

# Experimental Realisation of a Thermal Squeezed State of Levitated Optomechanics

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We experimentally squeeze the thermal motional state of an optically levitated nanosphere, by fast switching between two trapping frequencies. The measured phase space distribution of the center-of-mass of our particle shows the typical shape of a squeezed thermal state, from which we infer up to 2.7 dB of squeezing along one motional direction. In these experiments the average thermal occupancy is high and even after squeezing the motional state remains in the remit of classical statistical mechanics. Nevertheless, we argue that the manipulation scheme described here could be used to achieve squeezing in the quantum regime, if preceded by cooling of the levitated mechanical oscillator. Additionally, a higher degree of squeezing could in principle be achieved by repeating the frequency-switching protocol multiple times.

While squeezing a quantum state of light [1] has a long history of experiments, the squeezing of a massive mechanical harmonic oscillator has so far not seen many experimental realisations. The first demonstration of squeezing in a classical mechanical oscillator was by Rugar et.al [2]. Squeezing of classical motional states in electromechanical devices by parametric amplification and weak measurements has been subsequently proposed [3], and experimentally demonstrated in an optomechanical system [4]. Schemes relying on sinusoidal modulation of the spring constant have also been proposed and discussed by numerous authors [5–8]. In optomechanical cavities Genoni *et al.* suggested that squeezing below the ground-state fluctuations (quantum squeezing for brevity) may be attainable via continuous measurements and feedback [9]. Quantum squeezing of a high-frequency mechanical oscillator has only been experimentally demonstrated very recently, in a microwave optomechanical device [10, 11]. Also only very recently a hybrid photonic-phononic waveguide device has shown the correlation properties of optomechanical two-mode squeezing [12]. Another interesting method of generating squeezing, of relevance to this Letter, relies on non-adiabatic shifts of the mechanical frequency. Such method was initially discussed in relation to light fields [13, 14]. Similar ideas, utilising impulse kicks on a mechanical oscillator, have been discussed [15, 16]. In this Letter we report the first experimental demonstration of mechanical squeezing via non-adiabatic frequency shifts, thus realising a useful tool to manipulate the state of a levitated optomechanical system.

*Theory*– In what follows we shall present a quantum mechanical treatment of our squeezing protocol, in anticipation of future experiments that may achieve quantum squeezing. Due to linearity of the Heisenberg equations of our system, it should be pointed out that formally identical results may be obtained through classical statistical mechanics [17]. We consider a nanosphere of mass

$m$  trapped in a harmonic potential. Along the  $z$  axis, we can manipulate the system by switching between two Hamiltonians  $\hat{H}_1, \hat{H}_2$ , where  $\hat{H}_j = \frac{\hat{p}_z^2}{2m} + \frac{1}{2}m\omega_j^2\hat{z}^2$ ,  $\hat{z}, \hat{p}$  denote the  $z$ -components of the canonical position and momentum operators, and the trapping frequency may assume two distinct values:  $\omega_1$  or  $\omega_2$  (In our experiment, we adopt  $\omega_2 < \omega_1$ ). As we shall see shortly, our squeezing protocol relies on the rapid (i.e. *non-adiabatic*) switching between the two Hamiltonians [13, 14]. It is instructive to write down the annihilation operators, say  $\hat{a}$  and  $\hat{b}$ , corresponding to the two trap frequencies ( $\hbar = 1$ ):

$$\hat{a} = \sqrt{\frac{m\omega_1}{2}} \left( \hat{z} + \frac{i\hat{p}}{m\omega_1} \right), \quad \hat{b} = \sqrt{\frac{m\omega_2}{2}} \left( \hat{z} + \frac{i\hat{p}}{m\omega_2} \right). \quad (1)$$

