This is a post-peer-review, pre-copyedit version of an article published in Instructional Science. The final authenticated version is available online at
http://dx.doi.org/10.1007/s11251-016-9380-2

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Please cite this article as:

Vogel, F., Kollar, I., Ufer, S., Reichersdorfer, E., Reiss, K., & Fischer, F. (2016). Developing argumentation skills in mathematics through computer-supported collaborative learning: the role of transactivity. *Instructional Science, 44*(5), 477-500. doi: 10.1007/s11251-016-9380-2

Developing Argumentation Skills in Mathematics through Computer-Supported Collaborative Learning: The Role of Transactivity

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This research is part of the project ELK-Math, funded by the Deutsche Forschungsgemeinschaft (DFG) under grant RE 1247/9-1 and FI 792/7-1. This contribution contains work that is part of the doctoral dissertations of Freydis Vogel under the supervision of Prof. Dr. Frank Fischer and Prof. Dr. Ingo Kollar.

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Abstract

Collaboration scripts and heuristic worked examples are effective means to scaffold university freshmen's mathematical argumentation skills. Yet, which collaborative learning processes are responsible for these effects has remained unclear. Learners presumably will gain the most out of collaboration if the collaborators refer to each other's contributions in a dialectic way (dialectic transactivity). Learners also may refer to each other's contributions in a dialogic way (dialogic transactivity). Alternatively, learners may not refer to each other's contributions at all, but still construct knowledge (constructive activities). This article investigates the extent to which constructive activities, dialogic transactivity, and dialectic transactivity generated by either the learner or the learning partner can explain the positive effects of collaboration scripts and heuristic worked examples on the learners' disposition to use argumentation skills. We conducted a 2×2 experiment with the factors collaboration script and heuristic worked examples with $N = 101$ math teacher students. Results showed that the learners' engagement in self-generated dialectic transactivity (i.e., responding to the learning partner's contribution in an argumentative way by critiquing and/or integrating their learning partner's contributions) mediated the effects of both scaffolds on their disposition to use argumentation skills, whereas partner-generated dialectic transactivity or any other measured collaborative learning activity did not. To support the disposition to use argumentation skills in mathematics, learning environments should thus be designed in a way to help learners display dialectic transactivity. Future research should investigate how learners might better benefit from the dialectic transactivity generated by their learning partners.

Keywords:

transactivity; collaboration scripts; heuristic worked examples; argumentation; mathematics

Developing Argumentation Skills in Mathematics through Computer-Supported Collaborative Learning: The Role of Transactivity

Argumentation skills are required for individuals to be able to participate in apparently open-ended discussions not only in political and philosophical domains, but also in domains that are usually more driven by formal rules and deductive approaches such as mathematics. An engagement in meaningful activities like mathematical argumentation and proof is increasingly postulated as a goal of mathematical education (National Council of Teachers of Mathematics 2000). Boero (1999) proposed a process model of mathematical proof that starts from finding a conjecture and proceeds to the final formulation of a proof through phases of exploration as well as phases of systematization. This model includes iterative steps of balancing informal explorations and corresponding mathematical arguments that can collaboratively be discussed with others (Reiss, Heinze, Renkl, & Groß 2008). The discussion with others is considered a promising way to strengthen the approach to solving the proof task, since others may bring in new ideas and perspectives to refine original ideas and arguments.

Thus, social-discursive argumentation skills are required at different points either to construct sound arguments (Pease, Smaill, Colton, & Lee 2009) or to be able to engage in an argumentative dialogue, as mathematicians indeed “prove things in a social context and address them to a certain audience” (Thurston 1994, p. 175). For instance, social-discursive argumentation can already be an important part in Boero’s (1999) first step, which includes investigation of the problem space to come up with a reasonable conjecture. Within this step, two learners might investigate different parts of the problem space (e.g., negative and positive numbers) by different methods and come up with different conjectures. Then the learners need to discuss how reasonable the different conjectures are and which conjecture should further be followed to prove it. In this article, we focus on this social-discursive component of argumentation skills, which includes the skills to formulate structurally sound arguments (Toulmin 1958) and to engage in argumentative discourse with a learning partner, consisting of sequences of arguments, counterarguments, and syntheses (Leitão 2000).

Yet, engaging in meaningful argumentation is challenging for students. As prior research has shown, they rarely justify claims and often do not take counter-arguments into account (Sadler 2004). Within the mathematical domain, Heinze, Reiss, and Rudolph (2005) found that high school students failed to produce logical chains of more than one argument in geometry proof tasks. Prospective mathematics teachers (i.e., students in teacher education) are an especially reasonable target group to support in learning argumentation in the mathematical context. These students might not yet be able to engage in meaningful mathematical argumentation, but they will be expected to teach the corresponding skills in their future employment as teachers (Forman, Larreamendy-Joerns, Stein, & Brown 1998; National Council of Teachers of Mathematics 2000).

We might expect students in teacher education to already possess social-discursive argumentation skills to a certain degree, yet it is likely that they have used these skills more in other contexts such as everyday discussions or political debates than in a mathematical or scientific context. Therefore, supporting teacher students in argumentation in the mathematical context might have two consequences. Of course, they might learn new argumentation skills that they did not previously possess. Yet, studies have shown that learning argumentation skills can be a matter of several years of regular exercise (e.g., Kuhn & Crowell 2011). What might happen instead in a short-term intervention as the one that is described in the study at hand is that learners reconfigure components of argumentation skills they already possess (Fischer, Kollar, Stegmann, & Wecker, 2013) and learn how to transfer them to a new context such as mathematics or physics. They thereby change their disposition to use their argumentation skills in a new domain rather than actually acquiring and developing new argumentation skills. It is reasonable, that this not only helpful for argumentation processes in this new context, but also an important step in the long-term development of general argumentation skills. Even though the acquisition of argumentation skills might take years it shouldn't keep us from searching ways and methods to support argumentation, and it is our aim to find out how instructional means should be designed to foster argumentation skills or at least the disposition to use argumentation skills.

From a cognitive perspective, in our study it is important what the individual can learn within the collaborative learning scenario. Therefore, in the following we describe how collaborative learning activities can change the learner's individual dispositions and help to acquire different knowledge and skills.

The Potential of Collaborative Activities for Learning to Argue in Mathematics

To change students' disposition towards argumentation, collaborative learning activities that are in line with deep cognitive processing need to be activated. A number of studies have focused on the role of collaborative learning activities within educational settings (e.g., Choi, Land, & Turgeon 2005; De Vries, Lund, & Baker 2002; Schwarz & Linchevski 2007; Schwarz, Schur, Pensso, & Tayer 2011). Research about collaborative learning activities has especially attributed high potential to so-called *transactive activities*. Transactive activities are described as discourse moves by which one learner actively builds on a learning partner's contributions to further construct knowledge, an example of which would be answering the learning partner's questions (Teasley 1997).

Transactive activities can be contrasted with *constructive activities*, which are characterized by self-construction, or producing knowledge beyond the information the learner can decode from the learning material, but without taking the learning partner's contribution into account (Chi 2009; Chi & Wylie 2014). An example for a constructive activity would be to explain a text in one's own words. Transactive activities are assumed to be more relevant than constructive activities for reaching high-level individual learning outcomes through collaborative learning (Chi 2009). This assumption is due to the fact that transactive activities make it necessary to thoroughly elaborate on the learning partner's contributions and to monitor one's own previous contributions in light of the partner's possible criticism (e.g. De Wever, Van Keer, Schellens, & Valcke 2010; Ismail & Alexander 2005).

