

Zippering and Unzipping in String Networks: Dynamics of Y-junctions

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We study, within the Nambu-Goto approximation, the stability of massive string junctions under the influence of the tensions of three strings joining together in a Y-type configuration. The relative angle β between the strings at the junction is in general time-dependent and its evolution can lead to zippering or unzipping of the three-string configuration. We find that these configurations are stable under deformations of the tension balance condition at the junction. The angle β relaxes at its equilibrium value and the junction grows relativistically. We then discuss other potential “unzipping agents” including monopole/string forces for long strings and curvature for loops, and we investigate specific solutions exhibiting decelerated zippering and unzipping of the Y-junction. These results provide motivation for incorporating the effects of realistic string interactions in network evolution models with string junctions.

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I. INTRODUCTION

In the context of grand unified theories, phase transitions followed by spontaneously broken symmetries may leave behind cosmic strings [1–4], as false vacuum remnants. Cosmic strings are generically formed at the end of hybrid inflation [5–8]. In addition, brane interactions in the context of string theoretic cosmological models, can lead [9–13] to fundamental (F) strings, one-dimensional Dirichlet branes (D-strings) and their bound (FD) states, collectively known as cosmic superstrings [14–16], which may play a cosmological rôle as cosmic strings. In particular, cosmic superstrings are copiously formed at the end of brane inflation [9, 17–19].

Unlike ordinary abelian field theory strings which can only interact through intercommutation and exchange of partners with probability of order unity [20], collisions of cosmic superstrings typically happen with smaller probabilities and can lead to the formation of Y-junctions¹ at which three strings meet [21–23]. This characteristic property is of particular interest because it can modify dramatically the network evolution [24–28] leading to potentially observable phenomenological signatures, and thus providing a potential window into string theory [15, 27, 29–31].

The effect of junctions on the evolution of string networks was the central subject of several numerical [32–36] and analytical [24–28, 37–40] investigations. In particular, in Refs. [37–39] and [41] the authors studied in detail the kinematics of junction formation, under the assumption² that string dynamics is well-described by the Nambu-Goto action, and were able to find kinematic conditions under which junctions form. These kinematic constraints were later incorporated into network evolution modelling in Refs. [28, 40] and were shown to play an important rôle in determining the relative number densities of the dominant string species, thus affecting quantitatively any potential observational signals from these networks [28, 31]. However, an additional potentially significant source of uncertainty in this type of network models remains, as it is not well-understood under which conditions these junctions continue to grow and stabilise, or alternatively shrink resulting in “unzipping” of the heavier (bound) string states. Indeed, the simulations of Refs. [32, 33], studying field theory models in which zippering can occur, have found evidence supporting that heavier bound states can actually unzip, leading to a lower abundance of heavy strings in the network than what one would naively expect.

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¹ This property is also shared by non-abelian field theory strings.

² Remarkable agreement between the Nambu-Goto and field theory descriptions in this context has been demonstrated in Refs. [35, 36, 41].

The purpose of this paper is to study the dynamics of junctions in a Nambu-Goto approximation and investigate the conditions which could lead to spontaneous unzipping of the heavier, composite string states. We do this by assigning to the junction a mass, thus allowing for cases in which the tensions of the three strings joining at the junction are not balanced, and we study the dynamics of the junction under the influence of these tensions. We find that the straight string configurations under consideration, i.e. with a massive junction that allows for a non-trivial force, are stable under perturbations. More specifically, they exhibit a damped oscillating behaviour around the balance condition solution. Therefore unzipping does not occur and we need to allow for extra forces exerted on the junction. Considering such forces originating from monopole and string forces we indeed find decelerating solutions in the case of straight strings. In the case of loops, unzipping generically occurs as a result of local curvature near the junction.

