

A reliability-based approach to the robustness of corroded RC structures

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Abstract

Currently, decisions on infrastructural assets maintenance and repair, in particular on structures, are based, mostly, on the results of inspections and the resulting condition index, neglecting systems robustness, and, therefore, not making optimal use of the limited available funds. This paper presents a definition and a measure of structural robustness in the context of deteriorating structures, compatible with asset management systems for optimal maintenance and repair planning. The proposed index is used in defining the robustness of existing RC structures to rebar corrosion. Structural performance and the corresponding reliability index are assessed using combined advanced reliability and structural analysis techniques. Structural analysis explicitly includes deterioration mechanisms resulting from corrosion such as reinforcement area reduction, concrete cracking and bond deterioration.

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The First Order Reliability Method, combined with a Response Surface algorithm, is used to compute the reliability index for a wide range of different corrosion levels, resulting in a fragility curve. Finally, structural robustness is computed and discussed based on the obtained results. Robustness comparison of different structures can then be used to determine structural types more tolerant to corrosion and these results can be used for maintenance and repair planning.

Keywords: Robustness, Reliability, Damage, Reinforced Concrete, Corrosion

1. Introduction

Maintaining safety and serviceability of existing structures and bridges by making better use of available resources is one of major challenges of transportation agencies in most developed countries since the number of structures reaching the design life-time is growing year after year [1]. Strategies, as giving priority to the poorest condition, are clearly insufficient as do not take advantage of structural robustness and tolerance to damage. Currently, decisions on maintenance and repair are reactive and based, mostly, on the results of visual inspection and the resulting condition index. The condition index is a convenient indicator of the deterioration of a structure, but provides little information regarding the structural safety, as neither the initial (intact) safety nor the impact of deterioration on safety is considered. Experience has shown that different structures can, for similar deterioration levels, present significantly different safety reductions and safety levels, with a dramatic effect on the need to repair and on the optimal allocation of funds

16 in a network.

17 This paper presents a framework to assess robustness of structures under
18 deterioration. Considering that a detailed safety assessment of every exist-
19 ing structure is impossible due to financial limitations and to the uncertainty
20 related to the real deterioration, the robustness concept proposed herein can
21 serve as an approximated measure of the mean loss in safety independent
22 of the deterioration level for a given bridge type. The proposed robustness
23 framework can then be combined with the bridge deterioration information
24 to obtain a better indication of current and future safety loss due to deteriora-
25 tion and, therefore, to define an optimum maintenance policy. For instance,
26 the robustness indicator may help the decision-maker to take a wise decision
27 regarding the maintenance operations to be delivered on two bridges with
28 equal or similar condition rating.

29 Although a robustness analysis is also complex, the robustness of similar
30 structures is believed to be relatively uniform, allowing a classification of
31 structures in a network based on the detailed analysis of a limited number
32 of structural typologies. This classification can be used in conjunction with
33 the observed or predicted condition state to define the need or urgency of
34 maintenance, considering explicitly the structural properties of a particular
35 structure. This allows a clear distinction between structures which, although
36 presenting similar deterioration levels in specific main components, have very
37 different safety levels as a result of different geometry or critical failure paths,
38 among others.

39 Focus is also given to reinforced concrete structures (due to representa-
40 tiveness of this structural type worldwide) under reinforcement corrosion as

41 this is one of the major causes of structural deterioration.

42 **2. Structural robustness**

43 Research on robustness has focused on extreme events, such as terrorist
44 attacks. However, the concept can also be very useful in the context of
45 structural aging and deterioration, in particular in the asset management
46 field. Robustness of some structural types can be crucial to plan and design
47 future infrastructures, requiring less repair and maintenance actions during
48 service lifetime.

49 In what respects to corrosion of reinforced concrete structures, although
50 the mechanisms responsible for rebar corrosion are relatively well known [2],
51 the prediction of future deterioration is associated with very large uncer-
52 tainty. For this reason, deterioration of reinforced concrete structures can be
53 analyzed in a robustness framework, considering corrosion as unpredictable
54 and assuming levels within a wide range. This approach is useful for both new
55 and existing structures, as it indicates, on one hand, the structural designs
56 less susceptible to corrosion and, on the other hand, the existing structures
57 for which higher whole life repair costs can be expected.

58 Although robustness is a desirable property, a consensual definition and
59 a framework to assess it still do not exist [3]. Significant work has been done,
60 in particular under COST¹ Action TU-0601 - Robustness of Structures, but
61 no unanimous methodology has yet been found.

62 Some authors suggest robustness to be a structural property [4, 5, 6, 7]

¹COST - European Cooperation in the field of Scientific and Technical Research

63 while for others robustness depends also on the surrounding environment
64 [8]. In this case, Robustness is a much broader concept, since it accounts
65 with indirect consequences of failure which depend on several aspects such
66 as social and economical. A deep discussion on the robustness concept can
67 be found in [7].

68 In this paper the perspective of robustness being a structural property is
69 adopted, in order to characterize the damage tolerance of existing structures
70 to deterioration. The proposal of [7] is considered since it is sufficiently
71 generic to be applied to most structural types and damage scenarios and
72 can be applied in a probabilistic or deterministic framework. Robustness
73 is defined as a structural property which measures the degree of structural
74 performance remaining after damage occurrence. This relation can take many
75 different forms, depending of the limit state (from service to ultimate limit
76 state) that is adopted in the structural evaluation. Damage can vary from
77 simple degradation to a more serious damage scenario as a local failure.

78 In order to assess robustness, it is fundamental to define a measure of
79 structural performance f and a damage D causing performance decrease.
80 The next step is to define the performance function of the damaged structure
81 $f(D)$ for the complete damage spectrum. The maximum value of damage
82 in the spectrum corresponds to the maximum expected loss of performance
83 during service life. This is important when comparing robustness of different
84 structural types, where the performance profile can be highly different as a
85 function of the damage level or, alternatively, the service life. In the final step,
86 both damage and performance indicator are normalized and the robustness

87 indicator R_D is computed as follow:

$$R_D = \int_{D=0}^{D=1} f(x)dx \quad (1)$$

88 For null robustness structures, a small level of damage produce a total loss
89 of structural performance and vice-versa.

