EXAMINATION OF ACOUSTIC INSULATION OF COMPOSITE METAMATERIALS WITH NEGATIVE STIFFNESS ELEMENTS INCLUSIONS

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Damping is one of the most important properties of materials, being responsible for increased durability and conmfort of the structures. In this paper the damping enhancement of materials with internal oscillators is examined. Wave Finite Element (WFE) method is used to get the wave propagation of an two-dimensional layered mechanical material having embedded nonlinear internal oscillators acting as negative stiffness element. The acoustic properties of the panel are studied in a Statistical Energy Analysis (SEA) scheme.

Keywords: damping, metamaterials, negative stiffness, composites

1. Introduction

Despite their superior structural characteristics, composite structures exhibit inferior dynamic and acoustic performance levels compared to conventional metallic ones. The large number and variety of elastic waves propagating within a layered composite, implies high structure-borne vibration transmissibility and high noise transmission. Research interest has therefore focused towards achieving increased structural damping through measures of reduced mass, volume and cost impact. A methodology receiving increasing attention due to its attenuation efficiency (particularly in the low-frequency range) is the inclusion of internal tuned resonators [1]. As demonstrated below, a number of authors has suggested that the inclusion of negative stiffness (NS) elements in the structural ensemble, can drastically amplify internal oscillations and consequently the structural damping performance of the configuration.

The beneficial impact of NS mechanisms on vibration isolation has been reported as early as 1957 in the pioneering publication of Molyneux [2]. It was followed by the milestone developments of the authors in [3, 4] where the design of mechanical isolating NS mechanisms was achieved through the use of discrete macroscopic elements. It was demonstrated that the exceptional performance of these systems was attained through reducing the strain energy stored within the system during oscillation (by reduction of the overall effective stiffness), thus increasing the loss factor of the oscillator defined as:

$$\eta = \frac{\text{energy dissipated per cycle}}{\text{maximum strain energy stored in the system}} \quad (1)$$

There exist generally two design streams of NS inspired structures; one of them focussing on the development of continuous metamaterials containing NS phase inclusions (see [5]) and the second

one dedicated to the design of systems containing discrete macroscopic NS elements. Both research streams share common reported benefits (high structural damping and vibration isolation performance levels) however not the same challenges. With regard to the design of macroscopic NS elements, a variety of designs has been proposed for the realisation of such configurations, incorporating various structural elements such as post-buckled beams, plates, shells and precompressed springs, arranged in appropriate geometrical configurations [6, 7, 8, 9, 10]. Experimental validation of NS vibration isolation mechanisms with their designs based on Euler post-buckled beams have been presented in [12, 14, 15, 16].

In this work Wave Finite Elements (WFE) method was employed for the wave dispersion analysis of the structure. WFE method has, also, been used to examine the banded behaviour of a periodic beam in [17]. The WFE has recently found applications in predicting the vibroacoustic and dynamic performance of composite panels and shells [18, 19, 20, 21, 22, 23, 24] , with pressurized shells [25, 26] and complex periodic structures [27, 28, 29, 30] having been investigated. The variability of acoustic transmission through layered structures [31, 32], as well as wave steering effects in anisotropic composites [33] have been modelled through the same methodology.

In this paper the authors propose the design of a novel layered mechanical metamaterial having implemented NS inclusions. The paper is organized as follows: In Sec.2 the design of the mechanical NS inclusions is discussed and a number of numerical parametric results are provided on a variety of NS designs. In Sec.3 a periodic layered structure having NS inclusions is Finite Element (FE) modelled. The characteristics of the elastic waves propagating within the structure are sought through a two-dimensional periodic structure theory scheme. The damping loss factor associated with each propagating wave type is computed, as is the sound Transmission Loss (TL) of the structural panel. Conclusions on the exhibited work are eventually drawn in Sec.4.