This paper is organised as follows: In Section II, we present a brief overview and update of the currently known Y-junction configurations and their kinematics within the Nambu-Goto approximation; we pay special attention to the local geometry of strings near the junction. In Section III, we outline the basic formalism describing the dynamics of Y-type configurations of three Nambu-Goto strings ending at a massive junction. We introduce the general setup and obtain the evolution equations for the junctions. In Section IV, we study the kinematics of junction formation for two moving straight string segments colliding at angle α , following Refs. [37–39, 41]. We pay particular attention to the local angle, say β , at the (massive) junction and move on to study its dynamics in the formalism of Section III. We find that the evolution of β is given by damped oscillations around the equilibrium (critical) value β_{crit} (found in Refs. [37–39]) for which the vector sum of the three *effective* tensions vanishes. Thus, even if we deform the tension balance condition, the local angle quickly relaxes to β_{crit} and the junction grows relativistically, as has been assumed in most string evolution models. In Section V, we investigate other possible unzipping mechanisms, namely monopole forces, string forces, and string curvature for loops with junctions. We find solutions which exhibit decelerated zipping, eventually leading to the unzipping of the Y-junction configuration. We round up our conclusions in Section VI.

II. MASSLESS STRING JUNCTIONS: REVIEW AND UPDATE

Let us start by reviewing the kinematic condition for junction formation derived in Ref. [37], hereafter CKS. The authors considered a configuration of two straight infinite string segments with tensions μ_1 and μ_2 , moving towards one another each with speed v and colliding at an angle 2α at time $t = 0$. Choosing coordinates such that the two strings collide in the (x, y) -plane, each at an angle $\pm\alpha$ off the x -axis (Fig.1, *Top*), the two strings in this configuration for $t \leq 0$ can be parametrised in terms of two parameters (σ, t) as

$$\mathbf{x}_{1,2} = (-\sigma\gamma_v^{-1} \cos \alpha, \mp \sigma\gamma_v^{-1} \sin \alpha, \pm vt) , \quad (1)$$

with $\gamma_v^{-1} = \sqrt{1 - v^2}$. This is a solution of the Nambu-Goto equations of motion expressed in the conformal temporal gauge, where the spacelike worldsheet parameter σ is the *invariant length*, $d\sigma = d|\mathbf{x}|/\sqrt{1 - \dot{\mathbf{x}}^2}$, and the timelike worldsheet parameter is identified with background time t . Note that $\dot{\mathbf{x}}_{1,2} = (0, 0, \pm v)$ is the physical velocity, which is transverse to the string tangent $\mathbf{x}' \equiv \partial_\sigma \mathbf{x}$, and since v is constant the quantity $|\sigma|\gamma_v^{-1}$ in Eq. (1) simply measures the *physical length* along the string. In the chosen sign convention σ increases towards the vertex.

As a result of the collision at $t = 0$, a new string segment (a “link” or “zipper”) of tension μ_3 can be formed, giving rise to two trilinear Y-shaped junctions connecting it to the original strings (Fig.1, *Bottom*). Far away from the junctions the strings retain their original motion and orientation, so this 2-junction configuration is connected to the solution (1) for $t > 0$ through four kinks, moving at the speed of light along the original strings and away from the point of string intersection. Here, we concentrate on the simplest case, where the original strings have equal tension $\mu_1 = \mu_2 \equiv \mu$. Symmetry then implies that the newly formed “zipper” segment lies either along the x -axis or along the y -axis and that it stays in the (x, y) -plane at all times $t > 0$. For small angle α we may expect the zipper to be formed along the x -axis, as shown in Fig.1, *Bottom*.

