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Evaluation of the fracture performance of different rubberised bitumens based on the essential work of fracture

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ABSTRACT: The fracture performance of rubberised bitumen in addition to one pre-treated with a Warm Mix Additive (Sasobit[®]) was investigated using different test methods measuring different damage mechanisms. Two Recycled Tyre Rubber (RTR) modifiers together with two base binders were blended in the laboratory to produce various combinations of Recycled Tyre Rubber Modified Bitumens (RTR-MBs). The first RTR is a standard recycled polymer derived from discarded truck and passenger car tyres by ambient grinding. The second RTR consists of 100% recycled truck tyres derived by cryogenic grinding and pre-treated with special oil and WMA to allow further decrease of asphalt mixture production temperatures. A fracture mechanics testing approach was used for evaluating the binder fatigue resistance; consisting of the double-edge-notched tension (DENT) test. The critical tip opening displacement (CTOD) obtained from the DENT test was used for fracture characterization of the binders within the ductile failure region. The study applied the partitioning concept of the total energy of bituminous binders to have a more reliable parameter that could be independent of the stress state of the ligament. The results show that generally the addition of RTR improves the fracture properties of binders indicating better fatigue performance. Pre-treatment with Sasobit[®] makes the materials more brittle and hence more susceptible to fracture.

KEYWORDS: Fracture, fatigue, rubberized bitumen, CTOD, recycled tyre rubber

1. Introduction

It is well recognized that the cracking resistance of hot-mix asphalt (HMA) mixtures is significantly related to the properties of bituminous binders. Fatigue cracking usually starts and propagates within the binder or the mastic. Therefore, characterizing the fatigue resistance of binders and improving this property by the means of modification has been a topic of intensive studies for many years. Although, many studies have shown that crumb rubber modified asphalt mixtures have superior fatigue characteristics, only limited studies have considered characterizing the binders on their own. Another challenge is to find the most representative binder tests and parameters that can best describe the binder contribution to fatigue damage resistance.

Many studies have suggested that characterizing the binders at small strains within the linear viscoelastic region, as in the case of the SHRP fatigue parameter G*sin\delta, does not necessarily reflect the true binder performance related to asphalt mixture or pavement performance [1-6]. It is believed that the main drawback of the SHRP fatigue parameter is that it neglects the damaging circumstances that would take place in the pavement during the fracture process [7, 8]. These damaging conditions are normally accompanied with high strains level and yielding within the nonlinear viscoelastic range. Thus, the resistance properties of materials under these circumstances should be considered in order to develop fundamental and more related performance based characterizations. In response to this, researchers at Queen's University proposed the double-edgenotched tension (DENT) test which is based on the concept of essential work of fracture (EWF) of materials under ductile failure [8]. The binder ranking based on this method showed a strong correlation to the observed fatigue cracking performance under accelerated loading facility (ALF) conditions and exactly the same ranking as the push-pull asphalt mix fatigue test [1, 9].

The DENT test was also used to study the effects of RAP sources on fracture performance of polymer modified binders (PMB) [10]. The results of this study showed that both the essential work of fracture and CTOD decreased with the addition of RAP indicating that the fracture resistance of PMB is reduced when mixed with RAP. In other study, the effect of adding different chemical modifiers to different bitumens was investigated by means of strain tolerance as measured by DENT test [11]. The results of DENT test showed significant differences for binders

of comparable Superpave grades, and the addition of waxes exhibited a pronounced and negative effect on the strain tolerance [11].

In this study, the fracture characteristics of binders in the ductile state were studied by the means of EWF needed to generate new surfaces using the DENT test. This test allows the materials' resistance to fracturing on notched samples under high levels of strains, yielding and fracture processes to be evaluated.

2. Background

2.1 Essential work of fracture method

The EWF concept has been increasingly used to determine the fracture toughness in polymers. Yet, there are only few studies using this test on bituminous materials. Andriescu, Hesp [8] successfully applied this test on bituminous binders and found that no correlation exists between the fracture properties and SHRP fatigue parameter G*sin\delta. This means that binders with desirable fatigue properties, according to G* sin δ , do not necessarily have good fracture properties and vice versa. Therefore, it is important to characterize the exact fracture behaviour of materials for proper material selection.