2. Design of mechanical NS internal inclusions

In this work, an element that is stable and suitable for implementation in honeycomb cores is proposed as illustrated in Fig.1. Similar to a beam, the proposed tripod structure snaps through around the equilibrium point during operation, rendering it effectively a NS element around that region. Positive stiffness (viscoelastic straps) are attached to the element in order to render the ensemble of the inclusion stable, and facilitate its activation during operation.
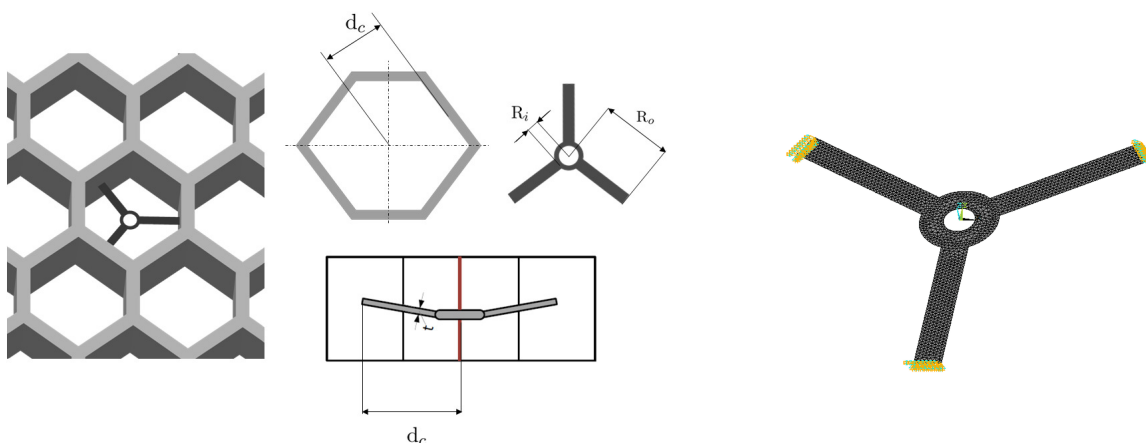

Figure 1: Left: The employed prestressed NS elements within the honeycomb core. Middle: Side view of the NS elements within a honeycomb cell. Positive stiffness elements and honeycomb cell are also illustrated, Right: FE model employed for the quasi-static analysis of the tripod mechanism.

A parametric survey is hereby performed in order to determine the sensitivity of the NS charac-

teristics of the proposed structure as a function of its design. For this reason, the inclusion is FE modelled as illustrated in Fig.1 and nonlinear quasi-static solutions are sought in order to determine its mechanical characteristics within the operational displacement regime. The design characteristics under investigation are the ratio of the mechanism's outer and inner dimensions $r_R = R_o/R_i$, the thickness t as well as the prestress factor equal to the ratio of the outer dimension of the mechanism R_o and the actual size of the honeycomb cell d_c expressed as $p_f = R_o/d_c$. A two-step, nonlinear quasi-static analysis was performed. The structure was initially prestresssed to the position corresponding to it being fitted inside the honeycomb cell. A small perturbation load is applied in order to facilitate the convergence of the FE solution close to the bifurcation point of the analysis (i.e. ensure a stable deflection of the tripod structure upwards during precompression). The second step consists of applying displacement constraints at the central ring of the tripod in order to trigger the snap-through process. The reaction forces measured at the supports are exhibited below.

The general trend for the displacement/force characteristics of the snap-through region of the proposed configuration are shown in Fig.2a. It is observed that a lower r_R factor will not only imply reduced weight for the device but it will also result in a more pronounced NS region and a higher NS coefficient. For higher r_R factors the NS region of the displacement/force characteristics tends to be minimised. A parametric study on the influence of the p_f design parameter on the snap-through
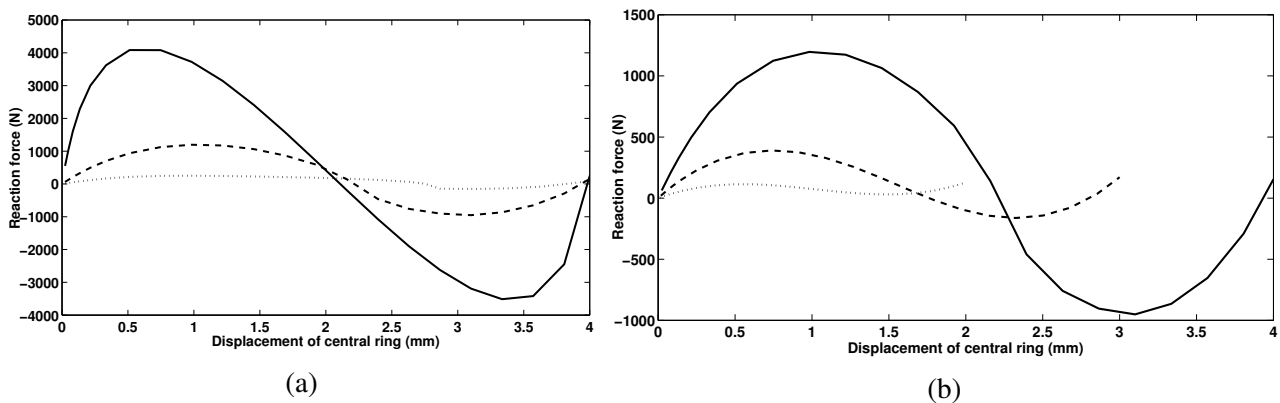

