Deduction of ultimate equilibrium limit states for concrete gravity dams keyed into rock mass foundations based on large displacement analysis

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Abstract

Concrete gravity dams are mass concrete structures, often built on rock mass foundations, conceived to rely upon their weight for stability. To prevent sliding, these structures are usually keyed/embedded into the foundation, a good construction practice particularly relevant in medium to high intensity seismic zones. In stability analysis, the extra strength obtained by keying the dam into the foundation is usually either neglected or taken as a passive resistance, which, such as explored in this paper, do not reflect the real structural response in pre-collapse situations. Limit state philosophy requires the ultimate equilibrium conditions to be expressed as accurately as possible.

In this paper, the rigid-body equilibrium of a wedgy model representing the dam and a downstream rock wedge is analyzed according to the large displacement regime. Failure mechanisms were identified, analytically described and numerically validated. Application to two Portuguese large concrete gravity dams led to safety factors considerably larger than those computed assuming the usual practice. The proposed approach is intended to support probabilistic and/or semi-probabilistic methodologies for safety assessment of concrete gravity dams, in the design and feasibility phases, in which the limit state approach

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is inherently followed.

Keywords: Concrete gravity dams, keyed profiles, stability analysis, analytical modeling, limit state approach, ultimate equilibrium, large displacement analysis

1. Introduction

Concrete gravity dams are mass concrete structures conceived to rely upon their weight for stability. These structures are often built on rock mass foundations which shall be capable of withstanding stresses within the range of 3 to 4 MPa [1] without excessive and uneven settlement. Consequently, excavation works are sometimes needed in order to reach the bed rock. In practice, however, the rock mass foundation is even excavated below in order to embed/key the dam and prevent dam sliding. In stability analysis, the extra strength obtained in this case is either neglected in order to account for a possible excavation downstream or taken as a passive resistance [2, 3, 4, 5, 6, 7], given by the Rankine's theory [8] such as considered in soil mechanics. The latter presupposes that the overall instability is attained when the shear strength of the dam base while the rock wedge downstream only contributed with an extra passive resistance. As shown in laboratory tests performed on physical models [9, 10], intended to study the dam collapse by failure along two planar discontinuities, that situation only describes a primary failure stage but new equilibrium configurations are immediately found. Global instability is only achieved in a subsequent stage.

The aforementioned simplification is comprehensibly adopted within a predominantly deterministic approach to structural safety, only considering a portion of the real structural capacity as a conservative strategy to handle inherent uncertainties. However, for probabilistic approaches, based on the limit state philosophy, the ultimate equilibrium conditions must be expressed as accurately as possible, which demand further investigations. In fact, although theoretical investigations for the adoption of probabilistic-based approaches to dam safety have long been encouraged [11], the dam engineering community has still serious objections regarding its validity [12]. On one hand, large dams are, among the class of structures whose failure may lead to catastrophic consequences, those whose structural behavior is predicted with the highest degree of uncertainty [13]. Analytical models would then require such assumptions that conclusions from a probabilistic assessment would not have any practical value [11]. On the other hand, the classical deterministic approach, combining conservative design principles and continuous monitoring, has been able to produce satisfactory results [14]. Nonetheless, recent developments have been made attesting the pertinence of probabilistic-based approaches. A probabilistic model-code for concrete dams [15] have recently been proposed and several works [16, 17, 18] succeeded in applying reliability methods to estimate the probability of failure of concrete gravity dams.

Within the framework of probabilistic-based approaches, limit states, defining the boundary conditions between desired and adverse states, represent performance requirements. In particular, ultimate limit states represent immediate or progressive collapse, resulting in human, economical and environmental losses. For the equilibrium of concrete gravity dams keyed into the foundation, there is no evidence that the former strategy to address the problem, by considering the contribution of keying the dam as a downstream passive resistance, has changed. It is therefore instrumental to study the kinematics of the problem and to model the failure of concrete gravity dams by loss of stability, such that the obtained ultimate limit states can represent the real ultimate equilibrium conditions and be used in design and feasibility phases.

In this paper, analytical descriptions of the failure mechanisms identified, which can be directly used in both deterministic and probabilistic stability calculations, are deduced and validated through equivalent numerical models. This task is part of a recent effort to explore the necessary steps towards the adoption of probabilistic principles in safety assessment and re-assessment of Portuguese large concrete dams, such as modeling the relevant sources of uncertainty [19, 20]. Furthermore, in some situations, such as in medium to high seismic intensity zones, where the typical gravity profile may not be sufficient to ensure stability conditions [21], addressing the problem such as proposed leads to a virtual increase of the safety conditions which may even render a specific structural solution feasible. This advantage is appreciated invoking, as examples, two Portuguese large concrete gravity dams, whose contribution of the keyed depth plays a more crucial role to stability than typically assumed.

2. Safety of concrete gravity dams keyed into the foundation

2.1. Structural idealization

Since they are divided into independent monoliths, separated by vertical transverse contraction joints, the safety analysis of concrete gravity dams is advantageously conducted based on two-dimensional representations, neglecting conservatively any three-dimensional effect. Furthermore, due to their dimensions, internal stresses in concrete gravity dams are, in general, much smaller than the concrete strength. The safety of concrete gravity dams is thus considered virtually independent on the mechanical strength of the concrete, since sliding or overturning failure modes generally occur before a conditioning stress field is achieved [22]. Therefore, safety evaluation of concrete gravity dams require, first and foremost, the performance of stability analysis of the cross-section profile, often considering rigid-body mechanisms without loss of representativeness. For that, any potential failure surface shall be tested, either in the dam body or within the rock mass foundation. Although they could attain some relevancy for large concrete dams, lift joints are not usually conditioning on the stability evaluation, since good construction strategy shall ensure adequate resistance properties. Rock joints may compromise the overall stability mostly in cases when unsafe geometrical/mechanical properties are identified.

The possibility of dam construction is only materialized if foundation characteristics are such that do not represent an uncontrolled risk to dam stability. Consequently, the structural solution to be adopted is often conditioned on the dam stability analysis along the concrete-rock interface provided that the rock mass foundation presents proper characteristics. However, with the implementation of construction quality control measures and preventive construction techniques, such as, for instance, preparing the weak surfaces in order to increase their shear strength, the potential destabilization planes above the foundation have actually a competent behavior regarding sliding mechanisms, ensuring a satisfactory material continuity. Even when dangerous geometrical/mechanical properties are identified within the uppermost strata of rock mass foundations, given the impossibility of finding a more suitable site with adequate geometry and/or mechanical properties to found the dam on, the rock mass foundation can be reliably strengthened by grouting, concreting, anchorage or other methods, also ensuring some continuity into the rock mass foundation. Given that, the most plausible structural failure of concrete gravity dams involves the loss of stability of multiple wedges, whether previously existing [2, 4] or formed due to excessive stresses [23, 24], in the uppermost strata of the rock mass foundation.

