A Note on University Admission Tests: Simple Theory and Empirical Analysis*

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Abstract

University admission mechanisms are often quite complex. This paper examines one effect of their design on the students' incentives to exert effort in preparation for the test. We adapt a multi-unit all-pay model of auction to draw the conclusion that abler students work harder: this conclusion is in line with the behaviour of a sample of students who apply for admission to the Greek university system with the complex rules newly introduced in 2013.

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1 Introduction

As every lecturer knows, the effectiveness of university teaching depends both on the students' attitude to learning and on the amount of preparatory study undertaken by those enrolled prior to the start of their university career. That these are at best imperfectly correlated has been recognised since at least early in the twentieth century: in 1926 "written, curriculum-based examinations designed to assess student learning in college preparatory subjects" were replaced in the US by the Scholastic Aptitude Test (SAT) designed as "an easily scored, multiple-choice instrument for measuring students' general ability or aptitude for learning." (Atkinson & Geiser 2009).

In order to design an admission system which strikes the desired trade-off between general learning ability and discipline specific pre-admission knowledge, it is essential to understand the response of students to the incentives generated by the allocation mechanism. When the exam is a single test, students will simply equate the expected cost, monetary or in term of disutility of effort, of achieving a certain score in the test, and the expected benefit of being admitted to the institution of choice. In more complex testing environments, for example when multiple tests combine to determine the probability of acceptance at several institutions at once, the specificities of the mechanism come to matter more, and it may be very hard to determine in principle the effects of the details of the tests on students' behaviour, and hence on the outcome of the admission process. We study here a mechanism where multiple tests are combined into a single score, which, unlike most of other such mechanisms, has the distinguishing feature that the students know the outcome of one set of tests before beginning the preparatory study for the next: they can, and hence in general will, condition their effort for the later test on the result of the earlier ones.

While other aspects of the admission mechanism, such as the quality of the matching between students and institutions has been extensively studied,[1] relatively little attention has been paid to understanding the students' response to the incentives created by the details of a complex multidimensional testing mechanism.[2] In this paper we contribute to this understanding. We derive some theoretical prediction from auction theory (Barut et al. 2002) on the effect on students effort of the entrance criteria for the highly competitive admission to Greek universities introduced in 2013. Data limitation prevents us from studying more than a single aspect of the model, and yet our empirical results are in line with the predictions from the theoretical analysis, and thus can be seen as one piece of evidence that support the use of theoretical tools to determine the effects of specific details of complex admission mechanisms.

In Section 2, we borrow from Checchi et al.'s analysis (2021) of the choice of effort by Italian academics, to set the econometric specification which we use in Section 3 to find that the theoretical predictions of the model tally with the behaviour displayed by the applicants to Greek universities in 2013.

2 A stylised model of university admission

In 2013, students' admission to public universities in Greece was determined by a ranking built from two separate measures: x_s, the grade point average (GPA) of the last three years of high school (Lyceum), with weights $0.2, 0.35, 0.45$,[3] and x_π, the mark obtained in the nationwide university entrance examinations (the

[1] Spearheaded by Gale & Shapley's seminal contribution (1962), which has anyway been shown to be conceptually different from the university admission (Roth 1985), much of the analysis has taken a mostly theoretical turn, concentrating on the idiosyncrasies of the matching mechanism (for example on the trade-offs between centralised and decentralised mechanisms, Che & Koh 2016) in a rich variety of environments. Some empirical work on the extent of mismatch highlights both the risk that low ability students may find themselves out of their depth and perhaps unable to complete their degree if they are enrolled in a too selective institution, and on the converse downside of the potential of high ability students to remain unrealised if they are enrolled in a less demanding university unable to challenge them (Smith et al. 2013, Bo et al. 2019).

[2] Silva (2020) is a detailed description of how a highly complex mechanism which combines several exams into a nationwide matching mechanism can be made to work in practice.

[3] These weights are different from those used in previous years.

Panhellenic examination), held annually in May. Once the exam results are made public, students may apply to any of the country's universities for which they are qualified, ordering them by their preference. These preference rankings and the combination of the exam result and the school GPA of all the students who have applied to a given university determine a threshold combined score needed to gain admission to that university. This threshold is set to ensure both that the given number of places allotted to each university by the Ministry of Education are filled, and that no student is admitted to an institution which rejected a student with a better aggregate score and who listed that institution as her first choice.

