Adoption incentives and environmental policy timing under asymmetric information and strategic firm behaviour

Alessio D'Amato and Bouwe Dijkstra[1]

Abstract. We consider the incentives of a single firm to invest in a cleaner technology under emission quotas and emission taxation. We assume asymmetric information about the firm's cost of employing the new technology. Policy is set either before the firm invests (commitment) or after (time consistency). Contrary to conventional wisdom, we find that with commitment (time consistency), quotas give higher (lower) investment incentives than taxes. With quotas (taxes), commitment generally leads to higher (lower) welfare than time consistency. Under commitment with quadratic abatement costs and environmental damages, a modified Weitzman rule applies and quotas usually lead to higher welfare than taxes.

Keywords: asymmetric information, commitment, time consistency, emission taxation, quotas

JEL classification: D62, D82, Q28

[1]D'Amato (corresponding author): University of Rome Tor Vergata, Rome and SEEDS, Ferrara. Dijkstra: University of Nottingham and CESifo, Munich. We thank Marco Caffera, Matti Liski, Alex Possajennikov and Marko Terviö and seminar participants at the universities of Stirling and Helsinki for valuable comments.

Adoption incentives and environmental policy timing under asymmetric information and strategic firm behaviour

Abstract. We consider the incentives of a single firm to invest in a cleaner technology under quotas and emission taxation. We assume asymmetric information about the firm's cost of employing the new technology. Policy is set either before the firm invests (commitment) or after (time consistency). Contrary to conventional wisdom, we find that with commitment (time consistency), quotas give higher (lower) investment incentives than taxes. With quotas (taxes), commitment generally leads to higher (lower) welfare than time consistency. Under commitment with quadratic abatement costs and environmental damages, a modified Weitzman rule applies and quotas usually lead to higher welfare than taxes.

Keywords: asymmetric information, commitment, time consistency, emission taxation, quotas

JEL classification: D62, D82, Q28

1 Introduction

Which environmental policy instrument gives the highest incentives to develop and adopt cleaner technologies? Which environmental policy instrument is best able to deal with asymmetric information about abatement costs of newly introduced production techniques? Should environmental policy be set before the firms have chosen their abatement technology (commitment) or afterward (time consistency)? These are among the most crucial and widely analyzed issues in environmental policy. To the best of our knowledge, however, so far only D'Amato and Dijkstra (2015) have addressed all three questions together, and only under the assumption of non-strategic firms. In this paper, we extend the analysis to strategic firm behaviour.

We model the technology adoption choice by a single polluting firm which is subject to regulation through price-based or quantity-based controls. Environmental regulation of a single firm, with increasing marginal damage of pollution, may seem unrealistic. One could think of a large polluter by a small lake, or on a small island. More importantly, this is the simplest setup that allows us to study a strategic regulatory environment, where one firm's decisions make a large difference to the aggregate outcome. This is the polar opposite of D'Amato and Dijkstra's (2015) multi-firm industry. Regulation of an industry with several large firms, the more realistic case that falls in between the two extremes, can be studied next. We will assume increasing marginal damage to allow for a non-trivial comparison between taxation and emission quotas. If marginal damage were constant, the regulator would always be able to implement the first best with taxation (under commitment and time consistency) by setting the tax rate equal to marginal damage.

We find that with emission quotas, except for very specific fixed costs ranges, commitment leads to higher welfare than time consistency, as the former generates larger incentives to invest than the latter. Indeed, time consistency under emission quotas effectively punishes investment, because it results in a lower quota. Surprisingly, under time consistency the regulator does not gain much from the information learnt from the firm's investment decision. Conclusions are reversed when dealing with emission taxes: time consistency is shown to yield higher welfare in most of the fixed costs ranges under scrutiny; this can be explained accountig for the larger incentives to invest taxation provides. Finally, we also compare quotas and taxes for quadratic abatement cost and damage functions. Concerning commitment, we find that quotas yield, in most of the scenarios concerning fixed costs, higher welfare than taxes. However, we cannot rank the instruments for the time consistency scenario.

Our analysis is linked to several strands of the literature. Since Downing and White (1986) and Milliman and Prince (1989), an attempt has been made to compare the relative merits of different environmental policy instruments in terms of their incentives for R&D into and adoption of new abatement technologies. This literature has been surveyed by Jaffe et al. (2003) and Requate (2005a).

Weitzman (1974) was the first to systematically address the relative performance of price and quantity regulation under uncertainty in environmental policy or indeed any area of policy.[2]

Moledina et al. (2003) compare taxes and tradable permits with grandfathering in a two-firm industry. The regulator does not know the firms' abatement cost and does not take into account that the firms will try to manipulate her beliefs and policy. The authors show that firms will underabate under taxation in order to obtain a lower tax rate. The result for tradable permits is less clearcut. On one hand, both firms benefit from a high permit price, because this will prompt the regulator to issue more permits. On the other hand, the permit buyer (seller) prefers a low (high) permit price.

The literature on the timing of government policy, starting with Kydland and Prescott (1977) and Fischer (1980), has almost unanimously found that with perfect information, commitment is always at least as good as time consistency. This result has been challenged by several papers on environmental policy (Amacher and Malik, 2002; Arguedas and Hamoudi, 2004; Requate, 2005b; Moner-Colonques and Rubio, 2015).

Let us now consider papers that combine at least two of the three issues just reviewed. Combining innovation and policy timing, Requate and Unold (2001, 2003) study the case where the regulator sets the emission tax rate or the number of tradable emission permits either before (commitment) or after (time consistency) the many small firms in the industry have chosen between the conventional and a new abatement technology. Both instruments yield the first best in both scenarios when firms are heterogeneous (Requate and Unold, 2001), but commitment to a tax rate does not always yield the first best when firms are homogeneous (Requate and Unold, 2003).

Amacher and Malik (2002) also compare commitment and time consistency, but only for emission taxation of a single firm choosing its abatement technology. They show that welfare may be higher with time consistency. Amacher and Malik (2001) show that unlike emission taxation, an emission quota always implements the first best with commitment. Our paper builds on Amacher and Malik (2002) to include asymmetric information. We find that with taxation, time consistency usually yields higher welfare. Due to asymmetric

[2] Recent papers in this vein include Coria and Hennlock (2012), who focus on policy reactions to technological development in the presence of transaction and political costs and Ambec and Coria (2013) who analyze the control of two pollutants with asymmetric information about their interdependent abatement costs. Goodkind and Coggins (2015) take corner solutions into account. In a two-country model, Weitzel (2017) analyses how an abatement cost shock in one country affects both countries.

information, commitment does not implement the first best with an emission quota, and time consistency can yield higher welfare in this case as well.

Combining asymmetric information and innovation, Mendelsohn (1984), Krysiak (2008) and Storrøsten (2014) examine the choice between price and quantity instruments under commitment. In all three papers, technology choice is continuous: A firm can invest to reduce the intercept and (in Krysiak (2008) and Storrøsten (2014)) the slope of its Marginal Abatement Cost (MAC) curve. Mendelsohn (1984) considers a single firm, with asymmetric information about marginal abatement costs and investment costs. Krysiak (2008) models an industry with many ex-ante identical small firms who discover their marginal abatement costs after their investment decision. Storrøsten (2014) adds product demand uncertainty to this. All three papers find that endogenous technical change reduces the slope of the long-run MAC curve, making quantity regulation more attractive. In our paper, with linear MAC and marginal environmental damage functions, we also find that quotas are generally better than taxes under commitment. This is because quotas offer higher investment incentives, and the MAC curve is relatively flat.

Yao (1988) assumes that asymmetric information concerns the firms' innovation capacity. The game consists of two periods and involves a single player (the "industry"). In period one, the regulator sets the period-one emission standard. Then industry chooses a research investment level. The game is repeated in period two. Yao (1988) finds that the industry underinvests in period 1 in an attempt to reduce the regulator's confidence in its ability and to obtain a more lenient standard in period 2. The regulator partially counteracts this effect by setting a stricter standard in period 1.

Finally, we shall discuss three papers that combine all three elements of innovation, uncertainty and policy timing. Malik (1991) compares commitment and time consistency for quota setting for a single firm in a two-period model where the period-2 damage function is revealed in period 2. The advantage of time consistency is that the regulator has perfect information when she sets the quotas. The disadvantage is that the firm underinvests in abatement capital in period 1 to obtain a more lenient period-2 quota.

Tarui and Polasky (2005) study a simplified version of Malik's (1991) game with only a single period and without costly enforcement. However, they analyze taxes as well as quotas. Commitment would result in the first best if there were no uncertainty about damages, because the firm has a continuous investment decision. With time consistency, the result is again that the firm underinvests with quotas and overinvests with taxes. When abatement costs and damages are quadratic, taxes are welfare-superior to quotas.

D'Amato and Dijkstra (2015) introduce asymmetric information into the Requate and Unold (2001, 2003) model discussed above: Firms differ in their fixed costs of adopting the new abatement technology, but all firms in the industry either have high or low variable cost of using the new technology. The cost realization is revealed to the firms, but not to the regulator. Time consistency allows the regulator to infer the cost realization both with emission taxation and with tradable permits, and to implement the first best, as with complete information (Requate and Unold, 2001). However, unlike with complete information, the regulator cannot implement the first best with commitment. In this case, the welfare comparison follows a modified Weitzman (1974) rule: Tradable permits lead to higher welfare than emission taxation if the weighted slope of the MAC curve is flatter than the Marginal Environmental Damage curve. Intriguingly, the slope of the high-cost MAC is weighted with the probability that the costs are low, and vice versa.

We apply D'Amato and Dijkstra's (2015) model to the regulation of a single firm. The game structure in our paper is also close to Tarui and Polasky (2005), but uncertainty enters in the form of asymmetric information on abatement costs, as in Moledina et al. (2003). Unlike Moledina et al. (2003), we assume the regulator realizes that the firm may try to manipulate her beliefs and policy.

The rest of the paper is organized as follows: Section 2 shows the main structure of the model. In Section 3 we compare commitment and time consistency with quotas. In Section 4 we do the same for emission taxation. In Section 5 we compare quotas and emission taxation. Section 6 concludes.

2 The model

We model the behaviour of a single polluting firm, currently using abatement technology 1 with no fixed cost and variable cost $C(a)$, where a is the abatement level. The firm must choose whether or not to invest in a cleaner technology. The new technology has lower marginal abatement costs,[3] but involves fixed investment costs ($F > 0$). The variable cost of the new technology is $\theta C(a)$, where θ is the firm's type, that can take two values:

- high ($\theta = h$), implying that the firm is inefficient in using the new technology,
- low ($\theta = l$), implying that the firm is efficient in using the new technology.

Parameter θ is known by the regulated firm, while the regulator only knows its *a priori* distribution, according to which $\theta = h$ with probability $p \in (0, 1)$ and $\theta = l$ with probability $1 - p$. By definition, $l < h < 1$, implying that for both types of firm, the new technology has lower variable and marginal cost than the existing technology. We can say that there are three technologies ϕ ($\phi = l, h, 1$) and the firm of type θ (or firm θ for short) can choose between technologies 1 and θ, $\theta = l, h$.

The cost function satisfies $C'(0) = 0$, $C'(a) > 0$ for $a > 0$, and $C''(a) > 0$. An example is given in Figure 1 with marginal abatement costs MAC_1 for the current technology and MAC_l (MAC_h) for the new technology with low (high) cost.[4]

Environmental damage from pollution is increasing and convex in emissions and therefore decreasing and convex in abatement. Damages are given by $D(e - a)$, where e are (exogenous) unabated emissions, with $D' = 0$ for $e - a = 0$, $D' > 0$ for $e - a > 0$, and $D'' > 0$. MED in Figure 1 is an example of a marginal environmental damage curve.

The objective of the environmental regulator is to minimize social costs. We assume that for all policy equilibria we consider, there is a unique interior cost minimum, and the second order conditions hold globally.