Through simple algebra one may notice that  $\hat{a}$  and  $\hat{b}$  are related by a squeezing transformation of the form  $\hat{b} = \cosh(r)\hat{a} - \sinh(r)\hat{a}^\dagger$ , with  $r \equiv \frac{1}{2} \log\left(\frac{\omega_2}{\omega_1}\right)$  the squeezing parameter. We may exploit the mathematical relationship between modes  $\hat{a}$  and  $\hat{b}$  to generate mechanical squeezing, as follows. Let the particle be initially prepared in an arbitrary state (in our experiment, this will be a thermal state of  $\hat{H}_1$ ). At time  $t = 0$  we suddenly change the trapping frequency from  $\omega_1$  to  $\omega_2$ , such that the Hamiltonian becomes  $\hat{H}_2$ . We then let the system evolve until a time  $t = \tau$  (the *squeezing pulse duration*), before rapidly switching back to Hamiltonian  $\hat{H}_1$ . In the Heisenberg picture, this amounts to a simple harmonic evolution  $\hat{b} \rightarrow \hat{b}e^{-i\omega_2\tau}$  for the operator  $\hat{b}$ . In terms of the quadratures  $\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ ,  $\hat{P} = -i(\hat{a} - \hat{a}^\dagger)/\sqrt{2}$ , however, the transformation is nontrivial:  $(\hat{X}, \hat{P})^\top \rightarrow M(\hat{X}, \hat{P})^\top$ , where the matrix

$$M = \begin{pmatrix} \cos(\omega_2\tau) & e^{2r} \sin(\omega_2\tau) \\ -e^{-2r} \sin(\omega_2\tau) & \cos(\omega_2\tau) \end{pmatrix} \quad (2)$$

embodies a combination of rotation and squeezing in the phase space of mode  $\hat{a}$ . Note that, in general, the squeezed quadrature will be a linear combination of  $\hat{X}$

