

The Design of a Position-Based Repetitive Control for Speed Ripple Reduction in PMLSMs

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Abstract—Periodic speed errors can occur in permanent magnet linear synchronous machines for two reasons: 1) a periodic reference signal; 2) cogging force and friction. For reducing such periodic errors, iterative learning control or repetitive control approaches, used in conjunction with more common control actions, can be strongly effective. However, the design of the stability filter, robustness filter and other parameters for a traditional repetitive controller can be a complex task and may need to be adjusted when the frequency of such periodic error varies. Existing solutions tend to develop more adaptive tuning methods for repetitive controller to enhance the whole control system. This paper shows that the performance of a traditional speed loop can be enhanced with a repetitive controller without complicating the tuning of the repetitive controller. Consequently, a position-based repetitive control combined with deadbeat current control method is proposed. Simulation results show that the proposed method is effective for reducing speed ripple at difference frequencies without necessarily adjusting its parameters.

Index Terms—repetitive control, deadbeat control, speed ripple reduction, permanent magnet linear synchronous machine

I. INTRODUCTION

Despite different design approaches, iterative learning control (ILC) and repetitive control (RC) can be considered essentially the same type of control method [1]. There is no doubt that control methods such as ILC and RC represent the best solution for tracking periodic references or rejecting periodic errors. Taking benefit from their inherent learning capability, iterative learning controllers and repetitive controllers can be designed without necessarily knowing the parameters of the plant (only the frequency of the target periodic error is required). However, the more information about the plant is known, the easier the controller can be tuned.

Permanent magnet linear synchronous machines (PMLSM) are widely used in many industrial applications for undertaking periodic tasks. Hence, the implementation of ILC or RC in PMLSM drives is also widely discussed in existing literature. A common approach is to use ILC or RC as a feedforward controller to enhance the performance of a PID feedback controller [2].

Many efforts have been taken in order to make the iterative learning controller or repetitive controller adaptive and robust.

[3] presents two solutions for increasing robustness to measurement noise when using ILC for PMLSMs. A filtered ILC is developed for the case when the frequency of this noise is outside the desired output spectrum. Otherwise, an ILC with decreasing learning gain can be used, however, convergence time would increase. [4] presents an ILC with a learning gain that can be updated online. The solution is claimed to be adaptive when the reference profile and disturbance are iteration varying (i.e. along the learning path, the reference trajectory and disturbance do not remain constant). Some other online tuning methods for RC and ILC can be found in [5] [6]. A RC integrated with an adaptive robust control law is proposed in [7].

As mentioned above, RC or ILC is generally used as an enhancement of a main control system, and therefore they are designed relatively independently from the rest of the system. Therefore, the existing solutions generally tend to modify RC or ILC to cope with the uncertainties (such as reference profile changes, disturbance variations) occurring in the rest of the system. However, this is not the only way. None of these papers have mentioned that the rest of the control system can be designed to cope with RC or ILC in a better way. Consequently, the overall control system can naturally be more adaptive to such uncertainties, and the parameters of RC or ILC can possibly be universal.

The aim of this paper is to introduce a new approach of designing a RC for PMLSM. The key point is to show how the rest of the control system can cope with the RC. For this purpose, the relationship between the parameters of RC (including its gain, length of delay chain, stability filter, robustness filter as shown in Fig. 1) and the rest of the control system is explained. Consequently, a control topology combining a position-based RC and a deadbeat current control (DBCC) loop is proposed. The reason of implementing RC based on position (instead of traditionally time-based) is mainly due to the fact that the cogging force of PMLSM is a function of mover position. Deadbeat current controller has been used for PMLSM in [8] [9], the combination of RC and DBCC for PMLSM has never been presented to authors knowledge. The most important reason of choosing DBCC is due to its fixed delay nature, a properly designed DBCC loop can be seen as a delay or two. This feature can simplify the filters in RC.

