Verification of a rigorous 2D model of rough surface scattering

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Summary
A rigorous two-dimensional (2D) model of electromagnetic surface scatter has been developed, based on a boundary element method (BEM) established by Simonsen [1]. Simulated far-field scatter is compared to that measured from a laser scatterometer for a sinusoidal grating, with a mean difference of 3% of the peak intensity.

Introduction
As part of a project to develop a fast in-process method to measure the surface topography of stochastic, structured, machined or additive surfaces, we have developed a rigorous 2D model that predicts the angular scattering distribution of an incident electromagnetic wave. Here we verify the first stage in the development of a theory for this technique.

Discussion
Analytical solutions to scattering problems rely on approximations, such as the so-called "slightly rough surface assumption", or the Kirchhoff-tangent approximation. However, machined or additive metal surfaces may have RMS surface heights of tens of micrometres and autocorrelation lengths of hundreds of micrometres, which when using visible light are easily beyond these approximations. Numerical and hybrid approaches have made progress to accurately compute scattered fields while accounting for non-linear surface effects such as shadowing, but few are capable of this over relatively large areas (in our case, 1 mm^2). For the more general optically rough surfaces considered here, multiple scattering is inherent and a rigorous vector solution of Maxwell's equations that properly accounts for polarisation, surface plasmons, etc. is required. For this purpose, we expand the scattered field in terms of a bi-layer of discrete dipoles that can be thought of as a surface equivalent of the coupled dipole method [2]. The following model essentially follows that presented by Simonsen [1], based on earlier work by Maradudin et al. [3].

It is simple to confirm that our model gives realistic results for surfaces for which we can predict the results, for example with a flat surface, with sinusoidal surfaces with amplitudes a lot less than the source wavelength (optically smooth), and for Mie scattering. In all cases, the model gives the predicted results from simple Fourier optics. Comparisons have also been made with a non-rigorous model reliant on the Kirchhoff-tangent approximation, its implementation following that of Beckmann and Spizzichino [4], giving near identical results in the regime where there are no expected multiple reflections, and as expected, worsening at the point where multiple reflection would be predicted by geometrical optics. However, the surfaces we are interested in may have spatial wavelengths and amplitudes that are much larger than the source wavelength (optically rough), therefore, there are no simplifications that can be applied to give an alternative "reference" result, even for deterministic surfaces.

To verify the theory for a 2D surface we have used a sinusoidal grating produced using nickel electroforming, which has a mean amplitude of 10.05 µm and a surface wavelength of 134.8 µm. The bidirectional reflectance distribution function (BRDF) from this grating has been measured with a high-accuracy angular scatterometer at the University of North Carolina at Charlotte (UNCC), named CASI [5]. The incident illumination is a p-polarised beam produced by a laser source (λ = 632.8 nm) which illuminates the grating at an angle of incidence of +5° from the grating's surface normal and with a spot size of roughly 1.5 mm. A scan of BRDF values is taken along the semi-circular path that lies on the plane of incidence. Our computational model calculates the far field angular scattered intensity distribution (ASD) for a virtual grating with properties matching that of the real one. The scattering problem for the grating is solved, and the far-field field strength calculated across 50,000 uniformly distributed angles between −80° and +80°; a convolution of these values with an aperture function is then necessary to appropriately imitate the measuring process of CASI.

Figure 1 shows a comparison between the forms of our BEM simulation and the experimental measurement after normalisation, with a mean absolute difference between the two of 0.22 AU, equivalent to 3% of the peak intensity. Some of the differences present are believed to be noise produced by reducing the beam width with a pinhole to 1.5 mm and imperfections due to the grating manufacture. Where

Fig.1. The BRDF as measured by CASI, compared to the ASD obtained from our BEM model.

the intensity varies rapidly, aliasing effects are present in the CASI data, which could be mitigated by using a smaller angular sampling step size.
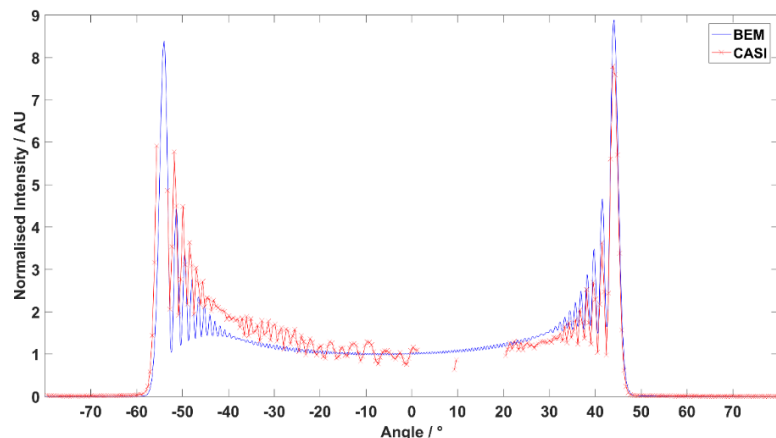
Conclusions

We have developed a 2D rigorous BEM solution to Maxwell's equations that will be used to predict scattering from a rough surface. We compared its predictions with the measurement data from a high-accuracy scatterometer with good agreement, especially when differences in the scatterometer set-up and the physical modelling are considered. Future work will experimentally verify the theory in the multiple reflection regime and develop a three dimensional solver. Funded by EPSRC grant EP/M008983/1.

References
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