However, previous research on collaborative learning activities displays two gaps: (1) the paucity of related studies to investigate the role of distinct types of transactivity that might differ in their predictive power for learning; and (2) the lack of research on whether a learner benefits only from his or her own social-discursive activities during collaboration, or (also) from the learning partner's social-discursive activities.

Regarding the first gap, by comparing Vygotskian theory with other sociocultural traditions, Wegerif (2008) stressed the differential impact that dialectic and dialogic activities may have on learning. Whereas learning in dialectic activities is a result of overcoming differences, in dialogic activities the differences themselves constitute what is learned. In dialectic transactivity, learners exchange arguments or argument components that build on the learning partner's contributions by addressing possible disagreements or conflicts directly, or that target overcoming a conflict. An example of dialectic argumentation would be that one learner proposing to solve a proof task by a general statement using a specific example, and her learning partner opposing this suggestion and pointing out that an example is valid only to refute a general statement. In contrast, dialogic transactivity refers to cases in which learners agree and build on the same position together by paraphrasing, completing or extending the partner's contribution. An example here would be one learner getting stuck in constructing a sequence of arguments, and her learning partner taking over and building upon these ideas without referring to their validity.

The dialectic-dialogic distinction for assessing learning activities raises the question of whether dialectic or dialogic activities are better for promoting individual learning outcomes of collaborative learning. For instance, in their study about undergraduates learning about evolutionary theory in dyads, Asterhan and Schwarz (2009) found that students learned better conceptual (i.e., domain-specific) knowledge when they engaged in “dialectical argumentation” compared to when they were engaged in (dialogic) “consensual explanation.” In this study, we thus differentiate between dialectic transactivity and dialogic transactivity. According to Wegerif’s (2008) conceptual work and the distinction made in the study by Asterhan and Schwarz (2009), we assume that an engagement in dialectic transactivity should yield higher individual learning gains than an engagement in dialogic transactivity.

In this study, we analyze to what extent the different kinds of transactivity contribute to the disposition to use domain-general social discursive argumentation skills and how they mediate the effects of specific instructional support (heuristic worked examples and collaboration scripts). In contrast, Asterhan and Schwarz (2009) studied the effect of similar learning activities on domain-specific (evolutionary theory) learning outcomes under one common instructional condition, which encouraged all students to engage in dialectical argumentation.

The second gap lies in the fact that research on collaborative learning activities so far has largely ignored possible differences in the relevance of collaborative activities that are produced by the learner him- or herself and the collaborative activities that are produced by the learning partner for one’s own skill acquisition. To generate transactivity, a learner needs to elaborate the partner’s contributions deeply. Only then will the learner be able to critique or argue for a partner’s contribution. In a short-term intervention, this deep elaboration can be assumed to go along with an increased disposition to use argumentation skills. The situation is different, however, if a learner is confronted with contributions from a learning partner; in that case, the learner might not deeply elaborate (or even notice) the partner's contributions, or react to them in a transactive way. In fact, Jiménez-Aleixandre, Bugallo Rodríguez, and Duschl (2000) showed that the quality and quantity of contributions that are made in a small group within a collaborative learning process are not necessarily equally distributed among the learning partners, so it is plausible to differentiate between the partners' own and others' contributions in a learning group. Moreover, studies with different designs and learning outcomes suggest that students' individual learning outcomes might be predicted by argumentative contributions that are self-generated rather than by argumentative contributions that are generated by a learning partner (e.g., Asterhan & Schwarz 2009). Furthermore, one learner might use the other learner as a model in terms of argumentation skills and learn by observing the partner's activities (van Gog & Rummel 2010). This scenario, however, requires high-quality arguments that are explicit enough to serve as a model, which is seldom observed spontaneously in collaborative argumentation. Moreover, to learn from models it often requires additional stimulation to ensure that the model is actually used (van Gog & Rummel 2010). This raises the question of whether all learning partners benefit equally from all social-discursive learning activities or whether their benefit is reduced to the activities they actually produce themselves as compared to those only observed in the behavior of the learning partner.

Scaffolding Argumentation in the Domain of Mathematics

Assuming that the disposition to use argumentation in mathematics is a valuable learning goal for prospective mathematics teachers (because they will have to teach their future students how to engage in mathematical argumentation and proof), a legitimate concern is how their acquisition and use of argumentation skills might be effectively supported. While domain-specific scaffolds such as providing a flow chart that depicts the steps of a Boero's (1999) mathematical proof process focus on providing support for structuring the content, collaboration scaffolds support the sequence of social-discursive exchange between learning partners (Leitão 2000). In the following section, we describe in detail heuristic worked examples as domain-specific scaffolds (Reiss & Renkl 2002) and collaboration scripts as domain-general collaboration scaffolds (Fischer, et al. 2013).

Heuristic worked examples

Traditional worked examples present a task description, the single solution steps, and the correct solution to a problem (e.g., Atkinson, Derry, Renkl, & Wortham 2000). They were shown to be helpful for the acquisition of skills needed to solve well-defined problems in well-defined domains such as probability theory. Yet, to be beneficial in ill-defined domains that allow for the selection of different solution paths and heuristics, worked examples need to offer flexible access to the heuristic strategies that underlie the process of solving ill-defined problems (e.g., Paas & van Merriënboer 1994). To adapt traditional worked examples to the needs of solving ill-defined mathematical proof problems, Reiss and Renkl (2002) introduced heuristic worked examples, which include a description of how an imaginary learner solves ill-defined problems in a way that matches the assumptions of a domain-specific experts' model. Heuristic worked examples demonstrate a set of heuristic strategies that vary in the degree to which they are immediately productive for finding a solution among different proof tasks.

So far, research about heuristic worked examples has focused more on individual learners and less on collaborative learning activities, which are relevant when learners are asked to study heuristic worked examples in teams (Hilbert, Renkl, Kessler, & Reiss 2008). Therefore, how learners can benefit from learning collaboratively with heuristic worked examples is still unknown. Nevertheless, by displaying heuristic strategies by which some may lead to impasses and others lead to a successful solution (e.g., in a proof task), collaborators may be triggered to discuss the pros and cons of different solution strategies and in that way engage in high-level argumentation. However, the collaborative use of heuristic worked examples might be challenging for learners, which is why more direct guidance on how to argue with each other in a group might additionally be necessary to lead to an engagement in high-level collaboration and eventually individual learning. Such guidance may come in the form of collaboration scripts.