(Variable) Social Costs $(V)SC$ for technology ϕ ($\phi = l, h, 1$) are:

$$SC_\phi(a) = F_\phi + VSC_\phi(a) = F_\phi + \phi C(a) + D(e - a) \tag{1}$$

with $F_1 = 0$ and $F_\phi = F > 0$ for $\phi = h, l$. The socially optimal abatement level a^*_ϕ is implicitly defined by the first order condition:

$$\phi C'(a^*_\phi) = D'(e - a^*_\phi) \tag{2}$$

[3] We abstract from the possibility that technological change increases MAC for high levels of abatement (Amir et al. (2008), Bauman et al. (2008), Bréchet and Jouvet (2008), Perino and Requate (2012)).

[4] All figures in this paper assume that marginal abatement costs and marginal environmental damage are linear in abatement. However, our formal analysis is not limited to this case. The MAC and MED curves in the figures are drawn for illustrative purposes and do not always satisfy all conditions we impose on them in the paper.

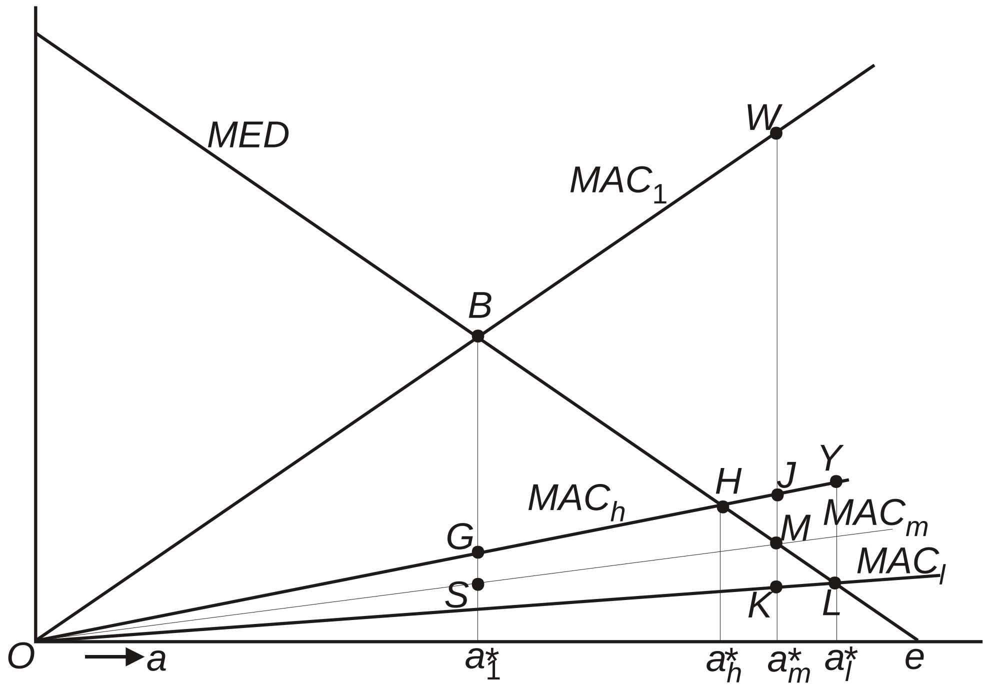

Figure 1: Abatement targets

In Figure 1, social costs under technology 1, h and l are given by the areas OBe, $OHe+F$ and $OLe+F$ respectively. Totally differentiating (2) with respect to ϕ, we find:

$$\frac{da_\phi^*}{d\phi} = \frac{-C'(a_\phi^*)}{\phi C''(a_\phi^*) + D''(e - a_\phi^*)} < 0 \tag{3}$$

Thus, as we see in Figure 1:

$$a_1^* < a_h^* < a_l^* \tag{4}$$

We define F_θ^* ($\theta = h, l$) as the level of fixed costs that equates social costs for the existing and the new technology at the optimal abatement levels a_1^* for the existing technology and a_θ^* ($\theta = h, l$) for new technology θ. From (1):

$$F_\theta^* \equiv C(a_1^*) + D(e - a_1^*) - \theta C(a_\theta^*) - D(e - a_\theta^*) \tag{5}$$

Totally differentiating with respect to θ and applying (2) yields:

$$\frac{dF_\theta^*}{d\theta} = -C(a_\theta^*) < 0 \tag{6}$$

Since $F_h^* = 0$ in (5) for $h \to 1$ and $l < h < 1$, this implies $F_l^* > F_h^* > 0$. In Figure 1, F_h^* is given by the area OBH and F_l^* by OBL.

Following Amacher and Malik (2002, p. 897), we will limit the number of cases to be examined by assuming that adoption of the new technology is socially desirable, in the sense that the minimized social costs are lower with the new technology, even if its cost are high. Thus we assume $SC_h(a_h^*) < SC_1(a_1^*)$ or equivalently $F < F_h^*$.[5]

In Figure 1, suppose the linear MAC_1 is very steep. Then when ϕ is very low, the marginal abatement cost curve is almost horizontal and total abatement costs at the social optimum a_ϕ^* are very low. When ϕ rises, total abatement costs at a_ϕ^* also rise, until they reach a maximum at $a_\phi^* = e/2$. When ϕ rises above $e/2$, abatement costs start to decrease. When $\phi = 1$, the marginal abatement cost curve is almost vertical and total abatement costs at a_ϕ^* tend to zero again. Thus with quadratic abatement cost and environmental damage functions, abatement costs at the social optimum a_ϕ^* are an inverse U-shaped function of ϕ.[6] This result holds for a large variety of other functional forms. In fact, we shall only consider functional forms for which it holds:

Assumption 1. *The function $\phi C(a_\phi^*)$, with $0 < \phi < 1$ and a_ϕ^* defined by (2), is unimodal with the maximum denoted by $\phi = \tilde{\phi}$.*

In the course of our analysis we shall make a number of additional assumptions intended to maximize the number of equilibria that can occur and to facilitate the presentation.[7] We prefer to introduce these assumptions when we have defined the relevant variables.

We analyze two environmental policy instruments in two policy regimes. The two environmental policy instruments are emission taxation and emission quotas. The emission quota specifies the maximum allowed level of emissions. Since emissions are given by

[5] Thus in our setting investment can never be larger than the socially optimal level. The issue of overinvestment is addressed in several papers, including Requate and Unold (2001, 2003) and Tarui and Polasky (2005).

[6] This can be seen as follows. Let $C(a) = \frac{1}{2}a^2$ and $D(e-a) = \frac{d}{2}(e-a)^2$, making MAC and MED linear in e. By (2), a_ϕ^* solves $\phi a = d(e-a)$, so that $a_\phi^* = de/(\phi+d)$ and

$$\phi C(a_\phi^*) = \frac{\phi}{2}\left(\frac{de}{\phi+d}\right)^2$$

Maximizing this with respect to ϕ yields $\phi = d$ and thus $a_\phi^* = e/2$.

[7] These are $F_l^h > 0$ in (15), $F_l^l < F_h^*$ and $\bar{F}_l < F_h^*$ in (5), (15) and (34), and Assumptions 2 and 3 in Appendix B.

the difference between exogenous business-as-usual emissions and abatement, the emission quota translates straightforwardly into an abatement target. We shall use the terms "(emission) quota" and "(abatement) target" interchangeably. The two policy regimes we consider are commitment and time consistency, the difference between them occurring in stages one and two of the game between the regulator and the firm.

In stage zero of each game, nature draws the firm's type θ. The type is revealed to the firm, but not to the regulator. All other parameters are common knowledge.

Under commitment, the regulator sets the abatement target or the emission tax rate in stage one. In stage two, the firm chooses a technology. This order is reversed under time consistency.

Finally, in stage three the firm chooses its abatement level. With abatement targets, it simply complies with the target.

For these sequential games with incomplete information, Perfect Bayesian Equilibrium (PBE) is the standard equilibrium concept (e.g. Fudenberg and Tirole (1991), section 8.2; Tadelis (2013), Ch. 15). A PBE requires that each player is sequentially rational, i.e. at a given information set she takes the expected utility maximizing choice given her system of beliefs. A system of beliefs assigns a probability to each state of the world in each information set. In our game, it specifies the probability that the regulator assigns to the firm being of type h, given its investment decision. Beliefs have to be consistent, i.e. formed according to Bayes' rule. Under commitment, where the regulator cannot learn anything from the firm's investment behaviour, the PBE reduces to the Subgame Perfect Nash Equilibrium (SPNE). For the sake of brevity, and where not needed to explain results, we will refer to the relevant equilibrium concept simply as "equilibrium".

Anticipating our analysis in the following sections, it is easily seen that (unlike in the Amacher and Malik (2001, 2002) models with perfect information) the regulator cannot achieve the first best under any of these scenarios. Given that $F < F_h^*$, the first best is for each type of firm to invest and for type θ to abate a_θ^* given by (2). It is clear from Figure 1 that firm l has lower marginal environmental damage in this case and should be set a stricter target or a lower tax rate.[8] Under commitment however, the regulator has

[8] This is formalized by equations (3) and (27) respectively.

to set the same target or tax rate for both types. When both types invest under time consistency, the regulator cannot infer whether the firm is of type h or l and will again have to set the same policy for both.

We can also establish:[9]

Lemma 1 *The regulator would be able to achieve the first best under commitment as well as time consistency, if she could verify the firm's choice between the current and the new technology, and offer the firm a contract. This contract would specify a payment T from the firm to the regulator depending on the firm's choice of abatement and (if necessary) technology.*

Though theoretically feasible, the first best contract implies the regulator can perfectly verify technology choice by the firm and make lump sum transfers. In order to focus on realistic contracts, we shall assume in the following that the regulator cannot offer the firm such a contract, either because the firm's technology choice cannot be verified, or because of constraints on the type of instrument that the regulator is allowed to use.

3 Emission quotas

In this section we establish the Perfect Bayesian Equilibria for emission quotas under commitment (subsection 3.1) and time consistency (subsection 3.2), and compare the two policy regimes to each other (subsection 3.3). The emission quota specifies the maximum allowed level of emissions. Alternatively, we can say that the regulator sets an abatement target specifying the minimum required level a of abatement.

3.1 Commitment

Under commitment, the regulator sets the emission quota in stage one before the firm chooses whether to invest or not in stage two.

Starting the analysis in stage two, given that the regulator has set the abatement target at a and the firm is of type θ, the firm invests if and only if $F < F_\theta(a)$ where:

$$F_\theta(a) \equiv (1-\theta)C(a) \tag{7}$$

[9] The proofs of all lemmas and of Propositions 4 and 6 are in Appendix A.

Since $h > l$ we have $F_h(a) < F_l(a)$. Also note that $dF_\theta(a)/da > 0$ since $C' > 0$.

In stage one, the regulator can set the quota such that $F < F_h(a)$ and both types of firm invest, $F_h(a) < F < F_l(a)$ and only firm l invests, or $F > F_l(a)$ and the firm does not invest. Let us examine which quota the regulator would like to set in each of these scenarios.

If the regulator sets a such that $F < F_h(a)$, both types of firm will invest. If both types of firm are going to invest, the regulator would like to set the quota that mimimizes:

$$E\left[SC_{hl}(a)\right] = F + D(e-a) + mC(a) \tag{8}$$

where

$$m \equiv E(\theta) = ph + (1-p)\,l \tag{9}$$

The optimal target a_m^* is then implicitly defined by the first order condition:

$$mC'(a_m^*) = D'(e - a_m^*) \tag{10}$$

It follows from (3) and $h > m > l$ that:

$$a_h^* < a_m^* < a_l^* \tag{11}$$

In Figure 1, the curve MAC_m represents the LHS of first order condition (10) and $E\left[SC_{hl}(a_m^*)\right]$ is given by the area OMe.

