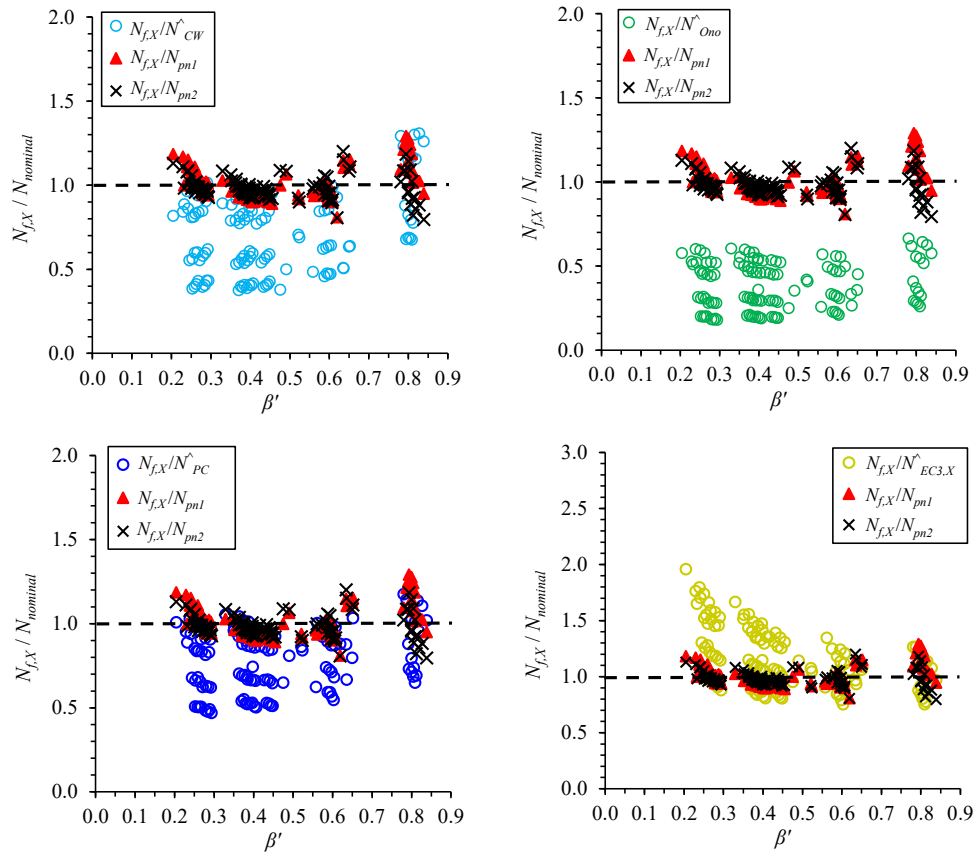


Figure 15. Distributions of joint failure strength comparisons ratios for DBB X-joints.

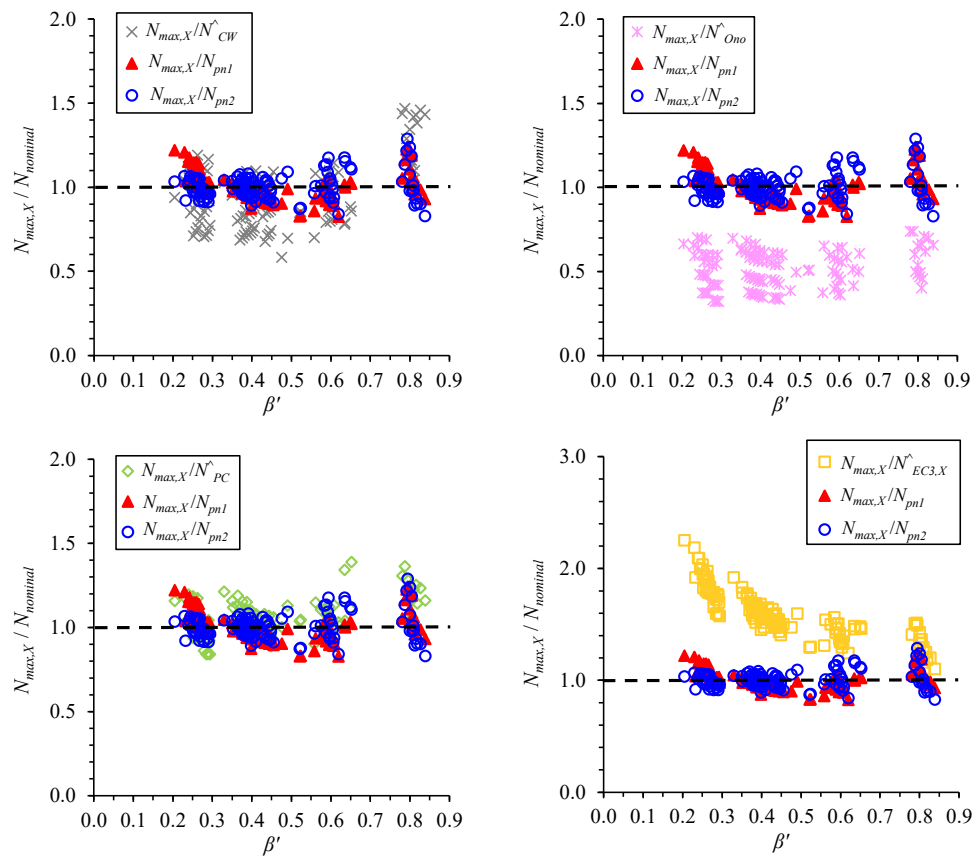


Figure 16. Distributions of joint ultimate capacity comparisons ratios for DBB X-joints.