

# Integrated Output Filter Inductor for Permanent Magnet Motor Drives

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**Abstract** – This paper presents a novel approach to integrate the output filter inductor in permanent magnet synchronous motor (PMSM) drive system. The integrated output filter inductor is based on utilizing the motor magnetics as a filter inductance instead of introducing a separate filter inductor. Thus, eliminating added filter inductor losses and associated weight and volume. The vector controlled model, taking modulation and switching effect in to account, has been developed using MATLAB/Simulink tool for the proposed integrated output filter inductor. The currents obtained from MATLAB/Simulink model are then injected into the Finite Element model to validate the concept. The performance of the proposed and conventional system is analyzed in terms of mean electromagnetic torque, torque ripple, motor losses, inductor losses, weight and volume.

**Index Terms**-- EE Core Inductor, Integrated Inductor, Integration of Passives, Finite Element Analysis (FEA)

## I. NOMENCLATURE

$K_f$	Filter branch factor
$K_m$	Motor branch factor ( $1 - K_f$ )
$V_c$	Converter voltage
$i_1$	Filter branch current
$i_2$	Motor branch current
$\omega_e$	Electrical input frequency
$\omega_r$	Mechanical rotor speed
$T_{em}$	Electromagnetic torque
$T_{load}$	Load torque
$J$	Motor inertia
$R_m$	Motor winding resistance
$R_f$	Damping resistance
$R_1$	Filter branch resistance
$R_2$	Motor branch resistance
$C_f$	Filter capacitance
$L_m$	Motor winding inductance
$L_1$	Filter branch inductance
$L_2$	Motor branch inductance
$X_{L1}$	Filter branch inductive reactance
$X_{L2}$	Motor branch inductive reactance
$X_c$	Filter capacitive reactance
$\theta_e$	Electrical rotor position w.r.t stator phase ( $\omega_e t$ )
$\psi_1$	No-load flux linkage of filter branch ( $K_f \psi_m$ )
$\psi_2$	No-load flux linkage of motor branch ( $K_m \psi_m$ )
$E_1$	Filter branch back-EMF ( $K_f \omega_e \psi_m$ )
$E_2$	Motor branch back-EMF ( $K_m \omega_e \psi_m$ )
$p$	No. of pole pairs
$\alpha$	Phase shift between $i_1$ and $i_2$

## II. INTRODUCTION

THE passive components associated with the drive system such as capacitors, inductors or transformers occupy a significant amount of space and have the added penalties of potential higher weight and loss. Traditionally, filters are designed and introduced after the drive system components have been defined which results in a discrete sub-system for passives. The integrated drives have been a focus of research in power electronics industry. There are different possibilities in drive system to integrate the passive elements. The use of integration in drive system allows functional and structural integration of different drive components into one component that result in a compact design, reduced cost, mass and size reduction, and fewer manufacturing process. Therefore, applications where high power density is needed, an alternative technology for integration have to be looked for [1].

A novel integrated output filter inductor is proposed, which uses the existing motor magnetics as a filter inductance while eliminating the need of a separate filter inductor. The mathematical model of the proposed concept is validated using MATLAB/Simulink. The MATLAB/Simulink simulation results with and without integrated filter are presented and using these simulated results, the operation of the motor with integrated filter is verified through finite element analysis (FEA).

## III. LITERATURE REVIEW

### A. Integrated Filter Inductor Designs

In recent times, the passive integration has been focused in power electronics and drive applications that have resulted in an overall compact and power dense system. In [1-4], the perspective on the integrated filter inductors are presented that motivates the drive integration on a system level. The design of integrated filter inductor for power factor correction application is presented in [1]. The paper modified the stator laminations to increase the stator back iron, which act as an integrated filter inductor. This modification increases the outer diameter of the motor.

The stator back iron is used as a magnetic component by integrating toroidal winding which drives alternating magnetic flux in the complete loops through the back iron of the stator core. If the stator back iron of the machine is operating in linear region then the presence of ring flux due to inductor winding will not affect the main machine flux [2-3].