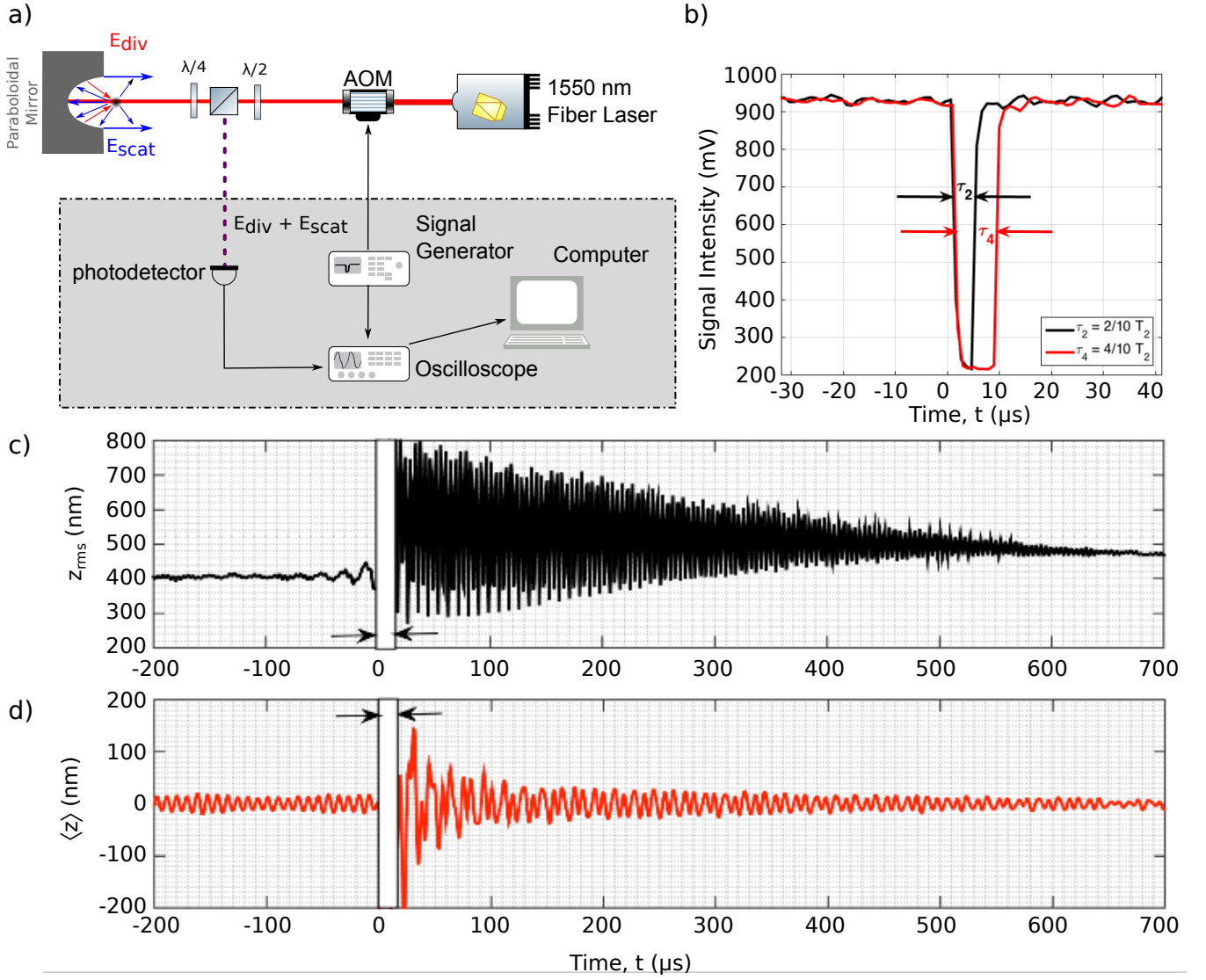


FIG. 1. **Experimental implementation of squeezing levitated optomechanics.** **a)** Schematic of the squeezing setup. The paraboloidal trapping system demonstrates the  $E_{div}$ , divergence field and the  $E_{scat}$ , Rayleigh scattered field from the trapped particle. The grey region is for homodyne detection, as well as, pulse application. **b)** Negative square pulse for squeezing generation, as seen by the photodetector. Two different pulse durations are shown,  $\tau_2$  and  $\tau_4$ . **c)** Root mean square position as a function of time,  $z_{rms}(t) = \sqrt{\langle (z - \langle z \rangle)^2 \rangle}$ , obtained from 1500 pulse sequences applied to the same particle. Oscillations for  $t < 0$  are due to band pass filtering. **d)** Time-dependence of the mean position  $\langle z \rangle$  (center of the thermal distribution). This quantity also shows oscillations at  $\omega_1$  after the squeezing pulse.

and  $\hat{P}$ . The associated squeezing parameter  $\lambda(\tau)$  is encoded in the singular values of  $M$ , and can be found as follows. Since  $\det(MM^\top) = 1$ , we can parametrize the eigenvalues of  $MM^\top$  as  $(\mu, 1/\mu)$  for some parameter  $\mu > 0$ . Note that  $\sqrt{\mu}$  quantifies the deformation of the *standard deviations* of the rotated quadratures. The mechanical squeezing parameter thus reads (in dB units)

$$\lambda(\tau) = 10 |\log_{10}(\sqrt{\mu})|. \quad (3)$$

The analytical expression for  $\lambda(\tau)$  is unwieldy if  $\tau$  is left generic. It is however readily verified that maximum

squeezing can be obtained by setting  $\omega_2\tau = \frac{\pi}{2}$ , in which case  $\lambda_{\max} = 10 \log_{10}(\omega_1/\omega_2)$ .

*Experiments*— We trap a silica nanosphere of radius 32 nm ( $\pm 5$  nm) and mass of  $3.1 \times 10^{-19}$  kg ( $\pm 1.4 \times 10^{-19}$  kg) in an optical dipole trap. The size of the particle is evaluated from fitting a Lorentzian to the power spectral density of the signal, as described in [18] and shown in Fig. 3b, from this the mass is obtained as well. We use a 1550 nm laser, directed into a parabolic mirror which focusses the light to a diffraction limited spot, where the particle is trapped. Experiments are performed in a vac-

