Out of hours workload management: Bayesian inference for decision support in secondary care

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Abstract

Objective: In this paper, we aim to evaluate the use of electronic technologies in Out of Hours (OoH) task-management for assisting the design of effective support systems in health care; targeting local facilities, wards or specific working groups. In addition, we seek to draw and validate conclusions with relevance to a frequently revised service, subject to increasing pressures.

Methods and Material: We have analysed 4 years of digitised demand-data extracted from a recently deployed electronic task-management system, within the *Hospital at Night* setting in two jointly coordinated hospitals in the United Kingdom. The methodology employed relies on Bayesian inference methods and parameter-driven state-space models for multivariate series of count data.

Results: Main results support claims relating to (i) the importance of data-driven staffing alternatives and (ii) demand forecasts serving as a basis to intelligent scheduling within working groups. We have displayed a split in workload patterns across groups of medical and surgical specialities, and sustained assertions regarding staff behaviour and work-need changes according to shifts or days of the week. Also, we have provided evidence regarding the relevance of day-to-day planning and prioritisation.

Conclusions: The work exhibits potential contributions of electronic tasking alternatives for the purpose of data-driven support systems design; for scheduling, prioritisation and management of care delivery. Electronic tasking technolo-

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gies provide means to design intelligent systems specific to a ward, speciality or task-type; hence, the paper emphasizes the importance of replacing traditional pager-based approaches to management for modern alternatives.

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1. Introduction

Secondary healthcare systems around the world are under increasing pressure ([1, 2]). Patient admissions are rising ([3]), and the number of available beds is falling ([4]); simultaneously, the complexity of conditions and treatments is increasing ([5]). Hence, healthcare systems must undergo major changes and optimise the use of limited resources.

This situation is especially acute during the *out of hours* (OoH) setting. For 75% of the working week hospitals are staffed by a skeletal team ([6]), and care must be delivered by a small and often junior group of clinicians over a wide range of medical specialities; frequently in large and complex sites ([7]). Decreasing budgets ([8]), tighter controls of working hours ([9]) and the desire for separation of work and private life ([10, 11]) have lead to a shrinkage of OoH working teams. Hence, in order to deliver safe healthcare of a consistently high quality the provision of this service is frequently revised ([12–14]), often without underlying comprehensive data or understanding of the demand placed on clinical teams.

Extensive research has been concerned with the study of expert and knowledge-based systems in healthcare management. This includes logistics, resource scheduling or estimation of service demands, and we refer the reader to [15–17] (and references therein) for some examples of this work. In particular, quantitative demand-forecasting studies focused on patient volumes (e.g. [18]) have confirmed that seasonal patterns and serial correlation structures play important roles in understanding demand loads. Additionally, the study of explanatory covariates in admission volumes has proved helpful in order to identify social pressures on workload (e.g. [19]). However, research restricted to global admission and consultation counts is insufficient in order to inform policy on local staff management; note that different patients receiving unrelated treatments over several medical disciplines put disparate workload pressures on specialist staff groups and grades; moreover, such pressures may vary drastically within distant geographical regions.

Hence, there is a need to employ modern embedded technologies for the design of effective management support systems that can target local facilities and specific working groups ([20]); examples of such work include [21] and [22]. In this paper, we explore the use of electronic task-management for the study of OoH workload in secondary facilities. The purpose is twofold; on one hand, to exhibit the additional value that such tasking data provides in combination with modern machine learning methodology, for supporting intelligent scheduling,

prioritisation and management of care delivery. On the other hand, to draw and validate conclusions with managerial relevance not restricted to the time-window and medical facilities covered in this study. We note that task-demand and completion numbers offer a better representation of workload, as opposed to admission or consultation counts; a lack of available sources has precluded previous quantitative studies of this kind.

To address these matters, we employ a state-space graphical model allowing for the extraction of patterns of cyclic variation in task-demand. This provides a useful framework to treat long series of observed multivariate counts, assuming independent observations conditionally on the values of a latent process. It allows to not underestimate true serial dependencies and control for discreteness and over-dispersion ([23]); moreover, link functions may allow for intuitive interpretations of covariates' effects (e.g. [24]). Thus, by means of a latent parameter-driven model, we show it is possible to draw inference on contemporary and serial correlations on demand, over different medical and surgical specialities within a local facility. We also exhibit the ability to quantify future demand pressures, and we compare results with approaches relying on common methodology. Finally, we offer a summary of relevant conclusions as scrutinized by local medical staff, consultants and nation-wide healthcare organizations.

The data collected for this purpose was gathered from two major university hospitals, which combined provide secondary healthcare to over 2.5 million residents in the United Kingdom. In both hospitals involved, tasks for the team are requested, assigned and managed via web and mobile device interfaces; and the data is collected at each stage of this process, allowing work-demand to be monitored and analysed. The dataset used for this work contained 652,585 task requests and covers the period from January 2012 to December 2015.

Main results in the paper identify shared characteristics of OoH workload and display a significant split between medical and surgical specialities. Also, strong serial dependencies in demand series and a fast-decreasing predictive power over increasing time-windows emphasize the importance of short-term scheduling and prioritisation. Moreover, results support claims relating to both the importance of data-driven local staffing and work-demand forecasts serving as a basis to intelligent scheduling support. Patient and administrative needs vary significantly according to the day of the week or shift; notably, weekend planning must account for the variation between medical and surgical wards and bank holidays need to be treated as weekends; yet, workforce should (subject to few exceptions) be similar all year-round.

The rest of the paper is organised as follows; in Section 2 we review literature on state-space models with applications in time-series analysis, and explain the model employed in this study. Section 3 offers a discussion on alternative approaches and related work within a medical context. A description and summary of the data is offered in Section 4. In Section 5 we report results in the paper, and finally we offer conclusions and discuss policy recommendations in Section 6.

2. State-space models for multivariate count data

State-space models are a class of probabilistic graphical models (see [25]) that have found applications in time-series analysis for supply chain planning [26], text and music analysis ([27]), econometrics ([28]) or political science ([29]), to name a few. In particular, they allow us to describe the dependence between continuous latent state variables and discrete observed counts in terms of some probabilistic distribution; hence, they pose a useful mean to relate electronic tasking information with patterns of workload in healthcare facilities.

