ON THE CONSTITUTIVE MODELING OF DUAL-PHASE STEELS AT FINITE STRAINS – A GENERALIZED PLASTICITY BASED APPROACH

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ABSTRACT: In this work we propose a general theoretic framework for the derivation of constitutive equations for dual-phase steels, undergoing continuum finite deformation. The proposed framework is based on the generalized plasticity theory and comprises the following three basic characteristics:

1. A multiplicative decomposition of the deformation gradient into elastic and plastic parts.
2. A hyperelastic constitutive equation
3. A general formulation of the theory which prescribes only the number and the nature of the internal variables, while it leaves their evolution laws unspecified. Due to this generality several different loading functions, flow rules and hardening laws can be analyzed within the proposed framework by leaving its basic structure essentially unaltered.

As an application, a rather simple material model, which comprises a von-Mises loading function, an associative flow rule and a non-linear kinematic hardening law, is proposed. The ability of the model in simulating simplified representation of the experimentally observed behaviour is tested by two representative numerical examples.

KEYWORDS: Dual-Phase Steels, Generalized Plasticity, Finite Strains

1 INTRODUCTION

Dual-Phase (DP) steels are a sub-group of advanced high strength steels comprising a soft ferrite matrix and a hard martensite phase in the form of islands. The combination of the unique formability properties of the ferrite matrix and the high strength properties of the martensite microstructure gives rise to a highly efficient steel alloy in terms of ductility, drawability and formability (see, e.g., [1]). As a result DP steels are increasingly used in the automotive industry, and in particular in sheet metal forming processes. Nevertheless, their widespread application is still limited due to their extremely complex behaviour, which can affect spring-back after forming. For these reasons, there is a great need for a better understanding of these materials and accordingly for the development of accurate constitutive models in order to fully exploit and apply their potential. Moreover, the large strains which occur during metal forming processes call for the development and the implementation of a finite deformation constitutive theory.

The basic objective of this study is the introduction of a general theoretical framework which in turn can be used as a basis for the constitutive modelling of DP steels, within the finite strain regime. The framework is based on the theory of *generalized plasticity* (see, e.g., [2-5]; see also the recent developments given in [6], where emphasis is paid in the present case). Generalized plasticity is a local internal variable theory of rate-independent behaviour which is physically motivated by loading-unloading irreversibility and is mathematically founded on set theory and topology. This mathematical foundation provides the theory the ability to deal with some "non-standard" cases. Such cases may comprise non-conventional elastic-plastic response, which may appear during a loading re-

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versal and/or a (proportional or non-proportional) change of the deformation path.
This work represents also a first attempt to show the applicability of the framework in the constitutive modelling of the materials in question. In order to accomplish this goal a rather simple material model is proposed. The model is implemented numerically and its ability in simulating several patterns of the experimentally observed behaviour is demonstrated by two representative numerical examples.

2 BASIC EQUATIONS

As a starting point we assume a continuum body made from a DP steel alloy, which - at time t - occupies a reference configuration $B \subset \mathbb{R}^3$, with particles labeled by $\mathbf{X} \in B$. A deformation of B is defined as the (time-dependent) mapping

$$\mathbf{x} : B \rightarrow b \ \mathbf{x} = \mathbf{x}(\mathbf{X}, \mathrm{t}), \tag{1}$$

which maps the points of the reference configuration B onto the points $\mathbf{x}$ of the spatial configuration b. The deformation gradient is defined as the derivative of the deformation (1), that is

$$\mathbf{F} = \mathbf{F}(\mathbf{X}, \mathrm{t}) = \frac{\partial \mathbf{x}(\mathbf{X}, \mathrm{t})}{\partial \mathrm{t}}. \tag{2}$$

The basic kinematic assumption consists of a local decomposition of the deformation gradient $\mathbf{F}$ into elastic $\mathbf{F_e}$ and plastic $\mathbf{F_p}$ parts (see, e.g., [3], [5], [7, pp. 301-303]), i.e.