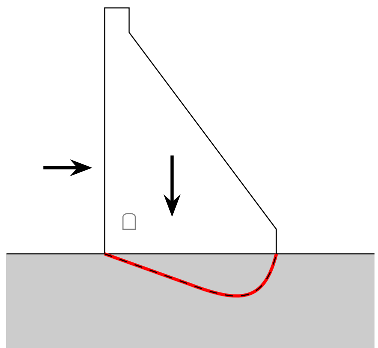
Keying the concrete gravity dam into the foundation is a measure frequently adopted in order to increase its stability in medium to high seismic intensity zones. This measure is also of good construction practice, even in all other situations, just to lock the foundation into the rock and prevent dam sliding. Also in this situation, foundation rock wedges must necessarily be mobilized to obtain kinematically possible mechanisms. In that case, a multi-wedge system analysis has been used for testing the dam sliding [25].

Some variants of the wedgy model are typically adopted (Figure 1). When assuming that dam and rock foundation behave as a single continuous body, in the absence of weakened surfaces, failures usually occur due to growing cracks developed at upstream and downstream portions of the foundation, following excessive tensile and compressive stresses, respectively [26]. At some point, when both cracks intersect each other, a continuous failure surface (Figure 1a) is formed followed by an overturning mechanism [24]. However, unless the rock mass foundations consist of very low-strength rocks without important discontinuities [27] which, in that case, would rarely be competent to found the dam on, weaker surfaces would invariably condition the structural solution to be adopted. Preferably, the definition of the wedgy model shall be based on geotechnical investigations, since a variety of situations with relevancy may exist. The greatest concern for the stability of concrete gravity dams keyed into the foundation is the potential for sliding along weak seams which can take any geometry, being typically planar or semi-planar. The basic case consists on the existence of a weak horizontal seam (Figure 1b). A generic hypothetical situation describes a three-wedge model (Figure 1c), formed after the development of vertical cracks below the dam heel and toe, in which the slope of each wedge is conditioned on obtaining the lowest safety factor [4]. The simpler case, even though frequently considered in dam stability analysis software [6, 28], considers a single rock wedge located just downstream from the dam that can serve as a ramp for the mechanisms take place (Figure 1d). In all those cases, investigations are needed to determine the contribution to stability from mobilizing such rock wedges.

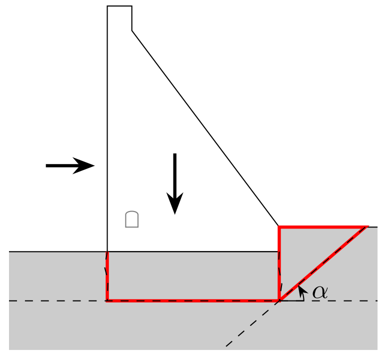
2.2. Stability analysis

Stability analysis shall be performed to assess the concrete gravity dam safety regarding loss of static equilibrium. The analytical methods mostly used differ according to the presumptions assumed: (i) in the shear-friction method, the maximum resisted load must be greater than the acting destabilizing load; while (ii) in the limit equilibrium method, the shear stress required for equilibrium must be smaller than the maximum mobilized strength. These methods are associated with the incremental load analysis and the shear reduction analysis, respectively. For the same structure, their conceptual differences result in safety factors with distinct order of magnitude, for inclined sliding surfaces [4]. However, according to the limit state philosophy, the ultimate equilibrium conditions from which static equilibrium is lost are not affected by the method used since a safety factor of one would be computed in any case. Thus, to describe the near-failure conditions that can be adopted to characterize ultimate equilibrium limit states, there are no practical differences between them.

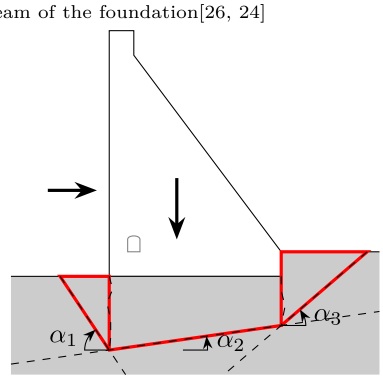
For the sliding stability analysis of a concrete gravity dam keyed into the foundation, considering the model illustrated in Figure 1d, the contribution of



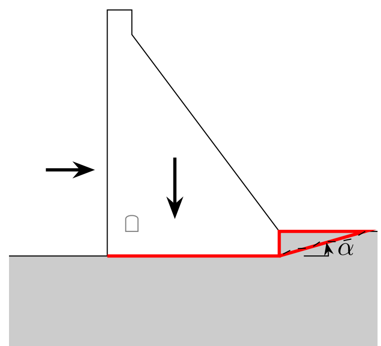
(a) Cracks propagated from upstream and downstream of the foundation[26, 24]



(b) Horizontal weak seam [2]



(c) Hypothetical three-wedge system with weaker seams [4]



(d) No weak seam, a downstream wedge must be formed [6, 28]

Figure 1: Variants of wedgy models of gravity profiles.

the keyed works is often assumed as a passive resistance applied by the downstream rock wedge (Figure 2). In this case, the shear-friction method should be preferably used [6, 28] once it allows to directly compute the safety factor SF, as,

$$SF = \frac{V \cdot \tan\phi_1 + c_1 \cdot L_1 + \dfrac{c_2 \cdot L_2/\cos\alpha}{1 - \tan\alpha \cdot \tan\phi_2} + W_w \cdot \tan(\alpha + \phi_2)}{H} \tag{1}$$

where H and V are the total horizontal and vertical forces, respectively, applied in the dam body; W_w is the weight of the downstream rock wedge; ϕ_i, c_i and L_i are the friction angle, cohesion and length, respectively, of the i-th sliding surface; and α is the downstream rock wedge base slope. Such formulation considers the sliding along the dam base to which the downstream rock wedge only apply a counteract force after its shear strength is fully mobilized. That is equivalent to assume a passive resistance given by the projection to the sliding direction of the weight of an unstable rock wedge. According to the Rankine's passive earth pressure theory [8], the critical situation occurs for $\alpha = 45° - \phi_2/2$.

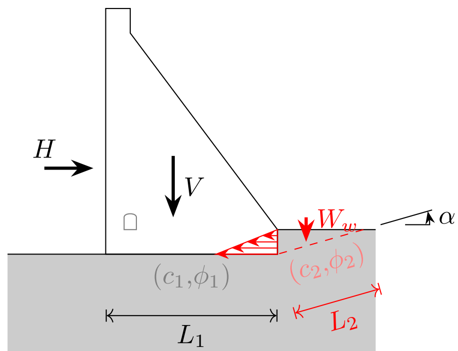


Figure 2: Passive wedge resistance [6, 28]

This problem can be generalized to a system of wedges that counteract the potential sliding movement by applying passive resistant forces to previous blocks [3, 4]. Accordingly, the maximum resisted load R that can be applied to a k-wedge system (Figure 3), considering again the shear strength of sliding surfaces described by Mohr-Coulomb failure envelopes, is, according to the passive resistance hypothesis, given by,

$$R = \sum_{i=1}^{k} R_i \cdot \cos\alpha_i + N_i \cdot \sin\alpha_i = \sum_{i=1}^{k} \frac{c_i \cdot L_i^{'}/\cos\alpha_i}{1 - \tan\alpha_i \cdot \tan\phi_i} + V_i \cdot \tan(\alpha_i + \phi_i) \quad (2)$$

where R_i is the sliding resisting force, assuming that it is at failure conditions along its entire extension, N_i is the normal component of the base reaction, V_i is the vertical force, c_i is the cohesion, $L_i^{'}$ is the non-cracked base length, α_i is the wedge base inclination and ϕ_i is the friction angle of the i-th wedge.