We find that:

Lemma 2 *Both types of firm will invest when the regulator has set a_m^*.*

Figure 1 illustrates this lemma, which is equivalent to $F < F_h(a_m^*)$ as given by (7) and (10). In Figure 1, $F_h(a_m^*)$ is area OWJ, which exceeds the maximum value of F given by $F_h^* = OBH$.

If the regulator sets a such that $F_h(a) < F < F_l(a)$, only firm l will invest. If the firm is only going to invest when it is of type l, the regulator would like to set the quota that minimizes:

$$E\left[SC_{1l}(a)\right] = pSC_1(a) + (1-p)SC_l(a) \tag{12}$$

The optimal target a_{1l} in this case is implicitly defined by the first order condition:

$$[p + (1-p)l]\, C'(a_{1l}) = D'(e - a_{1l}) \tag{13}$$

The following lemma shows that the regulator prefers a_m^* with investment by both types to a_{1l} with investment by firm l only:

Lemma 3 *$E\,[SC_{hl}(a_m^*)] < E\,[SC_{1l}(a_{1l})]$, with $E[SC_{hl}(a)]$ given by (8), a_m^* by (10), $E[SC_{1l}(a)]$ by (12) and a_{1l} by (13).*

The result follows from the assumption that investment is socially desirable for any firm's type.

Finally, if the regulator sets a such that $F > F_l(a)$, the firm will not invest. If the firm is not going to invest, the regulator would like to set a_1^* as defined in (2). We shall now see that the regulator prefers a_m^* with investment by both types to a_1^* without investment, which requires $F < F_m^*$ as defined by (5) and (9). It follows from (6) and $m < h$ that $F_m^* > F_h^*$. Since we have assumed that $F < F_h^*$, we also have $F < F_m^*$. In Figure 1, F_m^* is given by the area OBM. Since F is assumed to be smaller than OBH, it is also smaller than OBM.

Thus we can conclude:

Proposition 1 *Let the regulator set the abatement target a in stage one and let the firm make its investment decision in stage two. In the Subgame Perfect Nash Equilibrium, in stage one the regulator sets a_m^*, given by (10), and in stage two the firm invests, irrespective of its type.*

3.2 Time consistency

With time consistency, the firm decides in stage one whether or not to invest and the regulator sets the quota in stage two. If the firm does not invest in stage one, the regulator will set the abatement target at a_1^* given by (2) in stage two. In this case the firm's type is irrelevant and we do not need to specify the regulator's beliefs. However, we do need to specify the regulator's beliefs about the firm's type if it has invested.

Let us denote by q the regulator's posterior probability that the firm is of type h given that it has invested in stage one. There are three possible equilibrium values for q which we shall discuss in the following three subsections. Subsection 3.2.4 concludes with a statement of all equilibria in this scenario.

3.2.1 Firm h invests

In the first case, the regulator believes that both types of firm would have invested. Thus she does not learn anything when the firm has invested: her posterior probability q that the firm is of type h is the same as the prior probability p. In this case, the regulator sets the abatement target at a_m^* given by (10) in stage two.

In stage one, if firm θ anticipates that the regulator will set a_m^* in (10) when it invests, it will invest for:

$$F < F_m^\theta \equiv C(a_1^*) - \theta C\left(a_m^*\right) \tag{14}$$

with a_1^* given by (2). Since $h > l$, $F_m^h < F_m^l$. In Figure 1, F_m^h is given by $OBa_1^* - OJa_m^* = OBG - a_1^* GJa_m^*$ and F_m^l by $OBa_1^* - OKa_m^* = OBS - a_1^* SKa_m^*$. When $F < F_m^h$, both firm types invest, confirming the regulator's beliefs.

3.2.2 Firm h does not invest

In the second case, the regulator believes that only firm l would have invested. Now the regulator learns the firm type when she sees that the firm has invested: her posterior probability q that the firm is of type h equals zero. In this case, the regulator sets the abatement target at a_l^* defined by (2) in stage two.[10]

In stage one, if firm θ anticipates that the regulator will set a_l^* in (2) when it invests, it will invest for:

$$F < F_l^\theta \equiv C\left(a_1^*\right) - \theta C(a_l^*) \tag{15}$$

As with F_m^θ, $F_l^h < F_l^l$. In Figure 1, F_l^h is given by $OBa_1^* - OYa_l^* = OBG - a_1^* GYa_l^*$ and F_l^l by $OBa_1^* - OLa_l^* = OBS - a_1^* SLa_l^*$. When $F_l^h < F$, type h will not invest, confirming the regulator's beliefs. When $F_l^h < F < F_l^l$, only type l will invest. When $F_l^l < F$, type l will not invest either. We shall assume $F_l^l < F_h^*$ in order to include the latter equilibrium.

[10] We will assume that if the regulator believes neither type of firm would have invested, she would set a_l^* in the out-of-equilibrium event that the firm did invest.

3.2.3 Firm h mixes

There is a third possible case, where the regulator believes that firm l would always have invested and firm h would have invested with probability $\pi \in (0,1)$. Upon investment, the regulator updates the probability that the firm is of type h to q according to Bayes' rule (e.g. Tadelis (2013), section 19.4.3):

$$q = \Pr(h|investment) = \frac{\Pr(investment|h)\Pr(h)}{\Pr(investment)} = \frac{p\pi}{p\pi + 1 - p}$$

In stage two, the regulator sets the quota to minimize expected variable social cost given by:

$$E\left[VSC_{\pi l}(a)\right] = \frac{p\pi}{p\pi + 1 - p} hC(a) + \frac{1-p}{p\pi + 1 - p} lC(a) + D(e-a) \quad (16)$$

The first (second) term is the abatement cost of firm h (l) multiplied by the probability that the investing firm is a firm of type h (l). The optimal target a_r^* in this case is implicitly defined by the first order condition:

$$rC'(a_r^*) = D'(e - a_r^*), \qquad r \equiv \frac{p\pi h + (1-p)l}{p\pi + 1 - p} \quad (17)$$

We see that $r \in (l, m)$ is increasing in $\pi \in (0,1)$, with m defined by (9). It then follows from (3) that a_r^* is decreasing in π and:

$$a_l^* > a_r^* > a_m^* \quad (18)$$

because $l < r < m$.[11]

In stage one, if firm h expects the regulator to set a_r^* in (17), it will be indifferent between investing and not investing if:

$$F = F_r \equiv C(a_1^*) - hC(a_r^*) \quad (19)$$

As $C' > 0$, F_r is decreasing in a_r^*. It then follows from (18) that firm h can be made indifferent between investing and not investing, confirming the regulator's beliefs, for $F \in \left[F_l^h, F_m^h\right]$. Here F_l^h is given by (15) and F_m^h by (14), with $F_l^h < F_m^h$ by (18) and $C' > 0$. In Figure 1, $F_h^m = OBS - a_1^* SKa_m^*$ exceeds $F_l^h = OBS - a_1^* SLa_l^*$ by $a_m^* KLa_l^*$.

We can show that:

[11] There is no interval of F for which there is a mixed strategy PBE with firm l randomizing and firm h not investing. In this case, the regulator would set a_l^* given by (2) after investment. Firm l would only be indifferent between investing and not investing for $F = F_l^l$ given by (15).

Lemma 4 *$F_m^h < F_l^l$, with F_m^h given by (14) and F_l^l by (15).*

We shall assume $F_l^h > 0$. Combined with Lemma 4 and our assumption that $F_l^l < F_h^*$, this assures that we include all equilibria where firm h invests with probability $\pi \in (0, 1)$.

3.2.4 Equilibria

We know from the analysis above that the critical F values are F_m^h, F_l^h and F_l^l defined by (14) and (15). We have seen at the end of the previous subsection that $F_l^h < F_m^h$. The equilibria, illustrated in Figure 2, are then as follows:

Proposition 2 *Let the firm decide in stage one whether or not to invest and let the regulator set the abatement target a in stage two. Let $F_l^h > 0$ and $F_l^l < F_h^*$ with F_l^h, F_l^l given by (15) and F_h^* by (5). Then the Perfect Bayesian Equilibrium outcomes are:*

1. *When $F < F_l^h$, with F_l^h given by (15), both types of firm invest and the regulator sets a_m^* given by (10).*

2. *When $F_l^h < F < F_m^h$, with F_m^h given by (14), there are three equilibria. In the first equilibrium, both types of firm invest and the regulator sets a_m^*. In the second equilibrium, only firm l invests and the regulator sets a_l^*, given by (2), if the firm has invested. Finally, in the mixed strategy equilibrium, firm h invests with probability π while firm l always invests. If the firm has invested, the regulator sets a_r^*. The values of π and a_r^* as functions of F follow from (17) and (19).*

3. *When $F_m^h < F < F_l^l$, only firm l invests and the regulator sets a_l^* if the firm has invested.*

4. *When $F > F_l^l$, the firm does not invest.*

In each equilibrium, if the firm does not invest, the regulator sets a_1^*, given by (2).

We see that with emission quotas under time consistency, there are multiple equilibria, depending on fixed costs F. There is even a range of F where three equilibria overlap (Proposition 2.2). The firm prefers the equilibrium with the most lenient target, i.e.

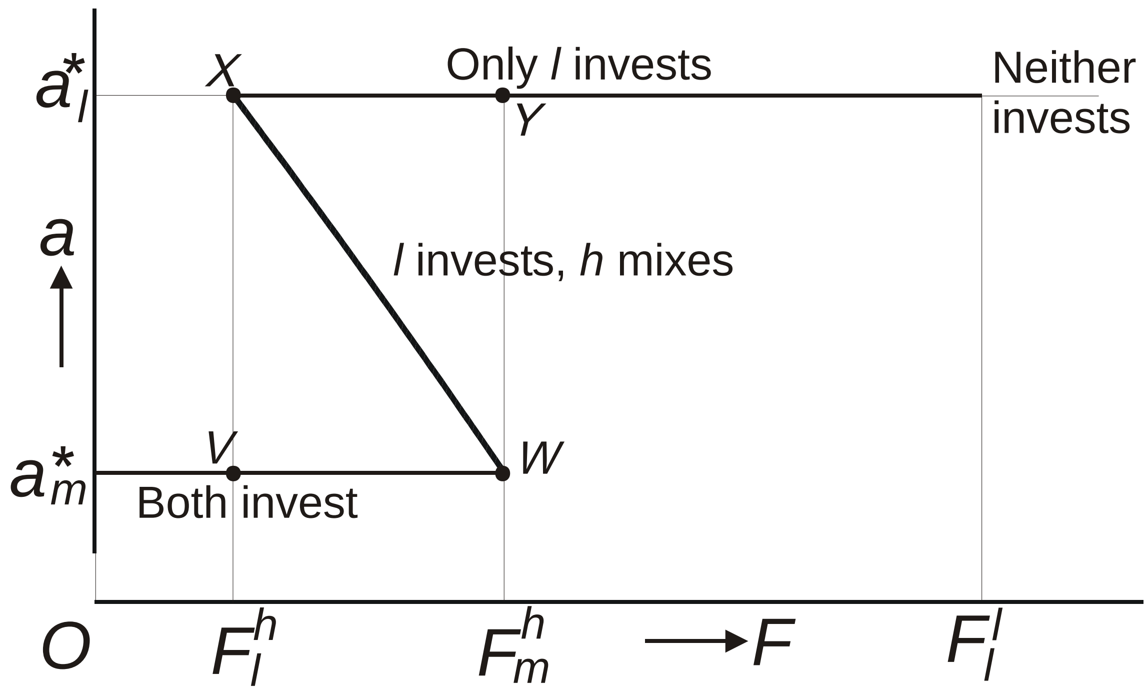

Figure 2: Abatement targets, time consistency: Equilibrium targets for the investing firm and investment decisions.

where both types invest and the regulator sets a_m^*.[12] One may wonder why the firm, as the first mover, does not choose to play this equilibrium. The reason is that in a Perfect Bayesian Equilibrium, the firm plays its best response to the regulator's system of beliefs, where the system of beliefs has to be consistent with the firm's behaviour. The firm does not get to choose the regulator's system of beliefs, only whether or not to invest.