90 The proposed index, R_D , is a generalization of the proposals of [4, 6] and
91 the damage based measure, $R_{d,int}$ proposed by [3], however with some ad-
92 vantages which appear to solve some of the limitations found in the referred
93 robustness measures. The proposal of [4] is not suitable to deal with con-
94 tinuous damage, which is the case of reinforcement corrosion. This problem
95 appears to be solved in the [6] proposal. Although this index considers con-
96 tinuous values for the damage variable, it results in different values for the
97 robustness index, depending on the damage level. These problems have been
98 solved by the proposed index, R_D , by considering normalized and continu-
99 ous values for both structural performance and damage. Additionally, since
100 all the damage domain is integrated, robustness is given by a unique value
101 independently of the damage level. Thus, robustness may result similar for
102 different structures even if one degrades continuously and the other reacts
103 brittle. However, this can be surpassed if a probabilistic approach is used to
104 measure the structural performance.

105 In this paper, the robustness of reinforced concrete structures subjected
106 to corrosion is analyzed. Damage inflicted to the structure is considered to
107 be the corrosion level on the reinforcement measured in terms of rebar weight
108 loss percentage. The difficulties in defining a probabilistic model for hazard,
109 in this case for corrosion, lead to the analysis under a range of different cor-
110 rosion levels. This strategy has been used in seismic engineering for instance,

111 where fragility curves resulting from exposing structures to different earth-
112 quakes intensities, have been used to characterize structural performance to
113 seismic events. However the concept can be extended to a wide range of
114 other hazards, as structural deterioration and in particular to reinforcement
115 corrosion.

116 In this paper, structural performance is measured through the reliability
117 index as this is a consistent measure of structural safety which takes uncer-
118 tainty into account.

119 **3. Corrosion of reinforced concrete structures**

120 *3.1. Corrosion process*

121 When reinforced concrete is exposed to environmental conditions, steel
122 bar corrosion and iron oxides formation are likely to occur due to the ener-
123 getic potential of the iron-carbon alloy. The iron oxides resulting from the
124 corrosion reaction do not have mechanical properties comparable to those
125 of steel and exhibit volume increase which can go to seven times the origi-
126 nal steel volume. The final result is the occurrence of several deteriorating
127 mechanisms which lead to a deterioration of the structural capacity.

128 During the lifetime of a reinforced concrete structure two periods concern-
129 ing corrosion can be distinguished [9]: the initiation period, respecting to the
130 stage where reinforcement is protected by a thin oxide layer. Within this pe-
131 riod corrosion takes place at a negligible rate and no deterioration effects are
132 expected. The second phase, the propagation period, starts when concrete
133 cover is contaminated and the passive oxide layer is destroyed. This results
134 in increased corrosion rate and deterioration of the structure condition.

135 Steel depassivation occurs mainly due to concrete carbonation and chlo-
136 rides contamination, typical of industrial and maritime environments, re-
137 spectively. In the first case, corrosion is likely to occur uniformly, along steel
138 bars length, while in the second case corrosion tends to be more localized and
139 pronounced, also called pitting corrosion. In both cases several deterioration
140 mechanisms are expected to aggravate the structure condition: reinforcement
141 effective area reduction; ductility reduction of steel bars; concrete cracking
142 and spalling of concrete; bond degradation between steel bars and surround-
143 ing concrete. The influence of these mechanism on the structural behavior
144 depends on several factors such as type of corrosion, reinforcement ratio,
145 concrete strength, loading, cross section geometry, among others [10]. In
146 general, steel bars effective area and ductility reduction are of more concern
147 in cases of localized or pitting corrosion [11, 12], while concrete cracking
148 and spalling and debonding effect play a more deteriorating role in cases of
149 general corrosion [13, 14, 15, 16].

150 Ductility reduction of steel bars is partly due to a chemical transformation
151 of the steel occurring during corrosion process, known as *hydrogen embrittle-*
152 *ment* [17, 18] and partly due to a localization phenomenon resulting from non
153 uniform corrosion [19]. The latter can explain the reason behind ductility
154 reduction of steel bars have been considered specially concerning in cases of
155 pitting corrosion. In these cases however, concrete cracking and spalling and
156 debonding of reinforcement are, in general, not critical, as steel bars can be
157 anchored in less corroded and non cracked zones [20]. However, if corrosion
158 attacks all bar length, spalling of concrete cover is likely to occur and loss of
159 bond between steel bars and concrete, compromising the composite behavior

160 of both materials, is expected. [21] have concluded the effects of localized and
161 generalized corrosion to be potentially more hazardous for bending ultimate
162 and service limit states of highway bridges, respectively. However, it must
163 be noted that the authors have assumed perfect anchorage of reinforcement
164 in the abutments. Even if hooks are provided at reinforcement ends, anchor-
165 age can be greatly impaired by the existence of lapped joints reinforcement
166 [22]. Additionally, it must be noted that corrosion rate is usually increased
167 in zones of reinforcement concentration or where it is bent. According to
168 [13] and [14] reinforcement debonding is the main cause of impaired flexural
169 behavior, if corrosion is found to be generalized and uniform.

170 This paper addresses generalized and uniform corrosion. Localized and
171 pitting corrosion stay outside the scope. From this stage onwards, and for
172 sake of simplicity, only the effects of concrete cracking and spalling, debond-
173 ing of steel bars and reinforcement effective area reduction will be considered.
174 Reinforcement impaired ductility and reduction of steel strength, including
175 the spatial variability of corrosion, are not considered herein, although it
176 is recognized, and as suggested by [11, 12, 23], that these are factors of
177 paramount importance in cases of localized corrosion, which is not the present
178 case.

179 *3.2. Methodology*

180 As discussed in the previous section, to adequately model the effects of
181 generalized corrosion it is necessary to take into account some undesirable
182 consequences of the oxidation process of rebars, including reinforcement net
183 area reduction and expansion due to corrosion products accumulation. This
184 last phenomenon leads to damage, cracking and splitting of the surround-

185 ing concrete and degradation of steel-concrete bond, responsible for stress
186 transfer between both materials.