2.3. Critical aspects

Such idealization of the ultimate equilibrium conditions obey the prerequisites of a small displacement analysis in which, by definition, the equilibrium of

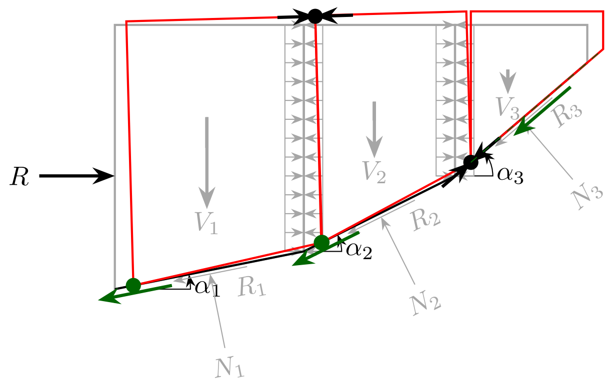


Figure 3: Illustration of the equilibrium conditions of a wedgy system.

forces is always done at the non-deformed structure (Figure 3 in gray). Nonetheless, the possibility for surface properties degradation is accounted when considering that the peak strength in different surfaces may not be additive, since deformation rates are often unequal [3, 4], which contradicts the small displacement principles. In fact, the small displacement idealization of the inter-wedge contacts and force transmission paths is incapable of simulating the conditions derived from the progressive degradation of the shear strength. For such degradation to take place, shear displacements must have occurred. In that case, as displacement progresses and the contact properties change, the initial non-deformed configuration is irrecoverable once the peak shear strength is overcome. Assuming rigid-body mechanisms, since that is one of the requirements for the analytical sliding safety assessment, after movement is initiated, surface contacts gradually change to point contacts, achieving new equilibrium configurations (Figure 3 in red) whose stability solutions approximate more realistically to the ultimate equilibrium conditions. Given that, the sliding process of the model presented in Figure 1d shall experience two consecutive shear responses (Figure 4): (i) an initial non-linear phase characterized by the progressive mobilization and degradation of the shear strength of both surfaces until a first yielding threshold is attained when both surfaces are at failure conditions, corresponding to the results of a small displacement analysis; and (ii) a hardening non-linear phase due to the continuous shear strength degradation, if non-residual values can still be mobilized at this point, with a gradual change from surface to punctual contacts possibly causing material crushing, until a second yielding threshold is attained.

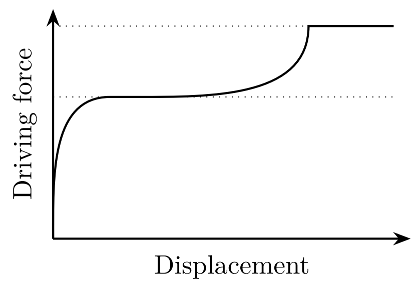


Figure 4: Expected shear response of the wedgy model illustrated in Figure 1d.

This situation was experienced in physical model tests performed to study the gravity dam collapse by sliding on a surface defined by two or more planar discontinuities, under static [9, 10] or dynamic loading conditions [29]. The movement of an equivalently scaled model was monitored using deflectometers placed at the dam toe as the external loading was being progressively increased [9], following a incremental load technique: Hydrostatic pressure at the upstream face, simulated using a fluid contained in a rubber bag, was gradually raised while the uplift pressures, simulated by concentrated vertical upward forces on the base using a pulley system, were kept constant. The obtained results are reproduced in Figure 5. After a linear response (first equilibrium state), a yielding threshold was attained. As loading increases from that point on, vertical movement initiates while horizontal displacement presents a similar but magnified response (second equilibrium state), i.e. the model starts riding up the inclined joint. The stepped response represents a progressive changing on the contact conditions. Ultimately, for the maximum resisted load, a new yielding threshold was attained such that an uncontrolled mechanism took place. At failure, the model was simply resting on two points, the dam heel and the dam toe.

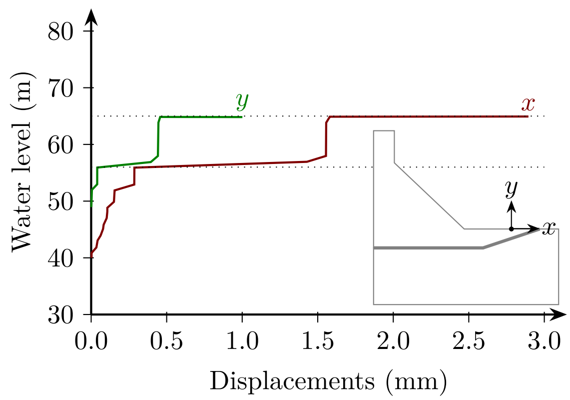


Figure 5: Gravity profile physical model's response during sliding along two planar discontinuities (adapted from [9]).

Note that, in these tests, only the frictional component of the shear strength was present. Since surfaces were planar and no degradation of the sliding surfaces was observed during preliminary tests on prisms [9], one can assume that the frictional strength was in fact residual. If considering the possibility of mobilizing a peak shear strength, in the first equilibrium phase, the structural response would have been more tortuous due to the coupled dilatancy-shearing mechanism and the external loading that leads to the first yielding threshold would have been higher. Nonetheless, once that stage is achieved, the surface must have degraded such that the frictional behavior is near residual. Since the initial goal of that study was indeed to test the gravity dam collapse by sliding, the frictional strength in a limit situation would be residual. Although a different transient response would be obtained, the maximum resisted external loading would have not been significantly different.

2.4. Limit state perspective

In the limit state approach, the ultimate equilibrium limit states refer to the conditions from which static equilibrium is no longer verified. More than concern about the structural integrity, the philosophy behind it points to the safety of direct users and people/assets indirectly affected once such limit state is violated. In large dams, ultimate limit states shall represent the boundary between the occurrence and avoidance of such adverse events that safety of people/assets in the downstream valley may not be ensured. Such events may then correspond to partial or global structural collapse, provided that the resulting dam break wave is sufficiently strong to threaten exposed elements. As mentioned, the small displacement idealization of the ultimate equilibrium conditions of the dam-foundation multi-wedge system, considering that a yielding threshold is attained in all sliding surfaces, describes a primary failure situation that do not instantaneously result in danger to potentially exposed elements. Instead, this situation is compatible with the definition of serviceability limit states, which limit the reversibility of potential structural damage. Following that, as mentioned above, a new stable configuration is progressively attained as deformation progresses and contact properties change. Ultimately, a new yielding threshold is achieved. From that point on, any increase of the loading conditions would result in the definitive loss of equilibrium. That limit situation shall then represent the truly limit equilibrium conditions that the application of the limit state approach to the structural safety of concrete gravity dams shall refer to.