3.3 Comparing commitment and time consistency

We can now compare the equilibrium outcomes under commitment and time consistency (Propositions 1 and 2 respectively) with respect to investment and expected social costs. We see that under commitment, the regulator always sets a_m^* and both types of firm invest. With time consistency, this is only one of the four possible equilibria. In a way, investment is punished under time consistency, because it results in stricter targets.

Let us now consider welfare. For $F_l^h < F < F_m^h$, there is a mixed strategy equilibrium under time consistency where firm l always invests and firm h invests with probability π.

[12] We shall see in subsection 4.3 that the regulator prefers the equilibrium where only firm l invests.

Expected social cost is:

$$\begin{aligned}
E_\pi(SC) &= p(1-\pi)SC_1(a_1^*) + [1-p(1-\pi)]\, E\left[VSC_{\pi l}(a_r^*) + F\right] \\
&= p(1-\pi)\left[C(a_1^*) + D(e-a_1^*)\right] + \\
&\quad + \left[(1-p)l + p\pi h\right] C(a_r^*) + \left[1-p+p\pi\right]\left[D(e-a_r^*) + F\right] \\
&= C(a_1^*) - (1-p)[h-l]C(a_r^*) + p(1-\pi)D(e-a_1^*) + \left[1-p(1-\pi)\right] D(e-a_r^*) \qquad (20)
\end{aligned}$$

The second equality follows from (1) and (16). The third equality follows from (19). Let us determine what happens to expected social cost as F rises from F_l^h to F_m^h. This means we are moving from point X to point W in Figure 2, with π rising from 0 to 1 while a_r^* falls from a_l^* to a_m^* according to $dE_\pi(SC)/da_r^* = 0$ in (17). Then, from the final line in (20):

$$\frac{dE_\pi(SC)}{d\pi} = \frac{\partial E_\pi(SC)}{\partial \pi} = p\left[D(e-a_r^*) - D(e-a_1^*)\right] < 0 \qquad (21)$$

The inequality follows from $a_r^* > a_1^*$ (by (4) and (18)) and $D' > 0$. Thus, expected social costs in the mixed strategy equilibrium are decreasing in π and (paradoxically) in F, and increasing in a_r^*.

We can now compare expected social costs in the three time consistency equilibria (one of which is also the commitment equilibrium) in the interval $F \in \left[F_l^h, F_m^h\right]$. Starting at F_l^h with the commitment and time consistency equilibrium where both types invest (point V in Figure 2), expected social costs rise at the rate of F as we increase F towards point W. At W we change to the mixed strategy equilbrium. By (21), expected social costs rise as we move up, with π decreasing from 1 to 0 and F decreasing, from W to X. At X we switch to the equilibrium where only firm l invests. Increasing F from point X to Y raises expected social costs at the rate of $(1-p)F$. Thus expected social costs rise continually along the trajectory $VWXY$. It then follows that for given F, expected social costs are lowest in the commitment equilibrium and highest in the time consistency equilibrium where only firm l invests, with the mixed strategy equilibrium in between.

For $F \in \left[F_m^h, F_l^l\right]$, the welfare comparison between commitment and time consistency is ambiguous. When $F > F_l^l$, the firm does not invest and abates a_1^* in the time consistency equilibrium. As we have seen in subsection 3.1, welfare in this equilibrium is lower than in the commitment outcome with both types of firm investing and abating a_m^*. We can conclude:

Proposition 3 *Comparing the commitment and time consistency equilibria under emission quotas:*

1. *Investment by both types of firm always happens in the commitment equilibrium. With time consistency, it occurs in the unique equilibrium for $F < F_l^h$ and in one of three equilibria for $F_l^h < F < F_m^h$. In such cases timing does not affect welfare.*

2. *For $F_m^h < F < F_l^l$, there is a unique time-consistent equilibrium where firm h does not invest and gets a target of a_1^*, while firm l invests and gets a target of a_l^*. In this equilibrium, expected social cost may be lower than with commitment.*

3. *For all other F values, expected social cost is higher in the time-consistent equilibria that are different from the commitment equilibrium.*

We see that with emission quotas, in all cases where we can unambiguously sign the welfare difference, commitment leads to higher welfare than time consistency (Proposition 3.3). This is because the investment incentive is higher under commitment. As we have seen, investment is punished under time consistency, because it results in stricter quotas. The fact that in the time consistency scenario, the firm's investment decision may reveal something about its type is not very helpful in this setup. In some equilibria, the firm reveals that it is of type h by not investing. However, the regulator would generally prefer firm h to invest.

4 Emission taxes

In this section we establish the Perfect Bayesian Equilibria for emission taxation under commitment (subsection 4.2) and time consistency (subsection 4.3), and compare the two policy regimes to each other (subsection 4.4). But first, in subsection 4.1, we discuss some elements of the game that are common between commitment and time consistency: the firm's choice of abatement in stage three, and the regulator's preferred tax rate for a given technology.

4.1 Preliminaries

In the third and final stage of the game, facing emission tax rate t, the firm with technology ϕ ($\phi = 1, h, l$) minimizes the sum of abatement cost and tax payment $\phi C(a) + t(e - a)$. The first order condition, defining the cost-minimizing abatement level $a_\phi(t)$, is:[13]

$$\phi C'[a_\phi(t)] = t \tag{22}$$

Total differentiation with respect to t yields:

$$\frac{da_\phi(t)}{dt} = \frac{1}{\phi C''[a_\phi(t)]} > 0 \tag{23}$$

The inequality follows from $C'' > 0$. Thus abatement is increasing in the tax rate.

Totally differentiating (22) with respect to ϕ yields:

$$\frac{da_\phi(t)}{d\phi} = \frac{-C'[a_\phi(t)]}{\phi C''[a_\phi(t)]} < 0 \tag{24}$$

The inequality follows from $C', C'' > 0$. As $l < h < 1$, this implies $a_1(t) < a_h(t) < a_l(t)$.

Let us now determine the welfare-maximizing tax rate for a given technology. If the regulator knew that the firm was using technology ϕ, she would set the tax rate to minimize variable social cost, where (variable) social cost $(V)SC_\phi(t)$ is given by (1):

$$SC_\phi(t) = F_\phi + VSC_\phi(t), \qquad VSC_\phi(t) = \phi C[a_\phi(t)] + D[e - a_\phi(t)] \tag{25}$$

with $a_\phi(t)$ given by (22). Differentiating with respect to t and using (22) implicitly defines the welfare-maximizing tax rate for a firm with technology ϕ:

$$t_\phi = D'[e - a_\phi(t)] \tag{26}$$

Totally differentiating (26) with respect to ϕ and noting that $a_\phi(t_\phi) = a^*_\phi$ given by (2):

$$\frac{dt_\phi}{d\phi} = -D''[e - a_\phi(t)]\frac{da^*_\phi}{d\phi} > 0 \tag{27}$$

The inequality follows from $D'' > 0$ and (3). Thus $t_1 > t_h > t_l$.

[13] We assume that there is always an interior solution for $a_\phi(t)$. Goodkind and Coggins (2015) show that when corner solutions can occur (either complete abatement or no abatement at all), taxes have an advantage over quantity controls.

If firm l invests and firm h has technology ψ, $\psi = 1, h$, the tax rate minimizes:

$$E\left[VSC_{\psi l}(t)\right] = pVSC_{\psi}(t) + (1-p)VSC_{l}(t) \tag{28}$$

with $VSC_{\phi}(t)$, $\phi = 1, h, l$, given by (25). The first order condition is, using (1) and (22):

$$p\left\{t_{\psi l} - D'[e - a_{\psi}(t_{\psi l})]\right\}\frac{da_{\psi}}{dt} + (1-p)\left\{t_{\psi l} - D'[e - a_{l}(t_{\psi l})]\right\}\frac{da_{l}}{dt} = 0 \tag{29}$$

We find that:

Lemma 5 *$t_l < t_{hl} < t_h$, with t_l and t_h given by (26), and t_{hl} by (29).*

4.2 Commitment

With commitment, the regulator sets the tax rate t in stage one and the firm chooses its technology ϕ in stage two. In stage three, which we have already analyzed in subsection 4.1, the firm sets the abatement level $a_{\phi}(t)$ given by (22).

For stage two, define $\hat{t}_{\theta}(F)$ and $F_{\theta}(t)$ as the tax rate and the fixed cost level, respectively, that make firm θ indifferent between the current and the new technology:[14]

$$\begin{aligned} C[a_1(\hat{t}_{\theta}[F])] - \hat{t}_{\theta}[F]a_1(\hat{t}_{\theta}[F]) &= \theta C[a_{\theta}(\hat{t}_{\theta}[F])] - \hat{t}_{\theta}[F]a_{\theta}(\hat{t}_{\theta}[F]) + F && (30) \\ F_{\theta}(t) &\equiv C[a_1(t)] - \theta C[a_{\theta}(t)] + t\left[a_{\theta}(t) - a_1(t)\right] && (31) \end{aligned}$$

with $a_{\phi}(t)$ given by (22).

Firm θ will adopt the new technology if and only if $t > \hat{t}_{\theta}(F)$ or equivalently $F < F_{\theta}(t)$. Totally differentiating (31) with respect to θ yields, using (22):

$$\frac{dF_{\theta}(t)}{d\theta} = -C[a_{\theta}(t)] < 0 \tag{32}$$

Thus, since $F_{\theta}(t) = 0$ in (31) for $\theta \to 1$, and $l < h < 1$, we have $F_l(t) > F_h(t) > 0$.

Totally differentiating (31) with respect to t yields, using (22) and (24):

$$\frac{dF_{\theta}(t)}{dt} = a_{\theta}(t) - a_1(t) > 0 \tag{33}$$

The inequality follows from (24).

Moving on to stage one, we can now state the equilibria for the whole game:[15]

[14] In the following, we will often write $\hat{t}_{\theta}(F)$ simply as $\hat{t}_{\theta}$.

[15] In Appendix B we present the additional assumptions that guarantee that the second and third equilibria coexist.

Proposition 4 *Let the regulator set the tax rate in stage one and the firm make its investment decision in stage two. Let F_h^* in (5) exceed $\bar{F}_l$ in:*

$$VSC_l[\hat{t}_l(\bar{F}_l)] + \bar{F}_l = SC_1[\hat{t}_l(\bar{F}_l)] \tag{34}$$

with $\hat{t}_l(\bar{F}_l)$ given by (30). Then the Subgame Perfect Nash Equilibria are:

1. *When $0 < F < F_h(t_{hl})$, with $F_h(t_{hl})$ given by (31), the regulator sets t_{hl} given by (29) and the firm invests, irrespective of its type.*

2. *When $F_h(t_{hl}) < F < \bar{F}_h$, with $\bar{F}_h$ given by*

$$pSC_1(t_{1l}) + (1-p)\left[VSC_l(t_{1l}) + \bar{F}_h\right] = pVSC_h(\hat{t}_h) + (1-p)VSC_l(\hat{t}_h) + \bar{F}_h \tag{35}$$

[with $(V)SC_\phi(t)$ given by (25) and $\hat{t}_h$ by (30)], the regulator sets $\hat{t}_h(F) + \varepsilon$ and the firm invests, irrespective of its type.