The principle of electromagnetic integration is used for integrating capacitor in the same magnetic component as that of the inductor. ‘‘The integrated filter is the planar integrated L-C winding, which consists of a dielectric substrate with conductor windings directly deposited on both sides. Thus, resulting in a distributed inductance and capacitance structure. Moreover, different equivalent circuits have been achieved by connecting the terminals of integrated LC structure in an appropriate manner’’ [4-9].

The same principle of [4-9] is applied for C-core EMI inductor in [11-12]. ‘‘The distributed capacitance is implemented in the conventional way, whereas the inductor is implemented by utilizing the cathode and anode foils of the capacitor to form the windings. The windings are then enclosed in a can, which has a hole in the middle for the magnetic core’’.

#### IV. NOVEL INTEGRATED OUTPUT FILTER INDUCTOR

The motor with integrated output filter inductor is based on utilizing the motor winding inductance as a filter inductance, instead of introducing a separate filter inductor. Fig. 1(a) and Fig. 1(b) show both conventional and integrated drive system respectively.

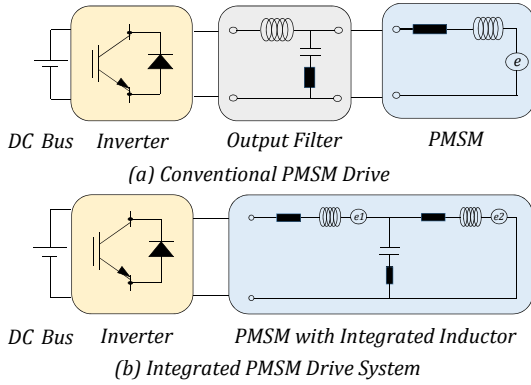


Fig. 1: Conventional and integrated PMSM drive system

It can be seen that the proposed motor with integrated inductor does not include a separate inductor, thus eliminating added filter inductor losses and associated weight and size. Filter capacitor and damping resistor are placed between the motor windings to form an integrated RLC output filter using inductance of motor winding.

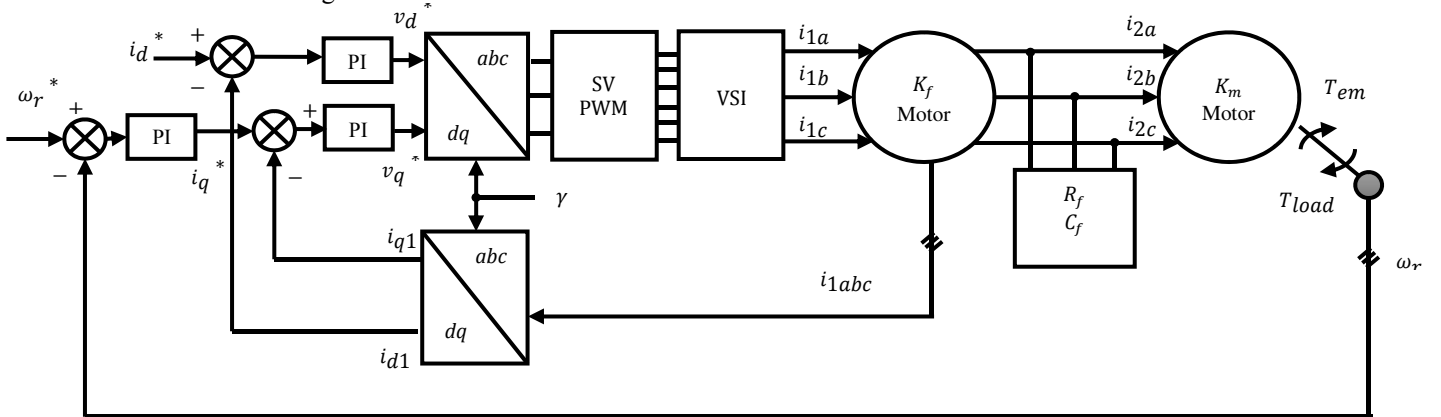


Fig. 2: Control block diagram of the proposed motor with integrated inductor

The value of the inductance can be varied by - varying proportion of motor winding used as filter and varying the capacitance of the integrated RLC filter for the same cut-off frequency. The motor is divided into two branches; filter branch and the motor branch. The motor branch windings will see the filtered sinusoidal currents, whereas PWM switching currents will flow through the filter branch winding, which is the fraction of the motor winding.