These models are either observation or parameter driven, and can extend generalized linear models by incorporating latent autoregressive processes within the conditional mean function (cf. [24, 30]); thus introducing both auto-regression and over-dispersion. Less common multivariate extensions can handle both contemporary and serial correlations, and therefore tackle questions not addressed by marginal models (cf. [31, 32]). In general, these models employ dynamic factors or vector auto-regressions at the latent level, and various technical examples of this work can be found in ([28, 31–33]) and references therein.

Let $\boldsymbol{y}_t \in \mathbb{N}_0^n$ denote a vector of task counts as observed at day t; in our application, this reflects counts over n different medical and surgical specialities within the two hospitals. We now formulate a model such that $\boldsymbol{y}_t$ is drawn from a family of conditionally-independent Poisson distributions, such that

$$y_{t,i}|h_t, \boldsymbol{\mu}_t \sim \mathcal{P}(h_t \cdot \mu_{t,i}), \quad \text{for } i \in \{1, \cdots, n\}, \tag{1}$$

and

$$\log(\boldsymbol{\mu}_t) = \sum_j \boldsymbol{f}_j(t) + \boldsymbol{\nu}_t, \quad \text{for } t \in \{1, \cdots, T\}.$$

Here, $\boldsymbol{\mu}_t \in \mathbb{R}_+^n$ denotes a latent rate vector of hourly-tasks, and $h_t \in \{16, 24\}$ indicates the working hours of the day. Thus, we aim to capture the relation within daily workload and temporal or autoregressive patterns by means of a log-link function. In the following, we discuss the components of the model along with a simplified example for the univariate series of global counts in Figure 1.
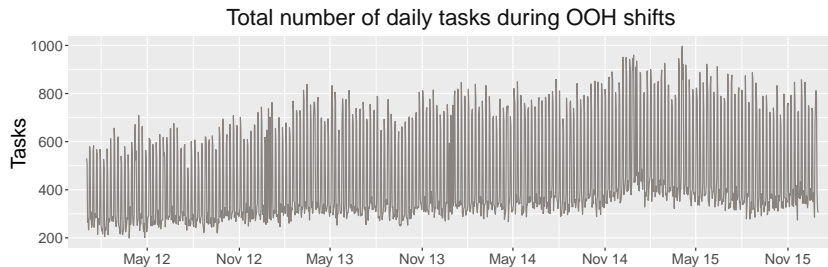

Figure 1: Joint time series of task-demand across both hospitals and specialities covered in the study.

4

2.1. Temporal trends in task-demand

The family $\{f_j(t) \in \mathbb{R}^n : j \geq 1\}_{t \geq 0}$ are vector sequences defined to capture cyclic trends of task-demand; we note that these are log-multiplicative factors scaling hourly-task rates and hence the expected demand in (1). Particularly, $f_1(t)$ includes $n \times 2$ coefficients β_1 that represent independent linear trends of workload over different specialities; for instance, in Figure 2 (Left) we observe a credible interval for a linear growing trend in global task-demand specific to the 4 years covered in this data set, there we notice an approximate 48% increase on demand. Next,

$$f_j(t) = \sum_{k=1}^{K_j} \beta_{j,k} \cdot \left(\cos\left(\frac{2\pi kt}{P_j}\right), \sin\left(\frac{2\pi kt}{P_j}\right) \right)', \quad j \in \{2, 3\}$$

are Fourier terms with periodicities $P_2 = 7$ and $P_3 = 365$. These are frequently employed (c.f. [28, 33]) in both frequentist and Bayesian settings in order to capture weekly and year-round patterns. It is noticeable in Figure 2 (center) that global OoH demand is highly influenced by the divide within weekday and weekend shifts (mostly explained due to further daylight hours during weekends); additionally, a mild yet significant decay is observed within the months of August and September (plot on the right).

Figure 2: Graphical summary of exponentiated scales on global hourly-demand, across both hospitals and specialities covered in the study. Credible intervals shown at a 95% level.

Also, $f_4(t)$ includes n coefficients β_4 allowing to further scale demand during Bank Holidays. In our worked example, this is a single parameter and its exponential has mean value 1.44. We finally note that

$$\beta = \{\beta_j ; j \in \{1, \cdots, 4\}\} \tag{2}$$

accounts for $n \times (3 + 2(K_2 + K_3))$ regression parameters.

2.2. Autoregressive trends in task-demand

In addition to seasonal trends, $\{\nu_t \in \mathbb{R}^n : t \geq 1\}$ defines a sequence of latent effects accommodating possible contemporaneous and serial dependencies

in counts, within a parameter driven framework. This is such that

$$\boldsymbol{\nu}_t \sim \mathcal{N}(\text{diag}(\boldsymbol{\delta}) \cdot \boldsymbol{\nu}_{t-1}, \Sigma), \quad \text{for } t \geq 2, \tag{3}$$

and $\boldsymbol{\nu}_1 \sim \mathcal{N}(\mathbf{0}, \Sigma)$. Thus, it defines a vector auto-regression with a basic serial dependence structure, controlling for likely epidemic departures in marginal workload trends. Here, Σ denotes a positive-definite covariance matrix incorporating correlation and over-dispersion within the model. Furthermore, in order to guarantee stationarity we impose $|\delta_i| < 1$ for all $i \in \{1, \cdots, n\}$, and note that matrix Θ with

$$\Theta_{i,j} = \Sigma_{i,j}/(1 - \delta_i \delta_j) \quad \text{for } i, j \in \{1, \cdots, n\}. \tag{4}$$

denotes the stationary covariance matrix in (3). In Figure 3, we observe the sequence of latent effects for global demand during 2012; note that in the univariate case, the above formulation reduces to an AR(1) model with no constant, single parameter and white noise. In the figure, we notice positive autoregressive dynamics; its factor has mean 0.79 with a 95% credible interval of $(0.74, 0.83)$.
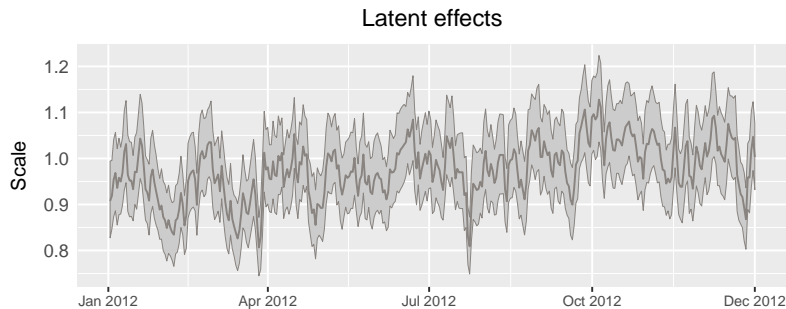

Figure 3: Series of latent effects in global hourly task-demand during 2012.