According to the EWF test when a notched ductile specimen (binder or bituminous mixture) is being loaded the total energy required for fracturing consists of two separated parts: an essential work (W_e) which takes place in the inner process zone of the progressing crack, and nonessential or plastic work (w_p) performed in the outer plastic zone [12], as shown in Fig. 1. The essential work (W_e) is the energy dissipated in the fracture region that is needed to create two new fracture surfaces. The EWF is considered a material constant property where it is more sensitive to materials integrity and modifications than the testing conditions [12]. The nonessential or plastic work is the energy dissipated in ductility, plasticity, and tearing. The essential work of fracture is proportional to the ligament cross-sectional area ($l \times B$), whereas, the plastic work is related to the plastic zone volume ($l^2 \times B$) multiplied by β which is a geometrical constant which depends on the shape of the plastic zone.



Fig. 1. Schematic representation of inner and outer zones for a typical DENT specimen

The total work of fracture (W_T) is expressed mathematically by the following simple relationship: $W_T = w_e \cdot l \cdot B + \beta \cdot w_p \cdot l^2 \cdot B$ (Eq. 1)

The above equation can be written in specific terms by dividing both sides by the ligament crosssectional area $(l \times B)$ as follows:

$$w_t = \frac{W_T}{l.B} = w_e + \beta.w_p.l$$
 (Eq.2)

Where: W_T is the total work of fracture in a DENT test as provided by the area under the forcedisplacement curve (J), w_t is the total specific work of fracture (J/m²), l is the ligament length (m), B is the sample thickness (m), β is a geometrical constant which depends on the shape of the plastic zone, w_e is the specific essential work of fracture (J/m²), and w_p is the specific plastic work of fracture (J/m³) [8, 12].

The test is performed on similar specimens with different ligament lengths (i.e. 5, 10, and 15 mm) as shown in Fig. 2. The total work of fracture W_T is obtained by measuring the area under the forcedisplacement curve (J). The total specific work of fracture is then calculated by dividing the later by the ligament cross-sectional area ($l \times B$).



Fig. 2. DENT test moulds

By plotting the w_t versus the ligament length l and using a linear fitting procedure a straight line can be produced as shown in Fig. 3. The intercept of the line represents the specific essential work (w_e) attained by extrapolation to zero ligament, and the slope is the geometry constant times the plastic work of fracture β . w_p . The literature that deals with EWF suggests that many assumptions and conditions need to be met in order to have intrinsic material properties [8, 13, 14]. These recommendations, conditions and assumptions are as follows:

- The ligament must be fully yielded before cracking initiates.
- Load-displacement (L-d) diagrams should be self-similar in appearance for all ligament lengths, verifying a common geometry of fracture.
- The sample must be yielded under a truly plane stress state of tension.



Fig. 3. Schematic sketch illustrating the relationship between w_t and ligament length l [12]

Generally, the first two requirements are easily satisfied, however, the third assumption is not always attained. The pure plane stress prevails over plane strain conditions in thin sections (small thickness to ligament ratios) and its influence gradually decreases as the ligament length reduces for a given thickness. The influence of thickness on the fracture toughness is illustrated in Fig. 4 [15].



Fig. 4. Schematic sketch illustrating the influence of thickness on the fracture toughness [15].

It can be seen from that when the thickness reaches a certain value B_c , pure plane strain conditions are thought to take place and the fracture toughness becomes independence of thickness. Also, there is an optimum thickness, B_o , at which the plane stress conditions are met. In the transition zone between B_o and B_c , the fracture toughness can be considered to be plane-stress/plane-strain (mixed mode). The thickness boundaries B_o and B_c may be estimated as follows:

$$B_o = \frac{K_{c1}^2}{3\pi\sigma_y^2} (Eq.3)$$
 and $B_c = 2.5 \left(\frac{K_{c1}}{\sigma_y}\right)^2 (Eq.4)$

Where K_{c1} is the fracture toughness, and σ_y is the tensile yield stress of the material. The influence of plane stress or plane strain condition modes on EWF can be seen in Fig. 5.