#### A. Vector Control Modelling

A detailed vector control model of the proposed motor with integrated inductor, taking switching effect and modulation into account, has been developed using MATLAB/Simulink tool. The control system block diagram of the motor with integrated inductor is shown in Fig. 2. 12 slots, 10 poles surface mounted permanent magnet motor is considered. The motor drive specifications are shown in Table I. The motor is operating at a speed of 2100rpm and load torque of 1.5Nm with corresponding peak current of 5.85A. The cross sectional view of the motor with integrated inductor is shown in Fig. 3.

TABLE I: MOTOR-DRIVE SPECIFICATIONS AND OPERATING CONDITIONS

PARAMETERS	VALUES	UNITS
DC Bus Voltage	200	V
Fundamental Frequency	0.175	kHz
Switching Frequency	10	
No. of Turns per phase	2×46	-
Operating Torque	1.5	Nm
Operating Speed	2100	RPM
Phase Resistance	2×0.21	Ω
Phase Inductance	2×1.29	mH
Peak Current	5.85	A

The mathematical model is based on abc reference frame and corresponding abc currents are transformed into d-q reference frame for controlling purpose. The filter branch currents are controlled to produce maximum torque of the filter branch. The filter capacitor and damping resistor are connected in between filter and motor branch. It should be noted that, inserting filter capacitance and resistance between the motor winding will introduce a phase shift between two branch currents, i.e.  $i_{1abc}$  and  $i_{2abc}$ .

The mathematical equations needed to model the proposed motor with integrated inductor can be obtained by using per phase equivalent circuit. The per-phase equivalent circuit of the motor with integrated inductor is shown in Fig. 4(a).

By applying KVL on filter branch we have,

$$V_{1a}(t) = R_1 i_{1a} + L_1 \frac{d}{dt} i_{1a} + \frac{d}{dt} \psi_1 \cos \theta_e + \frac{1}{C_f} \int (i_{1a} - i_{2a}) dt + R_f (i_{1a} - i_{2a}) \quad (1)$$

Converting Eq. 1 into s-domain by taking Laplace transformation on both sides we have,

$$V_{1a}(s) = R_1 i_{1a} + L_1 i_{1a} s - \omega_e \psi_1 \sin \theta_e + \frac{1}{C_f s} (i_{1a} - i_{2a}) + R_f (i_{1a} - i_{2a}) \quad (2)$$

$$\text{Solving for } i_{1a} = \frac{1}{sL_1} \left( V_{ca} + \omega_e \psi_1 \sin \theta_e - R_1 i_{1a} - R_f i_{1a} + R_f i_{2a} - \frac{1}{sC_f} i_{1a} + \frac{1}{sC_f} i_{2a} \right) \quad (3)$$

The current of phase b and c with 120° phase shifts are,

$$i_{1b} = \frac{1}{sL_1} \left( V_{cb} + \omega_e \psi_1 \sin \left( \theta_e + \frac{2\pi}{3} \right) - R_1 i_{1b} - R_f i_{1b} + R_f i_{2b} - \frac{1}{sC_f} i_{1b} + \frac{1}{sC_f} i_{2b} \right) \quad (4)$$

$$i_{1c} = \frac{1}{sL_1} \left( V_{cc} + \omega_e \psi_1 \sin \left( \theta_e + \frac{2\pi}{3} \right) - R_1 i_{1c} - R_f i_{1c} + R_f i_{2c} - \frac{1}{sC_f} i_{1c} + \frac{1}{sC_f} i_{2c} \right) \quad (5)$$

Similarly for the motor branch we have,

$$0 = R_2 i_{2a} + L_2 \frac{d}{dt} i_{2a} + \frac{d}{dt} \psi_2 \cos \theta_e + \frac{1}{C_f} \int (i_{2a} - i_{1a}) dt + R_f (i_{2a} - i_{1a}) \quad (6)$$