Overall, simulation results show that the proposed control

method is able to cope with reference of different frequencies without changing its parameters.

II. MECHANICAL MODEL OF PMLSM

Before any speed ripple reduction methods can be developed, proper mechanical model of PMLSM needs to be derived in order to include the main contributors of the ripple. For PMLSM, its inherent cogging force (due to armature slots and end effects), and friction (drag force) are the two main causes for the force ripple of the machine, and consequently generate speed ripple. In this paper, the mechanical model of PMLSM is modelled including these two effects as in (1).

$$F_e = M \frac{dv}{dt} + F_{cogg} + F_{fric} + F_{load} \quad (1)$$

where, F_e represents the electromagnetic force generated by the PMLSM, F_{cogg} denotes the cogging force of the PMLSM as in (2), F_{fric} denotes the friction as in (3), F_{load} represents the load force, M is the mass (kg) of the mover of PMLSM, v is the speed of the mover (m/s).

According to [10], the cogging force of PMLSM can be expressed as in (2).

$$F_{cogg} = \sum_{i=1}^N A_i \sin\left(\frac{2\pi i}{\tau} x + \varphi_i\right) \quad (2)$$

where, A_i is the amplitude (N) of the i^{th} harmonic, τ is the distance (m) between each pole pitch, x is the mover displacement (m), φ_i is the phase shift of the i^{th} harmonic. The equation shows that F_{cogg} is mover position x dependent. Also, as demonstrated in [11], the order of the cogging force depends on the number of slots and poles.

A widely known friction equation (3) including coulomb friction, viscous friction, and Stribeck effect can also be found in [4] [10].

$$F_{fric} = [F_c + (F_s - F_c)e^{-(\frac{v}{v_s})^2}] \text{sign}(v) + Bv \quad (3)$$

where, F_c is the minimum level of Coulomb friction (N), F_s is the static friction (N), v_s is the lubricant parameter (m/s) for the Stribeck effect, B is viscous friction factor (Ns/m).

III. EQUATION OF THE TRADITIONAL RC

The diagram of the traditional RC is shown in Fig. 1.

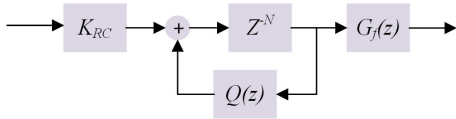


Fig. 1. Block diagram of the traditional repetitive controller

Its transfer function can be expressed as in (4).

$$G_{RC}(z) = \frac{K_{RC} z^{-N} G_f(z)}{1 - Q(z) z^{-N}} \quad (4)$$

where, the K_{RC} is the gain of RC, N is the length of the delay chain in RC, and it is normally chosen to be the closest integer to the ratio between the sampling frequency f_s and

the fundamental frequency of the target error. $G_f(z)$ is called stability filter, of which some commonly used options can be the phase lead compensator z^M , or the reverse of the plant. $Q(z)$ is known as robustness filter, of which some options can be the forgetting factor, or the moving average filter. In fact, the function of these parameters and filters in RC can be understood from the viewpoint of system delay. Taking the control topology in Fig. 2 for example, where, the structure of the repetitive controller is the same as in Fig. 1.

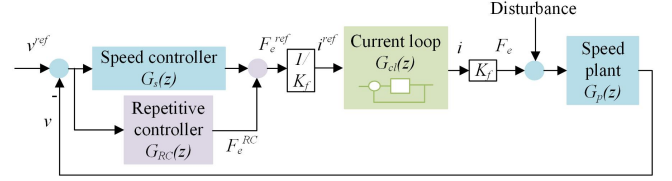


Fig. 2. Block diagram of an example topology with repetitive control

The key point of using RC to suppress periodic error caused by external sources (such as periodic references, periodic disturbances) is to generate the same periodic signal internally. Therefore, such periodic signal generated by RC needs to be synchronized with the target periodic error. And the key point for achieving such synchronization is to align the phase by considering all the system delays.