Collaboration scripts

Collaboration scripts distribute roles and activities among learners and then sequence activities and role changes to guide learners through a collaborative learning process that is beneficial for their learning (Fischer et al. 2013; King 2007; Kollar, Fischer, & Hesse 2006). Scripts that are designed to foster argumentation prompt the learners to fulfill adequate activities within each step of an argumentative discourse cycle (e.g., Hron, Hesse, Cress, & Giovis 2000; Kopp & Mandl 2011; Scheuer, McLaren, Weinberger, & Niebuhr 2013; Weinberger, Stegmann, & Fischer 2010) or distribute discussion roles among the learning partners with specific activities that are attached to each role (e.g., De Wever et al. 2010; Strijbos, Martens, Jochems, & Broers 2004). During the past decade, a large variety of argumentation scaffolds, and more specifically collaboration scripts for computer-supported collaborative learning, has been investigated (Noroozi, Weinberger, Biemans, Mulder, & Chizari, 2012). For instance, in a study by Stegmann, Wecker, Weinberger, and Fischer (2012), undergraduate educational science students learned with a collaboration script that sequenced their argumentation. The study revealed that students learning with the collaboration script outperformed students learning without the collaboration script in developing argumentation skills. Indeed, collaboration scripts may further be designed to have an impact on collaborative learning processes, especially on transactivity (Fischer et al. 2013; King 1997; Noroozi, Biemans, Weinberger, Mulder, & Chizari 2013), and – as explained – engaging in such activities should also have a positive impact on learners' disposition to use argumentation skills. This mechanism was recently formulated for learning with collaboration scripts in the transactivity principle of the script theory of guidance (Fischer et al. 2013), which states that "The more a given CSCL [Computer-Supported Collaborative Learning] practice requires the transactive application of knowledge, the better this knowledge is learned through participation in this CSCL practice" (p. 58). However, given the rather domain-general nature of the collaboration script concept, what seems promising is a combination with heuristic worked examples that take the peculiarities of the domain into account and thus give direction to the topics of discussion that seem fruitful to solve mathematical proof tasks (Vogel, Wecker, Kollar & Fischer, 2016).

The Current Study

Authors (2014) have investigated the effects of learning with heuristic worked examples and a collaboration script on different components of mathematical argumentation skills. In their study, the heuristic worked examples guided the learners through the process of mathematical proof (Boero 1999), and the collaboration script supported learners in producing certain kinds of argument sequences (Leitão 2000). The results showed that both scaffolds had a significant positive effect on learners' acquisition resp. disposition to use social-discursive mathematical argumentation skills (Authors 2014). The present article provides an in-depth process analysis of the data from the collaborative learning sessions collected by Authors (2014) in their study. This data source was not analyzed in the 2014 paper. In particular, this article investigates the role of transactive learning activities in explaining positive effects of learning with heuristic worked examples and collaboration scripts on the disposition to use argumentation skills

Research Question 1. To what extent are the effects of heuristic worked examples and collaboration scripts on the disposition to use argumentation skills mediated by self-generated collaborative learning activities (constructive activities, dialogic transactivity, and dialectic transactivity)?

Heuristic worked examples provide guidance for the elaboration of domain-specific content (which may specifically trigger constructive activities such as “explaining”), and construct arguments and thus produce “material” that the learning partners can refer to and build upon later (dialogic and dialectic transactivity). The collaboration script we used deliberately prompted learners to formulate arguments based on the provided mathematical content (constructive activities), to build on their learning partners’ contributions (dialogic transactivity), and to critique the learning partner’s contribution and integrate different positions (dialectic transactivity). We expected in particular that the stimulation of dialectic transactivity within the collaborative learning process would explain the positive effects of the heuristic worked examples and the collaboration script on changes in the learners’ disposition to use argumentation skills. When using dialectic transactivity, learners directly apply argumentation skills that are facilitated by the instructional support, which should positively affect their disposition to use argumentation skills through practice (Fischer et al. 2013). We also expected dialogic transactivity to mediate the effects of both kinds of instructional support on students’ disposition to use argumentation skills, although this type of contribution is not directly related to the practice of argumentation. In contrast to dialectic transactivity, learners in dialogic transactivity do not argue against each other from different positions. They instead engage in dialogic argumentation (Wegerif, 2008), i.e. they jointly construct explanations, argumentations, and problem solving together and thus continuously refine and develop the same position . This may facilitate the learners’ disposition to use argumentation skills as well. In constructive activities, learners elaborate and build upon the given learning material (Chi 2009). This elaboration may also surface during argumentation (Sadler 2004). Constructive activities form an important basis for social-discursive argumentation. Therefore, we also expected the constructive activities to mediate the positive effects of both types of instructional support, heuristic worked examples and collaboration scripts. However, we expected dialectic transactivity to have the strongest mediating effects between the two scaffolds and the learner's disposition to use argumentation skills.

Research Question 2. To what extent are the effects of heuristic worked examples and the collaboration script on the disposition to use argumentation skills mediated by partner-generated collaborative learning activities (constructive activities, dialogic transactivity, and dialectic transactivity)?

We also expected the partner-generated activities to mediate the positive effects of heuristic worked examples and the collaboration script on the disposition to use argumentation skills. For the same reasons as stated in RQ1, both scaffolds were expected to positively influence the activities for all learners. The learning partner's activities may then comprise an additional source of information for the learner to elaborate on and in turn practice argumentation (e.g., Clark & Sampson 2007). Beyond that, the learning partner's dialectic transactivity might function as a model for the learner to engage in argumentation as a collaborative practice (e.g., Ismail & Alexander 2005). However, a question arises as to whether the effects of both scaffolds would be mediated by the partner-generated contributions. In fact, one might be skeptical about the existence of such a mediating effect because learners might not deeply elaborate on the partner's contributions. Thus, we expected that the mediating effect of the partner-generated collaborative learning activities would be less pronounced than the mediating effect of the self-generated collaborative activities.

Method

Participants and Design

The study was conducted in two different universities as part of a voluntary two-week course for high school graduates who were beginning a math teacher university program.

Overall, $N = 101$ (57 female and 44 male) students were included in the study presented in this article. Their mean age was $M_{age} = 20.04$ ($SD_{age} = 2.41$), and their average final high school grade was $M_{grade} = 2.09$ ($SD_{grade} = 0.58$) with grades ranging from 1.00 = excellent to 3.50 = satisfactory. We established a 2×2 experimental design with collaboration scripts (with, $N = 48$ students vs. without, $N = 53$ students) and heuristic worked examples (with $N = 53$ students vs. without, $N = 48$ students) as the independent variables. Participants were randomly assigned to one of the four experimental conditions.

Instructional Setting

The study took place on five consecutive weekdays. Pre-test data were collected on the first day, and the post-test took place on the fifth day. On the second, third, and fourth days, the participants took part in one treatment session per day (45 minutes each). For each treatment session, new dyads (same and mixed gender) were formed to reduce the effect that a single specific participant or dyad might have. To avoid large discrepancies in learning prerequisites of the participants learning together in one dyad, the assignment of learners to the learning dyads was controlled by the median split of the participants' final high school grade ($Mdn_{grade} = 2.10$). Only participants within the same half of the sample determined by the median split (i.e., with similar final high school grades) were randomly assigned to a learning dyad.

Setting and Learning Environment

Students learned collaboratively in a computer-supported learning environment on three different mathematical proof tasks, one task per treatment session (see Table 1 for the three mathematical proof tasks and the conjectures that most learners generated during the sessions). Learners in the conditions with the heuristic worked examples were shown a solution that was suited for this specific conjecture. Learners in the conditions without the heuristic worked examples also were not provided with feedback, which could have helped them to learn about the expected conjecture, because the feedback would have been an additional type of instructional support. Nevertheless, in other parts of the introductory course, before the beginning of the intervention, participants created conjectures to a different mathematical topic in groups and received feedback from tutors. The learning partners of each dyad were seated on opposite sides of a table and were each equipped with one laptop and a graphics tablet that allowed them to draw and write into a graphical chat that was implemented in the computer-supported learning environment that was installed on the laptops. Face-to-face communication was allowed as well.