Converting Eq. 6 into s-domain by taking Laplace transformation on both sides we have,

$$0 = R_2 i_{2a} + L_2 i_{2a} s - \omega_e \psi_2 \sin \theta_e + \frac{1}{C_f s} (i_{2a} - i_{1a}) + R_f (i_{2a} - i_{1a}) \quad (7)$$

$$\text{Solving for } i_{2a} = \frac{1}{sL_2} \left( 0 + \omega_e \psi_2 \sin \theta_e - R_2 i_{2a} - R_f i_{2a} + R_f i_{1a} - \frac{1}{sC_f} i_{2a} + \frac{1}{sC_f} i_{1a} \right) \quad (8)$$

The current of phase b and c with 120° phase shifts are,

$$i_{2b} = \frac{1}{sL_2} \left( 0 + \omega_e \psi_2 \sin \left( \theta_e + \frac{2\pi}{3} \right) - R_2 i_{2b} - R_f i_{2b} + R_f i_{1b} - \frac{1}{sC_f} i_{2b} + \frac{1}{sC_f} i_{1b} \right) \quad (9)$$

$$i_{2c} = \frac{1}{sL_2} \left( 0 + \omega_e \psi_2 \sin \left( \theta_e + \frac{2\pi}{3} \right) - R_2 i_{2c} - R_f i_{2c} + R_f i_{1c} - \frac{1}{sC_f} i_{2c} + \frac{1}{sC_f} i_{1c} \right) \quad (10)$$

### B. Split Winding Configuration

The split winding configuration uses a part of motor winding as a filter inductance. This filter inductance forms a part of integrated RLC filter that attenuates the switching frequency component for a defined cut-off frequency. Fig. 3 shows the practical implementation of the split winding configuration.

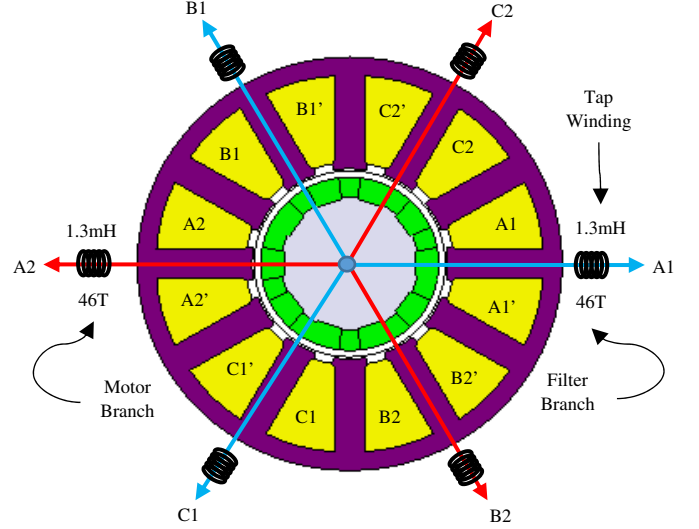
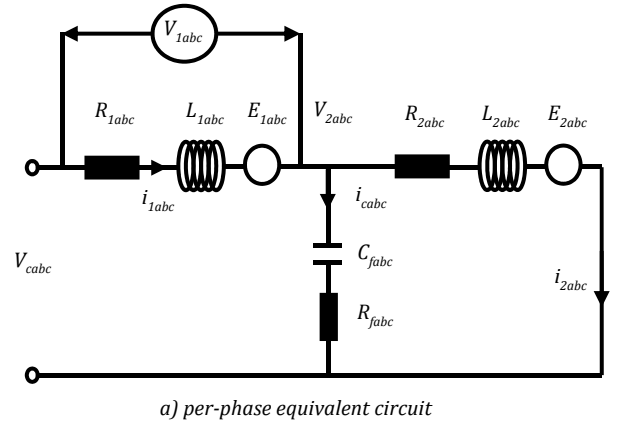


Fig. 3: Winding formulation of 12 Slots 10 Poles integrated permanent magnet synchronous motor

Initially, the motor has a total inductance of 2.6mH and 92 turns per phase. The required inductance of 1.3mH is considered for filter branch leaving 1.3mH for the motor branch. In Fig. 3, the vectors A1, B1 and C1 represent the filter branch, whereas the vectors A2, B2 and C2 correspond to the motor branch. The filter branch can be tapped based on the required filter inductance. In such a case, the number of turns in the filter branch winding will reduce through the tapings as per required inductance and the remaining turns would be added to the motor branch.