2.3. Graphical model, moments and parameter estimation

A graphical representation of the model in template notation is offered in Figure 4; in the case where $\boldsymbol{\delta} = \mathbf{0}$, the above formulation reduces to a variation of the model for correlated count data presented in [34]. Under this setting, it can be readily verified that

$$\hat{\boldsymbol{y}}_t := \mathbb{E}[\boldsymbol{y}_t | h_t, \boldsymbol{f}, \boldsymbol{\delta}, \Sigma] = h_t \cdot \exp\left(\frac{1}{2} \cdot \tilde{\Theta} + \sum_{j=1}^{4} \boldsymbol{f}_j(t)\right),$$

and

$$\text{Cov}(\boldsymbol{y}_t | h_t, \boldsymbol{f}, \boldsymbol{\delta}, \Sigma) = (\hat{\boldsymbol{y}}_t \cdot \hat{\boldsymbol{y}}_t') \circ (e^{\Theta} - 1) + \text{diag}(\hat{\boldsymbol{y}}_t),$$

for all $t \in \{1, \cdots, T\}$, where $\tilde{\Theta}$ stands for the entries in the main diagonal of Θ and $\circ$ denotes the Hadamard product. Thus, main moments can be approximated according to laws of iterated expectation, with samples from the full posterior distribution of autoregressive parameters, the covariance structure, and sequences

$$\{e^{f_j(t)} \; : \; j \in \{1, \cdots, 4\}, \; t \in \{1, \cdots, T\}\},$$

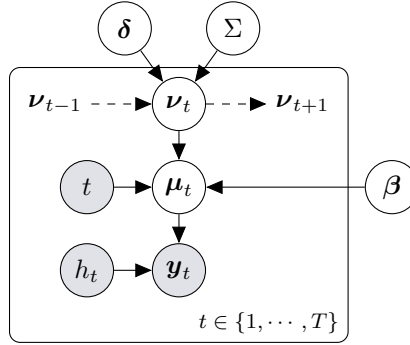which we recall are multiplicative scaling factors for $\boldsymbol{\mu}$ in (1).

Figure 4: Graphical representation of the relation within counts and the latent structure in the model.

In this work, the focus is given to (i) the study of distributional properties of model components, in order to unveil workload patterns during OoH shifts, and (ii) the assessment of the ability to quantify future task-demand, for providing support within intelligent management design. We note that analytical inference is intractable due to non-conjugacy between the exponential family and the normal latent process; thus, we resort to stochastic simulation and learning on latent states and parameters $\{\boldsymbol{\beta}, \boldsymbol{\delta}, \Sigma\}$ is carried out through Markov Chain Monte Carlo, employing Metropolis-Hastings (M-H) updates within blocked Gibbs steps. Starting values can be extracted ignoring the latent dynamic structure, and using standard optimization routines for classic likelihood-based estimation methods. Also, parameter priors for $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ in (2)-(3) are assumed uninformative, and a Wishart prior distribution is used for the precision matrix Σ^{-1} in (3). In summary,

a) Latent vector dynamics in $\boldsymbol{\nu}_t$ are updated by M-H, with target density

$$\pi(\boldsymbol{\nu}_t | \boldsymbol{\nu}_{\backslash t}, \boldsymbol{f}, \boldsymbol{\delta}, \Sigma) \propto \prod_{i=1}^{n} \exp\left(y_{t,i}\nu_{t,i} - h_t \exp\left(\sum_j f_{j,i}(t) + \nu_{t,i} \right) \right)$$

$$\cdot \exp\left(-\frac{1}{2}(\boldsymbol{\nu}_t - \boldsymbol{\mu})'\Omega^{-1}(\boldsymbol{\nu}_t - \boldsymbol{\mu}) \right),$$

for all $t \in \{2, \cdots, T-1\}$; cases $t \in \{1, T\}$ are straightforward. Here,

$$\Omega = \left[\Sigma^{-1} + d(\boldsymbol{\delta})\Sigma^{-1}d(\boldsymbol{\delta})\right]^{-1}, \quad \boldsymbol{\mu} = \Omega\left[\Sigma^{-1}d(\boldsymbol{\delta})\boldsymbol{\nu}_{t-1} + d(\boldsymbol{\delta})\Sigma^{-1}\boldsymbol{\nu}_{t+1}\right].$$

b) Parameters in $\boldsymbol{\beta}$ can be updated by M-H, with n independent blocks of medical speciality-specific regressors, each with target density

$$\pi(\boldsymbol{\beta}_{\cdot,i}|\boldsymbol{\nu}) \propto \prod_{t=1}^{T} \exp\left(y_{t,i}\sum_{j} f_{j,i}(t) - h_t \exp\left(\sum_{j} f_{j,i}(t) + \nu_{t,i}\right)\right),$$

for all $i \in \{1, \cdots, n\}$

c) The precision matrix Σ^{-1} is sampled from a Wishart distribution, such that

$$\Sigma^{-1}|\boldsymbol{\nu}, \boldsymbol{\delta} \sim \mathcal{W}_n(m, V),$$

with $m = m_0 + T$ degrees of freedom, and scale matrix

$$V = \left(V_0^{-1} + \boldsymbol{\nu}_1\boldsymbol{\nu}_1' + \sum_{t=1}^{T-1}(\boldsymbol{\nu}_{t+1} - d(\boldsymbol{\delta}) \cdot \boldsymbol{\nu}_t)(\boldsymbol{\nu}_{t+1} - d(\boldsymbol{\delta}) \cdot \boldsymbol{\nu}_t)'\right)^{-1}.$$

Here, m_0 and V_0 are the prior degrees of freedom and scale matrix for Σ^{-1}, respectively.

d) Finally, the vector of marginal autoregressive terms $\boldsymbol{\delta}$ is truncated normal, and may be updated so that

$$\boldsymbol{\delta}|\boldsymbol{\nu}, \Sigma \sim \mathbb{I}_{(-\mathbf{1},\mathbf{1})} \cdot \mathcal{N}(\boldsymbol{\mu}, \Omega),$$

where,

$$\Omega = \left[\sum_{t=2}^{T} d(\boldsymbol{\nu}_{t-1})\Sigma^{-1}d(\boldsymbol{\nu}_{t-1})\right]^{-1}, \quad \boldsymbol{\mu} = \Omega \cdot \sum_{t=2}^{T} d(\boldsymbol{\nu}_{t-1})\Sigma^{-1}\boldsymbol{\nu}_t.$$

In this paper, we omit cumbersome details regarding proposal distributions in M-H steps, and we resort to the general purpose JAGS sampler ([35]) for simplicity.