Fig. 5. Schematic sketch showing the influence of stress/strain state on w_e as a function of ligament length[12]

To examine the plane stress or strain conditions, the Hill criterion can be applied [16]. According to this criterion when a plot is made between the net section stress (maximum load divided by ligament cross section), σ_n , versus ligament length, l, a horizontal line should appear with $\sigma_n =$ 1.15 σ_y , where σ_y is the yield stress of the material. However, these conditions are not normally met in the case of bituminous binders. Bituminous materials are not as tough as polymers or metals materials, and having very thin samples to maintain a plane stress condition is not obtainable from a practical point of view. Therefore, the w_e from a mixed plane stress/strain mode is normally deduced in the case of bituminous binders. It should be noted that the plane strain value of w_e is also considered a valuable material property that is independent of sample geometry [8, 12].

The specific work of fracture in Eq.2 represents the energy required for full ligament yielding preceding the necking and tearing. However, research groups investigating the fracture of polymers have introduced the concept of energy partitioning by splitting the total energy of the load-displacement curves into two energies [14]:

the specific work of fracture required for yielding (w_y) and

the specific work of fracture required for necking plus tearing (w_{n+t})

It is believed that the load drop at full ligament yielding in the load-displacement curve corresponds to a clear transition of the process between crack initiation and the onset of crack propagation [17, 18]. The mathematical terms of Eq.2 after applying the partitioning concept become as follows:

$$w_t = w_{t(yielding)} + w_{t(necking+tearing)} = (w_{ey} + \beta_y \cdot w_{py} \cdot l) + (w_{en} + \beta_n \cdot w_{pn} \cdot l) (Eq.5)$$

Research studies on polymers have shown that the energy partitioning presented above may be a good technique to overcome some of the problems related to plane stress/strain conditions and also have more information about crack initiation and propagation fracture parameters [14, 18]. It is interesting to consider and apply the partitioning concept to bituminous binders. Therefore, the yield-related essential work w_{ey} will also be used in this study to have a more reliable parameter which should be independent of the stress state of the ligament.

Additionally, an approximation of the critical crack opening displacement (CTOD) can also be defined from the ratio of w_e over the net section stress. It is believed that a sufficient and complete yielding in the ligament section takes place at the smallest ligament, thus CTOD is approximated as $\delta_t = \frac{w_e}{\sigma_{net}}$ [15]. CTOD gives an indication about the strain tolerance of the binder in the presence of a crack and a high degree of stress concentration during the ductile regime. It is a useful parameter and has a very good correlation with the fatigue property where binders with large CTOD can better resist fatigue cracking. It was also successfully used to rank the fatigue performance of binders at different temperatures and rates of loading that cover the ductile state, and it is highly recommended by many researchers for performance grading of both binders and mixtures [1, 19-21].

3. Materials and testing programme

3.1 Materials

Two straight-run bituminous binders were used in this study, labelled H and S and two different source of RTRs, labelled as TRN and TRSE. The first one, TRN, is a standard recycled rubber, derived from discarded truck and passenger car tyres by ambient grinding. The second one, TRSE, consists of 100% recycled truck tyres; TRSE is pre-treated with a special oil and Fischer–Tropsch wax component. The special oil reduces the migration of the lighter components of the binder into the rubber and thus minimizes the effect of early ageing. The FT-wax component in TRSE allows

for a reduction in mixing temperature without running the risk of insufficient workability and compactability. The recycled tyre rubber modified bitumens (RTR-MBs) was manufactured by the wet process. The straight-run binder was preheated to 180°C and then 18% of recycled tyre rubber by the weight of the base bitumen was added gradually to the binder while mixing at 180°C using a Silverson L4RT high shear laboratory mixer. The mixing time was 120 minutes for RTR-MBs manufactured using both TRN and TRSE. Other physical and rheological properties of the straight-run binders and RTR-MBs are presented in Table 1.

Table 1. The properties of base bin	der and RTR-MBs used in this study
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Ageing states	Index	S	STRN	STRSE	Н	HTRN	HTRSE
Unaged binder	Penetration @25 °C, 0.1mm	200			40		
	Softening point °C	37.0	61.5	79.4	51.4	70.3	78.5
	Rotational viscosity, mPa.s						
	@135 °C	192			474	10829	1550
	Asphaltenes content	4.2%			15.2%		
	G* .sinδ @ 0 °C & 1.59Hz, kPa	14120	7815	9275	43960	18760	21180
	G* .sinδ @ 10 °C & 1.59Hz, kPa	2811	1677	3149	25140	10160	11420
	G* .sinδ @ 20 °C & 1.59Hz, kPa				7265	3656	3943