187 In order to model all these effects, an advanced Finite Element methodol-
188 ogy was used coupled with advanced constitutive models for modeling mate-
189 rials. Its capability to reproduce the behavior of corroded reinforced concrete
190 was demonstrated by comparing numerical results with results obtained ex-
191 perimentally [24]. The methodology employed considers a two-step analysis.
192 In the first step a finite element analysis of the structure cross section is car-
193 ried out, simulating the formation and accumulation of corrosion products
194 as an expansion of steel bars. In this phase, steel bars are modeled using
195 a linear elastic law and are coupled to concrete through an interface model
196 that regulates the shear stress transference between the two materials. For
197 sake of simplicity, corrosion is considered to attack uniformly around the bar
198 perimeter, although it is known that corrosion is more pronounced in outer
199 part of the steel bar. For concrete, an isotropic continuum damage model is
200 used enriched with kinematics provided by the strong discontinuities theory
201 [25]. The combination of these two approaches, for modeling concrete be-
202 havior, allows the simulation of crack development caused by corrosion and
203 expansion of rebars.

204 In the second step, results obtained during the cross section analysis are
205 then used to build a 2D structural model of the corroded structure used to
206 assess the impaired structural capacity. Reinforced concrete is modeled by
207 means of a composite material constituted by a matrix, representing concrete,
208 mixed with long fibers which represent steel bars, as proposed by [26]. This
209 is the main difference from the modeling strategy proposed by [24]. Whereas

210 [24] used a mesoscopic approach for the 2D longitudinal model, using different
211 finite elements for concrete, reinforcing bars and interface. In the homoge-
212 nized model used herein, a unique composite finite element is enriched to
213 reproduce the composite behavior of all the components. As an advantage,
214 the homogenized model requires much less computational resources, due to
215 the smaller size of the numerical model. This is an important aspect in this
216 case, since a large number of different analyzes are required to perform the
217 fragility curves. Additionally, the homogenized model seems to reproduce
218 better the global structural behavior since the interface between concrete
219 and steel bars is implicitly considered. In the mesoscopic approach, bond
220 effect is reproduced using interface elements. In this manner, results can be
221 affected by the mesh size, usually resulting in a less stiff global behavior.

222 *3.3. Cross section analysis*

223 This section depicts results obtained in the first step of the corrosion anal-
224 ysis methodology, obtained for a rectangular section (0.20m×0.40m) with
225 mean values properties of a C30/37 concrete and $2\phi 10$ and $2\phi 20$ reinforce-
226 ment steel bars (S400 grade) placed at the upper and bottom section surfaces,
227 respectively. Corrosion was simulated considering a volumetric expansion of
228 steel bars, with similar penetration rates on both bars. Resulting iron oxides,
229 as suggested by [27], were considered incompressible and to occupy twice the
230 initial iron volume. Figure .1 shows the effect of corrosion at a cross section
231 level. Figure .1 (a) shows damage map, d , on concrete due to expansion of
232 steel bars for a corrosion penetration depth, $X = 0.5\text{mm}$, which correspond
233 to an area percentage lost of $X_{P1} = 10\%$ and $X_{P2} = 20\%$ for bottom and
234 top reinforcement, respectively. Damage $d = 1$ means concrete had lost all

235 strength and cracking is eminent. Figure .1 (b) shows the corresponding
236 iso-displacement lines which concentration indicates crack development as
237 shown in Figure .1 (c).

238 [Figure 1 about here.]

239 Figure .2 shows width evolution of cracks (a) to (e) as corrosion increases.
240 Cracks (a) and (e) are those reaching the range of visible cracks ([0.1-0.2]mm)
241 for X_{P_1} and X_{P_2} equal to 1% and 2%, respectively, therefore consistent with
242 experimental results [10]. Figure .2 shows that, for corrosion X_{P_1} and X_{P_2}
243 above 5% and 10%, respectively, cracks width increase linearly and no addi-
244 tional cracks were detected. This allow the definition of the effective concrete
245 cross section as shown in Figure .1 (d). For sake of simplicity, corrosion of
246 transverse reinforcement was neglected [21], although it is recognized, on one
247 hand the respective positive confinement effect, and on the other hand the
248 additional negative contribution for the cross section deterioration.

249 [Figure 2 about here.]

250 3.4. Structural Analysis

251 Results obtained for the cross section analysis were used to build a 2D
252 structural model of the corroded structure. A simply supported 5.0m span
253 beam was used to illustrate the proposed methodology. Reinforced concrete
254 was modeled by means of a composite material constituted by a matrix,
255 representing concrete, mixed with long fibers which represent steel bars, as
256 proposed by [26]. Three types of composite material needed to be considered
257 (see Figure .3): concrete cover (unreinforced plane concrete); concrete on

258 the beam's web, transversely reinforced; and concrete surrounding flexural
 259 bars, longitudinally reinforced. As for the cross section analysis, and in order
 260 to be able to model crack development, in the longitudinal model the finite
 261 elements were also enriched with the strong discontinuities kinematics [25],
 262 and for concrete the isotropic continuum damage model was adopted [28].

263 [Figure 3 about here.]

264 For the embedded fibers, the objective was to simultaneously model rein-
 265 forcement behavior and debonding effect, resulting from corrosion. In order
 266 to achieve such goal, the slipping-fiber model proposed by [26] was adopted,
 267 which considers slipping-fiber ϵ^f strain as the sum of the fiber mechanical
 268 deformation and the deformation of interface.

269 [Figure 4 about here.]

270 Assuming a two-component serial system constituted by the fiber and the
 271 interface, the corresponding slipping-fiber stress σ^f is identical to the stress of
 272 each component. On both cases the stress-strain relation can be obtained via
 273 an one-dimensional elasto-plastic hardening/softening model. The resulting
 274 constitutive behavior, for the slipping-fiber, is also an elasto-plastic model
 275 with the following characteristics:

$$\sigma_y^f = \min(\sigma_y^d, \sigma_y^i) \quad (2)$$

$$E^f = \frac{1}{\frac{1}{E^d} + \frac{1}{E^i}} \quad (3)$$

277 in which E^d and σ_y^d are the steel Young's modulus and yield stress, respec-
 278 tively, E^i is the interface elastic modulus and σ_y^i is the interface bond limit

279 stress. Regard that, when $E^i \rightarrow \infty$ and $\sigma_y^d < \sigma_y^i$, the system provides only
 280 the mechanical behavior of the fiber, reproducing a perfect adhesion between
 281 concrete and reinforcement bars.