The computer-supported learning environment we used allowed for the implementation of the four experimental conditions. More information about how the computer-supported learning environment looked and how the different conditions were realized in the learning environment can be found in Authors (2014). On the left half of the computer screen, the learning environment presented the mathematical proof task, a calculator with basic functions, and domain-specific lecture notes (available in all conditions), as well as the heuristic worked examples (in the conditions with heuristic worked examples) or only the problem to be solved (in the conditions without heuristic worked examples). On the right half of the screen, the students were able to share text and drawings by using the text and graphic chat function (available in all conditions). The students were allowed to create an unlimited number of pages for their written communication and browse through the pages during the current treatment session. Since learning partners were co-present, they also were allowed to talk about their suggestions face-to-face. In the conditions with the collaboration script, the upper right side of the screen displayed the script prompts.

Independent Variables

Before learning in the computer-supported learning environment, the students in all experimental conditions watched an instructional video (20 minutes) that informed them about the sequence of argumentation and the rules for formulating arguments that they were supposed to follow in the subsequent treatment sessions. In all experimental conditions, students were requested to alternately work on the proof tasks individually and discuss their ideas with their learning partner, taking notes of the most important aspects.

Heuristic worked examples. In the conditions with heuristic worked examples, we presented a possible solution for each of the three proof tasks. The heuristic worked examples were split into the six steps of mathematical proofs (adapted from Boero 1999) and described how an imaginary peer applied different heuristic strategies to make progress within each of these steps. To increase the need for discussion, the two students of each dyad were presented with slightly different heuristic worked examples for each task. In contrast, in the conditions without heuristic worked examples, students had to work on the mathematical proof tasks by themselves through problem solving, without receiving guidance either on the steps of the mathematical proof or on heuristic strategies beyond the instructional video shown at the start of the first treatment session.

Collaboration script. In the conditions with a collaboration script, the collaborative discussion of the learning dyads was sequenced into three phases: (1) argument, (2) counterargument, and (3) integration. The sequence was displayed to the students via complementary written prompts, one for each learning partner per phase (e.g., "Please, formulate an argument supporting your position and share it with your learning partner," and "Please listen critically to the argumentation of your learning partner."). The prompts were presented in the computer-supported learning environment and were always adapted to the learners' specific phase in the argumentation sequence. The learning partners were instructed that once they felt they had completed what was prompted in one phase, they should click on a button to take them to the next phase. The prompts also encouraged the students to refer to their learning partner's contribution (Leitão 2000) and to formulate sound arguments according to Toulmin's (1958) model of argument construction (including claims, data, and qualifiers). In the conditions without a collaboration script, students were asked to discuss their ideas without receiving any guidance for their discussion (unstructured collaboration).

Dependent Variables

For the *pre- and post-test measure of the disposition to use argumentation skills*, we used a paper-pencil test that requested students to individually describe typical phases and activities they would expect to find in a discussion about a science topic (e.g, whether hot water freezes faster than cold water in the refrigerator). We deliberately chose a topic different from mathematics for these tests to see whether students would be able to transfer their disposition to use argumentation skills from mathematics to different domains, assuming that the argumentation skills that should be used in the learning environment would also be helpful in domains beyond mathematics. The test consisted of a prompt to describe typical phases and activities of a discussion and five empty fields in which students could describe the phases of a discussion in an open format. The students' answers were coded for the number of argumentative elements they included (pro-argumentation, counter-argumentation, integration of arguments, and response to other's arguments). After training, two independent coders coded the answers separately, reaching sufficient inter-rater reliability for coding each of the four elements on average ($M_{Cohen's\ \kappa}$ = .82; $\kappa_{minimum}$ = .76, $\kappa_{maximum}$ = .93). The correctly mentioned elements in each student's answer were summed to yield an overall measure of argumentation skills, with values ranging from 0 to 4 (for more detailed information about the test, see Authors 2014).

To analyze the *collaborative learning activities*, two independent raters coded the written contributions the students made during the treatment phases. Each contribution (turn) during collaboration within the three treatment sessions was categorized into one of the three collaborative learning activities (constructive activities, dialogic transactivity, dialectic transactivity) or a rest category (see Table 2 for exact descriptions and examples for each category). The rest category was not included into further calculations in the current study. After coding, the frequencies of contributions containing constructive activities, dialogic transactivity, and dialectic transactivity were summed for each learner separately. Inter-rater reliability was calculated based on the frequencies of contributions that were summed for each category. After training, a sufficient inter-rater reliability was achieved for a sample of the contributions of 22 participants across all conditions and treatment sessions (see Table 2 for exact *ICC* values of each category). The remaining turns were then coded separately by one trained coder.

Validity of the Measure for the Disposition to Use of Argumentation Skills

To test the validity and usefulness of the measure for the disposition to use argumentation skills as a predictor for the student's ability to engage in collaborative problem solving and discussion about mathematical proof tasks, we assessed the relation between the post-test disposition to use argumentation skills and the engagement in collaborative problem solving after treatment for each student. For this purpose, the students were paired together in new learning dyads in which they were requested to work on another mathematical proof task, similar to those that were used during the treatment phases. During this post-test task, the students used the same learning environment as in the treatment phases, but without any means of instructional support. Two raters practiced with a coding scheme to assess the number of turns in the chat protocol in which students engaged in the discussion about the given mathematical proof task. After this training, the raters achieved a good inter-rater agreement based on double-coding of > 20% of all included chat protocols (ICC_{unjust} = .94). The remaining turns were then assessed separately by one of the two raters.

A correlation analysis showed that the post-test measure for the disposition to use argumentation skills and the post-test measure for engagement in the discussion about the mathematical proof task were significantly positively correlated ($r = .31$, $p = .002$). A partial correlation of both measures that controlled for the pre-test measure of the use of argumentation skills also revealed a significantly positive correlation ($r = .30$, $p = .003$). Thus, the validity of our measure for the disposition to use argumentation skills might be confirmed because learners who showed a better performance in our test of the disposition to use argumentation skills were actually more engaged in collaborative discussions during an unguided mathematical proof task.

Statistical Analyses

We used linear regressions to determine the extent to which the different collaborative learning activities mediated the effects of heuristic worked examples and the collaboration script on the students' disposition to use argumentation skills. To test for mediation of the predictors in the linear regression model, we used the bootstrapping procedure recommended by Preacher and Hayes (2004). The calculations for the bootstrapping were conducted by the MEDIATE SPSS macro provided by Hayes and Preacher (2014).

Results

Preliminary Results

Using analyses of covariance, Authors (2014) showed that learning with heuristic worked examples and learning with the collaboration script in the context of mathematical proof had substantial positive effects on the disposition to use argumentation skills. Yet, there was no significant interaction effect between the two independent variables. For the reanalysis of Authors's (2014) data in the present article, a regression analysis confirmed that the pre-test disposition to use argumentation skills, the heuristic worked examples and the collaboration script were significant predictors for the student's post-test disposition to use argumentation skills (see Table 3). The interaction between heuristic worked examples and the collaboration script was excluded from the model because it was not a significant predictor for the disposition to use argumentation skills (β = .028, p = .86; ΔR < .001).

As both the heuristic worked examples and the collaboration script had significant positive effects on students' disposition to use argumentation skills, it is worthwhile to further investigate the collaborative learning processes to identify possible mediators of the effect of the heuristic worked examples and the collaboration script on the disposition to use argumentation skills as hypothesized in the research questions.