For example, the RC as in Fig. 2 needs to generate an additional force reference F_e^{RC} to cancel the speed error $(v^{ref} - v)$. Consequently, the phase of all the frequencies in F_e^{RC} should be synchronized with all the frequencies contained in $(v^{ref} - v)$. Therefore, the delay between the force reference F_e^{ref} applied and the speed response v of the plant need to be compensated. For this reason, the stability filter $G_f(z)$ is necessarily included in RC. This also explains why $G_f(z)$ can be chosen as the phase lead compensator Z^M or the reverse of the plant seen by the RC (i.e. $\frac{1}{G_{cl}(z)G_p(z)}$).

Besides the pre-mentioned synchronization issue, the noise in the speed feedback also needs to be considered. Therefore, robustness filter $Q(z)$ is used. $Q(z)$ can be a forgetting factor Q_{RC} (i.e. a constant value between 0 and 1) or a moving average filter.

What is more, gain K_{RC} of RC as in Fig. 1 and Fig. 2 is responsible for the amplitude of force reference F_e^{RC} , therefore, the choice of K_{RC} can refer to the ratio between F_e and v according to the speed plant $G_p(z)$.

Additionally, length N of the delay chain in RC is normally chosen to be the closest integer to f_s/f_d (where, f_s is the sampling frequency, f_d is the fundamental frequency of the target speed error).

After analysing the purposes of using $G_f(z)$, $Q(z)$, K_{RC} , and N , we can clearly see that $G_f(z)$ and N may need to be redesigned once conditions like speed error frequencies and system delays change, which is likely to happen once the speed reference changes. As a result, a more adaptive control topology is developed in the following section, which allows the RC to work at different frequencies without necessarily redesigning its parameters.

IV. DESIGN OF RC WITH DBCC

As mentioned above, the system delay is an important feature for the design of RC. The system delay seen by the RC, as in Fig. 2, consists of the delay of the current loop plus the delay of the speed loop plant. A straightforward way to simplify the system delay is to use deadbeat control for the current loop.

For a properly tuned deadbeat current loop as demonstrated in [12], its delay is fixed to be twice of the sampling period (i.e. $2T_s$). Therefore, $G_{cl}(z) = z^{-2}$, and the delay of such current loop can be easily compensated by a simple phase lead compensator z^2 . In such way, the control topology as shown in Fig. 3 is developed.

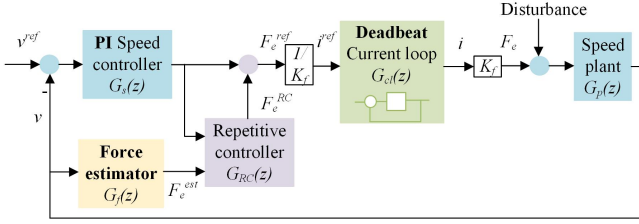


Fig. 3. Block diagram of the modified topology with repetitive control

As can be seen in Fig. 3, a force estimator has also been added. In this way, the RC would work with the force error instead of the speed error as in Fig. 2. Since the force estimator would be the reverse of the speed plant, the proposed topology is equivalent to dividing the stability filter $G_f(z)$ into two parts: one part is a phase lead compensator z^M inside RC, the other part is the reverse of the speed plant (i.e. the force estimator) outside of RC. During the $(k+1)^{th}$ sampling interval, the equation for the force estimator can be seen as in (5). In fact, the proposed control can possibly work without including $F_{fric}(k)$ in the force estimator. This will be discussed later in section V.

$$\begin{aligned} F_e^{est}(k) &= M \frac{v(k+1) - v(k)}{T_s} + F_{fric}(k) \\ &= M \frac{v(k+1) - v(k)}{T_s} \\ &\quad + [F_c + (F_s - F_c)e^{-(\frac{v}{v_s})^2}] \text{sign}(v) + Bv \end{aligned} \quad (5)$$