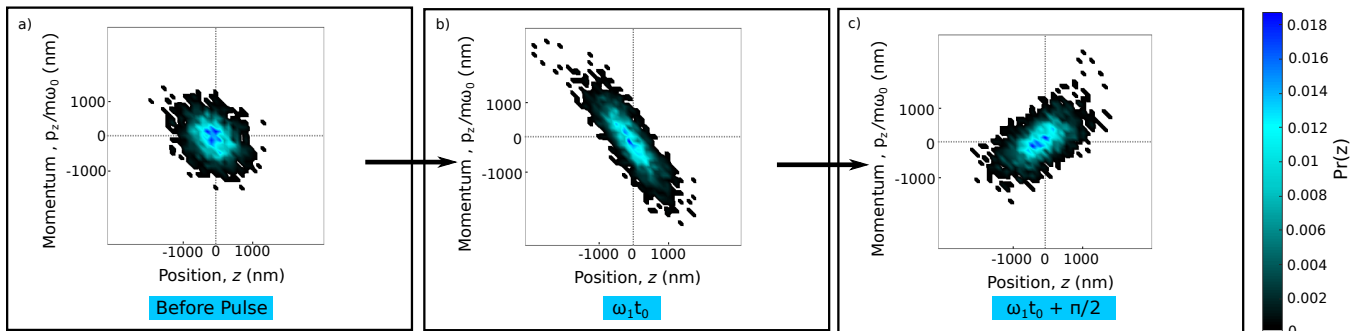


FIG. 2. Experimentally measured phase-space distributions of the mechanical state, before and after the squeezing pulse. The average displacement of the state has been subtracted (see Fig.1d). **(a-c)** Density plots of the phase space distributions for  $z$ -motion, at three different times, for a pulse duration,  $\tau = 3/10T_2$ . **a)** State of the particle motion before the pulse is applied. The former is well approximated by a Gaussian distribution, as typical for a thermal state. **b)** Phase space distribution shortly after the pulse has been applied (time  $t_0$ ). Note how it presents clear signatures of squeezing. **c)** Phase space distribution at time  $t_0 + \frac{1}{4}T_1$ . The squeezed state rotates in phase space while squeezing degrades with time, mainly due to background gas collisions that tend to restore the initial thermal distribution.

uum chamber at a pressure of  $1 \times 10^{-1}$  mbar. In this regime, the damping of the particle motion by random collisions with background gas is linear in the pressure  $p_{gas}$  and the related damping coefficient can be approximated by

$$\Gamma \approx 15.8 \frac{r^2 p_{gas}}{m v_{gas}}, \quad (4)$$

with  $m$  and  $r$  are the radius and the mass of the nanosphere, respectively, and  $v_{gas} = \sqrt{3k_B T / m_{gas}}$  is the mean thermal velocity of the background gas of mass  $m_{gas}$  [19]. We evaluate  $\Gamma = 2\pi \times 227$  Hz ( $\pm 2\pi \times 9$  Hz) for this experiment while the main uncertainty in mass comes from the pressure measurement ( $\sim 15\%$ ).

As shown in Fig.1a) the position of the single nanosphere is measured using an optical homodyne method. More details about the particle trapping and detection can be found elsewhere [18].

A short squeezing pulse of duration  $\tau$  is applied by switching between two different trapping laser powers  $P_1$  and  $P_2$  (see Fig.1b) using a free space acousto-optical modulator (AOM). The trapping frequency is given by  $\omega = \sqrt{k_0/m}$ , where  $k_0 = 8\alpha P / (c\pi\epsilon_0 w_f^4)$  for motion in  $z$ -direction, with  $\alpha$  the polarizability of the particle,  $c$  the speed of light,  $\epsilon_0$  the electric field constant and  $w_f$  the waist of the laser beam at the focal point. The laser power can be modulated by changing the voltage applied to the AOM; we switch between trap frequencies  $\omega_1 = 2\pi \times 112$  kHz and  $\omega_2 = 2\pi \times 49.3$  kHz. The timescale of the switch is determined by the AOM bandwidth, which is more than 1 MHz and therefore much larger than both trap frequencies. Hence, we model the switch as instantaneous. Here, evidently,  $\omega_2 < \omega_1$ . The condition  $\omega_2 > \omega_1$  may also be used, and it would result in squeezing of a different quadrature. Experimentally, we found it more practical to employ the tighter trapping potential (corre-

sponding to  $\omega_1$ ) most of the time, so as to minimise the probability of losing the particle.