The authors of Ref. [37] were able to confirm and quantify the last statement. By writing down and analysing the Nambu-Goto action for three strings $\mathbf{x}_i(\sigma_i, t)$; $i \in \{1, 2, 3\}$, joining at a vertex $X(t)$, they were able to find the kinematic conditions that must be satisfied for junction formation to be possible in the configuration (1). A first restriction is that the string tensions must satisfy the triangle inequalities. Further, it is possible to obtain a quantitative constraint depending explicitly on the tensions, the speed v and angle α . This can be done by thinking of the point of intersection at $t = 0$ (Fig.1, *Top*) as a “zipper” of tension μ_3 but zero length, which can grow for $t > 0$ resulting in the configuration of Fig. 1, *Bottom*. Let $s_i(t)$ be the coordinate of the junction on string i , that is $\mathbf{x}_i(s_i(t), t) = X(t)$ for all three strings. Then, for $\mu_1 = \mu_2 = \mu$, CKS found

$$\dot{s}_3 = \frac{2\mu\gamma_v^{-1} \cos \alpha - \mu_3}{2\mu - \mu_3\gamma_v^{-1} \cos \alpha} . \quad (2)$$

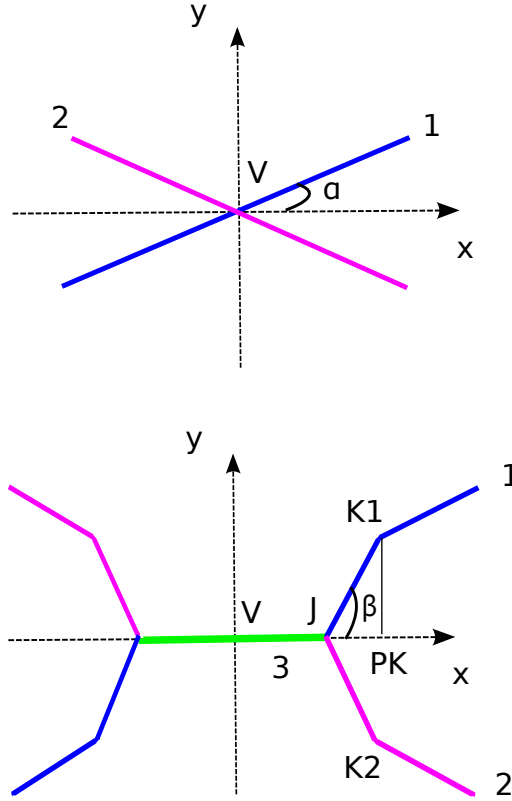


FIG. 1: *Top*: Two strings (denoted by 1, 2) colliding at time $t = 0$. *Bottom*: At time $t = \delta t$, a new segment (denoted by 3) has formed. The junction J is moving to the right and the zipper is growing.

The constraint for junction formation is that the zipper must grow, $\dot{s}_3 > 0$, which gives

$$\alpha < \arccos\left(\frac{\mu_3 \gamma_v}{2\mu}\right). \quad (3)$$

This defines the region of parameter space for which formation of a zipper along the x -axis is kinematically allowed. A similar constraint can be obtained for a zipper along the y -axis, which is expected to be the preferred configuration for large angles $\alpha < \pi/2$. Indeed, in this case one finds

$$\alpha > \arcsin\left(\frac{\mu_3 \gamma_v}{2\mu}\right), \quad (4)$$

confirming the above expectation.

The result (3-4) has important implications for the evolution of string networks with junctions, where one must consider collisions at all possible angles and velocities in a large ensemble of string segments. It tells us that string junctions only form in a subset of orientations $\alpha \in [0, \pi/2]$ defined by the union of the regions (3)-(4). In addition, there is a critical speed, depending on the ratio of the tensions, beyond which the junction cannot be formed. Indeed, (3)-(4) are well-defined for $\gamma_v \leq 2\mu/\mu_3$, whose saturation corresponds to a maximum speed. Although these results have been derived in the zero-width, Nambu-Goto approximation, they were found to be in remarkable agreement with Abelian-Higgs field theory simulations of straight string collisions [36]. Thus, these constraints, as well as their generalisation for unequal tensions [38] and cosmic superstrings [39], must be (and have been) incorporated in network evolution models, significantly affecting quantitative predictions [28, 40].