However, still, N needs to be changed if the fundamental frequency of the target force error changes. It can be told from the mechanical model (1-3) that, for variable speed, both speed-based (i.e. $F_{fric}(v)$) and mover position-based (i.e. $F_{cogg}(x)$) force ripple exists, while for constant speed, only mover position based ripple exists. Considering the case when only step changes are required in the speed reference v^{ref} profile, the speed demand v^{ref} is pulses or steps, i.e. when most of the time the speed is supposed to be constant, a position-based RC can be used. The implementation of RC can be found in [13], where an angle-based RC is proposed for rotational machines. This angle-based RC includes the time-to-angle conversion, main body of RC, and angle-to-time conversion. The key point is to memorize the target

error with respect to N selected rotor mechanical locations over one cycle (i.e. when the mechanical position equals to $2\pi/N, 4\pi/N, \dots, 2\pi$), length N of the delay chain for the traditional RC becomes therefore the number of the memory locations, which does not need to be changed when the rotor speed changes. The same equations of the angle-based RC in [13] can be applied for the PMLSM if converts the mover displacement x into an equivalent rotor position (i.e. $2\pi x/l$, where l is the full journey length (m) of the PMLSM).

Overall, the proposed control diagram is shown in Fig. 4. Ideally, the position-based RC is responsible for any position-based force error (i.e. cogging force error), whereas the speed PI controller $G_s(z)$ is responsible for the transient actions.

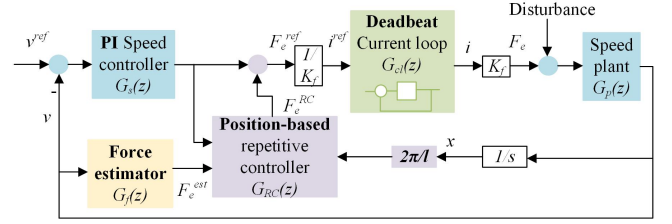


Fig. 4. Block diagram of the proposed topology with repetitive control

As discussed above, stability filter $G_f(z)$ has been chosen to be a phase lead compensator z^2 . It is also worth noting that the implementation of phase lead compensator z^2 in the angle-domain requires future position of the mover. The future position is predicted (as in [13]) using the speed feedback by assuming speed to be constant during the next $2T_s$.

The remaining parameters to be tuned for RC are the memory length N , its gain K_{RC} and robustness filter $Q(z)$, which will be discussed later in section V.

V. SIMULATION RESULTS

A simulation model is built using Matlab/Simulink. The PMLSM model includes the friction model and cogging force model as demonstrated in section II. The control loops are implemented as shown in Fig. 4. Again, the deadbeat current loop is implemented according to [12], the force estimator is as demonstrated in section IV, and the position-based RC shares the same equations with the angle-based RC in [13] (the mover displacement x is converted into an equivalent rotor position by times $2\pi/l$, l is the full journey length of PMLSM).

As noticed from Fig. 4, the proposed RC learns from the force error, and generates force reference. Therefore, the gain K_{RC} can be set to be one. $Q(z)$ is chosen to be a simple forgetting factor Q_{RC} . According to [14], the more Q_{RC} is close to one, the better is performance. Therefore, Q_{RC} is chosen to be 0.999. Key parameters for the simulation tests are as shown in Table I.

The aim of these tests is to investigate the performance of the proposed ripple rejection method by comparing the machine speed waveforms with/without the proposed RC, with N varies from 500, 1000, 2000 to 4000 respectively, with/without $F_{fric}(k)$ in the proposed force estimator, and