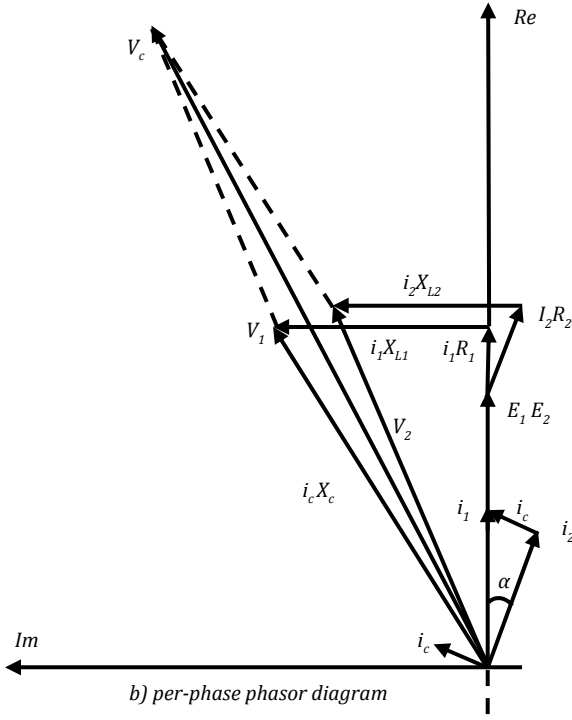


Fig. 4: Per-phase equivalent circuit and phasor diagram of integrated permanent magnet synchronous motor

### C. Per Phase Phasor Representation

The per-phase equivalent circuit and the phasor diagram of the proposed motor with integrated inductor are shown in Fig. 4(a) and Fig. 4(b) respectively. For steady state analysis, the voltage equations are:

$$\overline{V}_1 = \overline{i}_1 R_1 + \overline{i}_1 X_{L1} + \overline{E}_1 \quad (11)$$

$$\overline{V}_2 = \overline{i}_2 R_2 + \overline{i}_2 X_{L2} + \overline{E}_2 = \overline{i}_c R_f + \overline{i}_c X_c \quad (12)$$

The converter voltage is given by,

$$\overline{V}_c = \overline{V}_1 + \overline{V}_2 \quad (13)$$

$E_1$  and  $E_2$  are taken as reference vector. Since  $i_1$  is controlled,  $i_1 * R_1$  voltage drop is in phase with  $E_1$  while reactive voltage drop is at  $90^\circ$  with respect to  $i_1$  and the converter voltage  $V_c$  is the resultant phasor of all voltages in the filter branch. Since  $i_2$  is left uncontrolled,  $i_1$  is leading by  $i_2$  due to the addition of filter capacitor. The active and reactive drop for the motor branch is in phase and orthogonal with respect to  $i_2$  respectively and the voltage  $V_2$  is the resultant phasor of all voltage drops in the motor branch. Capacitor current  $i_c$  is leading  $V_2$  by less than  $90^\circ$  due to damping resistor. The phasor summation of  $i_c$  and  $i_2$  will make the total branch current  $i_1$ . Filter branch currents  $i_1$  is phase shifted (leading) by motor branch current  $i_2$  by an angle  $\alpha$ . The phase shift angle  $\alpha$  between two currents can be calculated as,

$$\alpha = \cos^{-1} \left( \frac{i_1^2 + i_2^2 - i_c^2}{2 i_1 i_2} \right) \quad (14)$$

### D. Electromagnetic Torque

The electromagnetic torque developed by the surface mounted permanent magnet synchronous motor is given by,

$$T_{em} = K \psi_m i_a \quad (15)$$

From the equivalent circuit of proposed model, shown in Fig. 4(a), Eq. 15 is not valid to calculate the electromagnetic torque developed by the motor with integrated inductor. There are now two torque components developed by the motor with integrated filter,