RQ 1: Mediation by the Learners' Self-Generated Activities

Descriptively, the analysis of the appearance of the different collaborative learning processes within each condition yielded that, in general, constructive activities were used the most, followed by dialogic transactivity, and dialectic transactivity (see Table 4 for the collaborative learning processes used per learner per condition). To answer the first research question, we conducted linear regression analyses and the bootstrapping procedure as recommended by Preacher and Hayes (2004). As predictors for the post-test disposition to use argumentation skills, we used the group indicators for heuristic worked examples and the collaboration script in the regression model. Additionally, the pre-test disposition to use argumentation skills was included as a covariate and we entered as mediators the frequencies of the three self-generated activities (constructive activities, dialogic transactivity, and dialectic transactivity).

Heuristic worked examples. The results showed that the heuristic worked examples served as a positive predictor for the frequencies of constructive activities as well as dialectic transactivity and as a negative predictor for the frequency of dialogic transactivity generated by the students during the learning process (see Fig. 1). By including the frequencies of the three self-generated activities into the regression model, the coefficient of the heuristic worked examples as a predictor for the disposition to use argumentation skills was reduced, which indicates the presence of a mediation effect. The bootstrapping procedure showed that neither constructive activities ($CI_{95\%}$: LL_{CI} = -0.03; UL_{CI} = 0.27) nor dialogic transactivity ($CI_{95\%}$: LL_{CI} = -0.06; UL_{CI} = 0.21) were significant mediators. Only the frequency of dialectic transactivity generated by the students during the learning process mediated significantly the effect of heuristic worked examples on the disposition to use argumentation skills ($CI_{95\%}$: LL_{CI} = 0.05; UL_{CI} = 0.46).

Collaboration script. The results showed that the collaboration script was a positive predictor for the frequencies of the three self-generated activities, namely, constructive activities, dialogic transactivity, and dialectic transactivity (see Fig. 1). By including the frequencies of the three self-generated activities into the regression model, the coefficient of the collaboration script as a predictor for the disposition to use argumentation skills was reduced, which indicates the presence of a mediation effect. Also for the collaboration script, the bootstrapping procedure showed that neither constructive activities ($CI_{95\%}$: LL_{CI} = -0.03; UL_{CI} = 0.28) nor dialogic transactivity ($CI_{95\%}$: LL_{CI} = -0.21; UL_{CI} = 0.05) were significant mediators. Only the frequency of dialectic transactivity generated by the students during the learning process mediated significantly the positive effect of the collaboration script on the disposition to use argumentation skills ($CI_{95\%}$: LL_{CI} = 0.03; UL_{CI} = 0.41).

RQ 2: Mediation by the Partner-Generated Activities

Because students were also exposed to the contributions made by their learning partners, RQ 2 asked to what extent partner-generated contributions also mediated the effects of heuristic worked examples and the collaboration script on the disposition to use argumentation skills. We again conducted linear regression analyses and the bootstrapping procedure for the mediator analysis. As predictors for the post-test argumentation skills, both scaffolds (heuristic worked examples and collaboration script) were included in the regression model. Additionally, pre-test disposition to use argumentation skills was included as a covariate, and this time the frequencies of the three partner-generated activities (constructive activities, dialogic transactivity, and dialectic transactivity) were included as mediators.

Heuristic worked examples. In parallel to the results for the self-generated activities, heuristic worked examples served as a positive predictor for the frequencies of partner-generated constructive activities as well as dialectic transactivity, and as a negative predictor for the frequency of partner-generated dialogic transactivity (see Fig. 2). The bootstrapping procedure showed that none of the three partner-generated activities were significant mediators for the effect of the heuristic worked examples on the disposition to use argumentation skills: not constructive activities ($CI_{95\%}$: $LL_{\text{CI}} = -0.02$; $UL_{\text{CI}} = 0.36$), nor dialogic transactivity ($CI_{95\%}$: $LL_{\text{CI}} = -0.13$; $UL_{\text{CI}} = 0.15$), nor dialectic transactivity ($CI_{95\%}$: $LL_{\text{CI}} = -0.24$; $UL_{\text{CI}} = 0.21$).

Collaboration script. The results showed that the collaboration script was a positive predictor for the frequencies of each of the three partner-generated activities, namely constructive activities, dialogic transactivity, and dialectic transactivity (see Fig. 2). Also for the collaboration script, the bootstrapping procedure showed that neither partner-generated constructive activities ($CI_{95\%}$: $LL_{\text{CI}} = -0.02$; $UL_{\text{CI}} = 0.34$), nor partner-generated dialogic transactivity ($CI_{95\%}$: $LL_{\text{CI}} = -0.14$; $UL_{\text{CI}} = 0.10$), nor partner-generated dialectic transactivity ($CI_{95\%}$: $LL_{\text{CI}} = -0.19$; $UL_{\text{CI}} = 0.16$) were a significant mediator for the effect on the disposition to use argumentation skills.

Qualitative Results

To illustrate what actually happened within the learning process, we compare here one transcript of a learner (Lisa) in the condition with heuristic worked examples and with the collaboration script, who showed a high learning gain in the disposition to use argumentation skills (see Table 5) to one transcript of a learner (Sebi) in the condition with heuristic worked examples and without the collaboration script, who had a rather low learning gain in the disposition to use argumentation skills (see Table 6). The excerpts are taken from the first treatment session that presented the proof task: “Choose some squared numbers. Calculate the differences per two squared numbers. Formulate a conjecture and prove it!” The excerpts depict more precisely the part of the learning process in which the learners in each dyad were presented with two different versions of how to display the first examples calculated within the given problem space. More specifically, one learner had access to a version, in which the examples were displayed by drawn circles (see Fig. 3), and the other learner had access to a version, in which the examples were displayed in a table (see Fig. 4).

A comparison of the two transcripts shows clearly that Lisa and her learning partner (Table 5) use the whole spectrum of different learning activities, especially dialectic transactivity, whereas Sebi and his partner (Table 6), use only constructive activities. The difference that can be seen between the learner with a rather high learning gain (Lisa) and the learner with a rather low learning gain (Sebi) depicts what we found to mediate the effect of the heuristic worked examples and the collaboration script on the disposition to use argumentation skills in the quantitative analysis of this study.

Discussion

This article addresses the question of which of three different types of collaborative learning activities (constructive activities, dialogic transactivity, and dialectic transactivity) mediates the effects of heuristic worked examples and a collaboration script in the context of mathematical proof tasks on the disposition to use argumentation skills . Furthermore, the study examined to what extent collaborative learning activities must be generated by the learners themselves or their respective learning partners in order to mediate the effects of both kinds of instructional support on the disposition to use argumentation skills.

The results showed that, overall, both the heuristic worked examples and the collaboration script had substantial (mostly positive) effects on the different types of learning activities generated by the students during the collaborative learning process. This finding aligns with results of previous studies that emphasized the importance of providing additional domain-specific scaffolds (e.g., content schemes; content-specific graphical representations) to take full advantage of the support with collaboration scripts (e.g., Ertl, Kopp, & Mandl 2006; Hron, Cress, Hammer, & Friedrich 2007; Vogel et al, 2016). Whereas the script prompted the students to directly perform argumentative moves by guiding them through an argumentation sequence of formulating arguments, counterarguments, and integrations (Leitão 2000), the heuristic worked examples provided rich domain-specific material to argue about (see Fig. 3 and Fig. 4), which may have increased the opportunities to engage in collaborative learning processes when compared to situations in which students had to develop solutions themselves. Also, the fact that the two learning partners received slightly different heuristic worked examples may have contributed to this effect. According to Dillenbourg and Hong (2008), such a distribution of learning material increases the collaborative effort for the learners to reach a shared understanding and thus may cause a higher amount of interaction between the learners as well as better learning (e.g., Clark & Sampson 2007; Molinari, Sangin, Dillenbourg, & Nüssli 2009).