282 For the slipping-fiber model characterization, pullout tests can be per-
 283 formed in order to assess the required parameters. In this paper, for the
 284 uncorroded state, perfect adhesion between steel bars and concrete is consid-
 285 ered and a rigid-plastic behavior for the interface is adopted. This hypoth-
 286 esis may be considered acceptable since it is considered that the anchorage
 287 lengths are respected and only ultimate limit states related to structural ca-
 288 pacity are under analysis. For corroded states, the bond limit stress σ_y^i was
 289 considered lower than the reinforcement yielding stress σ_y^d , and depending on
 290 the corrosion level X_p . This means that perfect adhesion it is not valid for
 291 the corroded states, as suggested by several researchers [29, 14, 15, 30, 16].
 292 The steel yield strength was considered not affected by corrosion, although
 293 reductions have been documented, in particular in cases of localized corro-
 294 sion. In order to characterize bond strength reduction as a function of the
 295 corrosion level the *M-pull* model proposed by [31] was adopted. This is an
 296 empirical model, based on several author's experimental tests (see Figure .5),
 297 thus the following results must be watched carefully. The *M-pull* gives the
 298 normalized bond strength reduction depending on the corrosion level:

$$\frac{\sigma_y^i(X_p)}{\sigma_y^i(X_p = 0)} = \begin{cases} 1.0 & \text{if } X_p \leq 1.5\% \\ 1.192 \cdot e^{-0.117X_p} & \text{if } X_p > 1.5\% \end{cases} \quad (4)$$

299 For sake of simplicity, bond degradation was considered uniform around the
 300 steel bars perimeter.

301 [Figure 5 about here.]

302 To build the 2D structural longitudinal model of the deteriorated structure,
 303 it was necessary to import the results obtained for the cross section corrosion
 304 analysis (see Figure .6 (a)). As referred, special attention was given to crack
 305 pattern, *i.e.*, when a crack crossed two cross section faces, the smaller section
 306 part was considered disconnected from the section core (see Figure .6 (b))
 307 and then, for simplicity, considered with damage $d = 1$ (see Figure .6 (c)).
 308 In this case, and as observed in the previous section, for advanced corrosion
 309 stages, concrete corners at both beam's top and bottom tended to split from
 310 the section core. The next step was to divide the cross section into thin
 311 horizontal slices and compute the average damage, d , for each slice (see Figure
 312 .6 (d)). Finally the damage values for each slice, as shown in Figure .6 (e),
 313 were projected on the 2D longitudinal structural model (see Figure .6 (f))
 314 defining the deteriorated structure.

315 [Figure 6 about here.]

316 4. Reliability Analysis

317 As previously referred, the reliability index, β , is the structural perfor-
 318 mance indicator chosen to assess robustness since it is a consistent measure
 319 of safety. However, the reliability of a corroding existing structure is a time-
 320 dependent problem, which can be expressed by the following equation:

$$P_f(t) = \int_{G[X(t)]} f_{X(t)}[X(t)]dx(t) \quad (5)$$

321 where $P_f(t)$ is the instantaneous probability of failure at time t , $X(t)$ is
 322 the random variables vector, $G[X(t)]$ is the limit state function and $f_{X(t)}$ the
 323 joint probability density function of the random variables. The instantaneous

324 probability of failure can be integrated over an interval of time, $[0; t]$, resulting
 325 in the probability of failure over that time period, $P_f(0, t)$. The random
 326 variables, $X(t)$, are time dependent and, thus, so is $P_f(t)$. The time t at
 327 which the limit state function, $G[X(t)]$, becomes zero is denoted time-to-
 328 failure and equation (5) correspond to a first-passage-probability, assessed
 329 with the out-crossing theory [32]. Time-integrated approaches for solving
 330 equation (5) are much simpler, as lifetime maximums distributions for loads
 331 are used as presented in equation (6)

$$P_f(0, t) = P\left(R(t) \leq S_{max}(t)\right) \quad (6)$$

332 where $R(t)$ is resistance and $S_{max}(t)$ is the maximum load effect for the time
 333 period $[0; t]$. However, as resistance is also time dependent, decreasing with
 334 deterioration, it is extremely unlikely that the maximum load effect coincides
 335 with the time of minimum resistance. By dividing structure lifetime into n
 336 limited time periods, for which resistance can be considered as time invariant,
 337 it is possible to approach the first-passage problem by equation (7):

$$P_f(0, t) = 1 - P\left(R_1 \geq S_{max,1} \cap R_2 \geq S_{max,2} \cap \dots \cap R_n \geq S_{max,n}\right) \quad (7)$$

338 where R_i respect to resistance at time interval $[t_{i-1}; t_i]$, considered as con-
 339 stant, and $S_{max,i}$ is the maximum load effects within the same period. Despite
 340 the independence of $S_{max,i}$ between time periods, the subset of events pre-
 341 sented in equation (7) still show some dependency as a result of the correla-
 342 tion between remaining involved variables. Establishing an analogy between
 343 different time periods and structural members of serial system, the probabilit-
 344 ity of failure in (7) can finally be approached by the narrow reliability bounds
 345 proposed by [33].

346 Thus, if relative short time periods are considered, attending to the cor-
 347 rosion rate, the probability of failure, given a certain level of corrosion, can
 348 be considered approximately as time-independent. The corresponding relia-
 349 bility index, β , is therefore used herein as the time-independent performance
 350 indicator and equation (1) results in:

$$R = \int_0^1 \frac{\beta(X_P = x)}{\beta(X_P = 0)} dx \quad (8)$$

351 Under severe deterioration, negative reliability indices might occur, mean-
 352 ing the structure is very likely to fail. Such high risk will significantly decrease
 353 the robustness index, indicating the high potential consequences of deteriora-
 354 tion. In order to compute the reliability index, the response surface method,
 355 RSM, is used to obtain an explicit approach for the structural response to
 356 allow the First Order Reliability Method, FORM, to be used [34, 35]. To
 357 depict the proposed methodology, the simply supported beam analyzed in
 358 the previous section is being used and considered to support a 0.075m depth
 359 and 1.25m wide concrete deck for pedestrians.

