

# Price vs. quantity competition in a vertically related market revisited

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**Abstract:** In a recent paper, Alipranti et al. (2014, Price vs. quantity competition in a vertically related market, *Economics Letters*, 124: 122-126) show that, in a vertically related market, Cournot competition yields higher social welfare than Bertrand competition if the upstream firm subsidises the downstream firm's production via negative wholesale input prices. However, the assumption of a negative input price is not economically viable as it encourages the downstream firms to buy an unbounded amount of inputs. We show that the welfare ranking is reversed once we introduce a non-negativity constraint on the input prices.

**Key Words:** Bargaining; Bertrand; Cournot; Two-part tariffs; Vertical pricing; Welfare

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## 1. Introduction

In a seminal paper, Singh and Vives (1984) show that Bertrand competition yields higher social welfare than Cournot competition if the goods are substitutes and the input markets are competitive. In a recent paper, Alipranti et al. (2014) show that when a monopoly input supplier (called upstream firm) bargains with the final goods producers (called downstream firms) over a two-part tariff vertical pricing contract, the upstream firm subsidises the quantity setting downstream firms via negative wholesale input prices. This creates a higher social welfare under Cournot competition compared to Bertrand competition.

The assumption of a *negative input price* is not economically viable as it will encourage the downstream firms to buy an unbounded amount of inputs, since the upstream firm would pay the downstream firms for each unit of input they purchase. We, therefore, impose a non-negativity constraint on the input prices. In contrast to Alipranti et al. (2014), we find that Cournot competition yields lower social welfare than Bertrand competition, thus supporting the findings of Singh and Vives (1984) even in a vertical structure.

## 2. The model and the results

We consider a model similar to Alipranti et al. (2014) where two downstream firms, denoted by  $D_i$ ,  $i = 1, 2$ , produce differentiated products.  $D_i$  purchases a critical input for production from a monopoly upstream firm,  $U$ , through a two-part tariff contract involving a fixed-fee,  $F_i$ , and a per-unit price,  $w_i$ ,  $i = 1, 2$ .  $U$  produces the input at a constant marginal cost of production  $c$ , which is assumed to be zero in line with Alipranti et al. (2014). We assume that  $D_i$  requires one unit of input to produce one unit of its output, and it can convert the inputs to the final goods without incurring any further cost.

Assume that the inverse and direct demand functions for  $D_i$ 's products are  $P_i = a - q_i - \gamma q_j$  and  $q_i = \frac{a(1-\gamma) - P_i + \gamma P_j}{1-\gamma^2}$ ,  $i, j = 1, 2$ ;  $i \neq j$  where,  $q_i$ ,  $P_i$  are respectively  $D_i$ 's output and price, and  $\gamma \in (0,1)$  measures the degree of product differentiation.

We consider the following game. At stage 1,  $U$  bargains with  $D_i$  to determine the terms of the two-part tariff contract. At stage 2,  $D_1$  and  $D_2$  determine their outputs (under Cournot competition) and price (under Bertrand competition) simultaneously and the profits are realised. We solve the game through backward induction.

### 2.1. Cournot competition

If the downstream firms compete in quantities, at stage 2,  $D_i$ ,  $i=1,2$ , determines  $q_i$  to maximise  $\pi_{Di}^C(w_i, w_j, q_i, q_j) = (a - q_i - \gamma q_j - w_i)q_i$ . Note that  $F_i$  is sunk at stage 2. The equilibrium output of  $D_i$  can be found as

$$q_i^C(w_i, w_j) = \frac{a(2-\gamma) - 2w_i + \gamma w_j}{4-\gamma^2}. \quad (1)$$

The gross equilibrium profit of  $D_i$  is  $\pi_{Di}^C(w_i, w_j) = [q_i^C(w_i, w_j)]^2$ .

At stage 1,  $U$  bargains with  $D_i$  to determine the two-part tariff contracts, taking as given the equilibrium contract  $(w_j^{C*}, F_j^{C*})$  with  $D_j$ ,  $i=1,2$ ,  $i \neq j$ . Like Alipranti et al. (2014), the fixed-fee and the per-unit input price are determined through a generalised Nash bargaining:

$$\text{Max}_{F_i, w_i} [\pi_U^C(w_i, w_j^{C*}) + F_i + F_j^{C*} - d(w_j^{C*}, F_j^{C*})]^\beta [\pi_{Di}^C(w_i, w_j^{C*}) - F_i]^{1-\beta} \quad (2)$$

where  $\beta$  (resp.  $1-\beta$ ) is the bargaining power of  $U$  (resp.  $D_i$ ) and  $\pi_U^C = \sum_i w_i q_i^C(w_i, w_j)$  is the upstream firm's profit. When bargaining between the upstream firm and  $D_i$  breaks down, the upstream firm's disagreement pay-off is denoted by  $d(w_j^{C*}, F_j^{C*}) = w_j^{C*} q_j^{mon}(w_j^{C*}) + F_j^{C*}$  and

$D_j$  produces  $q_j^{mon}(w_j^{C*}) = \frac{a-w_j^{C*}}{2}$  as a monopolist.