TABLE I
MACHINE AND CONTROL PARAMETERS

Symbol	Quantity	Value
τ	Pole pitch	0.016 m
L	Full journey length of PMLSM	0.096 m
$F_{cogg}(x)$	Cogging force	$10\sin(\frac{4\pi}{\tau}x)$ N
F_c	Minimum level of Coulomb friction	2.5 N
F_s	Static friction	5 N
v_s	Lubricant parameter	0.1 m/s
B	Viscous friction factor	10 Ns/m
M	Mass of mover	0.08 kg
f_s	Sampling frequency	10 kHz
T_s	Sampling period	100 μ s
N	Length of memory array	500/1000/2000/8000
K_{RC}	Gain of RC	1
Q_{RC}	Forgetting factor of RC	0.999

with the period of speed reference varying from 0.78s to 1.56s and then from 1.56s to 3.12s. The resultant speed waveforms are shown in Fig. 5. Further discussions and more results will be given in the following subsections from three aspects.

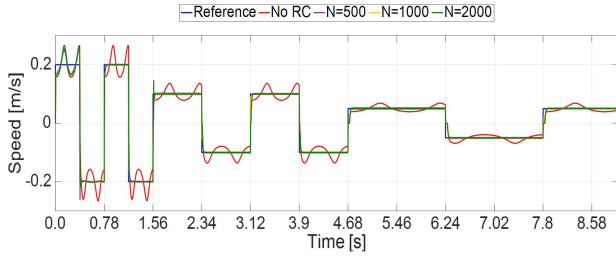


Fig. 5. Speed waveforms from the simulation tests with $N = 500, 1000, 2000$ and under variable frequency speed references (the force estimator is implemented as equation (5) assuming mechanical parameters M, B, F_c, F_s, v_s are known)

A. Influence of N on Performance

Fig.6 shows the zoom-in of Fig.5. It shows that the proposed control method is effective for reducing speed error from a peak to peak value of 54% (red line) to 1.8% (when $N=500$), 0.7% (when $N=1000$), or 0.1% (when $N=2000$). This can be easily understood since larger memory can offer better ripple learning accuracy, therefore leads to better performance.

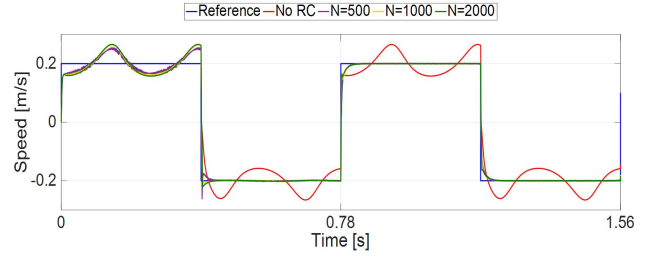
Fig.6(a) also shows that the proposed controller converges fast. As can be seen, the controller is able to remove the speed error after only half of a period (0.39s).

B. Performance under Variable Frequency Speed

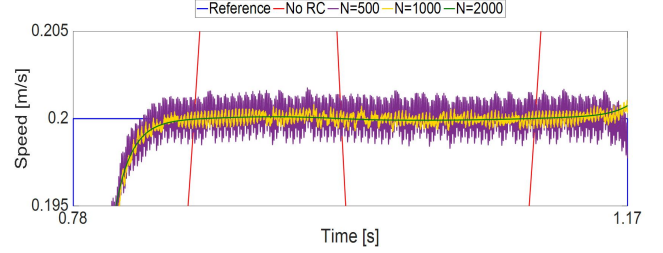
Fig.7 also shows the zoom-in of Fig.5. The results verify that the proposed control method can work for speed references of different frequencies without necessarily changing the control parameters.

C. Influence of Friction Model on Performance

Considering that parameters such as F_c, F_s, v_s for the friction force as expressed in (3) might be difficult to identified. It is worth to investigate the influence of the friction model on performance. A simulation test is carried out with the F_{fric}