$$\mathbf{F} = \mathbf{F_e F_p}. \tag{3}$$

By following Simo and Hughes [7, pp. 303-305] we consider the plastic right and the elastic left Cauchy-Green (deformation) tensors $\mathbf{C_p}$ and $\mathbf{b_e}$, respectively, which are defined as

$$\mathbf{C_p} = \mathbf{F_p^T F_p}, \ \mathbf{b_e} = \mathbf{F_e F_e^T}. \tag{4}$$

We note that both $\mathbf{C_p}$ and $\mathbf{b_e}$ are symmetric and positive-definite and accordingly they can serve as primitive measures (metrics) of plastic deformation. These metrics are related by: $\mathbf{b_e} = \mathbf{F C_p^{-1} F^T}$; one says that $\mathbf{b_e}$ is the *push-forward* (see, e.g., [5]) of $\mathbf{C_p^{-1}}$, that is $\mathbf{b_e}$ is $\mathbf{C_p^{-1}}$ as perceived in the spatial configuration.
The local state at the point $\mathbf{X}$ is assumed to be determined by the second Piola-Kirchhoff stress tensor $\mathbf{S}$, the tensor $\mathbf{C_p}$ and the hardening variables A and $\mathbf{Q}$. As usual, A is a scalar internal variable which serves as a measure of isotropic hardening, while $\mathbf{Q}$ is a tensorial internal variable which serves as a measure of directional hardening.
The flow rule in generalized plasticity is derived on the basis of a *loading/unloading postulate*, the *continuity of the material behavior* and the defining property of the *loading surface* (see, e.g., [2-5]) in the following form

$$\dot{\mathbf{C}}_\mathbf{p}^{-1} = H(F) \mathbf{\Lambda}(\mathbf{S}, \mathbf{C}, \mathrm{A}, \mathbf{Q}) \left\langle \mathbf{N} : \dot{\mathbf{S}} \right\rangle. \tag{5}$$

In Eq. (5), H is a scalar function of the mathematical expression for the loading surfaces which is assumed to be given as a one- parameter family of curves, that is $F = F(\mathbf{S}, \mathbf{C}, \mathrm{A}, \mathbf{Q}) = m$, $\mathbf{C}$ is the right Cauchy-Green tensor $(\mathbf{C} = \mathbf{F^T F})$ and $\mathbf{\Lambda}$ is a non-vanishing (tensorial) function of the state variables which is associated with the direction of the plastic flow. Next, in Eq. (4), $\langle \cdot \rangle$ stands for the Macauley bracket, i.e. $\langle x \rangle = (x + |x|)/2$, and $\mathbf{N}$ is the normal vector to the loading surface at the current stress point. The particular case $\mathbf{\Lambda} = \mathbf{N}$, corresponds *to normality or associative* plasticity. It is noted that the tensor $\mathbf{C}$ is used as a basic state variable in addition to $\mathbf{S}$, since it is included among the arguments of the functions F and $\mathbf{\Lambda}$; such a formulation is called the *convected representation* of the theory and offers several advantages for the development of a finite theory (see, e.g., [5], [7, p. 261]).
In view of Eq. (4) and in accordance with classical plasticity theories, the rate equations for the evolution of the hardening variables may be stated in the form

$$\begin{aligned} \dot{\mathrm{A}} &= H(F) M(\mathbf{S}, \mathbf{C}, \mathrm{A}, \mathbf{Q}) \left\langle \mathbf{N} : \dot{\mathbf{S}} \right\rangle, \\ \dot{\mathbf{Q}} &= H(F) \mathbf{P}(\mathbf{S}, \mathbf{C}, \mathrm{A}, \mathbf{Q}) \left\langle \mathbf{N} : \dot{\mathbf{S}} \right\rangle, \end{aligned} \tag{6}$$

where M and $\mathbf{Q}$ stand for a scalar and a tensorial function of the state variables.
To this end it is interesting to note that the decomposition (3) is a local one (see, e.g., [7, pp. 302-303]) and accordingly the (intermediate) configuration which appears upon unloading (by $\mathbf{F_e^{-1}}$), is in fact *a collection of local unloaded configurations*, that is a *manifold*. This can be endowed by a non-Euclidean metric - say $\bar{\mathbf{G}}_\mathbf{e}$ - so that $\mathbf{C_p}$ and $\mathbf{b_e}$ can be redefined - see [4, 8] for details - as follows

$$\mathbf{C_p} = \mathbf{F_p^T} \bar{\mathbf{G}}_\mathbf{e} \mathbf{F_p}, \ \mathbf{b_e} = \mathbf{F_e} \bar{\mathbf{G}}_\mathbf{e} \mathbf{F_e^T}. \tag{7}$$