1. Torque due to the filter branch winding,  $T_{a1}$  and,
2. Torque due to the motor branch winding,  $T_{a2}$

Therefore, the electromagnetic torque developed by the motor with integrated filter is,

$$T_{em} = T_{a1} + T_{a2} \quad (16)$$

$$T_{em} = 3 \frac{p}{2} (\psi_1 i_1 + \psi_2 i_2 \cos \alpha) \quad (17)$$

This electromagnetic torque will cause the speed response which is defined by,

$$\omega_r = \frac{1}{J_s} (T_{em} - T_{load}) \quad (18)$$

### E. MATLAB/Simulink and FEA Results

RLC output filter was designed [13] with cut-off frequency 2kHz and maximum frequency (to avoid the resonance between filter capacitance and motor inductance) 1.2kHz. The corresponding RLC values for the filter were selected as  $2\Omega$ , 1.29mH and 4.9 $\mu$ F respectively. Subsequently, the filter branch winding was fixed to 1.29mH leaving rest of the windings as motor branch.

Fig. 5(a) and Fig. 5(b) show the line voltage across the inverter terminal and the line voltage across the motor branch winding respectively. The current through the filter branch and motor branch are shown in Fig. 5(c) and Fig. 5(d) respectively. The filter branch currents have been injected in to the FEA model of the motor with integrated inductor to validate the effect of the integrated filter. The currents of the motor branch seen from the FEA model were identical to that of MATLAB/Simulink model and are shown in Fig. 5(d). The result shows that the half of the motor winding sees the filtered voltages. Thus, reduces the voltage stress and PWM losses in half of the motor winding. Moreover, the added losses, weight and size due to external filter inductor will also be eliminated.

It is important to note that the voltage with high gradient is dropped across the filter branch winding. The insulation of the filter branch can be made high to bear the PWM voltage stress. This will allow the integrated filter feasible for the applications where long cables are used between inverter and motor terminal.

The MATLAB/Simulink results of the motor without filter and with conventional filter are shown in Fig. 6 and Fig. 7

respectively. It can be seen from Fig. 5(c) and Fig. 7(c) that the filtering effect of the motor with integrated inductor and conventional RLC filter are alike.