The results concerning RQ 1 underpin the importance of transactivity in collaborative learning processes to explain individual learning (Chi 2009; Fischer et al. 2013; Noroozi et al. 2013; Teasley 1997). However, the results show that it is useful to differentiate among various aspects of transactivity. As the mediation analyses showed, the positive effects of the heuristic worked examples and the collaboration script on students' disposition to use argumentation skills were mediated by the amount of self-generated dialectic transactivity a student displayed during collaboration, but not by dialogic transactivity nor constructive activities. This means that providing domain-specific scaffolds such as the heuristic worked examples indeed plays an important role in helping learners to engage in dialectic transactivity during the collaborative learning process and thereby to acquire the disposition to use argumentation skills (Hron et al. 2007; Sadler 2004). At the least, learners can change their disposition to use their argumentation skills in the mathematical and science domains by reconfiguring their already learned components of argumentation skills (Fischer et al. 2013). Also, referring to the learning partner in an argumentative way (i.e., criticizing or integrating the learning partner's arguments), as was directly scaffolded by the collaboration script, is important for developing a disposition to use such argumentation skills (Fischer et al. 2013).

Learning from collaboration scripts might be criticized due to the suspicion that knowledge might be acquired only by memorizing the description of the prompts offered with the collaboration script rather than the actual realization of the scripted strategy (Wecker, Kollar, & Fischer 2011), especially if the post skill test is on a rather declarative level. The post-test disposition to use argumentation skills that was shown to be mediated by students' actual use of dialectic transactivity while learning collaboratively argues against this possible criticism. Rather, the results show that indeed the mechanism through which the disposition to use argumentation skills was acquired was that of performing what was suggested by the script – that is, by engaging in high-level collaborative learning activities. The kind of knowledge students acquired that way can then form the basis for the development of skills for engagement in argumentative discourse (Anderson 1996), which is also supported by the positive correlation between the learners' performance in the post-test on the disposition to use argumentation skills and the learner's engagement in an unsupported collaborative learning discourse.

Interestingly, the dialectic transactivity generated by the learning partner did not mediate the effect of the instructional support on one's own disposition to use argumentation skills (see results concerning RQ 2). Thus, for changing one's disposition to use argumentation skills it seems to be more important that learners generate dialectic transactivity themselves rather than merely to be exposed to a learning partner who is generating dialectic transactivity. This makes sense, as learners by necessity need to be actively and cognitively engaged in processing the partner's contribution when generating dialectic transactivity while a partner's transactive statement sometimes may (but often also may not) be processed with comparable effort (e.g. Asterhan & Schwarz 2009; Chi, Roy, & Hausmann 2008). This statement, of course, does not imply that the learning partner is not important. Dialectic transactivity by definition requires the learning partner's contribution to build on and refer to. On a more social and less cognitive level, being faced with a partner who constantly works with one's own statements may even serve as a model to act similarly. However, our results seem to imply that being exposed to this kind of modeling is not enough for one's own learning; instead, the power of transactivity for learning to argue lies primarily in modifying one's own thinking by reflecting on the thoughts of another learner.

Conclusions

Given these results, one implication for theory building concerns the transactivity principle stated in the Script Theory of Guidance (Fischer et al. 2013). Our results provide preliminary justification for a more specific reformulation of this principle, as they seem to imply the need to differentiate between the learners' self-generated and partner-generated dialectic transactivity, at least for the mathematics context used in this study. A reformulation of this principle might state that *the more a given CSCL practice for argumentation requires the generation of dialectic transactivity, the better the learner changes the disposition to use argumentation skills due to the dialectic transactivity generated by the learner her/himself rather than by the learning partner's dialectic transactivity the learner is exposed to*. Yet, more empirical evidence is needed to judge the validity of this principle. In case such evidence accumulates in future research, one interesting question for future studies would relate to how learners might be supported to also take advantage of the dialectic transactivity contributed by their learning partners.

Certainly, this study is not without limitations. First, the extent to which the results generalize to other domains is an open question. With respect to the importance of dialectic transactivity for the development of the disposition to use argumentation skills, generalization may be straightforward because the way we operationalized dialectic transactivity consisted more of domain-general categories (criticizing, integrating) that may easily be adapted to communication in other domains. Also, the fact that the disposition to use argumentation skills was measured in a different domain supports the assumption that students might be capable of transferring what they learned to other contexts. However, the effects of the two types of scaffolding on the collaborative learning process need to be considered with more attention when trying to generalize the results. The part of the effect that can be attributed to the heuristic worked examples might be rather difficult to transfer to a different context because the heuristic worked examples by definition provide domain-specific support, here based on a specific process model of mathematical proofs (Boero 1999). To make the heuristic worked examples approach work in other domains, adaptations are, of course, inevitable.

Another limitation is that the test measuring the students' disposition to use argumentation skills had a declarative format. Although we checked for the validity of this measure by comparing it to learners' actual engagement in collaborative problem solving in a post-test task, the positive correlation was rather low. Future studies preferably should use more performance-oriented tests to measure such social-discursive skills as argumentation. These performance-oriented tests, however, demand that two or more learners be tested together, which makes it methodologically challenging to estimate the learner's individual learning gain (Cress 2008).

A further limitation refers to how much the learners used dialectic transactivity throughout the treatment sessions. The fact that the use of dialectic transactivity was a rather rare event might have biased the probability for this variable to be a significant mediator. Therefore, our interpretation must be received carefully, and further studies are needed to support or refute our conclusion. Nevertheless, the learning partner's use of dialectic transactivity has been equally seldom, but did not turn out to be a significant mediator. Therefore, we see the result of self-generated dialectic transactivity being a mediator for the disposition to use argumentation skills as rather robust.

In conclusion, our study shows that the disposition to use domain-general argumentation skills can be enhanced in a domain that is not prototypical for a discursive domain. To do so, however, the learning environment needs to offer well-designed domain-specific support to provide learners with structured content to use in their argumentation. Moreover, both collaboration scaffolding and domain-specific scaffolding need to be designed with a specific focus on evoking dialectic transactivity during the collaborative learning process.