Figure 2: (a) Impact of the r_R design parameter on the snap-through characteristics of the proposed tripod mechanism. Displacement/force results for: r_R=10/8 (–), r_R=10/4 (- -), r_R=10/2 ($\cdots$). All results are for p_f=1.2, (b) Impact of the p_f design parameter on the snap-through characteristics of the proposed tripod mechanism. Displacement/force results for: p_f=1.2 (–), p_f=1.1 (- -), p_f=1.05 ($\cdots$). All results are for r_R=10/8

characteristics of the device are presented in Fig.2b. It is observed that employing a higher prestress factor leads to a more intense NS behaviour (as intuitively expected). The same choice leads to a longer NS operation region for the device which is also beneficial for maximizing internal energy absorption. It is noted that the curves in Fig.2b are interrupted just after the snap-through movement of the oscillator is completed (the displacement/force characteristics will evidently enter an intense positive stiffness behaviour after that). Last but not least the impact of the cell/tripod interface boundary conditions on the snap-through behaviour of the device was investigated. Relevant analysis revealed that when the device is clamped on the honeycomb cell walls (that is any rotation is constrained at the interface nodes) the NS performance of the device is heavily compromised. Instead, the restriction on the orientation of the interface nodes implied a pronounces increase of the positive stiffness behaviour of the device.

In short, it can be concluded that an increase of the p_f design factor as well as a decrease of r_R can be beneficial for extending the NS region of the proposed device. Both design parameters can be employed for tailoring the NS constant of the designed device. Caution should be exercised when selecting the implementation process of the inclusion, as restricting the rotation DoF at the interface of the device can be detrimental for its NS characteristics. As exhibited by the above results,

the displacement/force characteristics of the mechanisms can be approximated as linear around their snap-through position. A constant stiffness assumption will therefore be adopted in the following section.

3. Sound transmission through a layered metamaterial having NS inclusions

The configuration to be considered is a layered structural panel comprising two facesheets and a core layer (see Fig.3). In order to account for the impact of the core shear deformation on the elastic wave propagation characteristics, a 3D displacement field will be assumed and the panel will be modelled by FE. An illustration of the internal tripod devices as well as the equivalent modelled periodic structural segment are also exhibited in the same figure. NS and positive stiffness elements are modelled as translational spring FEs connected to mass m_2 (representing the mass of the added NS mechanism). It is noted that the static and dynamic stability of the internal oscillator has been discussed in [34]. It is noted that the NS inclusions are assumed to have the same periodicity in the x and y directions.
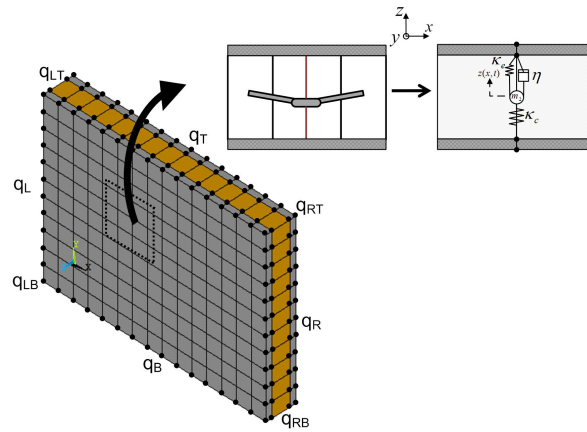

Figure 3: FE modelled composite layered panel with the interface edge and the interface corner DoF highlighted. The envisaged internal configuration of the inclusion, as well as the equivalent periodic structure (with m_2 the device's mass, κ_e the positive stiffness and κ_c the NS) are also illustrated.

In the following numerical case study, the metamaterial facesheets were assumed to have a Young's modulus E_f=70GPa, a Poisson's ratio v_f=0.3 and a mass density of ρ_f=3000kg/m^3. The thickness of the facesheets is equal to h_f=1mm. The core has a thickness equal to h_c=6mm and is made of a material having E_c=0.7GPa, v_c=0.2 and ρ_c=50kg/m^3. A constant damping loss factor $\eta_f = \eta_c$=1% was assumed for both materials. The dimensions of the two-dimensional modelled periodic segment are $l_x = l_y = 50mm$ while the total dimensions of the panel L_x, L_y are equal to $1.5m \times 2m$. With regard to the NS inclusions design, added mass m_2 was assumed to be equal to 5% of the total mass of the panel. Out of the entirety of the investigated device configurations (as described in Sec.2) it has been found that a NS device of κ_c=-10kN/m combined with a positive stiffness one of κ_e=11kN/m will provide maximum structural damping enhancement.