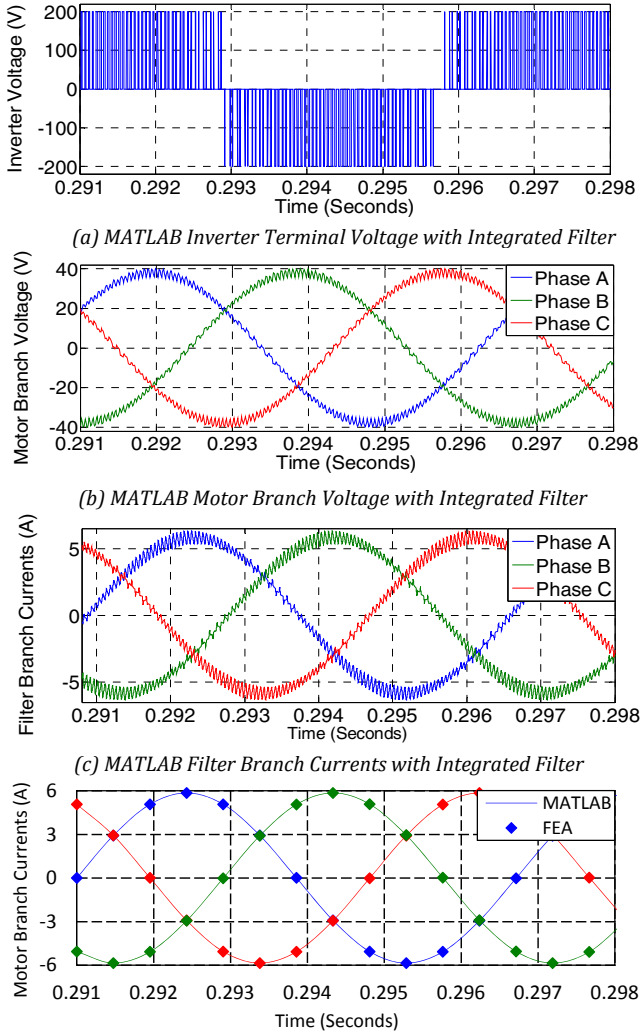


Fig. 5: MATLAB/Simulink results of the Motor with integrated inductor

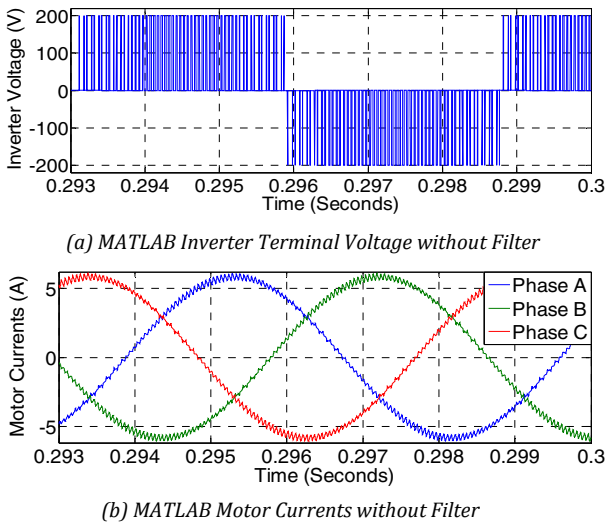


Fig. 6: MATLAB/Simulink results of the motor without filter

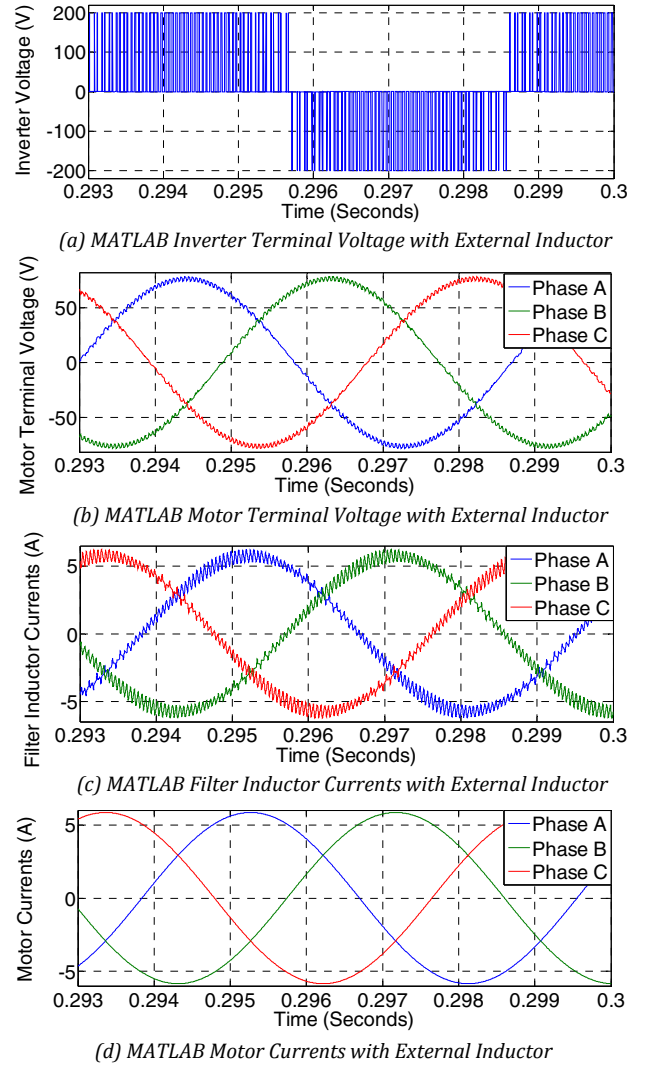


Fig. 7: MATLAB/Simulink results of the motor with external inductor

To get utmost benefit out of the proposed motor with integrated inductor, the filter branch has to be quarter or less of the entire motor winding so that a higher portion of the motor winding experiences the filtered currents. This will depend on the design of the output filter i.e. inductance and capacitance required to attenuate the switching component of the inverter. However, the reduction in filter inductance will increase the capacitance for the same cut-off frequency by the given relation,