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Figures

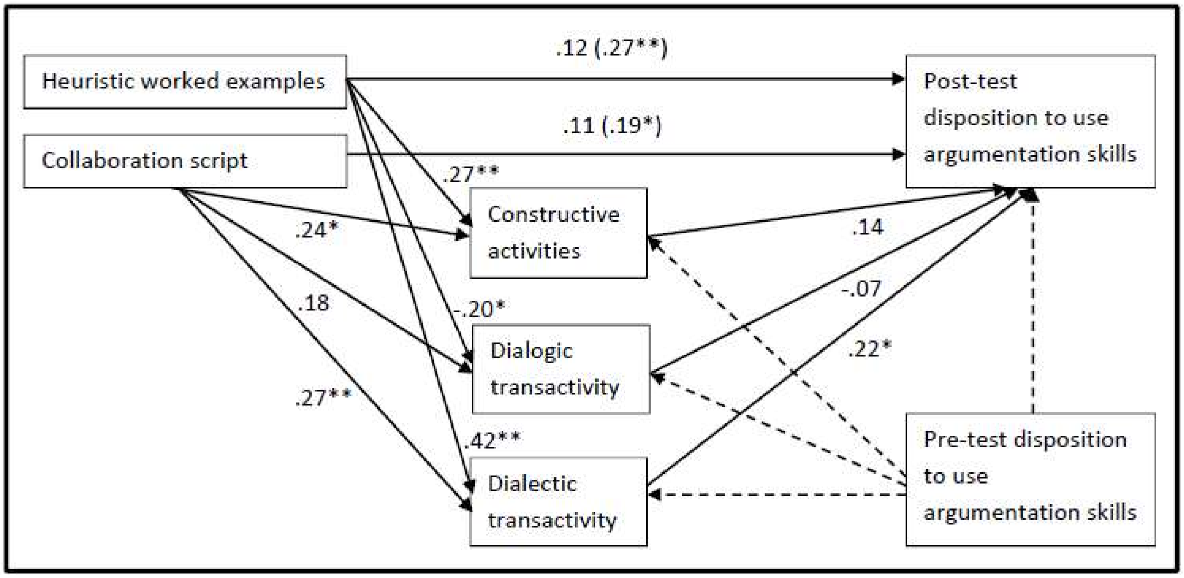


Fig. 1 Standardized beta weights in the path model of the mediator analysis for the effects of the heuristic worked examples and the collaboration script on the disposition to use argumentation skills with the self-generated collaborative activities included as mediators. The numbers in parentheses represent the direct effect of the independent variables on the post-test disposition to use argumentation skills before the mediator variables were added to the model. Note: *p < .05, **p < .01

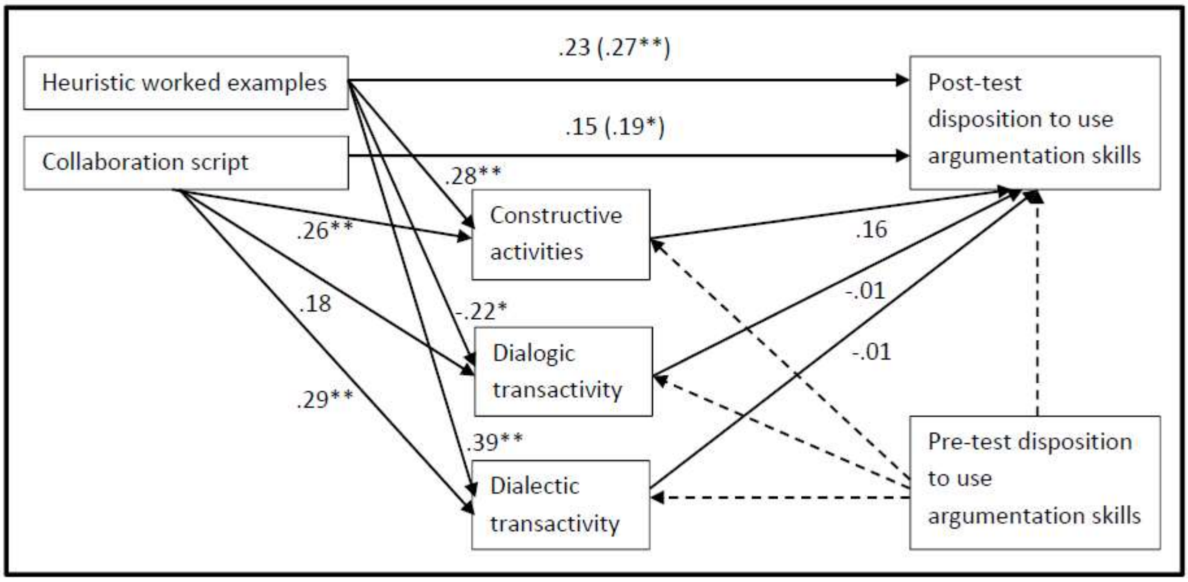


Fig. 2 Standardized beta weights in the path model of the mediator analysis for the effects of the heuristic worked examples and the collaboration script on the disposition to use argumentation skills with the partner-generated collaborative activities included as mediators. The numbers in parentheses represent the direct effect of the independent variables on the post-test disposition to use argumentation skills before the mediator variables were added to the model. Note: $^{*}p < .05$, $^{**}p < .01$

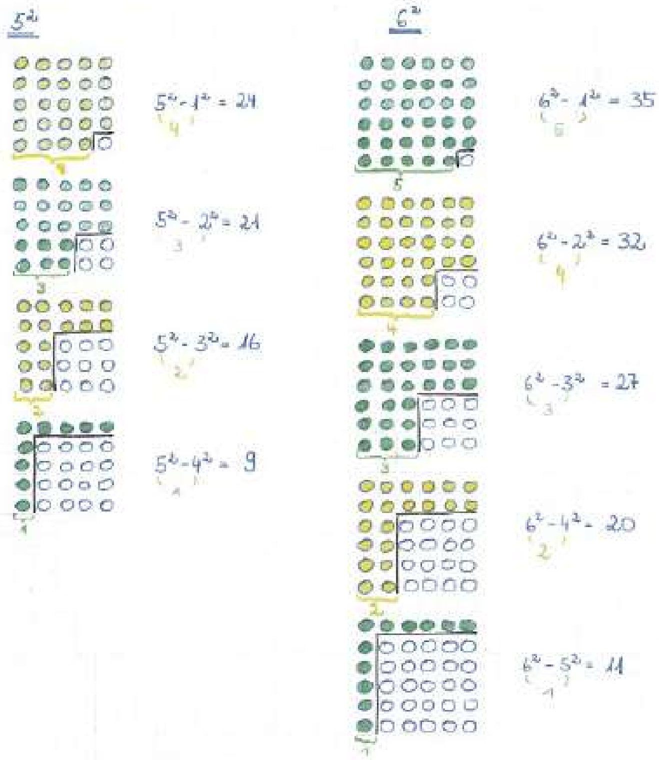
Investigating the problem by calculating examples

Jana starts to calculate some examples:

$$2^2 - 1^2 = 3 \qquad 4^2 - 2^2 = 12$$
$$3^2 - 2^2 = 5 \qquad 2^2 - 4^2 = -12$$
$$5^2 - 2^2 = 21$$
$$6^2 - 1^2 = 35$$

Jana: For the following investigation it is probably sufficient if I only look at the positive differences, i.e. if I subtract the smaller squared number from the larger one.

Since we are looking at squared numbers only, it could be possible to get a clearer view by drawing the differences of the squared numbers:



Jana: If the distance between the two numbers which are squared and after that subtract from each other is odd the result is also odd. Vice versa, if the distance is even the result is also an even number.

I think I can describe the even results more precisely. It seems like they are always divisible by 4.

Fig. 3 Part of the heuristic worked example presented to one learner from the dyad.

Investigating the problem by calculating examples

Janka starts to note some examples:

$2^2 - 1^2 = 3$ $\quad$ $4^2 - 2^2 = 12$

$3^2 - 2^2 = 5$ $\quad$ $2^2 - 4^2 = -12$

$5^2 - 2^2 = 21$

$6^2 - 1^2 = 35$

Janka: For the following investigation it is probably sufficient if I only look at the positive differences, i.e. if I subtract the smaller squared number from the larger one.

Maybe I can see more details if I put the examples in order:

⊖	0^2	1^2	2^2	3^2	4^2	5^2	6^2
0^2	0						
1^2	1	0					
2^2	4	3	0				
3^2	9	8	5	0			
4^2	16	15	12	7	0		
5^2	25	24	21	16	9	0	
6^2	36	35	32	27	20	11	0

Janka: If the distance between the two numbers which are squared and after that subtract from each other is odd the result is also odd. Vice versa, if the distance is even the result is also an even number.