In this environment, the student has control over three sets of variables, in this temporal order: (i) the effort exerted during the school years, (ii) the effort exerted to prepare for the Panhellenic examination, and (iii) which universities and degrees to apply for and how to rank them. Data limitation prevents us from drawing any inference regarding the first of these, and the absence of a limit on the number of applications implies that the last of these is independent of the first two choices and their outcome, and so it does not need to be considered. Thus we focus on the second of these sets of choices.

When faced with the decision of how hard to study for the Panhellenic examination, a student knows her school GPA, and this knowledge allows her to form a belief on the distribution of her expected aggregate score, and hence of the chance of admission to each university. This is important as it allows her to determine the marginal benefit of the effort exerted to prepare for the Panhellenic exam, and hence its optimal level; this trade-off depends on her school GPA. Formally, her decision process can be modelled as follows: if she has a high school score x_s, she will be admitted to her first choice university if $x_s + x_\pi \geq T$, where x_π is her score in the Panhellenic examination, and T is the admission threshold for her first choice university, which is endogenously determined in equilibrium and depends on the quality of all other candidates and the effort they choose. We assume her grade in the Panhellenic exam to be a random variable, given by the sum of her ability a, a continuous variable taking values in an interval of the real line, her effort e_π, and a random error ε distributed according to a differentiable (for simplicity) function $F(\varepsilon) \in [\underline{\varepsilon}, \bar{\varepsilon}]$, with $f(\cdot) = F'(\cdot)$. Formally, a student chooses her effort to maximise a payoff given by her probability of gaining admission to her preferred university, minus the disutility generated by exerting effort, $k(e_\pi)$, with $k'(e_\pi) > 0$:

$$P\big(x_s + a + e_\pi + \varepsilon \geq T\big) - k(e_\pi). \tag{1}$$

Each student solves the problem given by the maximisation of (1), that is she chooses her effort taking as given the choices of all other students. This set-up is a variation of the multi-unit all-pay auction (Barut et al. 2002). It is an auction because economic agents compete to win prizes, which are awarded to the highest bidders. In our case, the prize is acceptance by the chosen university, and the bidders are the students who have applied for admission. It is *multi-unit*, because a single bidding process awards many prizes, and it is *all-pay* because every bidder has to pay what they bid, not just the winners like in a standard English or Dutch auction. The "payment" is the disutility cost of the effort applicants exert to prepare for the Panhellenic examinations. The equilibrium is moulded by the uncertainty which besets students' choice-making.

Formally, the effort level chosen by the candidates is the Nash equilibrium of a game where the players are all the students applying to the university and their strategy space is the choice of effort. The formal model of the multi-unit all-pay auction is solved by Barut et al. (2002) (p 679). The present set-up has three differences with Barut et al. (2002): firstly, players are distinguished in their bidding cost, defined as the disutility of the effort needed to make a given bid, rather than their valuations; secondly, their strategy, the effort they exert, determines the bid for the auction with some uncertainty, the realisation of the random error term in the exam, ε, and, thirdly, the additional layer of uncertainty added by the interaction between the simultaneous admission processes of different universities, which implies that the random error of a student who is *not* applying to university X affects in general the chance of admission of the applicants to university X.[4] Checchi et al. (2021) modify Barut et al. (2002) to account for the first two differences in a model conceptually similar to our own.[5] Taking the third difference into account is straightforward, as it simply adds a layer of complexity to the distribution of the realisation of the competitors' errors. Thus we can borrow their Conjecture 1 (Checchi et al. 2021, p 789) and rephrase it in the terminology of the present paper.

Conjecture 1. *(Checchi et al. 2021) An increase in competition increases effort more for applicants with a higher school GPA. It may decrease effort for applicants with a very low school GPA.*

Conjecture 1 follows from the theoretical analysis and simulation of the complex first order conditions for optimal choice of effort and changes in values of the exogenous parameters.[6] The intuitive reason why it holds follows from the relative cost and benefits of effort for students with a good school GPA and those with a less good GPA. Consider a student with a good school GPA. If this student's first choice university has a low threshold, then the student can be admitted even with a fairly low score in the Panhellenic examination, and so working hard for this

[4]To be precise: suppose student A applies to university X only, student B to university X as first choice and to university Y as second choice, and student C to university Y only. In this case, students A and student C are not in competition for places. Nevertheless, student C may benefit from a poor exam performance by student A if this frees a place at university X, which is taken up by student B, thus in turn freeing a place at university Y, which student C may thus be offered.