Although these ultimate equilibrium conditions follow the principles of large displacement analysis, which can generally be modeled using discrete-element methods, they can also be analytically expressed considering a wedgy rigid-body model and a perfectly-plastic shear behavior of the sliding surfaces. On one hand, the actual semi-brittle behavior, due to a progressive degradation of the shear strength parameters, does not hold for large displacement when residual strength and plastic response are expected. On the other hand, even though the selection of the wedgy model shall be based on geotechnical investigations, the simpler situation illustrated in Figure 1d must be tested when explicitly evaluating the loss of stability along the concrete-rock foundation. For that, considering the formation of a downstream rock wedge, the following kinematically possible mechanisms (Figure 6) can be developed, depending on the direction of the total net force:

- Failure mechanism 1 (Figure 6a): occurring when the direction of the total net force intersects the dam-foundation interface (Plane A-B), such that the gravity profile slides along the dam-foundation interface and climbs the downstream slope, pushing the downstream rock wedge;
- Failure mechanism 2 (Figure 6b): occurring when the direction of the total net force intersects the Plane B-C, originating only compressive stresses, such that the gravity profile and the downstream rock wedge slide together along the downstream slope;
- Failure mechanism 3 (Figure 6c): also occurring when the direction of the total net force also intersects the Plane B-C but originating tensile stresses near the dam toe (point B), such that the gravity profile rotates around that point, pushing the downstream rock wedge; and
- Failure mechanism 4 (Figure 6d): occurring when the total net force passes above the point C, such that the profile rotates over the downstream rock wedge.

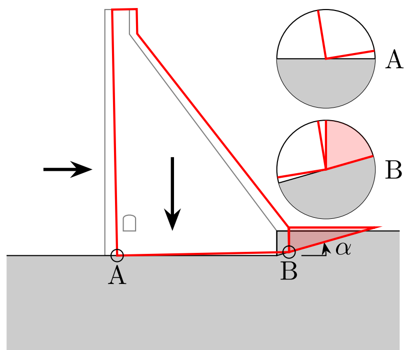
The analytical expressions describing these ultimate limit equilibrium conditions can be deduced simple from equilibrium of forces, since they represent static equilibrium situations. Their mathematical deduction is detailed and numerically validated in the following section.

3. Failure modeling

3.1. Analytical description

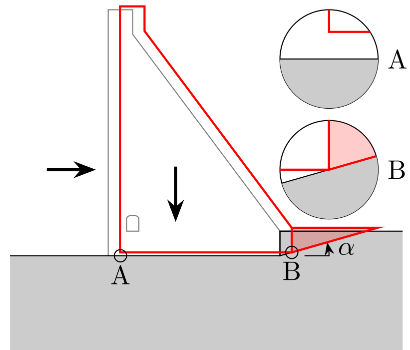
Stability analysis consists in the assessment of whether stabilization actions can counteract destabilization actions. This verification can be analytically performed whenever the problem is simple enough to find a mathematical solution, such as in this case as long as the loading conditions can be reduced to a total static net force. It is the location of such force that conditions the occurrence of a specific failure mechanism.

Dam slides along the interface, pushing the downstream rock wedge



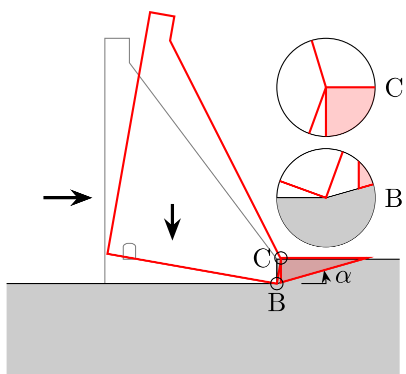
(a) Failure mechanism 1

Dam/rock wedge slide along the downstream rock slope



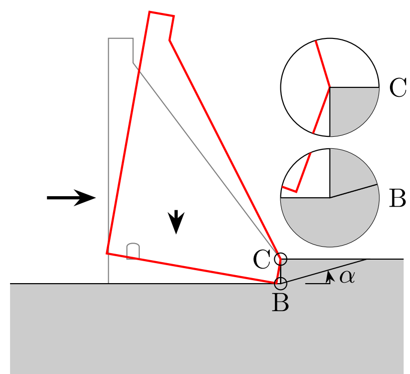
(b) Failure mechanism 2

Dam rotates around its toe pushing the downstream rock wedge



(c) Failure mechanism 3

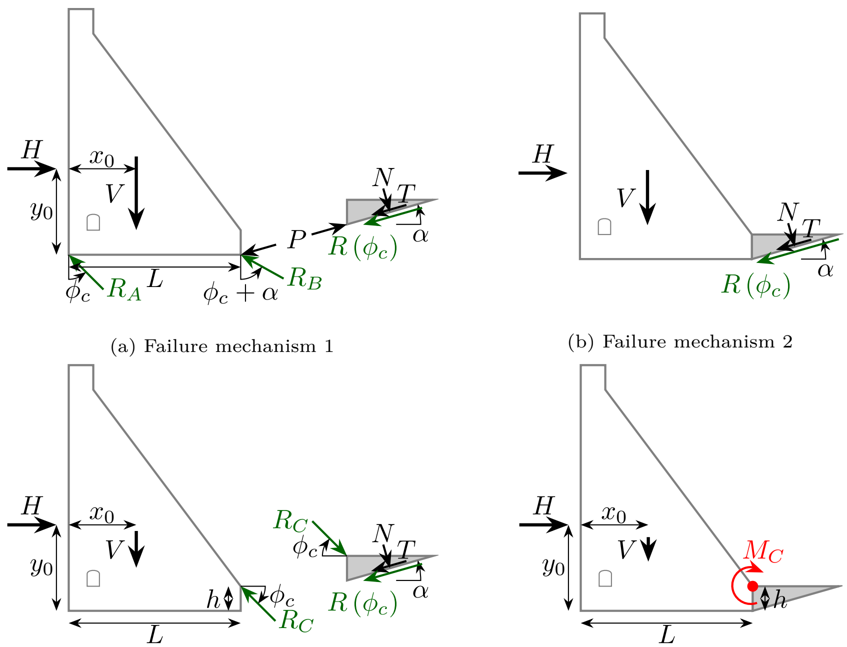
Dam rotates over the downstream rock wedge



(d) Failure mechanism 4

Figure 6: Ultimate failure mechanisms for gravity profiles keyed into the foundation.