I think I can describe the even results more precisely. In this table are only numbers that are divisible by 4. For instance, the number 6 does not exist in this table.

Fig. 4 Part of the heuristic worked example presented to the other learner from the dyad.

Table 1

Proof task and the formal conjectures the learners worked with collaboratively in the three treatment sessions.

Treatment session	*Proof task*	*Example for expected formal conjecture*
1	Chose some squared numbers. Calculate the differences per two squared numbers. Formulate a conjecture and prove it!	Taken the difference of two squared numbers: $(n+i)^2 - n^2 = 2ni + i^2$ with n, i being natural numbers, then $2ni + i^2$ is: • either an uneven number • or a number divisible by 4.
2	Chose some uneven numbers. Square each number and subtract 1 from it. What can be noticed? Formulate a conjecture and prove it!	If you subtract 1 from a squared number: $(2n + 1)^2 - 1 = 4n(n + 1)$, n being a natural number, then the result $4n(n + 1)$ is • divisible by 8.
3	Take an uneven amount of consecutive numbers and add them up. Repeat this and try to find regularities. Formulate a conjecture and prove it!	x and y should be two arbitrary natural numbers. The sum of an uneven number – or $2y + 1$ – of consecutive numbers: $x + (x+1) + \ldots + (x + 2y) = (1 + 2 + \ldots + 2y) + (2y + 1)x$ is • divisible by $(2y + 1)$.

Table 2

Descriptions and examples for constructive activities, dialogic transactivity, dialectic transactivity and the rest category during the collaborative learning process in the context of mathematical proof tasks.

Description	*Examples*	ICC_{unjust}
Constructive activities		.80
Contributions that elaborate on the content relevant to the given task but do not directly take the learning partner's contribution into account.	"My first idea is to write down all calculated results into a table" "My conjecture is that all numbers must be divisible by 3"	
Dialogic transactivity		.68
Contributions that are taking the learning partner's contribution into account either by using the same structure (e.g., calculating using the same formula as the learning partner), or or by building on the content of the learning partner's contribution (e.g., extending the learning partner's ideas), but without critiquing or integrating opposing positions.	"… a further pro for your claim is that displaying a table is more comprehensive" (extending the learning partner's contribution with own ideas but without criticizing or integrating opposing positions) "… 9 – 4 = 5" (calculating the differences of squared numbers by using the learning partner's results of $3^2 = 9$ and $2^2 = 4$)	
Dialectic transactivity		.92
Critiquing: Comments that tackle the approach to solve the problem or the solution itself and contain counterargumentation and/or criticism directly referring to the learning partner's contribution or	"… but your description of the problem space is less helpful because…" "…2 + 3 = 5" (opposing that the sum of two consecutive numbers is always even)	
Integrating: Comments that integrate previous contributions with at least one contribution made by the learning partner	"…the summary of the pros and cons we made is…" "Taking your criticism into account we could agree on distinguishing between cases when the numbers are even and uneven"	
Rest		.97
Off-topic contributions that are not clearly related to the task	"…blubblk…" "The weather is nice today."	

Table 3

Summary of stepwise regression analysis with pre-test disposition to use argumentation skills, heuristic worked examples and collaboration script, as predictors for students' post-test disposition to use argumentation skills.

	B	SE_B	*β*	*p*
Step 1				
Constant	1.58	0.18		.001
Pre-test disposition to use argumentation skills	0.40	0.09	.41	.001
Step 2				
Constant	1.26	0.20		.001
Pre-test disposition to use argumentation skills	0.38	0.09	.39	.001
Heuristic worked examples	0.69	0.22	.28	.002
Step 3				
Constant	0.99	0.24		.001
Pre-test disposition to use argumentation skills	0.42	0.09	.43	.001
Heuristic worked examples	0.67	0.21	.27	.002
Collaboration script	0.46	0.22	.19	.037

Note: R^2 = .17 for Step 1; ΔR^2 = .08 for Step 2; ΔR^2 = .03 for Step 3.

Table 4

Mean and standard deviation (in parentheses) for the amount of individual use of different collaborative learning processes per condition.

	without script		*with script*	
	without heuristic worked example *M* (*SD*)	with heuristic worked example *M* (*SD*)	without heuristic worked example *M* (*SD*)	with heuristic worked example *M* (*SD*)
constructive activities	6.27 (4.89)	11.93 (4.02)	11.64 (4.23)	12.39 (7.97)
dialogic transactivity	8.00 (5.85)	6.44 (4.72)	10.86 (5.69)	7.69 (6.99)
dialectic transactivity	1.54 (1.33)	1.93 (1.60)	1.32 (1.29)	4.15 (1.78)

Table 5

Excerpt from the learning process in the condition with heuristic worked examples and collaboration script.

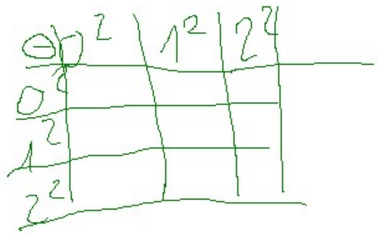
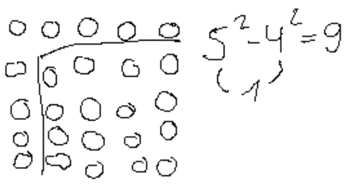
Learner	*Excerpt*	*Categorization*
	[...]	
Lisa	Jana first draws the squared numbers which she subtracts from each other as circles to investigate them more precisely. She suspects: if the distance of the two numbers is odd, then the result is also odd and if the distance is even, then [the result] is also even	Constructive activity
Partner	and how many circles does she draw?	Dialogic transactivity
Lisa	for example 9 circles for 3 squared	Dialogic transactivity
Partner	ah, yes and if she then subtracts 3 squared then she eliminates 3 circles no 9	Dialogic transactivity
	[…]	
Lisa	good :) what speaks pro Jana's approach [drawing circles] speaks that she tries to illustrate the task in order to simplify it	Constructive activity
Partner	but, in my opinion are circles not suitable for illustration because they are very cumbersome and complex for larger numbers in my proposal a better method is used to illustrate the problem Janka uses a table which looks like that  she then enters the results into the table this method is also suitable for relatively large numbers	Dialectic transactivity
Lisa	right I think it would be good to use the circle-method for small numbers and for larger numbers the table by this procedure the task would be illustrated  this is how it looks with the circles […]	Dialectic transactivity

Table 6

Excerpt from the learning process in the condition with heuristic worked examples and without collaboration script.

Learner	*Excerpt*	*Categorization*
	[…]	Constructive activity
Partner	She first tries to note some examples and then she tries to find commonalities and discrepancies within the calculated examples. Finally, she packs two approaches in so-called numbers-squares and finds her result after observations of these squares.	
Sebi	Janka first makes some exemplifying calculations. In her examples she finds that odd numbers and even numbers result in an odd number. Only even numbers result in an even number. Then she makes a table from 6^2 to 6^2 and can see that the distance between the numbers which are squared and then subtracted is crucial for the resulting number (odd or even). For an odd distance the resulting number is also odd, otherwise it is not. Advantage (approach with the table): The table helps through a large pool of resulting numbers --> can easily recognize commonalities Disadvantage (approach with the table: a large amount of calculations	Constructive activity
Partner	Advantage: Drawing is clearly arranged, easily understandable and one can see the commonalities within the squares […]	Constructive activity