[5]In Checchi et al. (2021), the players are professors vying for a set number of promotions to the higher rank, which correspond to students vying for admission to a set number of university places in our model, and the effort exerted determines the quality of their publications, which corresponds to the exam results. They have the added uncertainty of the importance of the publications' quality relative to other academic activities, such as teaching, which has no counterpart in our model.

[6]In Checchi et al. (2021), the effect is non linear, with very weak agents, who exert very little effort to begin with, reduce theirs by less: but in the current set up, we assume that these very weak students do not apply to university.

exam is not really necessary. But if the threshold is higher as competition for places increases, as better students will be "around" the threshold, more effort becomes necessary to avoid the risk of doing badly in the examination. However, a student seeking admission to a university with a low threshold who has a less good GPA will need to compensate by working hard in the Panhellenic examination to have any hope of admission to the preferred university. But conversely, when the university is very competitive, effort is less productive, because even by working very hard, success hinges on also having a great deal of luck in the exam, whilst the cost of effort must be incurred anyway, and so is very likely to be wasted. We can label this the *encouragement effect*: strong student are stimulated more than weaker ones by stiffening of competition.

Conjecture 1 creates a link between effort in the Panhellenic examination, e_π, and hence the probability of acceptance, and the the extent of the competition for admission, which we measure as the admission threshold of the university to which they are admitted. We can formalise this link in the following assumption.

Assumption 1. *A student who believes*[7] *to have ability a, and who expects her first choice university to have an equilibrium admission threshold T, exerts effort*

$$e_\pi(T, a) = \alpha - \beta T + \gamma a T, \tag{2}$$

where α, β, and γ are positive parameters.

From Assumption 1, we can therefore write the student's payoff maximising probability of admission as:

$$P\big(\varepsilon \geq T - x_s - a - e_\pi(T, a)\big) = 1 - F\big(T - x_s - a - \alpha + \beta T - \gamma a T\big). \tag{3}$$

[7] The student either knows her own ability, or, more plausibly and equivalently, believes it to be positively correlated with her school GPA. In some models, students have no (or imperfect) information about their own ability (Gary-Bobo & Trannoy (2008) and MacLeod & Urquiola (2015)).

This is the latent probability in a binomial regression model. Naturally, we require (i) $\frac{\partial P(\cdot)}{\partial T} = -f(\cdot)(1+\beta-\gamma a) < 0$, and (ii) $\frac{\partial P(\cdot)}{\partial a} = f(\cdot)(1+\gamma T) > 0$. That is, (i) it is more difficult to be admitted to a university where there is more competition for places and thus a higher threshold, and (ii) an abler student is more likely to gain admission. Both these requirement hold with the possible exception of (i) in the extreme case of a very strong encouragement effect, and then only for students with a very high school GPA score.

The key testable hypothesis of the model, embodied in Assumption 1 is

$$\frac{\partial^2 P(\cdot)}{\partial T \partial a} = \gamma f(\cdot) + f'(\cdot)\left(1+\beta-\gamma a\right)(1+\gamma T) > 0. \tag{4}$$

In words, stiffer competition is "less of a problem" for better students: as discussed above, they find an increase in effort less costly than their less able peers.

As a final point, note that the analysis assumes that a student's application to university i is determined only by her preference, not her school GPA: in other words a high admission threshold does not deter weak students from applying. This would be implausible if the mechanism had an upper bound on the number of applications (as in Portugal: Silva 2020), but here students have no opportunity cost in applying, and so they may as well rank all the universities in their sincere order of preference, even though they have little chance of admission.

3 Empirical analysis

The empirical counterpart to the analysis in Section 2, and in particular to the idea, embedded in Conjecture 1 and in expression (4), is that the responsiveness of a student's level of effort to the competitiveness of the admission process differ according to the student ability. Through the effort level, the link between competitiveness and the probability of admission to the preferred university also varies with ability. Ability of course is not observable, and so it needs to be proxied in

Table 1: Descriptive statistics

Variables	N	Mean	St. Dev.	Min	Max
Admitted to first choice university	1,059	0.592	0.492	0	1
GPA weighted average (x_s)	1,052	0.904	0.056	0.687	1
Admission threshold (T)	1,059	1.610	0.192	1.18	1.908

Note: Summary statistics for the variables used in the regression. The variable 'Admitted to first choice university' takes the value of 1 if admitted and 0 otherwise.

