Comments on: Extensions of some classical methods in change point analysis.

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1 Introduction

First and foremost, I should like to say that this paper is extremely interesting, and I was really grateful to have the opportunity to review it. As I try and explain below, I can foresee a lot of interest in this contribution from a very wide readership, in particular (in my opinion) in the field of econometrics. I believe that especially Chapter 3 of the paper provides a very interesting introduction to an alternative and more powerful way of dealing with breaks in a regression framework. As a more general comment, the changepoint problem has an enormous application potential, virtually in all disciplines: examples, to name but a few, include economics and econometrics (where "regime changes", e.g. following a recession or a major event such as an oil shock or the introduction of a common currency, play a pivotal role), climatology (the whole debate on climate change is, in essence, an application of the changepoint problem), engineering (where Page's procedure, referenced as the first example of changepoint analysis, was born in the context of quality control), and even linguistics (a famous example is based on identifying the number of translators in the Lindisfarne Scribes, which has also been studied in Horvath and Serbinowska, 1995).

In my comments, I focus mainly on two points: a discussion of what is common practice in the econometrics literature, and how this can benefit from the Darling-Erdos approach (Section 2), and some brief remarks on the issue of testing for the stability of covariance structures (Section 3).

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2 A further motivation for applications in econometrics

The econometric literature has considered the changepoint problem, mainly but not only in the context of regression analysis. Since the seminal contributions by Perron (1989) and Rappoport and Reichlin (1989), the literature has produced a wide set of results on the changepoint problem in a time series framework. However, as the Authors point out, the most influential contributions (primarily, Andrews, 1993) use, as a probabilistic tool, the weak invariance principle.

Whilst these procedures have the advantage of requiring relatively simple proofs, which are well adapted in the context of econometrics where models and assumptions aim at being quite general, however, as the Authors point out in Chapter 3, procedures suffer from lack of power versus breaks occurring close to either end of the sample. In order to better illustrate the problem, consider a standard change-in-mean problem, where the null of no change is tested by using a CUSUM procedure based on

$$\Lambda_T^* = \max_{0 \le \tau \le 1} \left| \frac{1}{\sqrt{T}} \left(\sum_{t=1}^{\lfloor T\tau \rfloor} X_t - \frac{\lfloor T\tau \rfloor}{T} \sum_{i=1}^{T} X_t \right) \right| = \max_{0 \le \tau \le 1} \Lambda_T(\tau),$$

where the X_ts are random variables with unit variance (for argument's sake) and some degree of dependence across t. We refer to Perron (2006) for a discussion of the possible assumptions on the X_ts. As mentioned by the Authors, a possibility to use Λ_T^* is to show a weak invariance principle for $\Lambda_T(t)$; under general assumptions, $\Lambda_T(t)$ should converge to a Brownian bridge. Thence, the Continuous Mapping Theorem could be applied to derive the limiting distribution of Λ_T^*.

The problem with Λ_T^* can be illustrated in terms of power versus the alternative of a break of finite magnitude occurring at a point in time t^*. It can be shown (the results are in Csorgo and Horvath, 1997, and have also been studied in a different context by Kao, Trapani and Urga, 2014) that tests based on Λ_T^* attain nontrivial power as long as t^* is strictly bigger than $O\left(\sqrt{T}\right)$. Thus, tests based on Λ_T^* are not powerful versus changes that occur relatively close to the beginning or the end of the sample. A possible alternative, considered in Andrews (1993) is to enhance the power of tests by normalising $\Lambda_T(\tau)$ by its asymptotic variance. Given that this is proportional to $\tau(1-\tau)$, this entails using

$$\tilde{\Lambda}_T^* = \max_{0 \le \tau \le 1} \left| \sqrt{\frac{T}{\lfloor T\tau \rfloor \times \lfloor T(1-\tau) \rfloor}} \left(\sum_{t=1}^{\lfloor T\tau \rfloor} X_t - \frac{\lfloor T\tau \rfloor}{T} \sum_{i=1}^{T} X_t \right) \right| = \max_{0 \le \tau \le 1} \frac{\Lambda_T(\tau)}{\tau(1-\tau)}.$$

However, when computing the test statistic at 0 or 1, some "divisions by zero" occur, whence the need for trimming the first and last few observations: again, tests based on $\tilde{\Lambda}_T^*$ do not have power versus breaks occurring close to either end of the sample. Although these results are sketched herein for the case of a change in the location of the data, the same results can be derived for changes in the slopes in a regression model, as the contribution by Hidalgo and Seo (2013) shows.

The usefulness of the Darling-Erdos approach is that it pushes the power of tests based on $\tilde{\Lambda}_T^*$ closer to the beginning/end of the sample. More precisely, it can be shown that, applying an Extreme Value theory to a suitably normalised version of $\tilde{\Lambda}_T^*$, tests are powerful versus breaks of finite size as long as t^* is strictly bigger than $O(\ln \ln T)$.

Such improvement comes at virtually no cost when it comes to the test having power versus breaks occurring at the middle of the sample. It can be shown that (again, see Csorgo and Horvath, 1997, and Kao, Trapani and Urga, 2014) when t^* is of size $O(T)$, tests based on the Darling-Erdos approach are powerful as long as the size of the break is strictly bigger than $O\left(\sqrt{\frac{\ln \ln T}{T}}\right)$. Conversely, when using trimmed versions of $\tilde{\Lambda}_T^*$, the test is powerful versus mid-sample alternatives of size $O\left(\frac{1}{\sqrt{T}}\right)$, which is virtually the same. From a technical viewpoint, however, the Darling-Erdos approach cannot be employed if only weak convergence is shown for $\Lambda_T(\tau)$ and its transformations: strong invariance principles are instead needed (Csorgo and Horvath, 1997, is an excellent reference). These are available for data that satisfy all the commonly employed assumptions made in the econometrics literature in terms of dependence (see e.g. Eberlein, 1986, and Ling, 2007).