In the failure mechanisms identified, the transmission of forces both between the dam and the foundation and between the dam and the downstream wedge is made on specific contact points. To express the corresponding limit equilibrium situations, these contact forces must be considered in the analysis, assuming that the contact points are at failure conditions. This hypothesis is assumed hereafter for the deduction of the analytical expressions that describe the ultimate limit equilibrium conditions, under the assumption that only the residual component of the shear strength can be mobilized in all sliding points, such as previously justified. By simple equilibrium of forces, the critical friction coefficient ($\tan \phi_c$) can then be obtained. This is supported on the free-body diagrams illustrated in Figure 7.



(a) Failure mechanism 1

(b) Failure mechanism 2

(c) Failure mechanism 3

(d) Failure mechanism 4

Figure 7: Free-body diagrams for the ultimate failure mechanisms of gravity profiles keyed into the foundation.

Failure mechanism 1 (Figure 6a) illustrates the limit equilibrium conditions characterized by the sliding of the gravity profile along the dam base and climbing the downstream wedge slope while pushing the downstream wedge. The gravity profile rests on two contact points (A and B) producing punctual reactions (R_A and R_B, respectively) which, at failure conditions, are inclined at ϕ_c to the direction of movement. This movement produces an unbalanced force (P) transmitted to the downstream wedge, which ultimately slides along the downstream ramp. The downstream wedge, applying normal (N) and tangential (T)

loads to the its base, would fail once the maximum mobilized shear strength (R) is exceeded. The critical friction coefficient ($\tan\phi_c$) can be obtained by solving a system of equilibrium equations, where the remain unknown variables are the vertical component of the reactions R_A and R_B and the unbalanced force P, canceling: (i) the moment around the instantaneous rotation center (a point O located at the intersection between the lines perpendicular to the movement of points A and B); (ii) the total horizontal and (iii) vertical forces acting on the dam body; and (iv) the total tangential forces acting on the downstream wedge, i.e.,

$$\sum M_0 = 0 \tag{3a}$$

$$\sum F_H = 0 \tag{3b}$$

$$\sum F_V = 0 \tag{3c}$$

$$\sum F_{T,w} = 0 \tag{3d}$$

which is reduced to a quadratic form given by,

$$c_6 \cdot \tan^2\phi_c + c_7 \cdot \tan\phi_c + c_8 = 0 \tag{4}$$

whose coefficients c_i are computed from the following expressions,

$$c_8 = c_2 \cdot \sin\alpha \tag{5a}$$

$$c_7 = c_4 \cdot \sin\alpha + (c_2 - c_5) \cdot c_3 \tag{5b}$$

$$c_6 = c_4 \cdot c_3 + c_5 \cdot \sin\alpha + N \cdot \tan^2\alpha \tag{5c}$$

$$c_5 = c_2 + T \cdot \cos\alpha + H \tag{5d}$$

$$c_4 = N/\cos\alpha - T \cdot \sin\alpha + V \tag{5e}$$

$$c_3 = \cos\alpha - \sec\alpha \tag{5f}$$

$$c_2 = c_1/(L \cdot \tan\alpha) - T/\cos\alpha \tag{5g}$$

$$c_1 = V \cdot x_0 - H \cdot (L/\tan\alpha - y_0) \tag{5h}$$

Failure mechanism 2 (Figure 6b) illustrates the limit equilibrium conditions characterized by the sliding of both the gravity profile and the rock wedge along the downstream wedge slope. In this case, the resultant net force lies within the dam-downstream wedge interaction surface, such that the dam and the downstream wedge move together, producing a levitation effect. Thus, the downstream wedge would fail once the maximum mobilized shear strength (R) is exceeded. The critical friction coefficient ($\tan\phi_c$) can then be deduced by considering the equilibrium of the dam-downstream wedge set which is simply expressed by the equation of tangential forces acting on the downstream wedge slope, i.e.,

$$\sum F_{T,w} = 0 \Leftrightarrow \tan\phi_c = \frac{H \cdot \cos\alpha - (V \cdot \sin\alpha + T)}{H \cdot \sin\alpha + (V \cdot \cos\alpha + N)} \tag{6}$$

Failure mechanism 3 (Figure 6c) illustrates the limit equilibrium conditions characterized by the rotation of the gravity profile around the dam toe pushing the rock wedge, which then slides along the downstream wedge slope. The rotation of the gravity profile implies that the contact between it and the rock wedge is made on point C producing a punctual reaction (R_C) which, at failure conditions, are inclined at ϕ_c to the horizontal. In this case, the external loading must be such that the resultant net force not only lies within the dam-downstream wedge interaction surface but also that tensile stresses are produced in the inferior portion of that surface such that the dam body and the downstream wedge separate. The downstream wedge would fail once the corresponding shear strength (R) is exceeded. The critical friction coefficient ($\tan\phi_c$) can be obtained by solving a system of equilibrium equations, where the other unknown variable is the horizontal component of R_C, canceling: (i) the moment around the dam toe; and (ii) the total tangential forces acting on the downstream wedge, i.e.,

$$\sum M_B = 0 \tag{7a}$$

$$\sum F_{T,w} = 0 \tag{7b}$$

which is reduced to a quadratic form given by,

$$(R_{h,C} \cdot \cos\alpha) \cdot \tan\phi_c^2 + (2R_{h,C} \cdot \sin\alpha - N) \cdot \tan\phi_c + (T + R_{h,C} \cdot \cos\alpha) = 0 \tag{8}$$

where the horizontal component of the reaction at C ($R_{h,C}$) is given by,

$$R_{h,C} = \frac{M_B}{h} = \frac{H \cdot y_0 - V \cdot (L - x_0)}{h} \tag{9}$$

Failure mechanism 4 (Figure 6d) illustrates the limit equilibrium conditions characterized by the rotation of the gravity profile over the rock wedge (around point C). This is a purely rotational failure mechanism makings it independent on the shear strength of any surface. This mechanism would occur if the total net force passes above point C. The limit equilibrium conditions are characterized by a null moment around point C, i.e.,

$$\sum M_C = H \cdot (y_0 - h) - V \cdot (L - x_0) = 0 \tag{10}$$

3.2. Numerical validation

The reasoning behind studying the stability conditions of concrete gravity dams keyed into the rock mass foundation is that, when the contribution of a downstream passive resistance is explicitly taken into account, this is often made under the assumptions of small displacement analysis. As mentioned, the real ultimate conditions that set the boundary between stability and instability situations cannot be analyzed unless admitting large displacements. The failure mechanisms identified and the mathematical description of the corresponding limit equilibrium situations can then be validated using discrete-element models, which are particularly appropriate for ultimate capacity studies involving the failure of discontinuous media. Since the contacts can be updated during the analysis, new equilibrium configurations can be found after relative sliding and/or separation of wedges, which is fundamental to model the large displacement regime [30].