order to perform an empirical analysis. We feel that is natural to proxy a student's ability with their school GPA. Students who are more capable are more likely to finish school with a higher GPA for the last three years than their less capable peers. Thus, we estimate the following nonlinear model of the conditional probability of admission, which includes, in parallel to equation (3), an interaction term between competitiveness, proxied by T, and ability, proxied by x_s:

$$P\left(u=1|x_s,T\right)=\Phi\left(\beta_0+\beta_1 x_s+\beta_2 T+\beta_3 x_s T\right). \tag{5}$$

We estimate (5), both as a logit and as a probit, using survey data collected from a sample of 1059 Greek undergraduates in the academic year 2014-2015. This survey is administered to admitted students, and so it does not include data for students who applied but were not admitted to any university: thus we define u as a dichotomous variable, which takes value 1 if the student is admitted to her first choice university course, and 0 otherwise. As to the remaining variables, x_s is the weighted average of the GPA in three years of high school, and T is the admission threshold of the university to which the student is currently enrolled. The descriptive statistics for these variables are in Table 1. We also control for the location of candidates and their subject, by including in (5) sets of fixed effects for the region where each university is located and for the academic department of admission, and their interaction.

Table 2 reports the results of the estimation of (5) with the data in our sample.

Table 2: Determinants of admission to first choice university

	(1)		(2)	
	Estimates	Marg. Eff.	Estimates	Marg. Eff.
Successful Application (u)	Logit		Probit	
GPA weighted average (x_s)	0.319***	0.056***	0.194***	0.058***
	(0.087)	(0.016)	(0.052)	(0.016)
Admission threshold (T)	-0.505*	-0.102*	-0.344**	-0.114**
	(0.281)	(0.054)	(0.170)	(0.054)
Interaction: $x_s \times T$	0.159*	0.034**	0.093*	0.033**
	(0.084)	(0.016)	(0.051)	(0.016)
Observations	1,052		1,052	
Pseudo R-squared	0.1602		0.1609	

Note: We report in parentheses robust standard errors. ***, ** and * indicate significance at the 1%, 5% and 10% level. x_s and T are normalised to have zero mean and unit standard deviation. To control for the impact of the location of candidates and for the difference in disciplinary fields, we include categorical variables indicating the regional location of each university and the academic department of admission, and their interaction. The estimates of these variables are not reported for brevity and they are available upon request. Columns (3) and (5) report the marginal effects. That for the interaction term was computed following Ai & Norton (2003).

In order to ensure their comparability, before carrying out the regressions we have normalised the GPA weighted average and the admission threshold to have mean 0 and unit standard deviation. Our findings are fully in line with the simple theoretical model in Section 2. A good performance in high school increases the probability admission: an improvement of one standard deviation in the student's GPA score increases, on average, the chance of admission by 6 percentage points, both because it constitute half of the overall score, and because it is likely to correlate with ability and hence the unobserved result in the Panhellenic examination. Competition for places, proxied by the admission threshold of the university to which the student is admitted, makes admission more difficult, as we argued above it should. Specifically, an increase in T of one standard deviation decreases the probability of success of the average student by 11.4 percentage points (slightly less for a logit).

The results for the coefficients x_s and T are natural, and should they not be confirmed, serious doubts would be cast on the reliability of the data. Instead, we can view the sign of the coefficient capturing their interaction as the main empirical test for the validity of our theoretical model. The positive sign that we report in Table 2 for the interaction term $x_s \times T$ is consistent with the conclusion that an increase in competition for places has a weaker negative effect for abler students. Specifically, for a student whose school GPA is one standard deviation above the average, the chance of admission to her preferred university is better than that of the average student by $1 \times 0.033 \times 1 = 0.033$, that is, it is only 8 percentage point lower, instead of 11.4. This is in line with (3) and the positive value of γ we posited following Checchi et al. (2021). In additional regressions, we performed robustness estimations controlling for gender, household income, parents education, the use of private tutoring prior to the exams, attendance to a private high school. None of these had a statistically significant impact. These estimations, and the fixed effects coefficients for the regression in Table 2, are available on request.

4 Conclusion

The model of the all-pay auction developed in Checchi et al. (2021) fits well data from the 2013 round of admission to Greek universities. We show that, as predicted by the theory, there is a complex link between students' effort and the competitiveness of admission to their preferred university. Specifically, our analysis suggests that abler students respond to higher admission thresholds by exerting more effort. As a consequence, the gap in achievement following the high school results is enhanced by the effort exerted in preparation for the Panhellenic examination.

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