360 The number of random variables considered in this study needed to be
 361 restricted to the most fundamental, due to demanding reliability analysis,
 362 sophisticated numerical models, and limited computational resources. Table
 363 .1 shows the distributions and parameters of the six random variables con-
 364 sidered as uncorrelated. The statistical properties of concrete [36, 37] and
 365 reinforcement bars have been considered. Live load is the result of people
 366 concentration and modeled through a exponential distribution with a 98%
 367 quartile of $7.0kN/m^2$ for the maximums distribution in a reference period
 368 of 50 years. This results in an exponential rate parameter, λ , of 1.1 and a
 369 mean value of $0.90kN/m^2$ for an annual occurrence rate. Thus the proba-

370 bility of failure to be computed will respect to the period of 1 year, and for
 371 the usual corrosion rates, the resistance of the deteriorating structure can be
 372 considered as constant. The width of the deck (1.25m) was considered on
 373 the surface loads.

374 [Table 1 about here.]

375 The limit state function, G , is defined as the resistance, R , minus the
 376 acting load, S , due to self weight and live load. The resistance is considered
 377 as the maximum uniform load that could be applied to structure until its
 378 failure in bending either defined by the steel bars yielding or the concrete
 379 crushing. The load effect, S , can be obtained through equation (9)

$$S = \theta_E \times [A_c^{beam} g + W (d_c^{slab} g + q)] \quad (9)$$

380 where W is the deck width equal to 1.25m. A_c^{beam} and d_c^{slab} are the beam cross
 381 section and the slab depth, respectively. The resistance can be computed
 382 through equation 10:

$$R = \theta_R \times R(f_c, f_y, X_P) \quad (10)$$

383 where $R(f_c, f_y, X_P)$ is the resistance obtained through the corrosion analysis
 384 methodology described previously, and explicitly approached by a response
 385 surface defined for each design point, d_P .

386 5. Discussion

387 5.1. Reliability analysis

388 Figure .7 shows the reliability index, $\beta(X_P)$, and the respective failure
 389 probability, $P_f(X_P)$, evolution with the corrosion level, X_P . The reliability

390 of the intact structure is 3.5 decreasing significantly as corrosion increases,
391 specially in the first 15% of reinforcement area lost. For corrosion levels
392 ranging from 15% to 40%, safety reduction is much less significant, and from
393 this stage onwards almost negligible. The residual reliability is 0.41 attained
394 for 60% of area lost.

395 Figure .7 also shows two additional fragility curves: $\beta(X_P)^*$, where the
396 debonding effect has been neglected; and $\beta(X_P)^{**}$, where only reinforce-
397 ment area reduction has been considered. The comparison between $\beta(X_P)$,
398 $\beta(X_P)^*$ and $\beta(X_P)^{**}$ shows that safety reduction due to reinforcement area
399 reduction is almost linear until corrosion reach about 80%. From this stage
400 onwards, the effective reinforcement area is below the minimum required to
401 avoid structural failure immediately after flexural cracks initiation. Cracking
402 effect is more significant for corrosion above 80%, as from this stage onwards
403 flexural strength is provided by the plain concrete section, which in this case
404 is deteriorated as shown in Figure .1 (d).

405 [Figure 7 about here.]

406 5.2. Robustness assessment

407 Figure .8 shows the normalized performance obtained through the ratio
408 between the reliability of the corroded structure and the intact one, as a
409 function of the normalized damage, in this case considered as the corrosion
410 level on bottom reinforcement. The maximum damage is limited to 50%,
411 as for existing structures such level of deterioration would trigger a repair
412 action and considering more advanced corrosion levels is clearly unrealistic.

413 [Figure 8 about here.]

414 Robustness computed according to (8) results in $R = 28\%$, showing that
415 tolerance to generalized corrosion is relatively low and safety reduction should
416 always be a concern. The mean normalized performance reduction is there-
417 fore 72%. This is a result of the lack of redundancy of a simply supported
418 beam, but also of the absence of a second layer of bottom reinforcement less
419 affected by corrosion.

420 Computing robustness of the remaining cases presented in Figure .7, re-
421 sults in $R^* = 75\%$ and $R^{**} = 82\%$ if debonding and debonding including
422 cracking have been neglected, respectively. Establishing the difference be-
423 tween the computed robustness indicators (Δf), provides the relative impor-
424 tance of each deteriorating mechanism for the lack in robustness. It results
425 that debonding effect is the main cause of structural deterioration producing
426 a mean safety reduction $\Delta f^1 = 47\%$, followed by reinforcement area lost and
427 then cracking, causing a mean performance reduction of $\Delta f^3 = 18\%$ and
428 $\Delta f^2 = 7\%$, respectively (see Figure .8).

429 5.3. Decision making based on robustness

430 Figure .9 shows the beam time-dependent probability of failure $P_f(0, t)$,
431 referred to the time period $[0, t]$, and considering a corrosion progression of
432 1% annually. The initiation period has been neglected and the time $t = 0$
433 respects to the onset of corrosion. The lower and the upper bounds of the
434 probability of failure resulted very narrow and overlapped in Figure .9 as weak
435 dependency was found between different time periods. Figure .9 also shows
436 the time-dependent probability of failure for a similar beam but considered
437 fully protected against corrosion, $P_f(0, t)^{***}$. The comparison between the
438 unprotected and protected beam shows the impact of corrosion on the time-

439 dependent safety indicating the former to be a case of concern, requiring
440 premature intervention. The probability of failure is approximately the same
441 within the periods of 5 and 50 years, for the unprotected and protected
442 beam, respectively. Figure .9 also shows the time-dependent probability of
443 failure when neglecting debonding effect, $P_f(0, t)^*$, and considering only the
444 effect of reinforcement area reduction $P_f(0, t)^{**}$. As mentioned, debonding
445 is the major cause for impaired robustness thus with major impact on the
446 time-dependent probability of failure.

447 [Figure 9 about here.]

448 Similarly, a longer time between periodic inspections could be adopted
449 depending on robustness. Figure .10 shows the time-dependent probability of
450 failure, for the same cases of Figure .9, given the observed corrosion level at
451 the inspection time and within the period of 3 years $P_f(3y|X_P)$. For exempli-
452 fication proposes, the mean time between periodic inspections was considered
453 herein equal to 3 years. As observed, the probability of failure within the
454 time between inspections is constant for the beam protected against corro-
455 sion due to full robustness. For the unprotected beam, the probability of
456 failure increases with the corrosion degree. Therefore a reduction of the time
457 between inspections is required over the beam lifetime.

458 [Figure 10 about here.]