3.2 Testing programme

The double-edge-notched tension (DENT) test was conducted on a force-ductility apparatus installed in a ductilometer. The elastic recovery specimen mould was modified by manufacturing new DENT inserts from 360 brass to have a space between the matching pair of notches equal to the three different ligament lengths of 5, 10, and 15 mm when fitted with the end pieces of the standard mould as shown in Fig. 2. It is important to make sure that no detachment happens between the sample and the end piece of the mould during the testing especially for large ligaments. The test was performed according to the following protocol:

- Specification: LS-299 (method of test for the determination of asphalt cement's resistance to ductile failure using double-edge-notched tension test (DENT))
- Temperatures: 0 °C and 10 °C for base binder S and its rubberized bitumens; and at 10 °C and 20 °C for base binder H and its rubberized bitumens.
- o Displacement rate of 50 ± 2.5 mm/min
- Ageing effects were not considered in this study, tests were conducted on unaged samples.

4. Results and discussions

4.1 The essential work of fracture and CTOD

Fig. 6 shows the typical force-displacement curves obtained from the DENT test for all materials and at different ligament lengths. The repeatability of the test is excellent and all ligament lengths have the same force-displacement curve but each material has its own unique failure mechanism. All materials have a clear maximum point which corresponds to the yielding around the ligament area. However, the rubberized bitumens produced with TRN show two yielding points. A behaviour known as strain-hardening which is found in some polymers could be responsible for the two yielding points. Researchers working on asphalt materials have noted that strain-hardening can also be found in polymer modified bitumens [23, 24]. Singh and Girimath [10] also noted that the addition of RAP to PMB has made the second peak of the load displacement graph for PMB disappeared indicating a damage to polymer interlinkage with inclusion of RAP. It happens due to the crosslinked network in the rubber acting as a two phase system when subjected to loads resulting in stretching to high strains. Johnson, Bahia [23] suggested that this phenomenon may explain the superior fatigue performance of SBS-modified pavements when tested using the Accelerated Loading Facility (ALF) as stiffening of the binder under higher strains can prevent extra damage. The two yielding points are very clear in STRN where the second yield point is even higher than the first. The very soft nature of the base bitumen S may explain why the second yield point prevailed over the first point while it is lower in HTRN. The effect of rubber modification on total fracture energy is clear when a comparison on the force-displacement curves is made between the base bitumens and RTR-MBs. The addition of recycled tyre rubber resulted in toughening of the materials, i.e. it made them stronger and more flexible which can be translated into better fracture properties. It can be seen from Fig.s 6c and 6f that RTR-MBs produced with TRSE are stronger but less able to stretch than neat binders. The pre-treatment by waxes might have made the binders less flexible due to the formation of a crystal lattice structure in the modified binder [25].



Fig. 6. Typical force-displacement curves for all materials

Fig. 7 shows the net section stress (Peak load/net section area) as a function of the ligament length. As the net section stress decreases with the increase in ligament length, it can be concluded that the materials were tested under plane-stress/plane-strain mixed mode conditions. All materials in this study underwent the same mixed mode stress conditions, therefore, the determined fracture parameters in the next sections should be able to reliably identify the differences in their fracture properties.



Fig. 7. Net section stress as a function of the ligament length for (a) S and its RTR-MBs @ 10 °C and (b) H and its RTR-MBs @ 20 °C and

Having the total fracture energy under the force-displacement curves determined and divided by ligament cross-sectional area (L x B), the w_t is obtained and plotted against the ligament length as shown in Fig. 8. By using a linear fitting procedure, the specific essential work w_e and the plastic work of fracture βw_p term are determined from the intercept and the slope of the line, respectively. The linear regression of data points in Fig. 8 demonstrates an acceptable fitting procedure indicating that the assumptions of the EWF approach are successfully met. Fig. 8 clearly shows that the plastic work of fracture term βw_p in both the neat bitumens is very small in comparison to the modified bitumens.

Fig. 9 shows both the essential work of fracture w_e and CTOD values for all materials and at different temperatures. The data of base bitumen H at 10°C is missing because it was not possible to fulfil a ductile state failure, i.e. the base bitumen H tended to fail in a brittle state without yielding the ligament section.