For that purpose, the Universal Distinct Element Code (UDEC) software [31] is used. As an example, a hypothetical 100-meter-high gravity profile (Figure 8), keyed at a depth of 10 meters, is considered. All blocks are considered as rigid bodies. To manipulate the direction of the total net force, the dead weight of the gravity profile (W) was fictionally applied at different inclinations according to an angle θ but not beyond a critical point, when the total net force intersects the point C, since, in that situation, failure mechanism 4 would occur. The same failure criteria is considered in all surfaces, characterized only by the friction component of the shear strength. The critical friction angle is obtained considering the strength reduction method, which gradually decreases a safe friction angle until static equilibrium is no longer ensured. To test different failure mechanisms, the downstream face slope (s) was varied from 0 to 2.50. The selection of the downstream wedge inclination angle α was subordinated on obtaining the highest critical friction angle, which was previously determined analytically. Besides the critical friction angle computed analytically (color according to the conditioning failure mechanism), Figure 9 also shows the corresponding solution of four specific numerical models (black circles) characterized by downstream face slopes of 0.25, 0.50, 0.75 and 1.00.

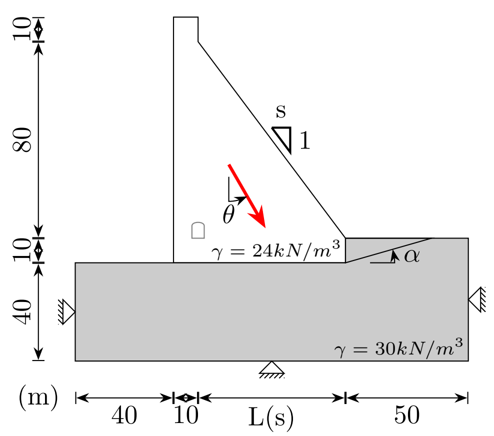


Figure 8: Illustration of the numerical model used to test the failure of gravity profiles keyed into the foundation.

Both the critical friction angle and the most conditioning failure mechanism deducted from analytical investigations of the stability of gravity dams keyed into the foundation match perfectly the results obtained in numerical analysis of equivalent models. When the resultant net force intersects the dam base, i.e. when $\theta\,(M_B) < 0$, failure mechanism 1 is the most conditioning, since the

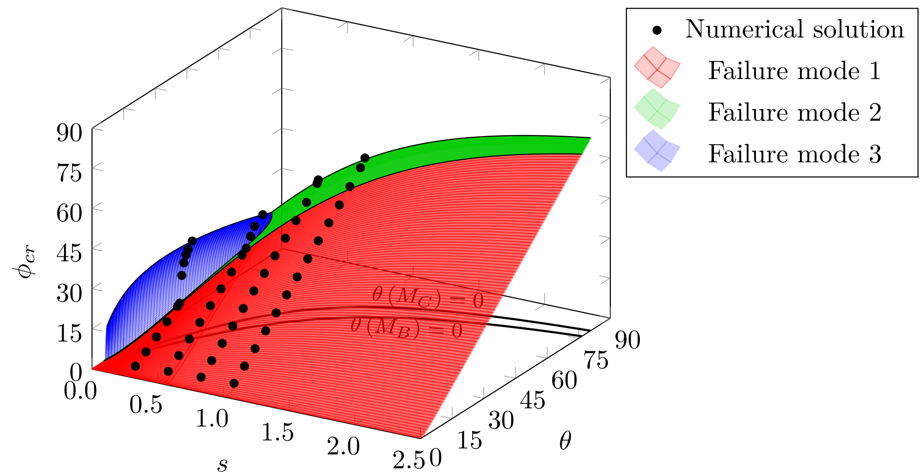


Figure 9: Analytical and numerical solutions for the ultimate equilibrium conditions of 100-meter-high concrete gravity profiles keyed into the foundation.

corresponding critical friction angle is higher than the one obtained for other failure mechanisms when analyzing the stability problem by analytical calculations. Once the loading conditions are such that the resultant net force lies within the dam-downstream wedge interaction surface, i.e. when $\theta\,(M_B) > 0$ and $\theta\,(M_C) < 0$, failure mechanism 2 or 3 are indeed the most conditioning on the stability analysis. In fact, failure mechanism 3 would only be the most conditioning in some rare cases with more inclined downstream face slope, producing tensile stresses on the lower portion of that surface. Only in profiles with downstream face slopes less than 0.57, clearly outside the range of practical values (0.70 to 0.80), the failure mechanism 3 could be conditioning. Finally, only under such extreme loading situations that the resultant net force passes above the point C, i.e. when $\theta\,(M_C) > 0$, failure mechanism 4 would occur, independently on the friction angle. These observations confirm what was expected and validate the analytical description of the failure mechanisms identified, representing the ultimate limit states related to loss of static equilibrium.

4. Case studies

4.1. Safety criteria

The Portuguese dam safety regulation [32] defines accident scenarios as extreme failure situations that may originate a flood wave towards downstream. Among the design situations that shall be plausibly considered, the occurrence of the maximum design earthquake (MDE) during normal operation periods is usually conditioning regarding the possibility for dam collapse. In that case, the structural (F_E) and dam-water interaction (I_E) effects due to the occurrence of such event shall be combined with the operational loads, namely the dead loads (W) and the water loads, both hydrostatic (I_H) and uplift (U) pressures, which are considered not to be changeable during the earthquake event due to its short duration. Structural safety must also be verified in post-earthquake conditions, given that this event may imply permanent degradation which can affect material properties and the uplift pressures, namely its spatial distribution due to cracking and/or loss of drainage effectiveness [33].

To analytically assess the stability conditions of a gravity profile, any loads must be simulated through static forces. In that case, the effect of an earthquake event can be simulated through the pseudo-static or pseudo-dynamic [34] methods. In the case studies below, the pseudo-static method, which considers inertia forces, applied at the rigid body's centroid, as the product between mass and acceleration, is used. These forces have a predominant horizontal component but also a vertical component, assumed 30% of the horizontal one, must be assumed. Generally, only a portion of the total effects (F_E+I_E) is considered (two thirds or 0.67) in this method, given the effective non-oscillatory peak accelerations. In the vertical component, this leads to a combination factor of 0.20.

Although, in specific cases of major importance, the MDE shall be determined through regional seismologic investigations, in general situations, the MDE can be quantified probabilistically through a return period, depending on a global seismic risk index [32]. For the least demanding situation regarding the potential consequences, such as assumed in the following examples, a return period of 1000 years shall be considered. The peak ground acceleration of such earthquake is obtained in the Portuguese standard for the design of earthquake-resistant structures [35].