3. Related work

Time series of count data are common in studies with diverse medical applications (for instance, see [36, 37] and references therein); hence, Bayesian modelling of such series has been a subject of previous research. Common scalable approaches generally ignore either serial or contemporaneous correlation structures within data; and include standard log-linear models, Gaussian

autoregressive approximations and observation-driven state space models. In this work, we will employ univariate Poisson regressions and observation-driven Poisson autoregressive models as a basis for drawing predictive comparisons and discussing congruency in results within Section 5. For references describing these methods, we refer the reader to [24, 33] or [37].

In summary, Poisson or log-linear regression models allow to model serially uncorrelated count data and will thus permit drawing comparisons with predictive results over large horizons. On the other hand, observation-driven specifications for Poisson autoregressive models offer straightforward and efficient ways to draw inference over short-horizons ([24, 38]). These benefit from easy to calculate likelihoods; however, stationarity and ergodicity properties are hard to derive, and they most importantly suffer from a lack of interpretability on covariates when compared to the parameter-driven alternative discussed in Section 2.

4. Data

The recently deployed electronic task-management system discussed in this paper allows a *senior nurse coordinator* to triage requests for clinical review and intervention over a team of doctors and clinical support workers; all within the *Hospital at Night* secondary-care setting in the UK. We note that different organizational models are used for providing OoH care internationally ([39]), and we refer the reader to [40, 41] for information on the setting and research in relation to the source of our data.

In brief, sourced data includes task requests outside the 9:00-17:00 Monday to Friday *in hours* setting, and incorporates all hours during Bank Holidays. Data belongs to two jointly coordinated hospitals used within the study; therefore, we have complementary sets offering precise information on the OoH secondary healthcare demand within a geographical region with over 2.5 million residents. Table 1 offers a summary on capacity, total workload and average staff on both sites. In this data set, we observe over 40 different kinds of task requests; most common observations include *cannulations, drug prescriptions, clinical reviews and management* or *blood results interpretations*, each accounting for over 10% of total demand. Also, we notice reasonable amounts of workload in relation to *fluids prescriptions, clerking, X-ray requests/reviews or blood test requests*.

We find a total of 652,585 entries ranging from January 2012 to December 2015, and with each of them we obtain attached information including request and completion times, associated ward and medical or surgical speciality, urgency level and assigned staff group. This constitutes a very detailed and complete set of data, rare within management and knowledge engineering studies of this kind. For the study, we isolate requests on the most relevant 14 specialities observed. Table 2 offers an overview of counts over different categories. There, we notice a divergence within median and mean values, along with a high spread in counts; this is due to varying numbers of working hours during different days of the week.

	Average staff in shift (Equivalent in both sites)			
	Senior doctors	Junior doctors	CSW	Nurse coord.
5pm (9am) - 10pm	1	6	1	1
10pm - 9am	1	4	1	1

	Bed capacity	Overall tasks in site			
		2012	2013	2014	2015
Hospital 1	1300	57869	76461	78349	78320
Hospital 2	1100	78353	81862	96229	105142

Table 1: OoH staff levels, capacity and overall workload in both Hospitals; note that day shifts during Weekends begin at 9am.

	Min	Median	Max	Mean & SD
Cardiology	1	17	58	19.29 (9.26)
Clinical haematology	4	18	59	18.44 (6.61)
Clinical oncology	5	23	79	25.94 (11.86)
Diabetes	5	22	102	30.94 (20.91)
General surgery	15	66	213	80.36 (38.37)
Care for the older people	10	37	170	54.72 (37.07)
Nephrology and transplants	4	19	72	21.27 (9.76)
Plastic surgery	0	9	36	10.14 (5.54)
Respiratory medicine	14	40	134	50.9 (26.21)
Rheumatology	0	10	68	14.26 (11.34)
Stroke assessment	4	25	85	28.97 (14.6)
Trauma and orthopaedic	1	21	71	23.71 (12.09)
Urology	0	9	42	10.03 (5.71)
Vascular surgery	0	11	57	12.44 (8.25)

Table 2: Summary of daily counts in different disciplines, for January 2012 to December 2015. Within parentheses we find standard deviation values.

5. Results

Results summarized in this work are obtained from a multivariate sample of 80,000 draws; 20,000 from each of the 4 independent chains run in parallel, all with a 100,000 burn-in phase and showing no significant pairwise correlation within parameters. In this case, $K_2 = 3$ and $K_3 = 5$, and starting values are identical across chains; however, different seeds are used within parallel streams of generated random numbers (for details, see [42]). For predictive purposes, the various models are fitted to data from 2012 to 2014, and results are reported for out-of-sample data.

In the following, we display results that expose the additive value of such electronic tasking information within the local facilities targeted in this study, for the purposes of understanding staff behaviour and managing the delivery of care. Additionally, we summarize conclusions that generalize in time and context, as surveyed among medical consultants across nation-wide healthcare institutions.