Maximising (2) with respect to  $F_i$  gives:

$$F_i^C = \beta \pi_{Di}^C(w_i, w_j^{C*}) - (1 - \beta) [\pi_U^C(w_i, w_j^{C*}) - w_j^{C*} q_j^{mon}]. \quad (3)$$

Substituting (3) in (2), we get the maximisation problem as:

$$\text{Max}_{w_i} [\beta^\beta (1 - \beta)^{1-\beta}] [\pi_U^C(w_i, w_j^{C*}) + \pi_{Di}^C(w_i, w_j^{C*}) - w_j^{C*} q_j^{mon}]. \quad (4)$$

Solving the first order condition gives the equilibrium per-unit input price as  $w_i^{C*} = -\frac{a\gamma^2}{2(2-\gamma^2)} < 0$ . Hence, the negotiated wholesale input price is negative. As per Alipranti et al. (2014), the monopoly input supplier subsidises downstream firms' production via negative input prices. However, the downside of this argument is that the downstream firms would want to buy an infinite number of inputs, since the upstream firm pays the downstream firms for each unit of input they purchase. Hence, a negative input price is not economically viable. Therefore, to make the analysis meaningful, we set  $w_i^{C*} = 0$ , which means that the equilibrium two-part-tariff only consists of a positive fixed-fee,  $F_i^{C*} = \beta \left(\frac{a}{2+\gamma}\right)^2$ .

Introducing the non-negativity constraint on the input price, we find the equilibrium output and the net profits of the downstream and upstream firms as  $q_i^{C*} = \frac{a}{2+\gamma}$ ,  $\pi_{Di}^{C*} - F_i^{C*} = \frac{a^2(1-\beta)}{(2+\gamma)^2}$  and  $\pi_U^{C*} + F_i^{C*} = 2\beta \left(\frac{a}{2+\gamma}\right)^2$ . The consumer surplus and social welfare (which is  $SW = \sum \pi_{Di} + \pi_U + CS$ ) are respectively  $CS^{C*} = \frac{a^2(1+\gamma)}{(2+\gamma)^2}$  and  $SW^{C*} = \frac{a^2(3+\gamma)}{(2+\gamma)^2}$ .

## 2.2. Bertrand competition

If the downstream firms compete in prices, at stage 2,  $D_i$ ,  $i=1,2$ , maximises  $\pi_{Di}^B(w_i, w_j, q_i, q_j) = (P_i - w_i) \left(\frac{a(1-\gamma) - P_i + \gamma P_j}{1-\gamma^2}\right)$ . The equilibrium price and output of  $D_i$  can be found as

$$P_i^B(w_i, w_j) = \frac{a(1-\gamma)(2+\gamma) + 2w_i + \gamma w_j}{4-\gamma^2} \quad \text{and} \quad q_i^B(w_i, w_j) = \frac{a(1-\gamma)(2+\gamma) - (2-\gamma^2)w_i + \gamma w_j}{(1-\gamma^2)(4-\gamma^2)}. \quad (5)$$

To avoid analytical repetition, we only report the equilibrium outcomes under Bertrand competition that are similar to Alipranti et al. (2014), i.e.,  $w_i^{B*} = \frac{a\gamma^2}{4}$ ,  $q_i^{B*} = \frac{a}{4} \left( \frac{2+\gamma}{1+\gamma} \right)$ ,  $\pi_{Di}^{B*} - F_i^{B*} = \frac{a^2(1-\beta)(2+\gamma)(4-2\gamma-\gamma^3+\gamma^4)}{32(1+\gamma)}$ ,  $\pi_U^{B*} + F_i^{B*} = \frac{a^2(2+\gamma)(2\beta(2-\gamma)+\gamma^3(1-\gamma)(1-\beta))}{16(1+\gamma)}$ ,  $CS^{B*} = \frac{a^2(2+\gamma)^2}{16(1+\gamma)}$  and  $SW^{B*} = \frac{a^2(2+\gamma)(6-\gamma)}{16(1+\gamma)}$ .

### 3. Results

Our main results are shown in the following propositions.