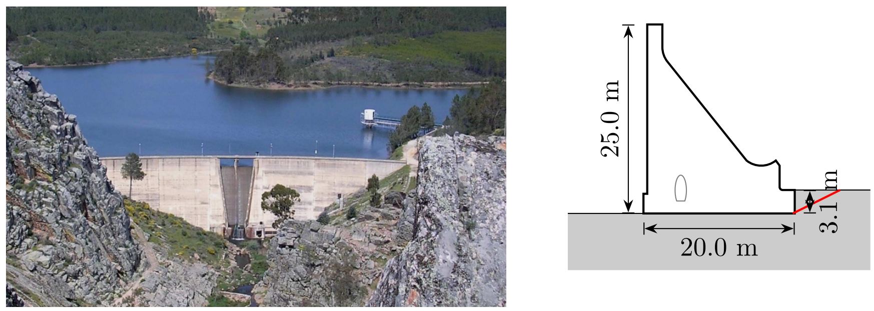
Several recommendations, namely admissible stresses, maximum discharge flow rates and safety factors, are made for the safety evaluation of concrete dams during accident scenarios. Concerning global instability, stresses at destabilization planes must fulfill the Mohr-Coulomb criteria with no cohesion and a prudent quantification of the residual friction coefficient ($\tan\phi_r$) minored by a factor larger than 1.2 [32]. Equivalently, a safety factor greater than 1.2, considering the residual shear strength with no cohesion, must be ensured. In the following examples, three situations are tested, namely: case 1) no contribution of the keyed depth is considered; case 2) the contribution is considered by a passive resistance (small displacement analysis); and case 3) the contribution is considered as proposed in this paper (large displacement analysis). The corresponding safety factors are computed, respectively, as,

$$SF = R/S = \begin{cases} V \cdot \tan\phi_r / H & \text{, for case 1} \quad (11a) \\ \left[V \cdot \tan\phi_r + W_w\left(\alpha\right) \cdot \tan\left(\phi_r + \alpha\right)\right]/H & \text{, for case 2} \quad (11b) \\ \tan\phi_r / \tan\phi_{cr}\left(\alpha\right) & \text{, for case 3} \quad (11c) \end{cases}$$

where $V = W - U - 0.2 \cdot F_E$ and $H = I_H + 0.67 \cdot (F_E + I_E)$ are the total vertical and horizontal net forces, respectively, W_w is the weight of the downstream rock wedge inclined at α and $\tan\phi_{cr}$ is the solution of equations 4, 6 and 8, for the failure modes 1, 2 and 3, respectively. For the case 2, $\alpha = 45° - \phi_r/2$, according to the Rankine's theory, and, for the case 3, α is again subordinated on obtained the maximum solution.

4.2. Case of Penha Garcia dam

Penha Garcia dam is located on Ponsul river, in the center interior of Portugal. Penha Garcia is a concrete gravity dam with 25 m of maximum height, operating since 1979 with a retention water level (RWL) at 22 m of height. Figure 10 shows the downstream view and the cross-section profile of the dam.



(a) Downstream view (b) Cross-section profile

Figure 10: Penha Garcia dam.

Penha Garcia dam has a light-weight cross-section profile (W=5836 kN/m) which, given its location on a medium intensity seismic zone, results in a small safety factor, when the contribution of the keyed depth is ignored, for the accident scenario corresponding to the occurrence of the MDE (a_g=1.48 m/s^2, for the seismic type 2). Therefore, the stability of this structure under those exceptional loading conditions (I_H=2374 kN/m, U=1790 kN/m, F_E=880 kN/m and I_E=386 kN/m) has been considered a cause of concern. In reality, the keying works play a crucial role to ensure stability. Figure 11 shows the safety factor obtained for three base assumptions (cases 1 to 3), considering different values of the friction coefficient.

Assuming the friction coefficient as 0.70 ($\phi_r = 35°$), erroneous conclusions regarding the safety conditions of Penha Garcia dam might be drawn whether ignoring or taking the contribution of the keyed depth as a passive resistance ($SF = 0.81$ for case 1 and $SF = 0.97$ for case 2). In this case, performing a large displacement analysis is crucial since safety criteria is only verified that way ($SF = 1.26$ for case 3).

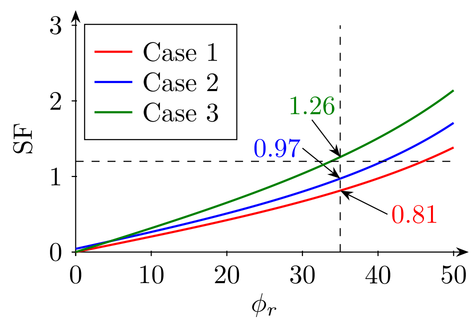
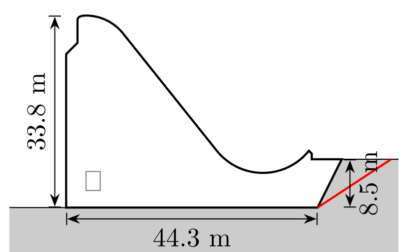


Figure 11: Penha Garcia dam: Safety factors obtained by considering no contribution of the keyed works (case 1), contributing as a passive resistance (case 2) or contributing as proposed in this paper (case 3).

4.3. Case of Pedrógão dam

Pedrógão dam is the first roller-compacted concrete (RCC) dam built in Portugal, located on Guadiana river, in the south of Portugal. It is a straight gravity dam with a maximum height of 43 m, operating since 2006. The dam has an uncontrolled spillway whose crest is located 33.8 m above the foundation, corresponding to the retention water level (RWL). Figure 12 shows the downstream view and the cross-section profile of the dam.

(a) Downstream view



(b) Cross-section profile

Figure 12: Pedrógão dam.

Pedrógão dam foundation, consisting of granite with small to medium-seized grains, is of good quality except in areas located near two faults in the main river channel. Although proper foundation treatment was provided, high discharges were recorded after the reservoir first filling, due to clogged drains with coarse sand [36]. Although further uplift/discharge reduction works were immediately executed, the stability of Pedrógão dam was tested, considering that no uplift pressure reduction was provided by an inoperative drainage system. This situation is particularly relevant since, given its wide base, the total uplift pressures (U=9517 kN/m) reduce considerably the normal stresses applied at the dam-foundation interface, even though the cross-section profile is considerably heavy (W=20317 kN/m). Furthermore, the dam is located on a higher intensity seismic zone, requiring other constructive dispositions to ensure stability for the accident scenario corresponding to the occurrence of the MDE (a_g=1.64 m/s^2, for the seismic type 2). Accordingly, the extra contribution provided by a large keyed depth (25% of the height of the uncontrolled spillway) is then crucial for that, under the corresponding exceptional loading conditions (I_H=5604 kN/m, F_E=3403 kN/m and I_E=1021 kN/m). Figure 13 shows the safety factor obtained for three base assumptions (cases 1 to 3), considering different values of the friction coefficient.