The same signal generator which is used to generate the squeezing pulse triggers an oscilloscope to record a time trace of duration of one second. The same single pulse sequence is repeated 1500 times for the same trapped particle, while allowing for one second between the pulses to restore the initial thermal state. The recorded time traces initially include signals from  $x, y$  and  $z$  motional degrees of freedom. However, the pulse scheme is only optimised for a single motional frequency, namely, the one in  $z$ -direction which is perpendicular to the mirror surface. This is primarily because the  $z$ -motion is predominant in our detection signal. We filter the signal around the  $\omega_z$  frequency peak to extract the impact of the pulses on the  $z$ -motion alone. The root mean square (rms) of the position of the particle  $z_{rms}$  is used to analyse the state of the motion, see Fig.1c). The entire experiment takes over ten hours, during which drifts in laser power (hence in trap frequency) may occur. Thus  $\omega_2$ , while known in principle, is taken as a free parameter in the fitting model.

*Results*– The Fourier transform of the oscillation in the mean position  $\langle z \rangle$  - i.e. the motion of the centre of the thermal distribution - shows that oscillations are dominated by frequency  $\omega_1$ , before and after the pulse, see Fig.1a and [17]. We can thus infer that our pulse imparts a small phase-space displacement to the particle. To correct for this we subtract the average displacement from the data, while we try to account for the remaining effects and experimental imperfections through an effective dephasing model (see supplement [17]).

Initially  $z_{rms}$  is constant as the phase of the oscillation is random between the 1500 individual pulse experiments. After the squeezing pulse the motion shows damped phase-coherent oscillations. The rms oscillation

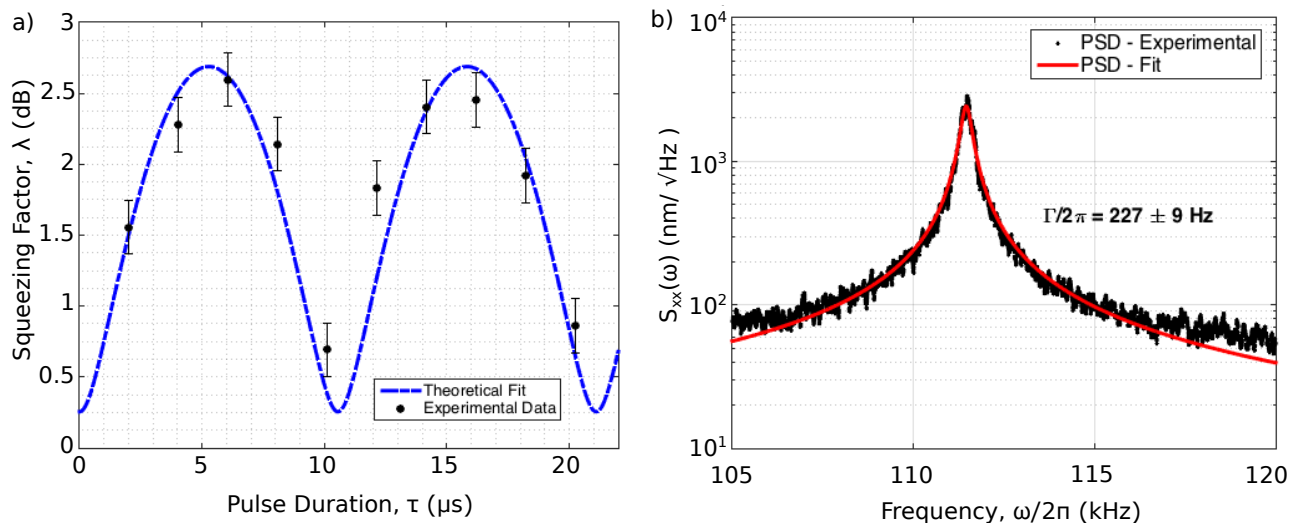


FIG. 3. **Quantitative analysis of the squeezing effect.** **a)** Squeezing factor  $\lambda$  as a function of pulse duration  $\tau$  (measured in seconds). This is extracted by comparing the minor axes of the phase-space ellipses (see e.g. Fig 2) before and after the pulse. The theoretical fit to the data (blue line) has been done according to Eq.(10) in the supplement [17]. **b)** Lorentzian fit to the power spectral density of the  $z$ -motion. This is used to extract the radius and mass of the particle, as well as, the collisional damping rate  $\Gamma$  according to equation 9 in [17].