**Proposition 1:** *Cournot competition generates higher downstream profits whereas the upstream profit is higher (lower) under Cournot competition compared to Bertrand competition for  $\beta > (<) \frac{8+4\gamma-6\gamma^2-5\gamma^3-\gamma^4}{16+6\gamma-6\gamma^2-5\gamma^3-\gamma^4}$ .*

**Proof:** We get that  $\pi_{Di}^{C*} - \pi_{Di}^{B*} = \frac{a^2\gamma^3(1-\beta)(16+6\gamma-6\gamma^2-5\gamma^3-\gamma^4)}{32(1+\gamma)(2+\gamma)^2} > 0$  and  $\pi_U^{C*} - \pi_U^{B*} = -\frac{a^2\gamma^3(8-16\beta+4\gamma-6\beta\gamma-6\gamma^2+6\beta\gamma^2-5\gamma^3+5\beta\gamma^3-\gamma^4+\beta\gamma^4)}{16(1+\gamma)(2+\gamma)^2} > (<) 0$  for  $\beta > (<) \frac{8+4\gamma-6\gamma^2-5\gamma^3-\gamma^4}{16+6\gamma-6\gamma^2-5\gamma^3-\gamma^4}$ . ■

Although the downstream profit ranking in our analysis is like Alipranti et al. (2014), the rationale behind our results are in line with Singh and Vives (1984), i.e., competition being less fierce under Cournot competition compared to Bertrand competition,<sup>1</sup> the former competition generates higher downstream profits than the latter.

The upstream profit ordering, on the other hand, is ambiguous and depends on the bargaining strength of the upstream firm. As explained in Alipranti et al. (2014), since the quantities are strategic substitutes and the prices are strategic complements, the upstream firm

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<sup>1</sup> Check that  $q_i^{C*} - q_i^{B*} = -\frac{a\gamma^2}{4(1+\gamma)(2+\gamma)} < 0$  and  $P_i^{C*} - P_i^{B*} = \frac{a\gamma^2}{4(2+\gamma)} > 0$ .

has a higher incentive to charge a relatively lower per-unit input price to  $D_i$  under Cournot competition compared to Bertrand competition. Thus, Cournot competition creates higher downstream profit which the upstream firm extracts through the fixed-fee. Since the non-negativity constraint on the per-unit input price in our analysis restricts the upstream firm's profit extraction under Cournot competition, its opportunistic behaviour remains (becomes less) pronounced under Cournot competition compared to Bertrand competition if the bargaining power of the upstream firm is sufficiently high (low), i.e.,  $\beta > (<) \frac{8+4\gamma-6\gamma^2-5\gamma^3-\gamma^4}{16+6\gamma-6\gamma^2-5\gamma^3-\gamma^4}$ ; resulting in a higher (lower) upstream profit under the former competition than the latter.

**Proposition 2:** *Cournot competition yields lower consumer surplus and lower social welfare than Bertrand competition.*

**Proof:** We get that  $CS^{C^*} - CS^{B^*} = -\frac{a^2\gamma^2(8+8\gamma+\gamma^2)}{16(1+\gamma)(2+\gamma)^2} < 0$  and  $SW^{C^*} - SW^{B^*} = -\frac{a^2\gamma^2(8-\gamma^2)}{16(1+\gamma)(2+\gamma)^2} < 0$ . ■

The above results are in stark contrast to Alipranti et al. (2014). The non-negativity constraint on the input price is responsible for this difference. Lower outputs and higher final-goods' prices under Cournot competition (see footnote 1) result in lower consumer surplus under Cournot competition compared to Bertrand competition. When  $\beta$  is sufficiently high, meaning that the opportunistic behaviour of the upstream firm is severe, the loss in consumer surplus under Cournot competition than Bertrand competition outweighs the gains in upstream and downstream firms' profits under the former competition than the latter, thus creating an overall welfare loss under Cournot competition compared to Bertrand competition. If  $\beta$  is sufficiently small, i.e., the opportunistic behaviour of the upstream firm is not so strong, the

losses in the upstream profit and consumer surplus under Cournot competition compared to Bertrand competition dominate the downstream firms' gain under the former competition than the latter, thus generating lower welfare under Cournot compared to Bertrand competition.

It is worth mentioning that the problem of negative input price persists even if the marginal cost of input production is positive, i.e.,  $c > 0$ . Imposing a non-negativity constraint on the input price, we show in our working paper (Basak and Mukherjee, 2016) that Cournot competition generates lower (higher) social welfare than Bertrand competition if the marginal cost of input production is sufficiently low (high).

#### **4. Conclusion**

Alipranti et al. (2014) show that Cournot competition generates higher welfare compared to Bertrand competition since the upstream firm subsidises the quantity setting downstream firms via negative wholesale input price. Once we impose the non-negativity constraint on the input prices, the welfare ranking is reversed.

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