5.1. Periodic patterns, speciality split and bank holiday effect

In Figure 5 we observe 90% credible intervals for weekly scaling factors on work-demand, over different medical and surgical specialities; these represent scales on estimated hourly rates of task generation within the studied workplace. There, we observe varying levels of significant departures from average demand in most specialities, specially over weekends.
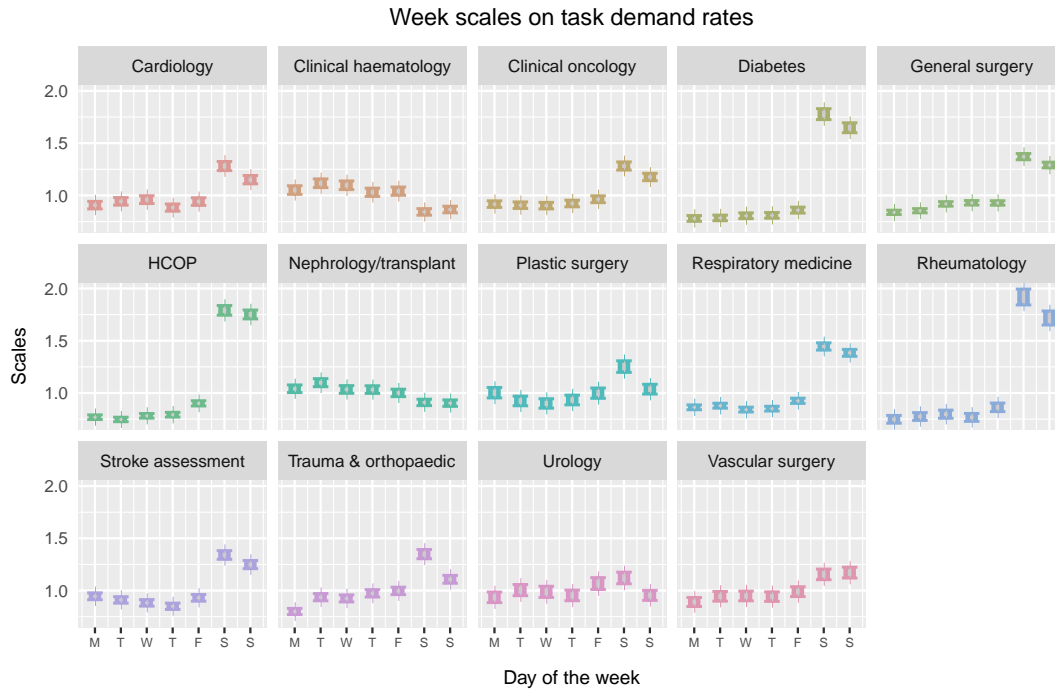

Figure 5: Credible intervals on weekly task-demand scaling factors accross different specialties.

An increase in OoH workload outside business days is mostly triggered by higher proportions of daylight hours; along with sets of leftover duties and routine care or ward round needs usually performed within the *in hours* setting during working days. Also, the rise in demand is generally more significant on Saturdays; partly, doctors on weekend shifts become familiar with patients and determine needs for treatment alterations during this day; also, Sunday duties

11

may be left for day teams. In general, results sustain claims suggesting that (i) staff behaviour varies according to different shifts/weekdays and (ii) workload is proportionally larger during daylight shifts (see [43] for a related discussion)

However, the variation within business days and weekends is highly dependent on speciality; in this regard, we notice a first split within medical and surgical disciplines. In our study, certain specialities such as *haematology* or *nephrology* have external consultants performing ward rounds of their inpatients over weekends, likely explaining some lack of week-round variation in demand. In addition, surgical specialities depend on independent teams dealing with some workload internally, thus not always reporting to centralized management teams. Nevertheless, treatments in surgical specialities are in general more elective (cf. [44, 45]), and thus allow for better planning; this explains most lack of variation within weekly workload. Extensive research (see [46, 47] and references therein) has raised global concerns regarding a lack of sufficient staff during OoH shifts over weekends; for instance, mortality rates are known to be proportionally higher. In view of this case study, we notice the ability of electronic tasking-data in order to quantify ward and facility specific needs; also, it is arguable that global weekend staffing should be discipline dependent.

In Figure 6 we observe 90% credible intervals for bank holiday scaling factors, these are further applied when estimating task-demand during holidays. The similarity between weekend and bank holiday effects is noticeable and not characteristic to this case study (cf. [48]); hence, similar considerations should be made when prioritizing demand and designing rotas.
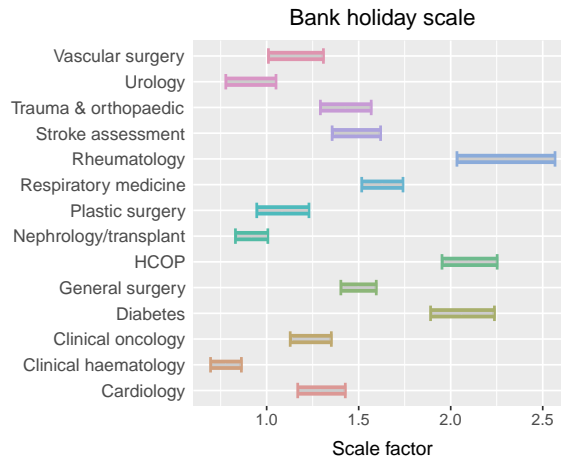

Figure 6: Credible intervals on Bank Holiday task-demand scaling factors accross different specialties

Next, we observe %90 credible intervals for year-round patterns of OoH task-demand in Figure 7. There, we again notice diverse levels of vulnerability to seasonality across specialities. The impact is of a lesser relevance to that in
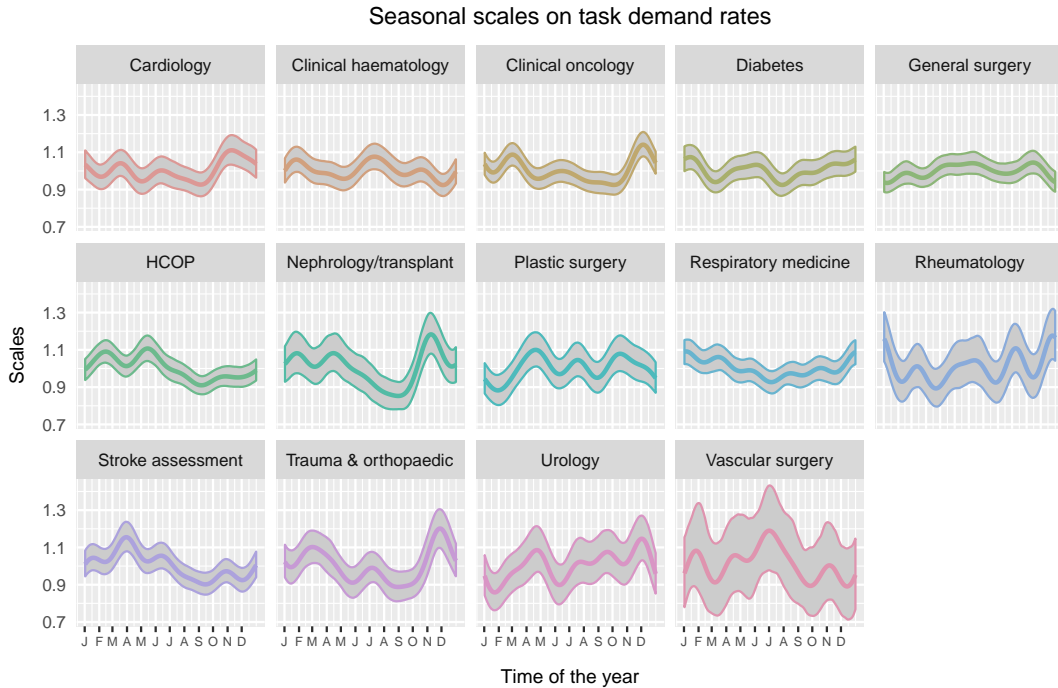
Figure 7: Credible intervals on year-round task-demand scaling factors accross different specialties.