decays within about  $680 \mu$ s to  $690 \mu$ s, which gives a rate of thermalization to the temperature of the background gas molecules between  $2\pi \times 230$  Hz and  $2\pi \times 234$  Hz. This is in good agreement with the value for  $\Gamma$  estimated via the Lorentzian fit.

We are operating in the classical regime, in that we observe quadrature variances that are several orders of magnitude larger than those in the quantum ground state. Therefore, we may estimate the particle’s momentum by simply taking the time differential of the position measurement. In passing we note that, in the quantum regime, our continuous measurement process would require a more rigorous treatment [20]. Applying the described strategy to our data set we generate the phase space distribution of the trapped particle motion. Fig.2a) shows the distribution of the system before the pulse is applied. Such initial distribution is nearly Gaussian, and its small asymmetry can be attributed to the non-linear response of the position measurement at large oscillation amplitudes [21].

Immediately after the pulse we observe the typical features associated with squeezing. It is evident that the applied pulse deforms the phase-space distribution of the particle, which then displays the typical oblong shape of a squeezed state: see Fig.2b). Following the pulse, the distribution rotates in phase space according to the harmonic oscillator evolution —see for instance Fig.2c). During such evolution the distribution progressively relaxes back towards a thermal one; we attribute this to thermalization via collisions with the background gas.

The measured degree of squeezing as a function of pulse duration is shown in Fig.3a). While the theoret-

ical prediction for the squeezing parameter  $\lambda(\tau)$  agrees qualitatively with the experimental results, the largest squeezing factor we achieve experimentally is 2.7 dB, lower than the expected  $\lambda_{\max} \simeq 3.56$  dB. We can obtain a reasonable fit of the data by assuming that the squeezing pulse is affected by some phase noise whose strength is  $\tau$ -independent [17], and which we assume is associated with the abrupt voltage changes. In a nutshell, this amounts to rescaling  $\langle \hat{b}^2 \rangle \rightarrow \eta \langle \hat{b}^2 \rangle$  at the end of the squeezing operation, where  $0 \leq \eta \leq 1$  quantifies the residual “phase coherence” [17]. For the best fit, as shown in Fig.3a) we obtain  $\omega_2 = 2\pi \times 47.9$  kHz ( $\pm 1.55$  kHz), and  $\eta = 0.73$  ( $\pm 0.10$ ). We have assumed that other sources of noise (e.g. thermalization with the background gas) can be neglected during the pulse, i.e., that the motion of the particle at short timescales during the squeezing operation is affected predominantly by such phase noise.

*Conclusion*– The demonstrated squeezing technique could be used for enhanced sensing and metrology based on levitated optomechanics such as for force sensing applications [22] and non-equilibrium dynamics studies [21]. Truly quantum squeezing may be approached by pre-cooling the motional state [23–25]. Centre of mass motion temperatures of trapped nanoparticles of below 1mK have been experimentally demonstrated [18, 19, 26] via parametric feedback, while alternative methods include quantum measurement techniques [20, 27] which have been successfully applied to membrane and cantilever optomechanical devices [28, 29]. Future work will include the investigation of multiple pulses to increase the achievable levels of noise reduction [30], and of methods to probe the non-classicality of mechanical oscillators.

Finally, we would like to comment our measurement scheme, which relies on a continuous monitoring of the particle's position. At first, this might appear to be undesirable in the future perspective of approaching the quantum regime, due to the well-known disturbance induced by the quantum measurement process. Yet it was recently shown that, if correctly accounted for, continuous monitoring may in fact improve the achievable mechanical squeezing [20].

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