3. *When $\bar{F}_h < F < F_l(t_{1l})$, with $F_l(t_{1l})$ given by (31), the regulator sets t_{1l} given by (29) and only the firm of type l invests.*

4. *When $F_l(t_{1l}) < F < \bar{F}_l$, with $\bar{F}_l$ given by (34), the regulator sets $\hat{t}_l(F)+\varepsilon$, with $\hat{t}_l(F)$ given by (30) and only the firm of type l invests.*

5. *When $\bar{F}_l < F < F_h^*$, the regulator sets $\hat{t}_l(F) - \varepsilon$, and the firm does not invest.*

Intuitively, when F is very low, both types of firm will invest, and the regulator can set the optimal tax rate t_{hl} given that both types invest (Proposition 4.1). When F gets higher, firm h no longer wants to invest at t_{hl}. Since the regulator still wants firm h to invest, she sets the tax rate $\hat{t}_h + \varepsilon$ that just induces investment by firm h (Proposition 4.2). For higher F, it is no longer optimal to induce firm h to invest. The regulator will then set the tax rate t_{1l} which is optimal given that only firm l invests (Proposition 4.3). For even higher F, mirroring Proposition 4.2, firm l no longer wants to invest at t_{1l}. Since the regulator still wants firm l to invest, she sets the tax rate $\hat{t}_l + \varepsilon$ that just induces investment by firm l (Proposition 4.4). Finally, for very high F, the regulator does not want firm l to invest. She therefore sets the tax rate $\hat{t}_l - \varepsilon$ that just discourages firm l from investing (Proposition 4.5).

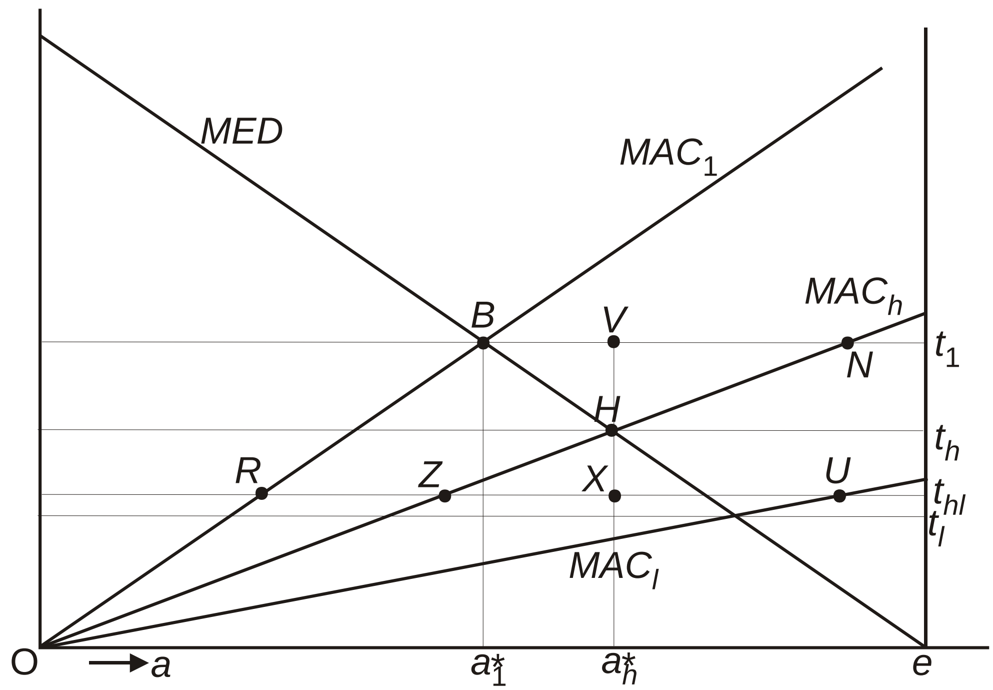

Figure 3: Emission taxation

4.3 Time consistency

With time consistency, the firm chooses its technology ϕ in stage one and the regulator sets the tax rate t in stage two. In stage three, which we have already analyzed in subsection 4.1, the firm sets its abatement level $a_\phi(t)$ according to (22).

Let us start the analysis in stage two. As under quotas, let us label the ex post probability that the firm is of type h as q. If the firm has not invested, the regulator sets the tax rate at t_1 given by (26). If the firm has invested and the regulator believes both types would have invested, then $q = p$ and she sets the tax rate at t_{hl} given by (29). If the firm has invested and the regulator believes only firm l would have invested, then $q = 0$, and she sets the tax rate at t_l given by (26). We will assume that if the regulator believes neither type would have invested, she will set the tax rate at t_l in the out-of-equilibrium event of investment.

In stage one, if firm θ expects a tax rate of t_{hl} upon investment, it will invest if:

$$F < F_\theta^{hl} \equiv C(a_1^*) - t_1 a_1^* - \theta C[a_\theta(t_{hl})] + t_{hl} a_\theta(t_{hl}) \tag{36}$$

Differentiating with respect to θ yields $dF_\theta^{hl}/d\theta = -C[a_\theta(t_{hl})] < 0$. Thus $F_l^{hl} > F_h^{hl}$. In

Figure 3, F_l^{hl} is given by area $OBt_1t_{hl}U$ and F_h^{hl} by $OBt_1t_{hl}Z$. Both exceed the maximum F of $F_h^* = OBH$:

Lemma 6 *$F_h^{hl} > F_h^*$, with F_h^{hl} given by (36) and F_h^* by (5).*

Thus both types of firm would invest when they expect a tax rate of t_{hl} after investment. If the firm expects t_l after investment, again both types of firm will invest. This is because obtaining a tax rate of $t_l < t_{hl}$ (by Lemma 5) is even more attractive than a tax rate of t_{hl}. There is then only one Perfect Bayesian Equilibrium:

Proposition 5 *Let the firm make its investment decision in stage one and the regulator set the tax rate in stage two. In the Perfect Bayesian Equilibrium, the firm invests, irrespective of its type, and the regulator sets the tax rate at t_{hl} given by (29).*

4.4 Comparing commitment and time consistency

Comparing the taxation equilibria under commitment (Proposition 4) and time consistency (Proposition 5), we find:

Proposition 6 *Comparing the commitment and time consistency equilibria under emission taxation:*

1. *Time consistency always results in both types of firm investing. With commitment, this only happens for $F < \bar{F}_h$ given by (35).*

2. *When $F < F_h(t_{hl})$ given by (29) and (31), commitment and time consistency result in the same outcome. When $F > F_h(t_{hl})$, expected social cost is lower in the time consistency equilibrium, except possibly for $F \in \left(\underline{F}_c, \bar{F}_c\right)$ where $\bar{F}_h < \underline{F}_c < F_l(t_{1l}) < \bar{F}_c < \bar{F}_l$, with $\bar{F}_h$ given by (35), $F_l(t_{1l})$ by (31) and (29), and $\bar{F}_l$ by (34).*

Time consistency always leads to investment by both types of firm, while commitment does not. Under time consistency, the firm is rewarded for investment by a lower tax rate. With commitment, the tax rate necessarily has to remain the same, whether the firm invests or not. Since we have assumed (by setting $F < F_h^*$) that investment in the new technology is socially desirable, the higher investment incentive of time-consistent policy means that it usually results in higher welfare.

5 Comparing the instruments

In this section we shall compare emission taxation and emission quotas given that the policy scenario is either commitment or time consistency. In subsection 5.1, we compare the investment incentives that the instruments provide. In subsection 5.2, we compare the instruments on welfare, given that the abatement cost and environmental damage functions are quadratic.

5.1 Investment incentives

It follows from Propositions 1, 2, 4 and 5 that the comparison of the investment incentives from emission quotas and emission taxation depends crucially on the order of moves:

Corollary 1.

1. *With commitment, quotas always lead to investment, while taxation does not;*

2. *With time consistency, taxation always leads to investment, while quotas do not.*

The second part of the corollary is the most straightforward to explain: taxation rewards investment, while quotas punish it. The regulator responds to the firm's investment by setting a lower tax rate, but a tighter quota.

Corollary 1.1 can be explained with the aid of Figure 4. Let us first consider the firm's investment incentives when the regulator knows the firm is of type h. We shall focus on the equilibrium where the regulator sets the optimal policy, correctly expecting firm h to invest. With quotas, she will then set a_h^* given by (2). The firm's abatement cost without investment is ORa_h^* whereas with investment it is $OHa_h^* + F$. The firm will invest if $F < F_h(a_h^*)$ as defined in (7), which is ORH in Figure 4. With taxation, the regulator sets t_h given by (26). The firm's total cost (abatement cost plus tax payment) without investment is OXt_he, whereas with investment it is $OHt_he + F$. The firm will invest if $F < F_h(t_h)$ in (31) which is OXH in Figure 4. We see that the incentive to invest under taxes is smaller than under targets, because $OXH < ORH$ in Figure 4. Formally, note first that by (23), $a_h^* = a_h(t_h) > a_1(t_h)$. It then follows from (7) and (31)

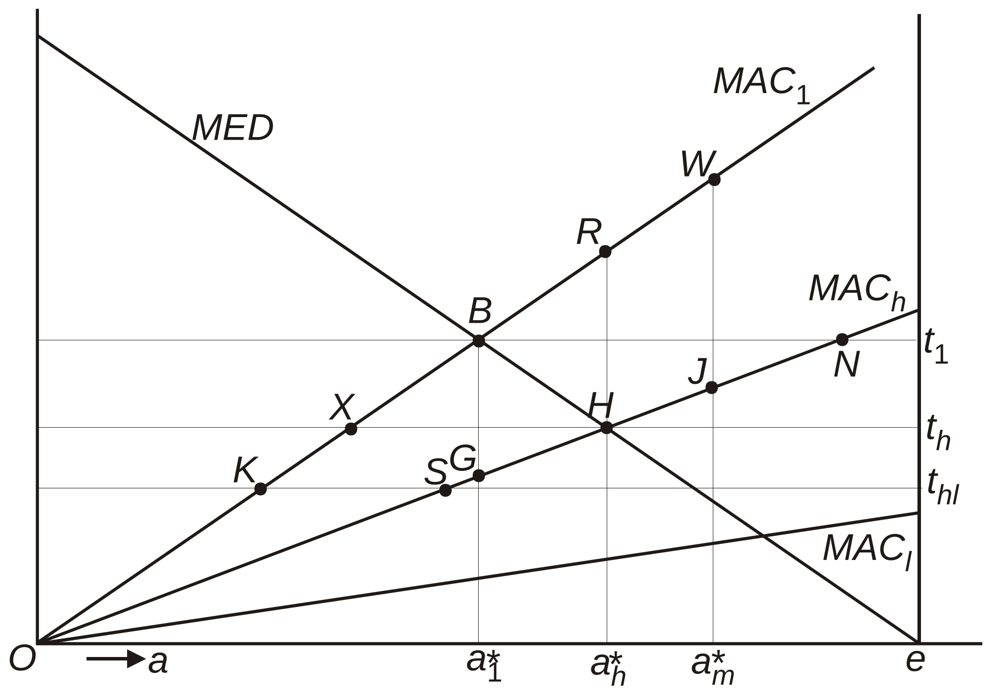

Figure 4: Investment incentives with commitment: targets and taxation compared

that $F_h(t_h) < F_h(a_h^*)$ if and only if:

$$t_h\left[a_h(t_h) - a_1(t_h)\right] < C[a_h(t_h)] - C[a_1(t_h)]$$

This inequality holds because $C'[a_1(t_h)] = t_h$ (by (22)) and $C'' > 0$.

This finding runs counter to the well-known textbook argument that taxes provide a larger investment incentive than quotas.[16] The argument there is that with quotas, investment only allows for the existing quota to be met at lower cost, while taxation gives the firm the option to abate more and to save on the tax bill. The textbook argument assumes that the instruments are set such that they result in the same abatement level pre-investment. For ease of comparison, we shall take the target to be a_1^* given by (2) and the tax rate as t_1 given by (26).[17] Then the firm would invest under targets if $F < F_h(a_1^*)$ defined by (7), which is OBG in Figure 4. The firm would invest under taxes if $F < F_h(t_1)$ given by (31), which is OBN in Figure 4. We see that $OBN > OBG$ or by (7) and (31):

$$F_h(t_1) - F_h(a_1^*) = t_1\left[a_h(t_1) - a_1^*\right] - h\left(C[a_h(t_1)] - C\left(a_1^*\right)\right) > 0$$

[16] E.g. Hanley et al. (2007, Figure 5.7, p. 162). The first graphical exposition appears to be Downing and White (1986, Figure 2, p. 20).