weekly scales; with exceptions such as *nephrology*, susceptible to suffer significant decreases in workload over summer-time, yet uniformly spread over both working and weekend days. There also exist predictable drops in summer workload within specialities where treatment is susceptible to being deferred by patients, such as *health care for the older people* or *diabetes*; and significant spikes are observed in categories where workload is vulnerable to admission levels or epidemic trends in certain illnesses. In general, the impact of such seasonal variation is mild (cf. [49]) when compared to day-to-day workload; hence, staffing levels must be reasonably balanced all year round, with small exceptions and a clear basis on the split within elective (mostly surgical) and non-elective (mostly medical) disciplines. The elective nature of many surgical operations allows for efficient scheduling; hence, it is likely to cause workload alterations aiming to balance year-round stress within the system. Table 3 offers an overview of mean absolute deviations in seasonality scales, around a central tendency of 1. These statistics allow for direct comparison within disciplines, and manifest the aforementioned variability split within specialities, with the noticeable exception of *general surgery*.

Finally, we note that seasonal effects on demand are likely to depend on the geographical location of each medical facilitiy; thus, studying localized solutions

	Year deviation	Week deviation
Cardiology	0.054 (0.011)	0.11 (0.009)
Clinical haematology	0.044 (0.009)	0.09 (0.008)
Clinical oncology	0.055 (0.008)	0.12 (0.009)
Diabetes	0.046 (0.010)	0.34 (0.009)
General surgery	0.036 (0.008)	0.17 (0.006)
Care for the older people	0.056 (0.009)	0.36 (0.008)
Nephrology and transplants	0.088 (0.017)	0.06 (0.009)
Plastic surgery	0.063 (0.013)	0.08 (0.011)
Respiratory medicine	0.046 (0.009)	0.21 (0.007)
Rheumatology	0.079 (0.015)	0.38 (0.014)
Stroke assessment	0.067 (0.011)	0.15 (0.009)
Trauma and orthopaedic	0.084 (0.014)	0.12 (0.009)
Urology	0.082 (0.017)	0.06 (0.011)
Vascular surgery	0.117 (0.025)	0.09 (0.012)

Table 3: Mean absolute deviations in seasonalities. Standard deviations in parentheses are obtained across samples.

for demand estimation is of special relevance with aims to design intelligent rota schedules. Thus, we again notice that electronic tasking provides the means for personalizing the study of workload patterns to meet medical facility or ward needs.

5.2. Serial dependencies and uncorrelated disciplines

In addition to periodic patterns, we analyse results on temporal and contemporaneous relations within task-demand counts. Figure 8 shows %90 credible intervals for autoregressive values $\boldsymbol{\delta}$ in (3), which measure latent serial dependencies on marginal trends of task-demand; there, we observe significant positive dependencies across all disciplines.

Such relation in marginal counts reflects the existence of temporally persistent departures from general trends of expected workload; and emphasizes the importance of short-term planning. Departures may reflect several things; on one hand, the presence of traditional epidemics and associated workload in a medical context, on the other hand, the impact that *in hours* staffing policy changes exercises on OoH workload. Note that an increase on team sizes during *in hours* shifts will likely cause a significant decrease in subsequent workload during OoH; on the contrary, downsizing leads to a higher demand in tasks. The inclusion of such autoregressive effects in the model allows to capture these phenomena, hence reducing the bias on inferred seasonality patterns of demand over different categories. Yet, we notice that high serial dependencies observed in Figure 8 may lead to wide intervals of confidence on seasonal patterns; such as in *vascular surgery* as seen in Figure 7.

Also, Table 4 offers an overview of the latent stationary correlation matrix obtained from (4). We observe a rather homogeneous structure of mostly

Figure 8: %90 Credible intervals for autoregressive parameters in vector $\boldsymbol{\delta}$.

unrelated pairs across all disciplines, with few exceptions showing very weak contemporaneous dependencies in workload. This suggests it is possible to employ scalable independent forecasts over different disciplines within the studied hospital facilities; yet, we note these relations may not be necessarily meaningful in the general context of OoH work, but rather respond to work management characteristics within the sites and team analysed in this study.

However, general daily counts of task-demand are highly correlated across disciplines; in view of these results, we conclude this is solely due to similarities on seasonal effects across groups of categories. Thus, alterations on staff numbers should respond to expected cyclic variations, along with spikes in workload within specific disciplines; yet, these must remain unaltered in view of workload trends within different specialities.

5.3. Predictability of local demand and scheduling horizons

There exist concerns regarding a growing need for data-driven methods serving as a basis to intelligent rota design; an open debate on the topic can for example be found in [50–52] and references therein. In this contribution, we discuss the ability of tasking data and forecasting methods for supporting local and ward level scheduling, prioritisation and management of care delivery.