459 6. Conclusions

460 In this work, a probabilistic framework for the evaluation of structural
461 robustness of structures, subject to continuous damage, is presented. In this

462 framework, damage is defined in terms of an unpredictable continuous vari-
463 able, making this robustness index particularly suitable for structural man-
464 agement systems allowing the analysis and comparison of different structural
465 types with the final objective of defining those requiring more and prior
466 maintenance.

467 The proposed robustness index can be used to estimate the need to repair
468 a structure when damage is identified, since it provides an estimate of current
469 and future structural safety. However, the inclusion of this index in existing
470 management systems will require a calibration process, including a large pool
471 of typical structures, where the condition index and the robustness index are
472 related with the remaining time before a safety threshold is reached.

473 The results obtained showed the ability of the proposed index to charac-
474 terize the robustness of a structure, from a structural viewpoint, in a single
475 indicator, independently of the structural safety of the undamaged struc-
476 ture. Robustness of the presented example resulted in 28% which shows
477 the structure tolerance to generalized reinforcement corrosion. The mean
478 performance lost is 72% of which 47%, 18% and 7% are caused by bond
479 deterioration, reinforcement area reduction and concrete cover cracking, re-
480 spectively. The comparison with a similar beam but fully robust (due to a
481 corrosion protection), shows that the unprotected beam, thus less robust, re-
482 quires sooner maintenance and shorter periods between periodic inspections.
483 For the sake of simplicity in introducing the concept of robustness index, the
484 example presented in this paper corresponds to a single simply-supported
485 beam. However, it is known that in statically indeterminate structures dam-
486 age effects include a redistribution of internal forces. How this redistribution

487 affects the final value of robustness because of redistribution of stresses and
488 activation of alternative loading paths, is the subject of future research.

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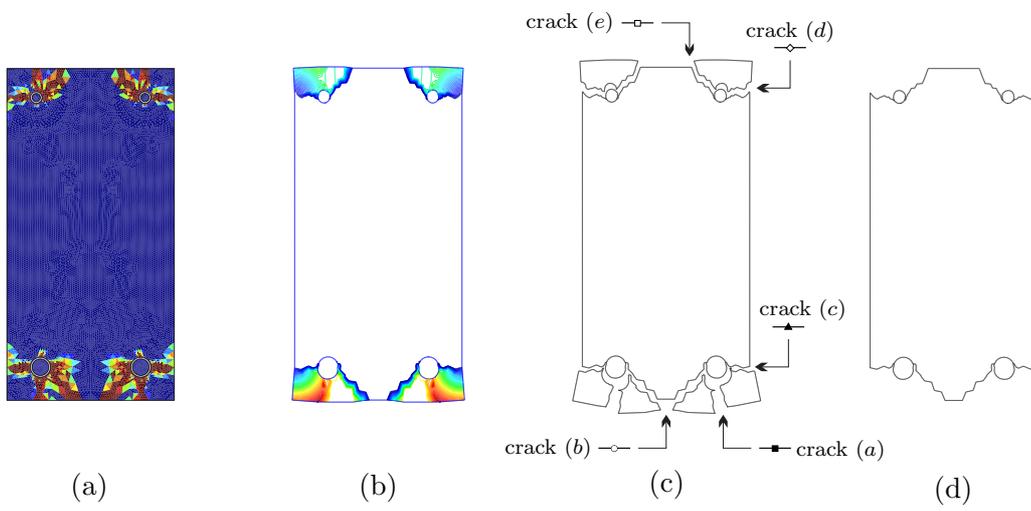


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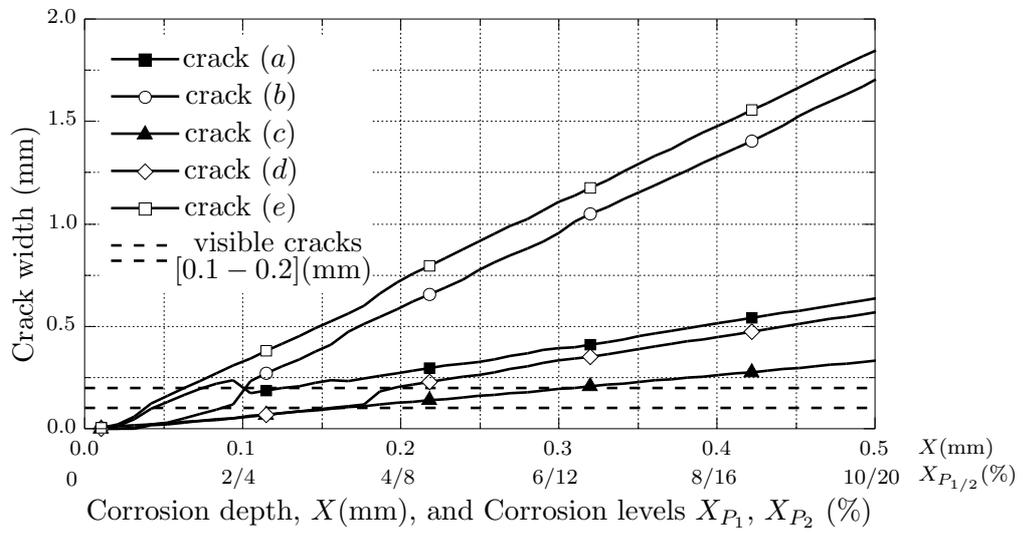


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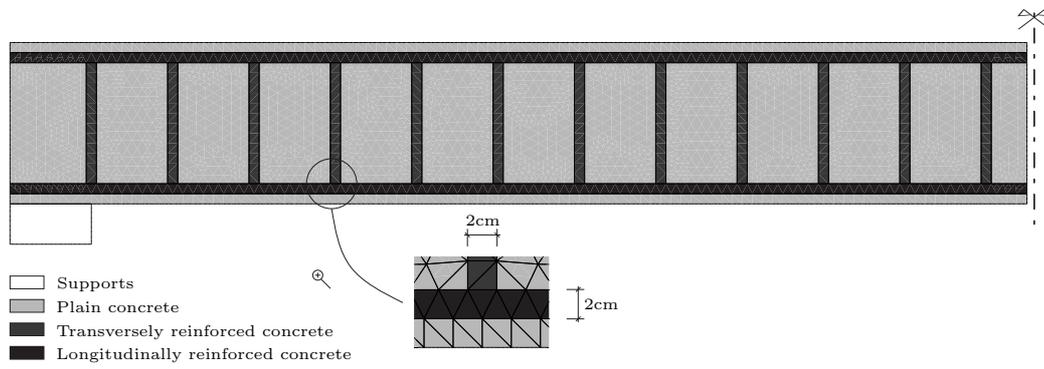