[17] The argument is the same for any tax rate $\bar{t}$ and abatement target $a_1(\bar{t})$ given by (22).

The inequality follows from $hC'[a_h(t_1)] = t_1$ by (22), and $C'' > 0$. Thus for a given pre-investment level of abatement, the investment incentive is higher with taxes.

Comparing the instruments for a given pre-investment level of abatement may make sense if it is assumed that the regulator is myopic, it takes time to change policy or the firm is small. However, we are looking at the case of a welfare-maximizing regulator that can set the policy just before the firm makes its investment decision. Also, we are focusing on the case of a large firm whose abatement decision has a significant impact on social welfare. In this case, if the regulator knows that the firm is going to invest, she should set the instruments such that they result in optimal abatement post-investment.

Now let us consider the effect of asymmetric information, in which case there is a possibility that the firm is not of type h, but of type l instead. Again we look at the case where the regulator sets the optimal policy, expecting the firm to invest even if it has high costs.[18] With quotas, the regulator sets a_m^* given by (10). Firm h will invest for $F < F_h(a_m^*)$ given by (7), which is OWJ in Figure 4. From (7), $F_h(a_m^*) > F_h(a_1^*)$ or $OWJ > ORH$ because $a_m^* > a_1^*$ by (4) and (11), and $C' > 0$. Thus, asymmetric information increases firm h's investment incentive under quotas. With taxation, the regulator sets t_{hl} given by (29), with $t_{hl} < t_h$ by Lemma 5, and the firm will invest for $F < F_h(t_{hl})$ given by (31), which is OKS in Figure 4. From (33) and Lemma 5, $F_h(t_{hl}) < F_h(t_h)$ or $OKS < OXH$. Thus asymmetric information decreases firm h's investment incentive under taxes.

We have found that under perfect information, quotas give more incentive to invest than taxes. Asymmetric information increases the investment incentive for quotas and decreases it for taxes. Thus asymmetric information makes the investment incentive gap even larger in favour of quotas.

5.2 Welfare

In this subsection, we compare welfare under quotas and emission taxation, given that the policy scenario is either commitment or time consistency. To achieve readable insights,

[18] We know from Proposition 4 that with taxation there is another equilibrium where both types invest for $F > F_h(t_{hl})$, with the regulator setting the tax rate $\hat{t}_h(F) + \varepsilon$ which just induces investment by firm h. However we also know from Proposition 4 that this requires the regulator to distort the tax rate by so much that she decides to give up on investment by firm h for $F > \bar{F}_h$.

we will assume that both abatement costs and environmental damages are quadratic.

First let us consider the case where both types of firm invest in the new technology and the regulator sets environmental policy optimally, given that they invest. This can occur under commitment as well as with time consistency.

Lemma 7 *Let the firm of type θ, $\theta = h, l$, have variable abatement costs $\theta C(a)$ where $C'(a)$ is linear with its slope normalized to 1. Let marginal damage $D'(e-a)$ be a linear function with slope d. Consider the case where both types of firm invest in the new technology and the regulator sets environmental policy optimally, given that they invest. That is, with emission quotas, the regulator sets the abatement target a_m^* given by (10) and with emission taxes, she sets t_{hl} given by (29).*

1. *Expected social cost is higher under taxes than under quotas if and only if:*

$$pl + (1-p)h < d \tag{37}$$

2. *This inequality always holds as $F_l^h > 0$ in (15).*

Lemma 7.1 is similar to the well-known Weitzman (1974) rule. Translated to our application, the rule says that under additive uncertainty (i.e. about the intercept of the Marginal Abatement Cost (MAC) curve), quotas are preferred to taxes if the MAC curve is flatter than the Marginal Environmental Damage (MED) curve. We consider multiplicative uncertainty, about the slope of the MAC curve. Like Weitzman (1974), we still find that the comparison depends on the relative slopes of the MAC and MED curves. However, condition (37) features a "reverse" probability weighting: the slope of firm l's MAC curve is weighted with the probability that the firm is of type h, and vice versa. Weitzman (1974, fn. p. 486; 1978) and Malcomson (1978) have previously analyzed multiplicative uncertainty, but without deriving reverse probability weighting.[19]

Our result for multiplicative uncertainty about MAC has previously been obtained by D'Amato and Dijkstra (2015) in a setting with a continuum of small firms. The main difference with D'Amato and Dijkstra (2015) is that in their paper the individual firms'

[19]See also e.g. Watson and Ridker (1984) and Hoel and Karp (2001) who rely mainly on simulations.

MAC curves with the new technology exhibit additive uncertainty. Endogenizing the share of adopting firms, this transforms to multiplicative uncertainty about the industry's MAC curve. By contrast, in our setting we assume multiplicative uncertainty about the firm's MAC with the new technology. As a result, in D'Amato and Dijkstra (2015), the reverse probability rule is all that is needed for the welfare comparison between the price and the quantity instrument under commitment. In the present paper, by contrast, the reverse probability rule only applies when the firm invests under both instruments.

In order to understand the reverse probability weighting, let us suppose the regulator is practically certain that the firm is of type h. She would then set the abatement target at a_h^* and the tax rate at t_h. If, against all expectations, the firm is of type l, then the related welfare loss depends on the slope of the MAC curve in the unlikely scenario that the firm is of type l.

Lemma 7.2 shows that quotas are better than taxes when both types invest, since (37) always holds as $F_l^h > 0$. F_l^h is firm h's gain from investing under time consistency when the regulator sets the quota at a_l^* after investment. In Figure 1, F_l^h is given by the area $OBG - a_1^* GY a_l^*$. Note that the flatter the MAC_h curve, the larger the area OBG and the smaller $a_1^* GY a_l^*$. In Figure 1, OBG is larger than $a_1^* GY a_l^*$, which requires MAC_h to be quite flat. Indeed, it has to be flatter than the MED curve. If MED is steeper than MAC_h, it is also steeper than any weighted average of MAC_h and MAC_l.

We now turn to comparing the instruments under commitment more generally. For quotas, the equilibrium described in Lemma 7 is the unique equilibrium (Proposition 1). For taxation, there are many equilibria (Proposition 4), but the equilibrium described in Lemma 7 generally has the lowest expected social cost of them all (Propositions 4 and 6.2). Using Lemma 7 we can then state:

Proposition 7 *Let marginal abatement cost $C'(a)$ and marginal damage $D'(e-a)$ be linear functions. Let the regulator set either the emission quota target or the emission tax rate in stage one and let the firm make its investment decision in stage two. Then, given that $F < F_l^h$ in (15), expected social cost is lower with quotas than with taxes, except possibly except possibly for $F \in \left(\underline{F}_t, \bar{F}_t\right)$ where $\bar{F}_h < \underline{F}_t < F_l(t_{1l}) < \bar{F}_t < \bar{F}_l$, with $\bar{F}_h$ given by (35), $F_l(t_{1l})$ by (31) and (29), and $\bar{F}_l$ by (34).*

The generally better performance of quotas is due to two factors. First, quotas offer higher investment incentives (Corollary 1.1), and investment is socially desirable. Secondly, where quotas and emission taxation both result in investment, quotas have lower expected social costs (Lemma 7).

The welfare comparison of quotas and emission taxation under time consistency is ambiguous, even if we assume quadratic abatement cost and environmental damage functions. On the one hand, taxation has the advantage of always resulting in investment by both types of firm (Corollary 1.2). On the other hand, however, taxation results in higher expected social costs when both instruments result in investment (Lemma 7). Even when both types of firm invest under taxation and they do not under quotas, taxation may still result in higher expected social costs.

6 Conclusion

The incentives provided for the adoption of cleaner technologies are a significant dimension when dealing with the design of environmental policy. The regulatory framework can be particularly complicated when the technology in question is relatively new, so that the regulator herself might not possess all the relevant information. We model such a situation by assuming that when the firm invests, the environmental authority cannot observe whether the regulated firm is efficient or inefficient in using the adopted technology. We address the performance of environmental quotas and emission taxes under two institutional settings, commitment and time consistency. With commitment (time consistency), the regulator sets environmental policy before (after) the firm has made its investment decision.

Like the present paper, D'Amato and Dijkstra (2015) [henceforth AD15] compare emission taxation and quantity regulation (emission quotas in the present paper, tradable emission permits in AD15) under commitment and time consistency when firms can choose between the current and a new abatement technology and abatement costs with the new technology are either high or low. When comparing our results to AD15, we should bear in mind the differences in setup between the two papers. In AD15 there is a continuum of small firms, and the first best is for some, but not all, firms to invest in the new technology.

The present paper features only one firm, and the first best is for this firm to invest, even if it has high costs.

It is also worth noting that previous papers have already found differences between the single-firm and the many-firm case under full information. With many heterogeneous firms, both taxation and tradeable permits implement the first best under commitment and time consistency (Requate and Unold, 2001). With a single firm, the only scenario where the first best is always implemented is with an emission quota under commitment (Amacher and Malik, 2001).

When there is asymmetric information in the single-firm (the present paper) or the multi-firm (AD15) case, the first best cannot be implemented under commitment, because the regulator cannot find out whether costs are high or low. The welfare comparison between the two instruments is guided by the reverse probability-weighted Weitzman rule in both cases (although in the present paper this only applies if the firm invests under both instruments).

The big difference between the papers occurs under time consistency, where both instruments implement the first best in AD15, but neither instrument achieves this in the present paper. In AD15, the regulator can infer the cost realization from the number of firms that invest in the new technology, and each firm takes the regulator's beliefs and policy as given when it makes its investment decision. As a result, asymmetric information is no obstacle to reaching the first best. The present paper shows that with asymmetric information the regulator cannot implement the first best in the single-firm case even when it was possible to do so with complete information. This is because the regulator can only see the investment decision by the single firm. If both types of firm invest, as they should in the first best, the regulator cannot infer the firm's type from its investment decision. Firms can signal their type through their investment decision. However, this signal is of a much lower quality with a single than with many firms.

Our paper sheds new light on the complexity of environmental policy design under asymmetric information about the effectiveness of a new abatement technology. There is no one-size-fits-all solution: the choice of the best instrument depends on the ability of the regulator to commit. Although the regulator can never reach the first best, we can still compare investment incentives and welfare under the two instruments, with some surprising results.

Our results are obtained in a very simple setting where, in particular, asymmetric information is modeled assuming only two possible efficiency levels in the use of the newly adopted technology, and only one firm is subject to regulation, yet marginal damage is increasing. This implies that some issues, mainly linked to firm heterogeneity and to the possibility of asymmetric choices, cannot be analysed and are left for future research. Yet we can show that the ordering of commitment and time consistency is crucially affected by the chosen environmental policy instrument and vice versa. Our analysis is policy relevant, as we assess how informational asymmetries related to a newly crafted technology affect the adoption patterns when the regulator can gain information on the regulated firm's costs by being time (in)consistent.

In order to limit the number of cases to be analyzed, we have assumed that investment is always socially desirable. We have also abstracted from the positive spillovers that the firm's investment could have in the form of knowledge and adoption externalities (Jaffe et al. (2003), pp. 471-474; Jaffe et al. (2005)). These positive externalities would change the regulator's behaviour under commitment. The regulator might want to set a stricter target or a higher tax rate in order to nudge the firm towards investment, thereby generating positive spillovers. The regulator's behaviour under time consistency would remain the same, because here she sets policy after the firm has made its investment decision. Taking these positive externalities into account, the firm's investment might well have the potential to improve overall social welfare, even if it has a negative effect on the sum of its own abatement costs and pollution damage. However, modeling this explicitly would lead to a further proliferation of cases to be analyzed. We leave this for future research.