Under the present framework, we note that it is possible to draw inference on expected work-demand across varied time-horizons. For that matter, either deterministic integration or simulation based forward filtering-procedures may be employed (cf. [26, 28]); thus tracking the evolution of latent dynamics $\boldsymbol{\nu}$ in (3). In both top plots in Figure 9 we observe the evolution of task-demand for disciplines *general surgery* and *rheumatology*; there, green bars represent out-of-sample demand during the beginning of year 2015. Additionally, superimposed gray lines represent point estimates for expected demand as inferred

15

	ClHae	ClOnc	Diabe	GeSur	HCOP	Nephr	PlSur	ReMed	Rheum	StAss	Traum	Urolo	VaSur
Cardi	0.059	-0.013	0.035	0.038	0.049	0.033	0.047	-0.002	0.000	0.003	0.031	0.066	0.011
	(0.038)	(0.038)	(0.038)	(0.036)	(0.037)	(0.038)	(0.041)	(0.037)	(0.040)	(0.039)	(0.037)	(0.041)	(0.035)
ClHae		0.068	0.016	0.001	0.033	0.055	-0.024	0.013	0.017	0.028	-0.006	0.008	-0.027
		(0.039)	(0.037)	(0.034)	(0.036)	(0.037)	(0.041)	(0.035)	(0.038)	(0.038)	(0.036)	(0.041)	(0.032)
ClOnc			-0.009	-0.009	-0.036	0.013	0.038	-0.020	0.007	0.010	0.001	-0.053	-0.004
			(0.037)	(0.034)	(0.036)	(0.035)	(0.040)	(0.035)	(0.036)	(0.037)	(0.036)	(0.040)	(0.032)
Diabe				0.065	0.066	-0.026	-0.067	**0.083**	**0.081**	0.014	0.019	-0.001	0.068
				(0.034)	(0.036)	(0.037)	(0.039)	(0.035)	(0.037)	(0.037)	(0.038)	(0.040)	(0.035)
GeSur					**0.080**	-0.029	-0.053	0.042	**0.071**	-0.034	**0.083**	0.003	**0.095**
					(0.033)	(0.035)	(0.036)	(0.034)	(0.035)	(0.034)	(0.034)	(0.036)	(0.033)
HCOP						0.033	-0.014	0.009	0.058	-0.001	0.043	0.022	0.021
						(0.036)	(0.039)	(0.035)	(0.036)	(0.035)	(0.035)	(0.037)	(0.033)
Nephr							0.075	-0.048	-0.034	0.009	0.028	**0.094**	-0.030
							(0.040)	(0.037)	(0.038)	(0.038)	(0.037)	(0.039)	(0.039)
PlSur								-0.036	-0.056	0.002	0.005	**0.204**	-0.055
								(0.037)	(0.042)	(0.040)	(0.040)	(0.041)	(0.038)
ReMed									0.020	0.016	-0.046	-0.041	0.061
									(0.037)	(0.036)	(0.035)	(0.038)	(0.035)
Rheum										0.023	0.032	0.003	-0.005
										(0.039)	(0.041)	(0.042)	(0.038)
StAss											-0.001	0.017	-0.035
											(0.036)	(0.040)	(0.033)
Traum												0.011	-0.027
												(0.040)	(0.037)
Urolo													-0.060
													(0.036)

Table 4: Latent stationary correlation structure obtained from (4). Standard deviations shown within parentheses are computed across samples. Significant correlations at a 5% confidence level are shown in bold numbers.

from the model in Section 2; predictions over out-of-sample demand correspond to estimates obtained right at the end of 2014. Finally, orange lines represent 1-day ahead predictions obtained from dynamically updating the distribution of the latent process $\boldsymbol{\nu}$, and represent work-demand predictions over the shortest possible horizon.

On the other hand, the two bottom plots in Figure 9 present the same evolution of task-demand; yet, superimposed lines offer estimates of task-demand obtained from (i) a Poisson regression model (gray lines) and (ii) an observation-driven Poisson AR(1) model (orange lines, 1-day ahead predictions). Furthermore, Table 5 offers varied measures of predictive accuracy for different fits within the full year 2015 and across all specialities in the study; these include correlation coefficients within estimations and observations, root relative squared errors and relative absolute errors. In order to compute error terms, demand observations and predictions are standardised across medical and surgical categories, thus removing the effect of severe variations in global trends of demand over different specialities.

We notice the ability of tasking information in order to produce reasonable estimates on future demand; most importantly, we recall that the granularity of such information does not only allow to draw facility-level inference on expected demand, but can also marginalise over sets of specific tasks within precise wards
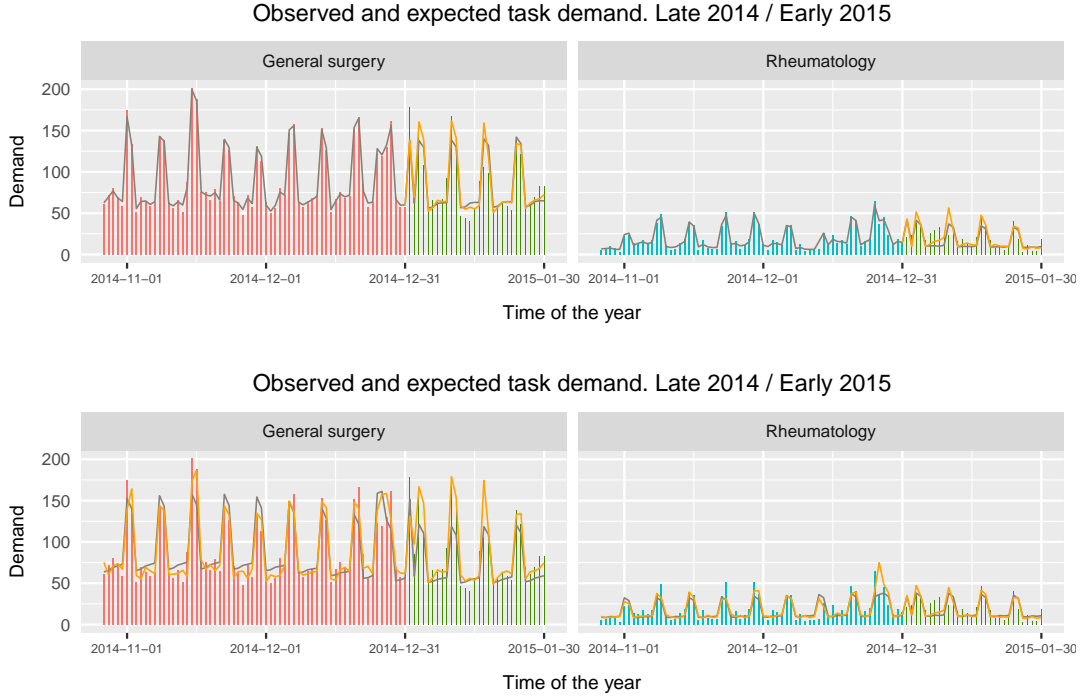
Figure 9: Subset of the evolution of task-demand within disciplines *general surgery* and *rheumatology*, green bars represent out-of-sample demand. In top figures, gray lines show estimate predictions drawn at the end of 2014, orange lines are 1-day ahead preditions. In bottom figures, gray lines represent a Poisson regression fit, orange lines 1-day ahead observation-driven AR-Poisson estimates.