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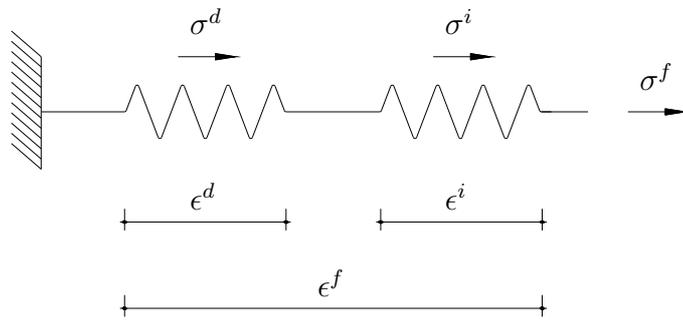


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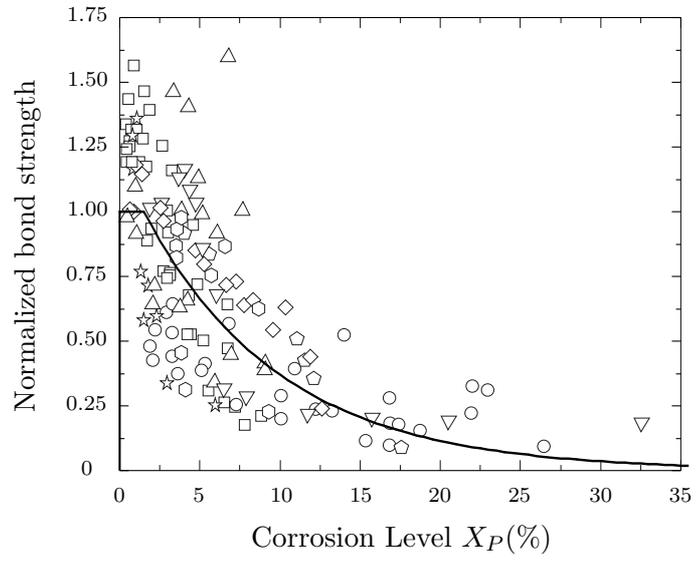


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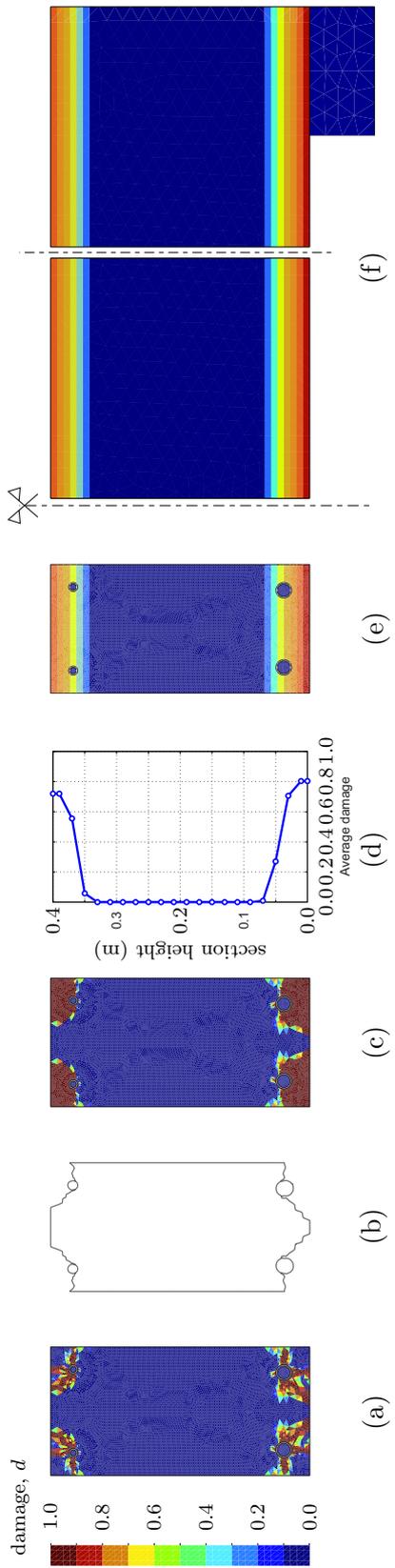


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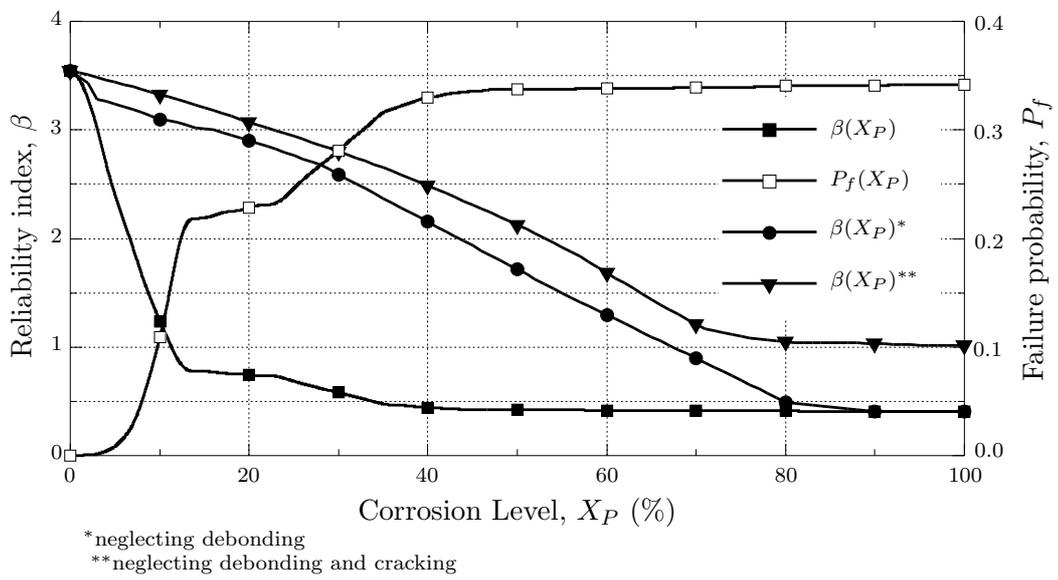