Equation (2) was derived by considering the string configuration (1) at $t = 0$, when the angle between the colliding segments and the x -axis at the junction is $\pm\alpha$. One may ask whether this equation is valid for $t > 0$ when, clearly, the angle between the strings and the x -axis at the junction (segments JK_1 , JK_2 in Fig. 1, *Bottom*) is $\pm\beta$, with $\beta > \alpha$. In fact, Eq. (2) is actually valid for all $t > 0$, as the solution has $\dot{s}_1 = \dot{s}_2 = -(\mu_3/2\mu)\dot{s}_3 = \text{const}$. For $t > 0$, Eq. (2) can be understood by considering the union of segments JK_1 and JK_2 as a “rigid” body subject to the tension μ_3 of the “zipper” segment VJ (applied on point J and pulling to the left) and the tensions of the two strings beyond

the kinks (which are applied at points K_1 , and K_2 at angles $\pm\alpha$ with the x -axis respectively) together pulling to the right. This was done in detail in Ref. [41]. The authors of Ref. [41] showed that the growth of the segments JK_1 and JK_2 leads to a rate of change of x -momentum:

$$\dot{p}_x = \dot{s}_3 (2\mu - \mu_3 \gamma_v^{-1} \cos \alpha) , \quad (5)$$

which is exactly balanced by the x -component of the sum of the external tensions:

$$T_x = 2\mu \gamma_v^{-1} \cos \alpha - \mu_3 . \quad (6)$$

Equating Eq. (5) and Eq. (6) we recover Eq. (2), which is a statement of energy-momentum conservation.

Thus, in the solution considered, where $\dot{s}_3 = \text{const}$, the “local” angle $\beta = \text{const} > \alpha$ and can be eliminated from the dynamics. The resulting configuration is an ever-growing zipper, $\dot{s}_3 > 0$. Motivated by numerical simulations [32, 33] which suggest that the zipper growth can be inverted in string networks (thus leading to string unzipping), we are interested in studying deviations from this model solution, allowing, in particular, non-trivial evolution of the local angle β . This will be done in the following sections, by considering perturbations around this solution and by introducing a massive junction, subject to external forces, allowing us to study junction dynamics. In the remaining of this section we will complete the above basic picture by describing the configuration of Fig. 1 near the junction, in terms of the angle β . Starting with the simplest possible geometric configuration, we will first assume that $v \rightarrow 0$ as $t \rightarrow 0$, ensuring that all strings stay on the (x, y) -plane for all $t \geq 0$. Once this configuration is fully described, we will then restore the z -velocity, v .

By symmetry it suffices to consider only one-half of Fig. 1, e.g. $x > 0$, the other half being just the mirror image. First, we note that the angle β can be easily determined from \dot{s}_3 and the fact that the kinks move at the speed of light at angles $\pm\alpha$ off the x -axis. In fact, even if we did not know that $\dot{s}_3 = \text{const}$, we could still determine β for sufficiently small time $\delta t > 0$ such that $s_3(\delta t) \simeq \dot{s}_3(0)\delta t$. This only assumes continuity of $\dot{s}_3(t)$ at $t = 0$, where $\dot{s}_3(0)$ is determined from Eq. (2). We are also assuming that the segments JK_1 , JK_2 are straight.

Let us determine β in the configuration shown in the bottom panel of Fig. 1. Since $\dot{s}_3(0) > 0$, the junction J is moving to the right and the zipper is growing. As mentioned above, causality requires that sufficiently far from the junction the solution (1) is still valid at time $t = \delta t$ and this solution is joined to the segments JK_1 , JK_2 at the moving kinks, K_1 , K_2 . Due to longitudinal Lorentz invariance the kinks move along the strings at speed $c = 1$ and so they are positioned at a distance $c\delta t = \delta t$ away from the original vertex V and along the original direction of the strings, i.e. at angles $\pm\alpha$. The junction J only moves at speed $\dot{s}_3 < 1$ along the x -axis so it is at a distance $\dot{s}_3(0)\delta t$ from the original vertex. This is the half-length of the zipper at $t = \delta t$ (the other half is in the mirror half of Fig. 1 *Bottom*, i.e., the one with $x < 0$, and involves a junction moving to the left):