By means of these definitions, several *curved material structures* may be analyzed without the requirement of testing every local neighborhood of the body. By using Eqs. (7), in place of Eqs. (3), the curved structure can be accounted for by means of the metric $\bar{\mathbf{G}}_{\mathbf{e}}$ of the (curvilinear) coordinate system of the local configuration. This approach may find application in DP steels, in the case where curved specimens, like the triangle or Nagajima specimens, are used in place of the standard cylindrical or rectangular ones.

The formulation of the theory is supplemented by the stress-deformation (strain) relations. Without loss of generality, it may be assumed that the stress response is *hyperelastic*, governed by an isotropic strain energy function (see, e.g., [5], [7, p.258]) in terms of the first (I_1) and the third (I_3) invariants of the tensor $\mathbf{C}\mathbf{C}_{\mathbf{p}}^{-1}$, that is

$$E(I_1, I_3) = \frac{\lambda}{4}(I_3 - 1) \\ -(\frac{\lambda}{2} + \mu)\ln\sqrt{I_3} + \frac{\mu}{2}(I_1 - 3),$$

where λ and μ are material parameters to be interpreted as Lame′ parameters. Then the stress response is determined by the standard relation (see, e.g., [7, p. 256]) $\mathbf{S} = 2\frac{\partial E}{\partial \mathbf{C}}$, which yields

$$\mathbf{S} = \frac{\lambda}{2}(I_3 - 1)\mathbf{C}^{-1} + \frac{\mu}{2}(\mathbf{C}_{\mathbf{p}}^{-1} - \mathbf{C}^{-1}). \tag{8}$$

Next we derive an equivalent assessment of the basic equations in the spatial configuration. These can be done in a straight forward manner by performing a push-forward operation (see, e.g., [5]) to the basic Eqs. (4), (5) and (7). The resulting equations read

$$\begin{aligned} L_{\mathbf{v}}\mathbf{b}_{\mathbf{e}} &= h(f)\boldsymbol{\lambda}(\boldsymbol{\tau}, \alpha, \mathbf{q}, \mathbf{F})\langle \mathbf{n} : L_{\mathbf{v}}\boldsymbol{\tau}\rangle, \\ \dot{\alpha} &= h(f)\upmu(\boldsymbol{\tau}, \alpha, \mathbf{q}, \mathbf{F})\langle \mathbf{n} : L_{\mathbf{v}}\boldsymbol{\tau}\rangle, \\ L_{\mathbf{v}}\mathbf{q} &= h(f)\mathbf{p}(\boldsymbol{\tau}, \alpha, \mathbf{q}, \mathbf{F})\langle \mathbf{n} : L_{\mathbf{v}}\boldsymbol{\tau}\rangle, \\ \boldsymbol{\tau} &= \frac{\lambda}{2}(i_3 - 1)\mathbf{I} + \mu(\mathbf{b}_{\mathbf{e}} - \mathbf{I}), \end{aligned} \tag{9}$$