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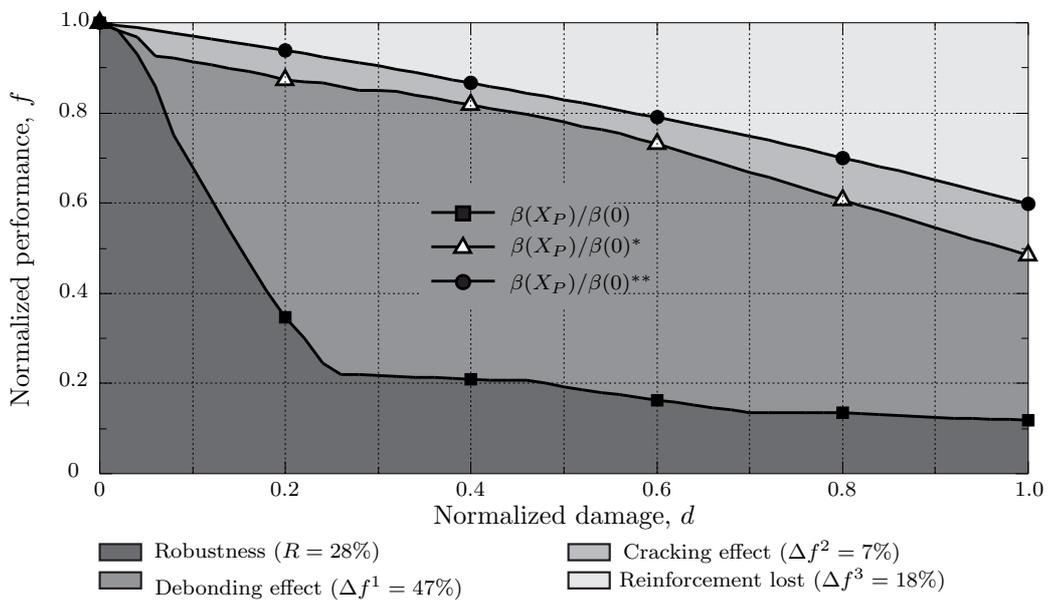


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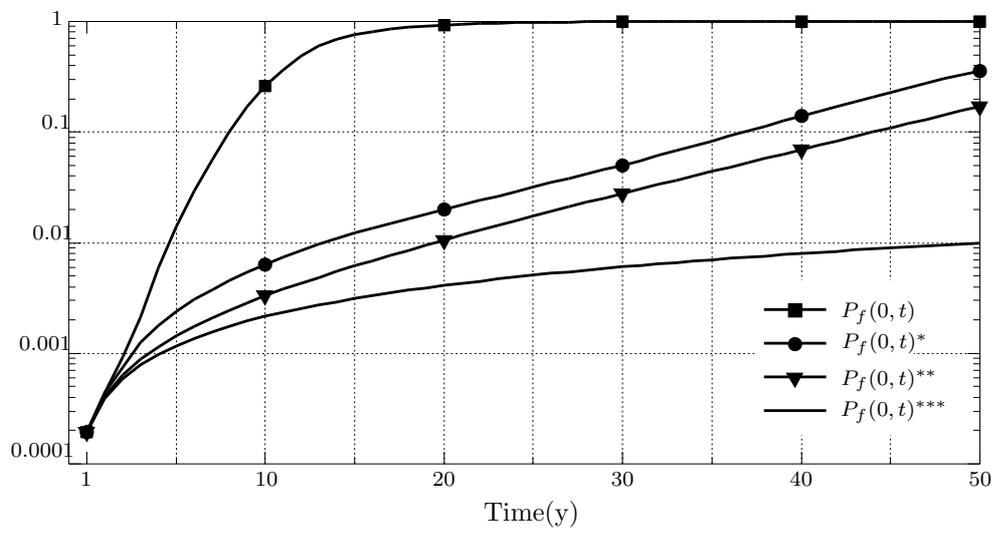


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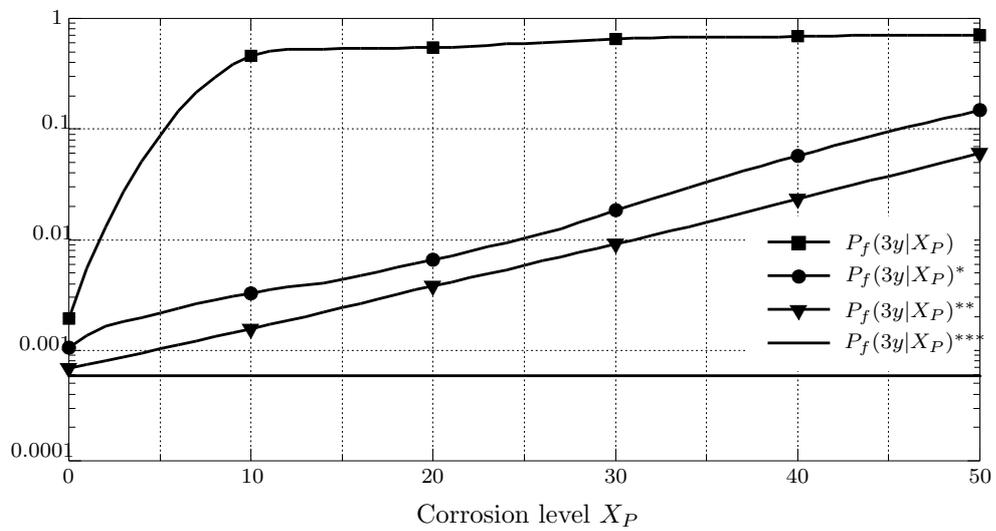


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Table .1: Random variables distributions and parameters

Random Variable	Dist.	Mean	Std. dev.
Concrete strength, f_c	logn	$38.5MPa$	$5.8MPa$
Steel yield stress, σ_y^i	norm	$460MPa$	$30MPa$
Concrete self-weight, g	norm	$25kN/m^3$	$0.75kN/m^3$
Live loads, q	exp	$0.90kN/m^2$	$0.90kN/m^2$
Resistance model uncertainty, θ_R	logn	1.1	0.15
Load model uncertainty, θ_E	logn	1.0	0.10