$$\omega_{cut-off} = \sqrt{\frac{1}{L_1 C_f}} \quad (19)$$

#### F. FEA Based Motor Performance

The motor performance depicted in Tables II and III of the proposed and conventional filter has been compared in terms of mean electromagnetic torque, torque ripple, total losses, inductor weight and volume. The external filter inductor is designed [14] and validated with FEA. The cross sectional view of external filter inductor is shown in Fig. 8 and the specifications are shown in Table IV. It is evident that the losses, weight and volume have been reduced in the motor with integrated inductor. It is also worth noting that the effect

on the mean electromagnetic torque due to angle  $\alpha$  is negligible. However, torque ripple become relatively higher when compared to the motor with external filter inductor due to the presence of switching current component in the filter branch. This can be observed from Fig. 9 which shows the electromagnetic torque with and without filter.

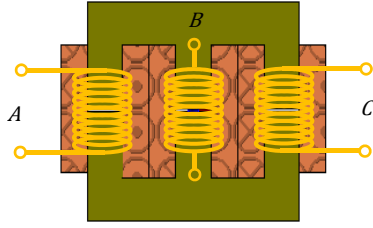


Fig. 8: Cross sectional view of external filter inductor

TABLE II: FEA BASED MOTOR LOSSES OF BOTH SYSTEMS

MOTOR WITH	Motor Losses (W)	External Inductor Losses (W)	Total Losses (W)	External Inductor Weight (Kg)	External Inductor Volume (cm <sup>3</sup> )
Integrated Inductor	36	0	36	0	0
External Inductor	26.1	40.67	66.77	1.52	338.7

TABLE III: FEA BASED MOTOR TORQUE RIPPLE OF BOTH SYSTEMS

MOTOR WITH	Mean Torque (Nm)	Torque Ripple (%) at $T_{load} = 1.5Nm$	Torque Ripple (%) at $T_{load} = 3Nm$	Torque Ripple (%) at $T_{load} = 4.5Nm$
Integrated Inductor	1.485	9.54	5.33	4.2
External Inductor	1.49	0.7	0.574	0.54

TABLE IV: INDUCTOR DESIGN SPECIFICATIONS

PARAMETERS	VALUES	UNIT
Material	M – 15 29 Ga	-
Required Inductance	1.3	mH
Peak Current	18	A
Turns per phase	20	-
Fill Factor	0.49	-
Conductor Diameter	2.62	mm
Peak Current Density	3.3	A/mm <sup>2</sup>

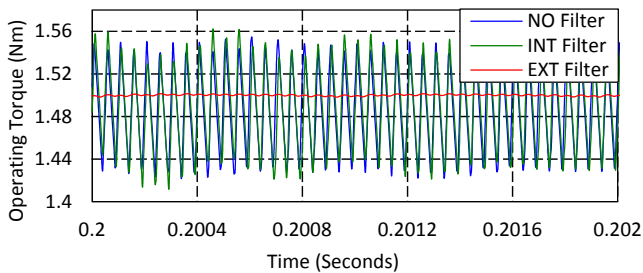


Fig. 9: FEA torque ripple comparison with and without filter

## V. CONCLUSIONS

An integrated filter inductor for permanent magnet motor drive is proposed in this paper. The proposed motor with integrated inductor eliminates the need of a separate filter inductor which reduces the overall losses, and associated

weight and volume. From MATLAB/Simulink results, it has been observed that the voltage stress and losses due to PWM are eliminated in the motor branch. The inductor losses, weight and volume related with the separate inductor are 40.67W, 1.52Kg and 338.7cm<sup>3</sup> whereas these losses, weight and volume are removed in the motor with integrated inductor. However, the torque ripple was relatively higher when compared to the motor with external inductor. This is due to the presence of switching component in the filter branch. The MATLAB/Simulink and FEA results show that this integrated approach is feasible and can be practically implemented. The proposed concept can be a suitable solution for high power AC drives where bulky and lossy inductors are required but this comes at the expense of relatively higher torque ripple.

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