where $\boldsymbol{\tau}$ is the Kirchhoff stress tensor $(\boldsymbol{\tau} = \mathbf{F}\mathbf{S}\mathbf{F}^{\mathbf{T}})$, $\mathbf{q}$ is the push-forward of $\mathbf{Q}$ and $L_{\mathbf{v}}(\cdot)$ stands for the Lie derivative (see, e.g. [5], [7, pp. 254-255]). Further, in Eqs. (8) α, f and h stand for the equivalent expressions of the *scalar invariants* A, F and H in terms of the spatial variables and $\boldsymbol{\lambda}$, $\mathbf{p}$ and $\mathbf{n}$ are the push-forwards of the functions $\boldsymbol{\Lambda}$, $\mathbf{P}$ and $\mathbf{N}$ in the spatial configuration. Finally, $\upmu$ is the spatial expression for M, i_3 is the third invariant of $\mathbf{b}_{\mathbf{e}}$ and $\mathbf{I}$ is the identity (rank -2) tensor. The presence of the deformation gradient $\mathbf{F}$ among the arguments of the functions $\boldsymbol{\lambda}$, $\upmu$ and $\mathbf{p}$ is noteworthy. This is due to the push-forward operation by which equations (8) are derived from their referential counterparts.

Eqs. (4), (5) and (6) - or equivalently Eqs. (8) - constitute a rather general approach to the constitute modeling of DP steels within the context of the multiplicative decomposition (3). In order to develop a particular model we have to specify:

1. The mathematical expression for the loading surfaces *F* (or equivalently *f*),
2. The expression for the scalar function *H* (or *h*),
3. The expressions for the state functions $\boldsymbol{\Lambda}$, M and $\mathbf{P}$ (or $\boldsymbol{\lambda}$, $\upmu$ and $\mathbf{p}$).

These will be specified in the forthcoming section where a rather simple model is proposed.

3 MATERIAL MODEL

From the aforementioned analysis it is concluded that a finite strain model may in principle be formulated equivalently with respect to the reference or the spatial configurations. Since we deal with large scale plastic flow, the kinematics of the problem, together with the concept of spatial invariance (see, e.g., [5]), suggest that a formulation in the spatial configuration is more fundamental. Moreover a careful observation of the basic equations, reveals that the spatial ones, although involving Lie derivatives, are in general simpler than their referential counterparts. Accordingly, we develop the model in the spatial configuration. An equivalent assessment of a related model in the reference configuration can be found in [6].

For this purpose we introduce a von-Mises type of expression for the loading surfaces,

$$f = \left|dev(\boldsymbol{\tau} - \mathbf{q})\right| - \sqrt{\frac{2}{3}}(\sigma_y + K\alpha) = m,$$

where $dev(\cdot)$ stands for the deviatoric operator, $|\cdot|$ is the Euclidean norm, σ_y is the uniaxial (Kirchhoff) yield stress and K is the isotropic hardening modulus. Note that the *initial loading surface* arising for $m = 0$ constitutes the *yield surface of classical plasticity* ([4], [6]; see also [7]). Note further, that more sophisticated expressions for the loading surface family which may comprise distortional hardening (see, e.g., [9, 10]) can be implemented in place of *f*, at some computational cost, without any conceptual difficulty.

The plastic flow is assumed to be governed by the associated flow rule $(\boldsymbol{\lambda} = \mathbf{n})$ i.e.

$$L_v \mathbf{b_e} = h(f)\mathbf{n}\langle \mathbf{n} : L_v \boldsymbol{\tau} \rangle,$$

while for the evolution of the isotropic hardening variable α, motivated by the infinitesimal theory (see, e.g., [7, pp. 90, 9])we assume a rate equation of the form

$$\dot{\alpha} = \sqrt{\frac{2}{3}} h(f) \langle \mathbf{n} : L_v \boldsymbol{\tau} \rangle.$$

For the evolution of the kinematic hardening variable $\mathbf{q}$ we consider an evolution law of Armstrong-Frederic type, that is

$$L_v \mathbf{q} = \frac{2}{3} \mathrm{H} L_v \mathbf{b_e} - L\dot{\alpha}\mathbf{q},$$

where H and L are the linear and non-linear kinematic hardening moduli, respectively.
Finally, for the scalar function h we assume an expression discussed in [11], which within the present (finite strain) context is expressed in a somewhat surprising format, as

$$h(f) = -\frac{1}{2} \frac{\langle f \rangle}{\beta(K + L - L) + R(\beta - f)},$$

where β and R are two additional model parameters.