It can be seen that both w_e and CTOD were improved by the rubber modification of TRN and for the two base binders H and S. On the other hand, it seems that the fracture properties of binders modified with TRSE were probably compromised by the wax pre-treatment as their fracture properties were inferior in comparison to the base bitumens. Several studies have indicated that the addition of wax could make the bituminous binders fragile at low temperatures and hence more susceptible to cracking [11, 25-29]. However, the fracture properties of HTRSE at low temperatures may be considered better than the neat bitumen H as the later was too brittle to be tested at 10 °C. Temperature decrease, as seen in Fig. 9, is accompanied by an increase in w_e and a decrease in CTOD. Temperature decrease makes the binders stiffer but less able to stretch; therefore, the relative effect between them (load and displacement) was different with respect to w_e and CTOD.



Fig. 8. Determination of the essential and plastic works of fracture analysis for and (a) S and its RTR-MBs @ 10 °C and (b) H and its RTR-MBs @ 20 °C.



Fig. 9. The essential work of fracture w_e and CTOD values for (a) S and its RTR-MBs and (b) H and its RTR-MBs.

As has been previously mentioned, applying energy partitioning may also be useful in determining reliable fracture parameters as it allows the resistance of materials to be evaluated while preventing existing "notched" cracking from being further propagated. Fig. 10 shows the linear fitting of w_t calculated from the force-displacement curves at the maximum load against ligament length. The results in Fig. 10 show that fracture energies dissipated during the first stage (yielding) are much

smaller (15 to 30% of total essential work) and the rest was used in the second stage (necking and tearing). Also, the plastic work of fracture term βw_p is almost negligible in the initiation first stage (3 to 5% total plastic work) which indicates that most of the plastic constrains occurred during the necking, propagating and tearing stage. The results of essential work of fracture w_{ei} and CTOD_i that are necessary for yielding are shown in Fig. 11. In that sense, CTOD_i represents here the ability of materials to elongate before existing cracks start propagating while the former CTOD represents the total elongation that materials can sustain after cracks have already propagated. It can be seen that the fracture parameters of HTRSE at 20 °C were the worst among the materials when the fracture energy was taken globally, however, it changes to be the best when the energy partitioning is applied. These results suggest that HTRSE could be more resistant to the onset of crack propagation, but, less resistant to post-yield fracture. However, ideal asphalt materials should resist both crack initiation and crack propagation.



Fig. 10. The essential and plastic works of fracture analysis based on partitioning concept.



Fig. 11. The essential work of fracture w_{ei} and CTOD_i values that are necessary for yielding (a) S and its RTR-MBs (b) H and its RTR-MBs.

5. Summary and conclusions

The fracture properties of different rubberised bitumens in addition to their base bitumens were evaluated based on fracture testing by the means of the DENT. In view of the results offered in this paper, the following conclusions can be drawn:

- The DENT test offers a simple test method with reproducible data to characterise the fracture properties of bituminous binders under ductile conditions. CTOD values obtained from the DENT test were reliably sensitive to the effect of temperature and can be considered a good discriminating parameter to quantify the fatigue performance of binders.
- 2. The concept of partitioning the total fracture work of energy was successfully applied on bituminous binders. It enables the determination and separation of the resistance of materials to fracture initiation in addition to fracture propagation resistance. Also, it was found that the plastic constrains had negligible effect during the yielding initiation stage which means that during this stage the energy of fracture was mostly dissipated in the inner zone.
- 3. The DENT test has successfully captured the detrimental effect of FT-waxes on fatigue properties.
- 4. The rubberised bitumens processed with TRSE appeared to have poorer fatigue resistance than the base bitumen when the fracture energy is taken globally. However, their ability to resist the propagation of existing cracking was better than the base bitumen and other rubberised bitumens according to the fracture parameters obtained from the energy partitioning concept.

- 5. When the different bituminous binders are ranked based on the SHRP parameter and DENT fracture parameters, the different test methods and parameters gave different rankings. This highlights the importance of characterising materials under different damage mechanisms which can result in totally different behaviour. For example, the SHRP parameter reflects the energy dissipation at very low strain within the linear viscoelastic region while the DENT characterises the material in the ductile state under high strains, yielding and tearing.
- 6. All the tests methods and parameters have shown that the addition of rubber results in better fatigue properties.

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