References

Amacher, Gregory S. and Arun S. Malik. 2001. Price and quantity regulation with discrete technologies. Department of Economics working paper, George Washington University.

Amacher, Gregory S. and Arun S. Malik. 2002. Pollution taxes when firms choose technologies. *Southern Economic Journal* 68: 891-906.

Ambec, Stefan and Jessica Coria. 2013. Prices vs quantities with multiple pollutants. *Journal of Environmental Economics and Management* 66: 123-140.

Amir, Rabah, Marc Germain and Vincent van Steenberghe. 2008. On the impact of innovation on the Marginal Abatement Cost curve. *Journal of Public Economic Theory* 10: 985-1010.

Arguedas, Carmen and Hamid Hamoudi. 2004. Controlling pollution with relaxed regulations. *Journal of Regulatory Economics* 26: 85-104.

Bauman, Yoram, Myunghun Lee and Karl Seeley. 2008. Does technological innovation really reduce Marginal Abatement Costs? Some theory, algebraic evidence, and policy implications. *Environmental and Resource Economics* 40: 507-527.

Bréchet, Thierry and Pierre-André Jouvet. 2008. Environmental innovation and the cost of pollution abatement revisited. *Ecological Economics* 65: 262-265.

Coria, Jessica and Magnus Hennlock. 2014. Taxes, permits, and costly response to technological change. *Environmental Economics and Policy Studies* 14: 35-60.

D'Amato, Alessio and Bouwe R. Dijkstra. 2015. Technology choice and environmental regulation under asymmetric information, *Resource and Energy Economics* 41: 224-247.

Downing, Paul B. and Lawrence J. White. 1986. Innovation in pollution control. *Journal of Environmental Economics and Management* 13: 18-29.

Fischer, Stanley. 1980. Dynamic inconsistency, cooperation and the benevolent dissembling government. *Journal of Economic Dynamics and Control* 2: 93-107.

Fudenberg, Drew and Tirole Jean. 1991. *Game Theory.* MIT Press, Boston MA.

Goodkind, Andrew L. and Jay S. Coggins. 2015. The Weitzman price corner. *Journal of Environmental Economics and Management* 73: 1-12.

Hanley, Nick, Jason F. Shogren and Ben White. 2007. *Environmental Economics in*

Theory and Practice, second edition, Palgrave Macmillan, Basingstoke/New York.

Hoel, Michael and Karp, Larry. 2001. Taxes and quotas for a stock pollutant with multiplicative uncertainty. *Journal of Public Economics* 82: 91-114.

Jaffe, Adam B., Richard G. Newell and Robert N. Stavins. 2003. Technological change and the environment, in K.-G. Maler and J.R. Vincent (eds), *Handbook of Environmental Economics*, Elsevier, Amsterdam, Vol I, 461-516.

Jaffe, Adam B., Richard G. Newell and Robert N. Stavins. 2005. A tale of two market failures: Technology and environmental policy. *Ecological Economics* 54: 164-174.

Krysiak, Franz. 2008. Prices vs. quantities: The effects on technology choice. *Journal of Public Economics* 92: 1275-1287.

Kydland, Finn E. and Edward C. Prescott. 1977. Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy* 85: 473-491.

Malcomson, James M. 1978. Prices vs. quantities: a critical note on the use of approximations. *Review of Economic Studies* 45: 203-207.

Malik, Arun S. 1991. Permanent versus interim regulation: A game-theoretic analysis. *Journal of Environmental Economics and Management* 21: 127-139.

Mendelsohn, R. 1984. Endogenous technical change and environmental regulation, *Journal of Environmental Economics and Management* 11: 202-207.

Milliman, Scott R. and Raymond Prince. 1989. Firm incentives to promote technological change in pollution, *Journal of Environmental Economics and Management* 17: 247-265.

Moledina, Amyaz A., Jay S. Coggins, Stephen Polasky and Christopher Costello. 2003. Dynamic environmental policy with strategic firms: Prices versus quantities, *Journal of Environmental Economics and Management* 45: 356-376.

Moner-Colonques, Rafael and Santiago J. Rubio. 2016. The strategic use of innovation to influence environmental policy: Taxes versus standards, *BE Journal of Economic Analysis and Policy* 16: 973-1000.

Perino, Grischa and Till Requate. 2012. Does more stringent environmental regulation induce or reduce technology adoption? When the rate of technology adoption is inverted U-shaped, *Journal of Environmental Economics and Management* 64: 456-467.

Requate, Till. 2005a. Dynamic incentives by environmental policy instruments - a survey. *Ecological Economics* 54: 175-195.

Requate, Till. 2005b. Timing and commitment of environmental policy, adoption of new technology, and repercussions on R&D. *Environmental and Resource Economics* 31: 175-199.

Requate, Till and Wolfram Unold. 2001. On the incentives of policy instruments to adopt advanced abatement technology if firms are asymmetric. *Journal of Institutional and Theoretical Economics* 157: 536-554.

Requate, Till and Wolfram Unold. 2003. Environmental policy incentives to adopt advanced abatement technologies - Will the true ranking please stand up? *European Economic Review* 47: 125-146.

Storrøsten, Halvor Briseid. 2014. Prices versus quantities: Technology choice, uncertainty and welfare. *Environmental and Resource Economics* 59: 275-293.

Tadelis, Steven. 2013. *Game theory: an introduction.* Princeton University Press, Princeton NJ.

Tarui, Nori and Stephen Polasky. 2005. Environmental regulation with technology adoption, learning and strategic behavior. *Journal of Environmental Economics and Management* 50: 447-467.

Watson, William D. and Ridker, Ronald G. 1984. Losses from effluent taxes and quotas under uncertainty. *Journal of Environmental Economics and Management.* 11: 310-326.

Weitzman, Martin. 1974. Prices vs. quantities. *Review of Economic Studies* 41: 477-491.

Weitzman, Martin. 1978. Reply to "Prices vs. quantities: A critical note on the use of approximations". *Review of Economic Studies* 45: 209-210.

Weitzel, Matthias. 2017. Who gains from technological advancement? The role of policy design when cost development for key abatement technologies is uncertain. *Environmental Economics and Policy Studies* 19: 151-181.

Yao, Dennis A. 1988. Strategic responses to automobile emissions control: A game-theoretic analysis. *Journal of Environmental Economics and Management* 15: 419-438.

A Appendix A: Proofs

Lemma 1. Recall that a_ϕ^*, $\phi = l, h, 1$, is defined by (2). For any $a \geq a_l^*$ (with the new or the current technology), normalize the firm's payment T to the regulator to zero. For any a with $a_h^* \leq a < a_l^*$ (again regardless of the technology choice), the firm pays T^* with

$$l\left[C(a_l^*) - C(a_h^*)\right] < T^* < h\left[C(a_l^*) - C(a_h^*)\right]$$

For any a with $0 \leq a < a_h^*$ and investment in the new technology, the firm pays $\hat{T}$ with $\hat{T} > T^* + hC(a_h^*)$. For any a with $0 \leq a < a_h^*$ without investment in the new technology, the firm pays $\bar{T}$ with $\bar{T} > T^* + hC(a_h^*) + F$. The firm will then invest, regardless of its type, and firm θ will abate a_θ^*.

Lemma 2. The lemma holds if and only if $F < F_h(a_m^*)$ as given by (7) and (10). We find from (5) and (7):

$$\begin{aligned} F_h(a_m^*) &= (1-h)\,C(a_m^*) > (1-h)C(a_h^*) = C(a_h^*) + D(e - a_h^*) - [hC(a_h^*) + D(e - a_h^*)] > \\ &> C(a_1^*) + D(e - a_1^*) - [hC(a_h^*) + D(e - a_h^*)] = F_h^* \end{aligned}$$

The first inequality follows from (11) and $C' > 0$. The second inequality follows from the fact that a_1^* minimizes SC_1. Since we have assumed that $F < F_h^*$, we also have $F < F_h(a_m^*)$, and both types of firm will invest when the abatement target is a_m^*.

Lemma 3. We see from (8) and (12) that $E[SC_m(a_m^*)] - E[SC_{1l}(a_{1l})]$ is increasing in F. Thus if $E[SC_m(a_m^*)] - E[SC_{1l}(a_{1l})] < 0$ for $F = F_h^*$, then the inequality holds for all $F < F_h^*$. Let us now determine the sign of $E[SC_m(a_m^*)] - E[SC_{1l}(a_{1l})]$ for $F = F_h^*$.

First, define $\bar{a}_{1m} > a_1^*$ implicitly by $SC_1(\bar{a}_{1m}) = SC_h(a_m^*)$, or using (5) for $F = F_h^*$:

$$SC_1(\bar{a}_{1m}) - SC_1(a_1^*) = SC_h(a_m^*) - SC_h(a_h^*) \tag{A1}$$

We shall now see that $\bar{a}_{1m} < a_m^*$. Suppose that $\bar{a}_{1m} \geq a_m^*$. Then we can write the LHS of (A1) as:

$$SC_1(\bar{a}_{1m}) - SC_1(a_1^*) = [SC_1(\bar{a}_{1m}) - SC_1(a_m^*)] + [SC_1(a_m^*) - SC_1(a_h^*)] + [SC_1(a_h^*) - SC_1(a_1^*)] \tag{A2}$$

The first term between square brackets on the RHS is nonnegative, because $\bar{a}_{1m} \geq a_m^* > a_1^*$ (the second inequality follows from (3) and $m < 1$) and $SC_1'(a) > 0$ for $a > a_1^*$.

The third term between square brackets on the RHS of (A2) is positive, because a_1^* minimizes $SC_1(a)$. Thus we have:

$$SC_1(\bar{a}_{1m}) - SC_1(a_1^*) > SC_1(a_m^*) - SC_1(a_h^*) \tag{A3}$$

Comparing the RHS of (A3) to the RHS of (A1) yields:

$$SC_1(a_m^*) - SC_1(a_h^*) - [SC_h(a_m^*) - SC_h(a_h^*)] = (1-h)\,[C(a_m^*) - C(a_h^*)] > 0 \tag{A4}$$

The inequality follows from $h < 1$, (11) and $C' > 0$. It follows from (A3) and (A4) that $\bar{a}_{1m} \geq a_m^*$ cannot hold. Thus $\bar{a}_{1m} < a_m^*$.

We can now write:

$$\begin{aligned} &[pSC_h(a_m^*) + (1-p)SC_l(a_m^*)] - [pSC_h(a_h^*) + (1-p)SC_l(a_l^*)] = \\ &pSC_1(\bar{a}_{1m}) + (1-p)SC_l(a_m^*) - [pSC_1(a_1^*) + (1-p)SC_l(a_l^*)] < \\ &< [pSC_1(a_{1l}) + (1-p)SC_l(a_{1l})] - [pSC_1(a_1^*) + (1-p)SC_l(a_l^*)] \end{aligned} \tag{A5}$$

The equality follows from (A1) and $F = F_h^*$ in (5). The inequality follows from $\bar{a}_{1m} < a_m^*$, which implies $\bar{a}_{1m} < a_{1l} < a_m^*$. Then $SC_1(\bar{a}_{1m}) < SC_1(a_{1l})$ since $a_1^* < \bar{a}_{1m} < a_{1l}$ and $SC_1'(a) > 0$ for $a > a_1^*$, and $SC_l(a_m^*) < SC_1(a_{1l})$ since $a_l^* > a_m^* > a_{1l}$ and $SC_l'(a) < 0$ for $a < a_l^*$.

Since the second term in square brackets in the first line of (A5) cancels out against the corresponding term in the last line by (5) with $F = F_h^*$, the Lemma follows.