3.1 Wave propagation and dissipation within the mechanical metamaterial

The mass, damping and stiffness matrices of the periodic segment $\mathbb{M}$, $\mathbb{C}$ and $\mathbb{K}$ are extracted using a conventional FE software. Time harmonic wave propagation is hereby considered within the layered structure in the x and y directions implying $z(x, y, \omega, t) = Z e^{i(\omega t - k_x x - k_y y)}$ with $k_x = k \cos(\theta)$ and $k_y = k \sin(\theta)$ where k the propagating structural elastic wavenumber and θ the propagation direction.

The problem was modelled by a WFE approach similar to the one exhibited in [35] which for the sake of brevity will not be developed here.

3.1.1 Damping loss factor calculation

Within a vibroacoustic analysis it is valuable to compute and employ the damping loss factor of the entire panel $\eta_{w,\theta}$ when excited by a certain wave type w propagating towards direction θ at an angular frequency ω. The loss factor associated with a given propagating wave is defined as

$$\eta_{w,\theta}(\omega,\theta) = \frac{P_{diss,w}}{\omega(U_w + T_w)} \tag{2}$$

with P_{diss} the time-average power dissipated by structural damping, U the time-average strain energy and T the time-average kinetic energy in the modelled structural segment. By employing the appropriately normalised computed displacements $\mathbf{x}$ as derived by the eigenproblem the above quantities can be expressed as

$$T_w = \frac{\omega^2}{2}\mathrm{Re}\left(\{\ \mathbf{q}_{w,\theta}\ \}^{\mathrm{H}}\mathbf{M}\{\ \mathbf{q}_{w,\theta}\ \}\right)$$

$$U_w = \frac{1}{2}\mathrm{Re}\left(\{\ \mathbf{q}_{w,\theta}\ \}^{\mathrm{H}}\mathbf{K}\{\ \mathbf{q}_{w,\theta}\ \}\right)$$

$$P_{diss,w} = \sum_n 2\omega\eta_n U_{n,w} \tag{3}$$
$$= \omega\sum_n \mathrm{Re}\left(\{\ \mathbf{q}_{n,w,\theta}\ \}^{\mathrm{H}}\eta_n\mathbf{K}_n\{\ \mathbf{q}_{n,w,\theta}\ \}\right)$$
$$= \omega^2\mathrm{Re}\left(\{\ \mathbf{q}_{w,\theta}\ \}^{\mathrm{H}}\mathbf{C}\{\ \mathbf{q}_{w,\theta}\ \}\right)$$

with n the number of structural layers, η_n the damping loss factor of layer n and

$$\mathbf{q}_{w,\theta} = \left\{\begin{array}{c} \mathbf{q}_{I,w} \\ \mathbf{q}_{B,w} \\ \mathrm{e}^{-i\varepsilon_y}\mathbf{q}_{B,w} \\ \mathbf{q}_{L,w} \\ \mathrm{e}^{-i\varepsilon_x}\mathbf{q}_{L,w} \\ \mathbf{q}_{LB,w} \\ \mathrm{e}^{-i\varepsilon_x}\mathbf{q}_{LB,w} \\ \mathrm{e}^{-i\varepsilon_y}\mathbf{q}_{LB,w} \\ \mathrm{e}^{-i\varepsilon_x-i\varepsilon_y}\mathbf{q}_{LB,w} \end{array}\right\} \tag{4}$$

being the displacements induced in the modelled segment by the propagating wave type w travelling in direction θ. By substituting Eq.3 in Eq.2 and after calculations we get the global frequency dependent damping loss factor corresponding to wave type w

$$\eta_w(\omega) = \frac{\int_0^\pi \eta_{w,\theta}(\omega,\theta)\mathrm{d}\theta}{\int_0^\pi \mathrm{d}\theta} \tag{5}$$