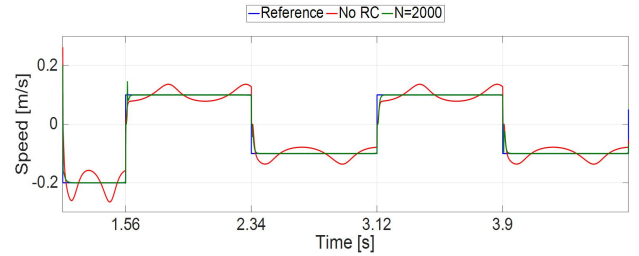


(a) Zoom in of Fig. 5

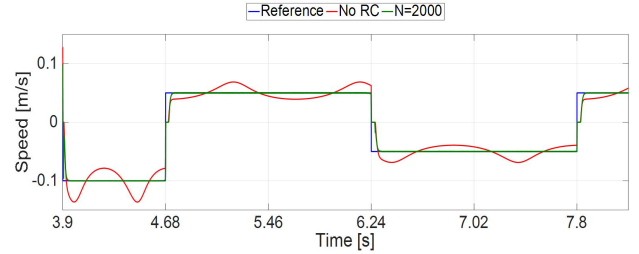


(b) Zoom in of (a)

Fig. 6. Performance of the proposed controller with different choices of N (the amplitude of speed reference is ± 0.2 m/s, period is 0.78s)



(a) When speed amplitude varies from ± 0.2 m/s to ± 0.1 m/s, and period varies from 0.78s to 1.56s



(b) When speed amplitude varies from ± 0.1 m/s to ± 0.05 m/s, and period varies from 1.56s to 3.12s

Fig. 7. Performance of the proposed controller with variable frequency speed reference ($N=2000$, zoom in of Fig. 5)

term being removed from the proposed force estimator as in (5). Comparing the speed waveforms with and without the friction model in the force estimator ($N=2000$), Fig.8(a) shows that the performance degrades without the friction model, especially after 4.68s when the speed reference is ± 0.05 m/s. However, as discussed above in subsection A, the performance can be improved by the using larger memory. Therefore,

another simulation test is carried out with $N=8000$ and without the friction term in the force estimator. As it can be seen from Fig.8(b), the performance is improved effectively, and becomes even better than when with the friction model and $N=2000$. This indicates that it may not be necessary to identify the friction model for applying the proposed control, which can be another benefit.

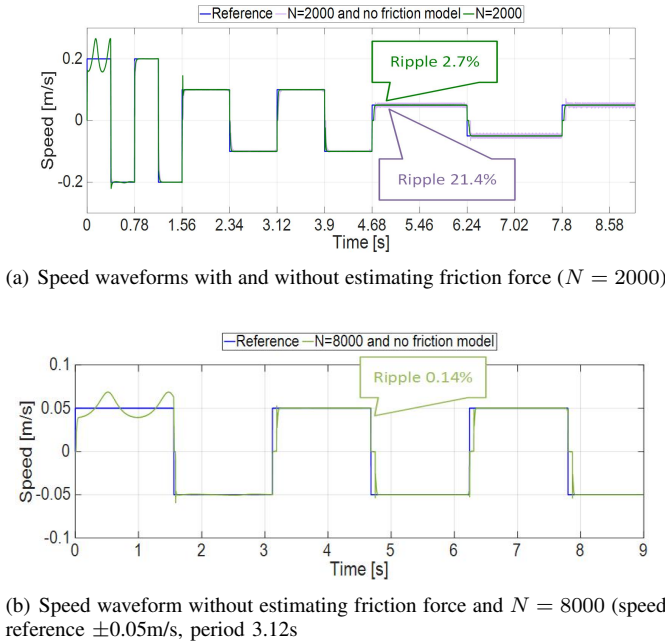


Fig. 8. Performance of the proposed controller with/without including the friction model in the proposed force estimator

VI. CONCLUSIONS

In this paper, a position-based repetitive controller is proposed for a linear permanent magnet synchronous motor application where the force ripple is high. In the preliminary analysis, it has been shown that, if combined with a current deadbeat control, the tuning of the repetitive controller can be rendered independent from the frequency of the periodic reference. The initial simulation results show good performance of the position-based RC which will be expanded in further works.

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