Lemma 4. F_m^h in (14) is decreasing in a_m^* (since $C' > 0$) and thus by (3) increasing in m. Since by (9) the highest value of m is h for $p \to 1$, the highest possible value of F_m^h is:

$$F_h^h \equiv C(a_1^*) - hC(a_h^*) \tag{A6}$$

We thus need to prove that $F_h^h < F_l^l$ which implies from (15) and (A6):

$$hC(a_h^*) > lC\,(a_l^*) \tag{A7}$$

Since $F_l^l > 0$ in (15) so that $C(a_1^*) > lC(a_l^*) > 0$, and also $l < 1$, $lC(a_l^*)$ must be on the increasing branch of $\phi C(a_\phi^*)$: $l < \widetilde{\phi}$ by Assumption 1. Then if h is between l and $\widetilde{\phi}$, it is also on the increasing branch and (A7) holds. If h is between $\widetilde{\phi}$ and 1, it is on the decreasing branch of $\widetilde{\phi}C(a_\phi^*)$, so that $hC(a_h^*) > C(a_1^*) > lC(a_l^*)$ and (A7) also holds.

Lemma 5. Suppose $t_{hl} \leq t_l$, then by (23) we would have $a_l(t_{hl}) \leq a_l(t_l)$ and $a_h(t_{hl}) < a_h(t_h)$, so that by $D'' > 0$ and (26), $D'\left[e - a_l(t_{hl})\right] \geq t_{hl}$ and $D'\left[e - a_h(t_{hl})\right] > t_{hl}$. Thus the first term in curly brackets on the LHS of (29) is negative and the second term is nonpositive. This, combined with (23) means that the LHS of (29) is positive, so that (29) cannot hold. In the same way, we can show that $t_{hl} < t_h$.

Proposition 4. In the **first equilibrium**, for very low values of F, in stage one the regulator sets the optimal tax rate of t_{hl} (defined by (29)), given that both types of firm will invest, and in stage two both types of firm invest. At t_{hl}, firm h will invest for all $F < F_h(t_{hl})$ given by (31). As we have seen following (32), $F_h(t_{hl}) > 0$.

In the **second equilibrium**, while firm h would no longer invest at t_{hl}, the regulator would still like to induce it to invest. Thus the regulator sets the tax rate at $\hat{t}_h(F) + \varepsilon$, with $\hat{t}_h(F)$ given by (30).

As F keeps rising, there comes a point at which the regulator prefers to see only firm l investing. In the **third equilibrium** the regulator sets the optimal tax rate of t_{1l} (defined by (29), given that firm l will invest, but firm h will not. At $F = \bar{F}_h$ in (35), the regulator is indifferent between the second and the third equilibrium:

When F grows larger, it will reach $F_l(t_{1l})$, defined by (31) as the point beyond which firm l no longer wants to invest at tax rate t_{1l}. However, in the **fourth equilibrium**, the regulator still induces firm l to invest by setting the tax rate at $\hat{t}_l(F) + \varepsilon$, with $\hat{t}_l(F)$ given by (30).

For even higher F, the regulator no longer wishes firm l to invest. When neither type of firm invests, the regulator would ideally like to set the tax rate at t_1. However, if in stage one the regulator sets t_1, firm h will invest in stage two. Figure 3 illustrates this: Firm h invests at t_1 for $F < F_h(t_1) = OBN$, which exceeds the maximum F of $F_h^* = OBH$.

Formally, comparing $F_h(t_1)$ to F_h^* in (5), we find:

$$\begin{aligned} F_h(t_1) - F_h^* = &\left\{t_1(a_h^* - a_1^*) - \left[D(e - a_1^*) - D(e - a_h^*)\right]\right\} + \\ &+ \left\{t_1[a_h(t_1) - a_h^*] - h\left[C[a_h(t_1)] - C(a_h^*)\right]\right\} > 0 \end{aligned}$$

The first term in curly brackets on the RHS is positive by (4), $t_1 = D'(e - a_1^*)$ by (26) and $D'' > 0$. This term is given by area BVH in Figure 3. The second term in curly brackets is positive because $a_h(t_1) > a_h(t_h) = a_h^*$ as $t_1 > t_h$ by (27) and by (23), $hC'[a_h(t_1)] = t_1$ by (22) and $C'' > 0$. This term is given by area VNH in Figure 3.

Since setting the tax rate at t_1 would not have the desired result, the **fifth equilibrium** features the regulator setting the tax rate as high as possible, while still discouraging firm l from investing. This means the tax rate will be $\hat{t}_l(F) - \varepsilon$, with $\hat{t}_l(F)$ given by (30). We have defined $\bar{F}_l$ in (34) as the level of fixed cost at which the regulator is indifferent between inducing firm l to invest and discouraging investment. We have assumed $\bar{F}_l < F_h^*$, in order for all five equilibria to occur for $F \in (0, F_h^*)$.

Lemma 6. From (5) and (36) we find:

$$F_h^{hl} - F_h^* = \{(t_1 - t_{hl})(e - a_h^*)\} + \{(t_1 - t_h)(a_h^* - a_1^*) - [D(e - a_1^*) - D(e - a_h^*)]\} + \\ + \{(t_h - t_{hl})\left[a_h^* - a_h(t_{hl})\right] - h\left(C(a_h^*) - C[a_h(t_{hl})]\right)\} > 0$$

The first term in curly brackets is positive because $t_1 > t_h > t_{hl}$ by (27) and Lemma 5. This is area $Vt_1t_{hl}X$ in Figure 3. The second term is positive by (4), $t_h = D'(e - a_h^*)$ in (26), $D'' > 0$ and $t_1 > t_h$ by (27). This is area BVH in Figure 3. The third term is positive because $t_h > t_{hl}$ by Lemma 5; $a_h^* = a_h(t_h) > a_h(t_{hl})$ by (23), (26) and $C'' > 0$. This is area ZHX in Figure 3.

Proposition 6. The first commitment equilibrium listed in Proposition 4 is the same as the only equilibrium under time consistency (Proposition 5) and thus yields the same expected social costs. The second equilibrium under commitment yields higher expected social costs than this, because while both equilibria have both types of firm investing, t_{hl} is the optimal tax rate given that they do. At $F = \bar{F}_h$ under commitment, the regulator is indifferent between the tax rates of $\hat{t}_h + \varepsilon$ (the second equilibrium) and t_{1l} (the third equilibrium). Thus by continuity, the third equilibrium under commitment also yields higher expected social costs than the time consistency equilibrium for F close to the lower bound of $\bar{F}_h$. For F close to the higher bound of $F_l(t_{1l})$ however, social costs could be lower under commitment.[20] Since $\hat{t}_l[F_l(t_{1l})] = t_{1l}$, social cost could also be lower in the fourth commitment equilibrium for F close to the lower bound of $F_l(t_{1l})$. However, at the higher bound of $F = \bar{F}_l$, social cost is higher in commitment. This is because in the fifth

[20] In Appendix B, we use a quadratic specification to show that it is indeed possible for social costs to be lower under commitment for $F = F_l(t_{1l})$.

commitment equilibrium, social cost is higher under commitment:

$$SC_1(\hat{t}_l + \varepsilon) > SC_1(t_1) = SC_1(a_1^*) > SC_h(a_h^*) = SC_h(t_h) > E\left[SC_{hl}(t_{hl})\right]$$

The first inequality follows from the fact that t_1 minimizes social cost, given that the firm does not invest. The second inequality follows from $F < F_h^*$ in (5). The third inequality follows from the fact that $E\left[SC_{hl}(t_{hl})\right]$ is decreasing in l with $l < h$.

Lemma 7. **Part 1.** A linear marginal abatement cost curve $C'(a)$ with slope 1 implies $C(a) = \frac{1}{2}a^2$, $C'(a) = a$. A linear marginal environmental damage curve $D'(e-a)$ with slope d implies $D(e-a) = \frac{d}{2}(e-a)^2$, $D'(e-a) = d(e-a)$. From (10), the quota a_m^* is given by:

$$a_m^* = \frac{de}{d + ph + (1-p)l} \tag{A8}$$

The corresponding expected social costs are:

$$E\left[SC_m(a_m^*)\right] = F + \frac{1}{2}\frac{de^2\left[ph + (1-p)l\right]}{d + ph + (1-p)l} \tag{A9}$$

From (22), the firm with technology ϕ responds to a tax rate of t by setting:

$$a_\phi(t) = \frac{t}{\phi} \tag{A10}$$

From (29), the first order condition for social cost minimization with respect to t implies:

$$t_{hl} = \frac{dehl\left[(1-p)\,h + pl\right]}{(1-p)\,h^2\,(d+l) + pl^2\,(d+h)} \tag{A11}$$

The corresponding (minimum) expected social costs at t_{hl} are:

$$E\left[SC_{hl}(t_{hl})\right] = F + \frac{1}{2}de^2\frac{dp(1-p)\,(h-l)^2 + hl\left[(1-p)h + pl\right]}{d\left[(1-p)h^2 + pl^2\right] + hl\left[(1-p)h + pl\right]} \tag{A12}$$

The differential gain in favour of quotas is, from (A9) and (A12):

$$E\left[SC_{hl}(t_{hl})\right] - E\left[SC_m(a_m^*)\right] = \frac{\frac{1}{2}d^2e^2\,(h-l)^2\,p(1-p)\left[d - (1-p)h - pl\right]}{\left[d + ph + (1-p)l\right]\left[(1-p)\,h^2\,(d+l) + pl^2\,(d+h)\right]}$$

which is positive if and only if (37) holds.

Part 2. (2) and (15) imply, in our quadratic setting:

$$F_l^h = \frac{1}{2}\left(\frac{de}{d+1}\right)^2 - \frac{h}{2}\left(\frac{de}{d+l}\right)^2 > 0$$

As F_l^h is increasing in $l < h$, a necessary condition for the inequality to hold is: $(d+h)^2 > h(d+1)^2$, that can be rewritten as $d^2(1-h) > h(1-h)$, or $d^2 > h$, since $h < 1$. If $d > 1$, then $d > h$, so that (37) always holds. If $d < 1$, then $d > d^2 > h$ and again (37) always holds.

B Appendix B: Additional assumptions for Proposition 4

In order to make sure that the **second equilibrium** in Proposition 4 exists, we must examine the regulator's and the firm's incentives at $F = F_h(t_{hl})$. If $t_{1l} > t_{hl}$, firm h would invest at a tax rate of t_{1l}, so that even if the regulator would prefer t_{1l} and no investment by firm h, this outcome is not feasible. If $t_{1l} < t_{hl}$, firm h would not invest at t_{1l} and the regulator might prefer to set t_{1l}. In that case, there would not be an equilibrium with $\hat{t}_h + \varepsilon$. To ensure existence of this equilibrium, we must thus assume:

Assumption 2. *If $t_{1l} < t_{hl}$, with t_{1l} and t_{hl} given by (29), then $E\left[SC_{hl}(t_{hl})\right] < E\left[SC_{1l}(t_{1l})\right]$ at $F = F_h(t_{hl})$, with $E\left[SC_{hl}(t)\right]$ and $E\left[SC_{1l}(t)\right]$ given by (28), and $F_h(t)$ by (31).*

From (28), $E\left[SC_{hl}(t_{hl})\right] - E\left[SC_{1l}(t_{1l})\right]$ is increasing in F. Thus Assumption 2 means that $E\left[SC_{hl}(t_{hl})\right] < E\left[SC_{1l}(t_{1l})\right]$ for all $F \leq F_h(t_{hl})$.

As for the **third equilibrium**, the switch from $\hat{t}_h$ to t_{1l} at $\bar{F}_h$ can only occur when firm h will not invest, but firm l will. We shall assume that this is the case:

Assumption 3. *$\hat{t}_l(\bar{F}_h) < t_{1l} < \hat{t}_h(\bar{F}_h)$, with $t_\theta(F)$ given by (30), $\bar{F}_h$ by (35), and t_{1l} by (29).*