The wave type transferring the dominant portion of energy during elasto-acoustic transmission will be the out-of-plane bending waves propagating through the structure. Following the wave propagation analysis, the frequency dependent damping loss factor for the bare panel (no inclusions) as well as for the modified panel incorporating NS elements has been computed and is presented in Fig.4.
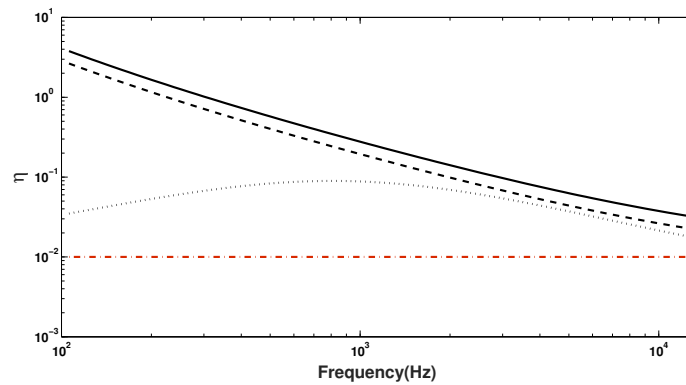
Figure 4: Frequency dependent damping loss factor associated with the out-of-plane flexural wave for the layered metamaterial: Reference bare layered panel (--), metamaterial with NS inclusions having a 30mm periodicity (–), metamaterial with NS inclusions having a 50mm periodicity (- -), panel with a mass-equivalent polyvinyl butyral (PVB) layer inclusion (· · ·).

As expected, the bare panel exhibits a constant damping behaviour with a loss factor equal to the one of the constituting materials, i.e. 1%. On the other hand, the results for two composite metamaterial panels are exhibited (with a 30mm and a 50mm periodicity respectively) both presenting an intensely frequency dependent loss factor. It becomes evident that the inclusion of the NS elements has a major impact in the low frequency range (η approximately two orders of magnitude higher), where the oscillation of the internal masses has a higher vibration amplitude. This is a major advantage compared to viscoelastic materials which start providing damping enhancement mainly in the high frequency range. The impact of the NS inclusions is gradually diminishing in the higher frequency range of the vibroacoustic analysis, with the benefits however still visible at 12kHz, where the modified panel has a loss factor more than three times higher. It can be observed that increasing the periodicity of the internal oscillators has a negative impact on the panel's global damping ratio. In this case study a maximum loss of damping of approximately 32% was observed at very low frequencies (100Hz).

3.2 Calculation of the acoustic transmission through the panel

It is now of interest to investigate how the enhanced damping properties of the composite metamaterial can impact its noise insulation properties. For this reason a wave-based statistical energy analysis scheme as exhibited in [24, 36] was employed, in which the computed wavenumber and damping loss factor values for the panel are input. The major vibroacoustic index that provides an important indication of a structure's acoustic opacity properties is its sound TL. The TL indices for the bare composite, as well as for the layered metamaterial are presented in Fig.5.

It is evident that the NS inclusions induce a major improvement of the insulation properties of the panel around its coincidence frequency range (where the acoustic wavenumber becomes equal to the structural one, implying efficient acoustic transmission and a pronounced drop of TL) with a maximum TL increase of about 20dB. In the high frequency range the advantages are also evident with a constant TL improvement of about 5dB. On the other hand, in the (mass and stiffness controlled) sub-coincidence range, despite the fact that the loss factor of the layered metamaterial was massively greater, the average TL improvement was inferior to 2dB. This further confirms the difficulty of increasing noise insulation in the very low frequency range without significant mass addition to the structural panel.
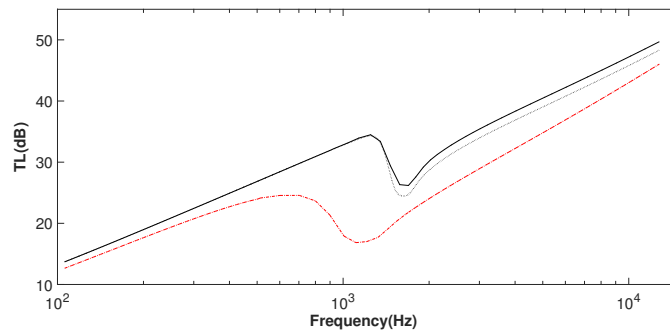
Figure 5: Broadband sound Transmission Loss for the composite panel: Reference bare layered panel (-··-), metamaterial with NS inclusions (–) having a 30mm periodicity, panel with a mass-equivalent polyvinyl butyral (PVB) layer inclusion (· · ·).

4. Conclusions

The design of mechanical oscillators exhibiting NS behaviour around their snap-through position was discussed. A design appropriate for implementation within a honeycomb structural core was presented and a parametric survey was performed in order to determine the design parameters having a notable impact on its mechanical characteristics. It was demonstrated that an increase of the prestress as well as an increase of the inner radius of the tripod structure can be beneficial for extending the NS region of the device. On the other hand, restricting the rotation of the device at its interface with the core can be detrimental for the NS performance. It was shown that the enhanced damping properties of the suggested metamaterial can lead to radical enhancement of its noise insulation properties in the mid and high frequency ranges, while in the very low acoustic frequency range it proves challenging to design an efficiently isolating panel without considerable added mass.

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