	Cor. coef.	rRSE	RAE
1 day ahead	0.7409	71.98%	64.76%
2 days ahead	0.7113	75.98%	67.90%
Since end of 2014	0.6932	78.33%	69.97%
Obs-driven AR(1), 1 day ahead	0.7341	72.92%	65.52%
Univ. Poisson regression	0.6837	79.53%	71.43%

Table 5: Measures of predictive accuracy for various fits in out-of-sample data.

covering particular specialities. In addition, estimates are consistent across models employed and predictive horizons; nevertheless, strong serial dependencies previously observed in Figure 8 force a fast-decreasing predictive power over increasing horizons; again, emphasizing the relevance of short-term scheduling and redesigning of staffing levels. Figure 10 shows 95% credible intervals on the evolution of dynamics $\boldsymbol{\nu}$ in (3) across *general surgery* and *rheumatology*, and

provides additional evidence regarding the loss of information caused by forecasting over large horizons. There, gray intervals represent the MCMC output accounting for data up to the end of year 2014; on the other hand, orange intervals offer 1-day ahead filtered values accounting for data up to the day before each estimate is drawn.
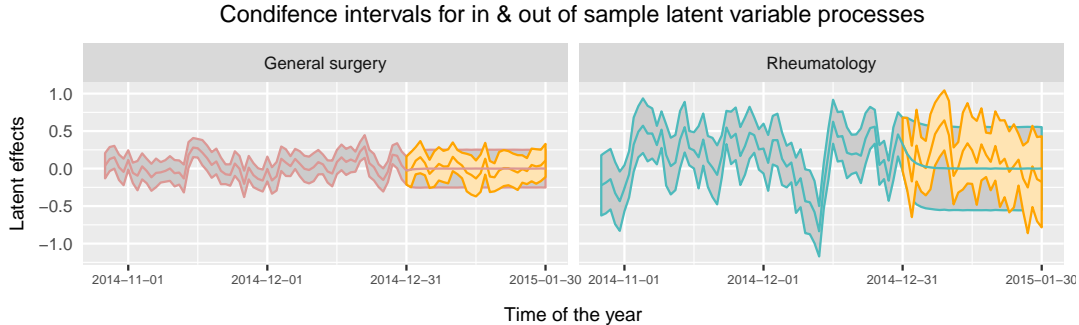

Figure 10: Credible intervals on the evolution of process ν for *general surgery* and *rheumatology*. Orange intervals represent 1-day ahead filtered processes; gray intervals provide MCMC output with data up to 2014.

Finally, we notice that real-world applications require constant updates in model fits for prediction. For instance, global growing demand previously observed in Figure 2 is characteristic to the time-interval analysed, and it is likely to die-out in the long term. Thus, weighted composite-likelihoods or reduced data sets should be employed, with aims to weight recent observations heavily.

6. Discussion

The work in this paper has explored the use of electronic task-management alternatives in combination with modern statistical and machine learning methodology, in order to study the potential for contributions in the design of decision support systems for OoH workload management in local healthcare facilities. For that matter, the work has examined the general impact of work-demand patterns and contemporaneous dependencies within two major hospitals over a 4-year time-window; and has drawn conclusions contributing to open debates discussing needs for intelligent data-driven rotas, speciality-specific staffing variations and the extent of workload disparity within different shifts or working days.

Major contributions in the paper have exhibited the capacity of such tasking data, for providing means to support intelligent data-driven methods for scheduling, prioritisation and management of care delivery. Replacing traditional pager-oriented methods for digital alternatives based on web and mobile interfaces provides means to collect information on workload that has the power of being specific to each ward, speciality, working group or task-type. Also,

quantifying demand variability over local facilities may allow for the study of suspected relations with cyclic patterns of staff sickness, norovirus effects on wards, lack of social beds and traditional winter pressures ([53, 54]).

Additionally, scrutinized results in this work have validated several conclusions with practical relevance not restricted to the facilities in the study. In summary,

- Levels of predictable variation in workload and with disparate characteristics within medical and surgical disciplines suggest such split should serve as a basis for weekend rota scheduling.

- Year-round staffing levels should be unaltered. In this regard, imbalances are either mild or statistically not significant, subject to few exceptions.

- Sharp variation within weekday, weekend and bank holiday demand poses a clear argument regarding patients and staff behaving differently according to the day or shift. Also, bank holidays need to be treated as weekends.

- The existence of significant serial relations in task-demand yield fast-decreasing predictive powers over growing horizons; this emphasizes the need for day-to-day organization and consideration of expected workload.

Finally, a technical limitation in the present modelling approach relates to the latent vector auto-regression structure, which could increase in complexity in order to accommodate a higher order or incorporate serial cross-dependencies within categories, yet increasing the computational requirements in such a high-dimensional problem. Also, it would be possible to further relax the restriction on the dispersion of counts using a negative binomial distribution (cf. [28]); or attempt to model contemporaneous correlations employing copulas ([31]). Additionally, while we have informed on the impact of seasonality, in order to draw inference on triggering factors (such as admission numbers or care complexity) hospital occupancy information is required.

In conclusion, the work presents a valuable working framework for the study of workload within secondary healthcare institutions during Out of Hours. For this matter, the paper resorts to tasking information from an electronic management system, and results contribute to the quantitative study of care fragmentation, scheduling and team management in healthcare (cf. [17–19]).

Acknowledgements

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