4 NUMERICAL SIMULATIONS

In this section we implement the proposed model numerically - see [5] for computational details - in order to test its ability in predicting some of the complex phenomena appearing in DP steels. In particular we consider two examples: one of simple shear and another one of uniaxial tension.

4.1 SIMPLE SHEAR

The simple shear problem constitutes a standard test within the context of finite deformation plasticity and is defined as

$$x_1 = X_1 + \gamma X_2,\ x_2 = X_2,\ x_3 = X_{3,}$$

where $\gamma = \gamma(\mathrm{t})$ is the applied shear. Our purpose in this example is to show the effect of the basic model parameters β and R in a typical DP steel alloy stress-strain curve. For this purpose we consider elastic perfectly-plastic behaviour i.e. we assume that $K = \mathrm{H} = L = 0.$ The remaining parameters are set equal to

$$E = 205GPa,\ \nu = 0.3,\ \sigma_y = 420MPa$$

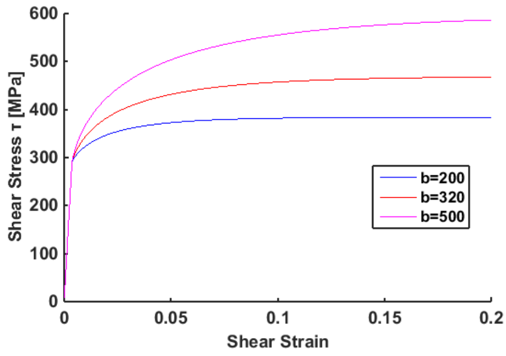

Fig. 1 *Simple shear: Typical monotonic stress-strain curves.*

The monotonic stress-strain curves predicted by the model for three different values of the parameter β are shown in Fig. 1.
By referring to this figure we note that the predicted stress-strain curves have the same qualitative characteristics with the monotonic curves reported by Tarigopula et al. in [12]. In particular they show a general a *smooth elastic-plastic transition* and finally the *stress convergences to the ultimate strength*. Moreover we note that higher is the value of β is, the higher is the predicted stress; accordingly it is concluded that β *has to be related directly to the ultimate strength of the steel*.
As a second illustration, we show the effect of the parameter R in a typical loading-unloading-reloading stress-strain curve $(\beta = 320)$. The results are shown in Fig. 2.

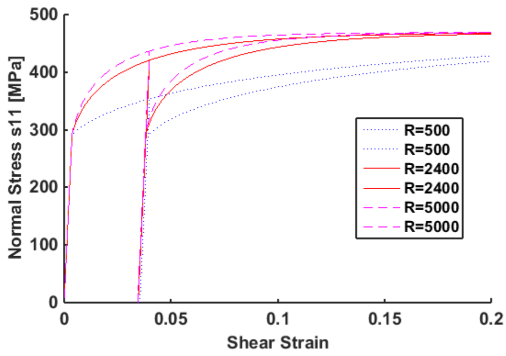

Fig. 2 *Simple shear: Typical loading-unloading-reloading stress-strain curve.*

In this case we note the ability of the model in predicting an unconventional response, according to which, the reloading, following (plastic) loading and subsequent (elastic) unloading, is not elastic up to the state where the unloading began, while the subsequent loading curve does not attain the (monotonic) loading one. This is the so called *long-term* or *permanent softening effect* (see, e.g. [13]). Moreover the ability of the model in predicting a very fast increase of the hardening rate upon reloading, which is characteristic in DP steels and is termed *transient (rapid) strain hardening*, is easily verified.

Since the permanent softening effect appears more intense in higher strength DP steels (see, e.g., fig. 6 in [13]), in view of fig. 2, we conclude that in *general a relative low value of R has to be chosen for a high strength DP steel alloy; on the other hand a relative large value of R corresponds to a lower strength one.*

4.2 TENSION-COMPRESSION TEST ALONG THE ROLLING DIRECTION

This one-component loading problem is defined as

$$x_1 = (1+\chi)X_1,\ X_2,\ x_2 = (1+\psi)X_2,$$
$$x_3 = (1+\psi)X;$$

where $1+\chi(\mathrm{t})$ and $1+\psi(\mathrm{t})$ are the principal stretches $(1+\chi)$ and $(1+\psi)$ (see, e.g. [7, pp. 241-242]) along the rolling and transverse directions respectively. Note that in the infinitesimal case, $\chi = \chi(\mathrm{t})$ and $\psi = \psi(\mathrm{t})$ are equal to the corresponding principal strains.