$$\frac{\ell_3(t = \delta t)}{2} = VJ = \dot{s}_3(0)\delta t .$$

Overall, the configuration at $t = \delta t$ involves the zipper segment VJ (with tension μ_3), two straight segments (with equal tensions μ) linking the junction J to the kinks K_1 and K_2 , plus the original string segments labeled by 1 and 2 beyond K_1 and K_2 , respectively. The angle β between JK_1 and the x -axis is given by (see, Fig. 1, *Bottom*)

$$\cos \beta = \frac{VK_1 \cos \alpha - VJ}{JK_1} = \frac{[\cos \alpha - \dot{s}_3(0)]\delta t}{\sqrt{\delta t^2 + \dot{s}_3(0)^2 \delta t^2 - 2\dot{s}_3(0) \cos \alpha \delta t^2}} = \frac{\cos \alpha - \dot{s}_3(0)}{\sqrt{1 + \dot{s}_3(0)[\dot{s}_3(0) - 2 \cos \alpha]}} . \quad (7)$$

Similarly, we also have (directly from Fig. 1, *Bottom*)

$$\sin \beta = \frac{K_1 P_K}{JK_1} = \frac{\sin \alpha}{\sqrt{1 + \dot{s}_3(0)[\dot{s}_3(0) - 2 \cos \alpha]}} , \quad (8)$$

and

$$\tan \beta = \frac{K_1 P_K}{VK_1 \cos \alpha - VJ} = \frac{\sin \alpha}{\cos \alpha - \dot{s}_3(0)} . \quad (9)$$

Since $\dot{s}_3(0)$ is positive, and also³ $\dot{s}_3(0) \leq \cos \alpha$ from (2) above, we have

$$\tan \beta > \tan \alpha .$$

³ Physically, this guarantees that the junction does not move to the right faster than the projection P_K of the kinks K_1 , K_2 on the x -axis.

Thus, for any $\alpha < \arccos(\mu_3/2\mu)$, Eq. (9) implies that the angle at the junction must change discontinuously from α to $\beta > \alpha$ at $t = 0$, when $\dot{s}_3 > 0$ is suddenly switched on. For $\alpha = \arccos(\mu_3/2\mu)$ we have $\dot{s}_3(0) = 0$ so the zipper does not form. It stays formally at zero length and the junction does not move, which is expected since $\alpha = \arccos(\mu_3/2\mu)$ corresponds to the balance of tensions at the junction, Eq. (6). In the other extreme, $\alpha = 0$, we get $\dot{s}_3(0) = 1$ and the junction moves to the right at the speed of light.

Using Eq. (2) we can re-write Eq. (9) as (recall we are assuming $v \rightarrow 0$ as $t \rightarrow 0$ from below)

$$\tan \beta = \frac{(2\mu/\mu_3) - \cos \alpha}{\sin \alpha}. \quad (10)$$

For $\alpha \rightarrow 0$ we get $\beta \rightarrow \pi/2$, while for $\alpha = \arccos(\mu_3/2\mu)$ we find also that $\beta = \arccos(\mu_3/2\mu)$. In other words, $\beta \in [\arccos(\mu_3/2\mu), \pi/2]$, as we may have expected from the geometry. This may at first appear counterintuitive — especially the statement that $\alpha = 0$ produces $\beta = \pi/2$ — but the picture is clear: $\alpha = 0$ corresponds to the two strings being aligned, which as we saw gives velocity $\dot{s}_3 = 1$ for the junction. However, the kink projection velocity $\cos \alpha$ for $\alpha = 0$ is also 1 (the kink is trivial and moves along the x -axis). Thus, both the kink and the junction move at the same speed along the x -axis, starting at the same point V . Formally, the segments JK_1 and JK_2 are at right angle to the x -axis, but they have zero length. More generally, small α leads to β near $\pi/2$. In the other extreme, $\alpha = \arccos(\mu_3/2\mu)$, we have $\dot{s}_3 = 0$ as explained above and the zipper does not grow, staying formally at zero length. The “kinks” are trivial and propagate up the original strings: there is no bend so $\beta = \alpha = \arccos(\mu_3/2\mu)$.