Finally, I was very pleased and impressed to see the extension of the theory to panel data (Chapter 6). The literature has shown that the cross sectional dimension can lead to better inference when tests for breaks are applied to multivariate data; for example, Bai, Lumsdaine and Stock (1998) show that the estimation of the changepoint in a Vector AutoRegression improves with the dimension of the VAR, due to the presence of cross sectional information. Thus, a natural development to enhance the power of tests for structural breaks is to use panel data models; however, the inferential theory on structural changes in panels is still underdeveloped. There are a few exceptions: Feng, Kao and Lazarova (2008) and Bai (2010) propose procedures for dating breaks in simple settings with no cross sectional dependence amongst units; Kim (2010, 2011) investigates the estimation of change points in panel time trend models with cross-sectional dependence; Breitung and Eickmeier (2011), Chen, Dolado and Gonzalo (2014) and Han and Inoue (2011) investigate testing for changes in the loadings of a panel factor model; Kao, Trapani and Urga (2013) consider testing for breaks in a cointegrating regression framework. All these contributions are based on extending the weak invariance principle to the panel context.

The contribution of the Authors is bound to help the panel data literature to consider the more powerful Darling-Erdos approach.

3 Testing for breaks in covariance matrices

In Chapter 3, the Authors have studied the CUSUM approach when testing for changes in the covariance matrix of an n-dimensional time series, y_t. A seminal contribution by Aue, Horvath, Hormann and Reimherr (2009) lays out the theory to test for changes in $\Sigma = E\left(y_t y_t'\right)$, using primarily weak invariance principles; Kao, Trapani and Urga (2014) extend the results to verifying the stability of the eigensystem of Σ, also considering Darling-Erdos-type results.

As the Authors point out in their contribution, a difficulty that arises when applying the Darling-Erdos approach is to prove strong invariance principles for a multidimensional process: the dimension n enters the rates of approximation of the strong invariance principles needed to use the Darling-Erdos theory - we refer to several results by Goetze and Zaitsev (2008). When the dimension of y_t, n, is finite (and consequently, the dimension of Σ is finite), at least theoretically the existing results can be applied, and Extreme Value theorems such as equation (3.8) in the Authors' contribution can be derived - in finite samples, I am not sure whether asymptotic theory will provide a good approximation, since the impact of n on the rate of approximation of the strong invariance principle can be quite severe. However, a very interesting question is what would happen when $n \to \infty$.

References

Andrews, D.W.K., 1993. Tests for parameter instability and structural changes with unknown change point. Econometrica 61, 821-856.

Aue, A., Hormann, S., Horvath, L., Reimherr. M., 2009. Break detection in the covariance structure of multivariate time series models. Annals of Statistics 37, 4046-4087.

Bai, J., 2010. Common breaks in means and variances for panel data. Journal of Econometrics 157, 78-92.

Bai, J., Lumsdaine, R. L., Stock, J. H., 1998. Testing for and dating common breaks in multivariate time series. Review of Economic Studies 65, 395-432.

Breitung, J., Eickmeier, S., 2011. Testing for structural breaks in dynamic factor models. Journal of Econometrics 163, 71-84.

Chen, L., Dolado, J.J., Gonzalo, J., 2014. Detecting big structural breaks in large factor models. Journal of Econometrics (forthcoming).

Csorgo, M., Horvath, L. 1997. Limit Theorems in Change-Point Analysis. Chichester: Wiley.

Eberlein, E., 1986. On strong invariance principles under dependence assumptions. Annals of Probability 14, 260-270.

Feng, Q., Kao, C., Lazarova, S., 2008. Estimation and identification of change points in panel models. Mimeo, Center for Policy Research, Syracuse University.

Götze, F., Zaitsev, A.Yu., 2008. Bounds for the rate of strong approximation in the multidimensional invariance principle. Theory of Probability and its Applications, 53, 100-123.

Han, X., Inoue, A., 2011. Tests for parameter instability in dynamic factor models. Mimeo.

Hidalgo, J., Seo, M.H., 2013. Testing for structural stability in the whole sample. Journal of Econometrics, 175, 84-93.

Horvath, L., Serbinowska, M., 1995. Testing for changes in multinomial observations: the Lindisfarne Scribes problem. Scandinavian Journal of Statistics, 22, 371-384.

Kao, C., Trapani, L., Urga, G., 2013. Testing for breaks in cointegrated panels. Mimeo.

Kao, C., Trapani, L., Urga, G., 2014. Testing for instability in covariance structures. Mimeo.

Kim, D., 2010. Common local breaks in time trends for large panel data. Mimeo. Department of Economics, University of Virginia.

Kim, D., 2011. Estimating a common deterministic time trend break in large panels with cross sectional dependence. Journal of Econometrics 164, 310-330.

Ling, S., 2007. Testing for change points in time series models and limiting theorems for NED sequences. Annals of Statistics 35, 1213-1237.

Perron, P., 1989. The great crash, the oil price shock and the unit root hypothesis. Econometrica 57, 1361-1401.

Perron, P., 2006. Dealing with breaks. Chapter 8 in the Palgrave Handbook of Econometrics: Vol. 1 Econometric Theory edited by T.C. Mills and K. Patterson. Palgrave Macmillan: Basingstoke.

Rappoport, P., Reichlin, L., 1989. Segmented trends and non-stationary time series. Economic Journal 99, 168-77.