This simulation concerns primarily the validation of the model against the actual experimental results of a DP 780 steel reported in the recent paper by Sun and Wagoner [13]. The corresponding model parameters are:

$E = 205{,}000$ (Mpa), $\nu = 0.3$, $\sigma_y = 320$ Mpa

β=320 (Mpa), $R = 1200$ (Mpa), $K = 50$ (Mpa)

$\mathrm{H} = 410$ (Mpa), $L = 1025$ (Mpa).

As in [13] the material is subjected to uniaxial (monotonic) tension and in tension-compression (up to zero strain)-tension at three different strain levels. The results are shown in Fig. 3.

As it is shown in this figure the model is able to describe actual experimental data. In particular the model can simulate in a very precise way, besides transient hardening and permanent softening phenomena, and the Bauschinger effect,, that is the *early re-yielding of the material in stress levels lower that the initial yield stress, upon a loading reversal*. This response appears alike in a tension-compression test (see Fig. 4)

5 CONCLUSIONS

In this work we have introduced a finite strain version of the theory of generalized plasticity and have shown it to constitute a convenient framework for modelling DP steel alloys. In particular:

1. Based on the multiplicative decomposition of the deformation gradient and the use of a hyperelastic constitutive equation we have presented the theory in an *invariant setting*, i.e. a setting where the basic equations can be written in an equivalent manner in both the reference and the spatial configuration. This approach has the advantage that one can find a configuration where the basic equations take their simpler form and then recast them in the dual configuration by just performing a standard push forward or pull-back (see, e.g. [5]) operation.
2. We have confined the theory in the simplest possible setting by considering a von-Mises type expression of loading surfaces, an associated flow rule and a nonlinear kinematic hardening law. We have shown that, although this consideration is simple, it is able to simulate several patterns of the complex behaviour of DP steels, upon a loading reversal.

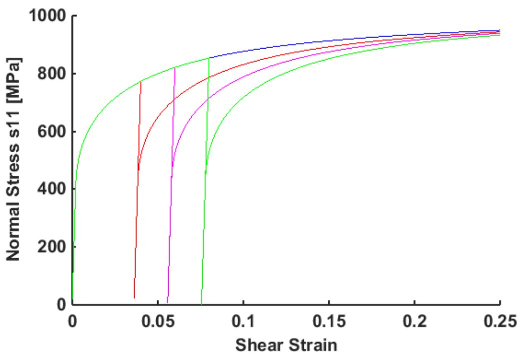

Fig. 3 *Uniaxial loading-unloading-reloading: Bauschinger effect, transient hardening, permanent softening for a DP 780 steel alloy*

However, the assumption of a von-Mises type expression for the loading surfaces constitutes an oversimplification and in general a more sophisticated expression has to be introduced. In a future work

1. We will extend the present model, to address more complex phenomena, e.g., cross-hardening and latent hardening appearing upon a change of the loading path. For this purpose a more complicated loading function, which (possibly) comprises distortional hardening will be considered within the present model,
2. We will implement the model within the context of the finite element method, where special emphasis will be given in spring-back predictions.

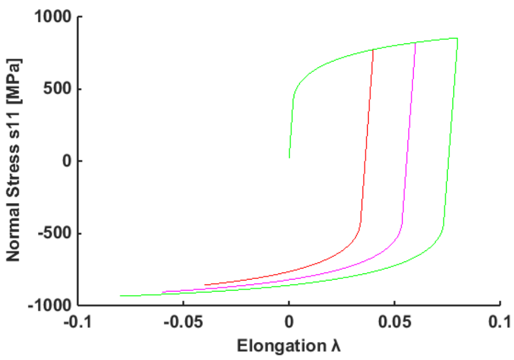

Fig. 4 *Uniaxial loading: Tension-compression*

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