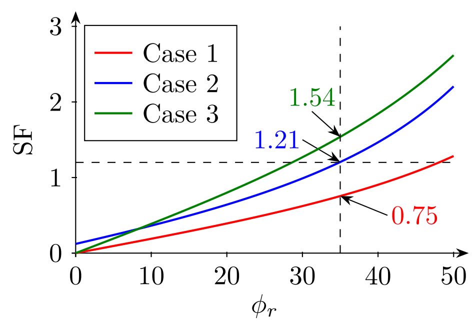


Figure 13: Pedrógão dam: Safety factors obtained by considering no contribution of the keyed works (case 1), contributing as a passive resistance (case 2) or contributing as proposed in this paper (case 3).

Assuming the friction coefficient as 0.70 ($\phi_r = 35°$), the safety criteria would not be verified if the contribution of the keyed depth is ignored ($SF = 0.75$ for case 1). However, Pedrógão dam has a much larger safety factor given its large keyed depth. By following a small displacement analysis, safety would already be verified ($SF = 1.21$ for case 2). However, by following a large displacement analysis, since this can represent the ultimate stability conditions, reflecting more realistically the real structural capacity, a much large safety factor is obtained ($SF = 1.54$ for case 3). In reality, there is a relevant safety margin which could not be noticeable by assuming the contribution of the keyed depth as a passive resistance.

5. Limitations and future research

The failure mechanisms identified, and the corresponding mathematical descriptions deducted, have been studied under the assumption of a rigid-body formulation. However, in failure mechanisms 1, 3 and 4, the dam is resting on simple points in which non-realistically high compressive stresses certainly develop, at least for such loading conditions that the dam stability may be jeopardize, causing invariably local material crushing. Furthermore, the large displacement principles that were invoked to justify the changes on contact properties could not be assumed to update the effects on the external actions that both the material crushing of support points and the corresponding rotational movement that takes place during mechanisms would cause. In fact, whether the concrete or the rock material itself would crush at points A and B, in failure mechanism 1 (Figure 6a), at point B, in failure mechanism 3 (Figure 6c), or at point C, in failure mechanism 4 (Figure 6d), forming a contact plane, with such extension that the stress installed would not exceed the corresponding material strength. That is possible to take analytically into account in the safety assessment of unkeyed gravity profiles during overturning mechanisms by successively adjusting the point of rotation as the crushing proceeds [24]. For gravity profiles keyed into the foundation, to take all that into account, the system of equilibrium equations must express the dependency of each equilibrium equation on the length of the crushed contact zone. However, a closed-form solution for the critical friction coefficient ($\tan \phi_c$) could not be obtained in that case, requiring a complex non-linear system of equations to be solved. Therefore, the mathematical descriptions of the failure mechanisms presented in this paper slightly overestimates the real structural capacity of the gravity profile.

Nonetheless, the benefits obtained with this approach to stability analysis are relevant. For the idealized structure in Figure 8, it can be showed that critical friction angles up to more 8° are obtained within the practical range of downstream face slopes (0.7 to 0.8). This simple change of perspective when analyzing the stability of a gravity profile can render a specific solution feasible, whereas, otherwise, it could not be considered safe. The safety margin implicitly assumed when analyzing this problem from a small displacement perspective, can now be fully taken into account. Although, as mentioned, this formulation slightly overestimates the real structural capacity regarding the dam stability, given its inability to account for the effects of local punctual crushing, it is certainly closer to the "exact" solution. Furthermore, other conservative assumptions are still made here, such as, for instance, disregarding any three-dimensional effect and considering the existence of a downstream wedge, which may thus compensate such overestimation. Nonetheless, further research can be done by using the discrete element method to study the fracture mechanism in concrete-rock contact points and to evaluate the actual ultimate load capacity of a keyed concrete gravity dam.

Extension to other geometric features, namely considering inclined failure surfaces, is also worth to be studied. Besides, the consideration of residual shear strength has been assumed in failure conditions, given that large displacements usually occur for the sliding mechanism of unkeyed gravity profiles. Although, the large displacement analysis is followed here, not so large displacement would occur to meet pre-collapse conditions, so the actual ultimate strength may not be truly residual. Solving the equilibrium problem assuming the peak shear strength (either linear or non-linear envelopes) could be interesting in order to obtain an upper bound of the safety factor or a lower bound of the probability of failure. In that case, a non-linear system of equations would be derived and a closed-form solution could not again be obtained so numerically solving procedures must be pursued.

6. Conclusion

Concrete gravity dams are mass concrete structures conceived to rely upon their weight for stability. These structures are often built on rock mass foundations which shall be capable of withstanding stresses within the range of 3 to 4 MPa [1] without excessive and uneven settlement. Consequently, excavation works are sometimes needed in order to reach the bed rock. In practice, however, the rock foundation is even excavated below in order to embed/key the dam and prevent dam sliding. In stability analysis, the extra strength obtained in this case is either neglected in order to account for a possible excavation downstream or taken as a passive resistance, given by the Rankine's theory [8] such as considered in soil mechanics. This hypothesis presupposes that the overall instability is attained when the shear strength of both the dam base and the joint located below a detached downstream rock wedge is fully mobilized. However, that situation describes a primary failure stage that do not necessarily lead to global instability, which is only achieved in a subsequent stage.

The aforementioned assumption is comprehensibly adopted within a predominantly deterministic approach to structural safety, only considering a portion of the real structural capacity as a conservative strategy to handle inherent uncertainties. However, for probabilistic approaches to structural safety, based on the limit state philosophy, the ultimate equilibrium conditions must be expressed as accurately as possible, which would demand further investigations. In that case, the redistribution of stresses and the reconfiguration of equilibrium states would be accounted. In this paper, those conditions were analyzed taking into account the large displacement regime. For the simpler case given by a keyed dam and a downstream rock wedge, four kinematically possible mechanisms were identified, whose prominence depends on the direction of the total net force. Mathematical descriptions of the failure mechanisms could be deducted by simple equilibrium of forces. Afterwards, those solutions were validated through comparison to equivalent discrete-element models. The obtained benefits for safety assessment were proved using two Portuguese large concrete gravity dams as examples, the

Penha Garcia and Pedrógão dams. Although it is located in a medium seismic intensity zone, Penha Garcia dam has a light-weight cross-section profile whose stability is a cause of concern. In the first reservoir filling, Pedrógão dam foundation had exhibited large discharges which clogged the drains. The dam stability in case of an inoperative drainage system was therefore tested. In both cases, the contribution of the keyed depth is crucial to ensure stability. However, that is only really noticeable when performing a large displacement analysis.

The formulation presented in this paper has limitations regarding its inability to account for the effects of local crushing. Nonetheless, the slightly overestimation of the real structural capacity regarding the dam stability is compensated by other conservative assumptions made, namely disregarding any three-dimensional effect, which is often present, and the formation of a downstream rock wedge. Accordingly, by taking the contribution of the keyed depth as proposed in this paper, the real ultimate equilibrium conditions can be assessed more accurately which is particularly suitable for the development of probabilistic and/or semi-probabilistic methodologies for safety assessment of concrete gravity dams, in the design and feasibility phases.

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