A source of potential confusion is that Eq. (2) is clearly not satisfied for the angle β , yet the configuration in Fig. 1, *Bottom*, appears to be identical to the one used for the derivation of Eq. (2). As we saw, to derive Eq. (2) one takes the $x > 0$ half of the configuration of Fig. 1, *Top*, and considers the vertex V as a zipper of tension μ_3 and zero length at $t = 0$. The difference from the configuration in Fig. 1, *Bottom*, appears to be the length of the zipper (zero vs finite length) and the angle at the junction (α vs β). How can it be that the same equation (2) is not valid for the angle β ? The answer is that in the configuration of Fig. 1, *Top*, the string segments only have velocity v in the z -direction (even though we are taking $v \rightarrow 0$ in this simplest example), while in that of Fig. 1, *Bottom*, the junction is moving with velocity \dot{s}_3 to the right so the segments JK_1, JK_2 must have non-zero transverse velocity, w , in the (x, y) -plane. Thus, the solution (1) used to derive Eq. (2) for the angle α does not apply to the configuration in Fig. 1, *Bottom*. The correct parametrisation of this configuration (up to the kinks K_1, K_2) for $t > 0$ is

$$\begin{aligned} \mathbf{x}_1 &= (-\sigma\gamma_w^{-1} \cos \beta + w \sin \beta t, -\sigma\gamma_w^{-1} \sin \beta - w \cos \beta t, 0) \quad , \quad -t \leq \sigma \leq s_1(t) \\ \mathbf{x}_2 &= (-\sigma\gamma_w^{-1} \cos \beta + w \sin \beta t, +\sigma\gamma_w^{-1} \sin \beta + w \cos \beta t, 0) \quad , \quad -t \leq \sigma \leq s_2(t) \\ \mathbf{x}_3 &= (\sigma, 0, 0) \quad , \quad 0 \leq \sigma \leq s_3(t) \end{aligned} \quad (11)$$

which is a solution of the Nambu-Goto equations in the conformal gauge. Note that for strings 1 and 2 (segments JK_1 and JK_2) we have only kept the physical *transverse* velocities, which corresponds to our choice of conformal gauge. The magnitude w of the transverse velocities for the two strings can be readily found to be $w = \dot{s}_3 \sin \beta$. Using this solution for $\mathbf{x}_1, \mathbf{x}_2$ and repeating the analysis of Ref. [37] we arrive at the following equation expressing \dot{s}_3 in terms of the local angle at the junction, β :

$$\dot{s}_3 = \frac{2\mu(\gamma_w^{-1} \cos \beta + w \sin \beta) - \mu_3}{2\mu - \mu_3(\gamma_w^{-1} \cos \beta + w \sin \beta)}. \quad (12)$$

Clearly, $\dot{s}_3 = \text{const}$, consistent with Eq. (2). As already mentioned, even though Eq. (2) was derived from the solution (1), valid only up to $t = 0$ but not later, it holds for all $t > 0$. This now becomes clear via the equation $\cos \alpha = \gamma_w^{-1} \cos \beta + w \sin \beta$, which can be verified with $w = \dot{s}_3 \sin \beta$. For $\alpha = \arccos(\mu_3/2\mu)$ we have $w = 0$ so $\beta = \alpha$, while in the other extreme, $\alpha = 0$, we have $w = 1$, $\beta = \pi/2$.

This complements the analysis of Refs. [37] and [41] for $\mu_1 = \mu_2$. Restoring the velocity v , the simple geometrical picture above in which all strings lie on the (x, y) -plane is lost, but it is still straightforward to construct the solution algebraically. The solution along each of the segments JK_1 and JK_2 can be represented in terms of a unit direction vector and a transverse velocity [41]

$$\mathbf{x}_i = \frac{\sigma}{\gamma_w} \mathbf{d}_i + t \mathbf{w}_i \quad , \quad i \in \{1, 2\} \quad (13)$$

such that $\mathbf{d}_i \cdot \mathbf{w}_i = 0$. This parametrisation again corresponds to choosing the conformal gauge, so that σ measures *invariant* length. Since the segments JK_i must be joined to the zipper VJ at the junction J and to the original solution (1) at the kinks K_1, K_2 , the vectors \mathbf{d}_i and \mathbf{w}_i can be determined by requiring continuity at $\sigma = s_3(t)$ and

$\sigma = -t$, as well as imposing the conditions $\mathbf{x}_i(-t, t) = \mathbf{x}_3(s_3(t), t) + J\vec{K}_i$. One then finds⁴ that the segments JK_1 , JK_2 and VJ still lie on a plane, albeit one which is rotated by an angle $\tan \phi = v\gamma_v \csc \alpha$ around the x -axis (Fig. 2). The local configuration at the junction is still described by the angle β discussed above, which for $v \neq 0$ is given by

$$\cos \beta = \frac{\cos \alpha / \gamma_v - \dot{s}_3}{\sqrt{1 + \dot{s}_3[\dot{s}_3 - 2 \cos \alpha / \gamma_v]}}. \quad (14)$$

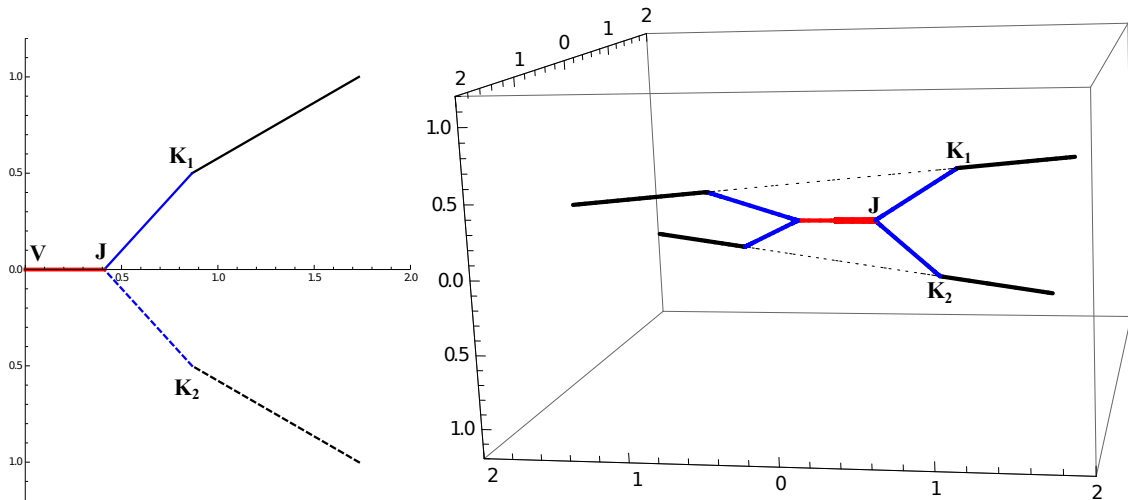


FIG. 2: String junction configurations for $v \rightarrow 0$ as $t \rightarrow 0$ (Left) and for $v = \text{const} > 0$ (Right).

Focusing on the local structure at the junction in terms of the geometric picture described, it is clear that for these solutions to make sense with $\beta = \text{const}$, $\dot{s}_3 = \text{const}$, the vector sum of the tensions $\mathbf{T}_i = \mu_i \mathbf{x}'_i$ of all strings at the junction must exactly balance the transfer of momentum due to the shrinking of the segments JK_1 and JK_2 , which happens for $\dot{s}_3 > 0$. This is simply a statement of energy conservation. The momentum carried by a segment of invariant length L and constant transverse velocity \mathbf{u} is $\mathbf{p} = \mu L \mathbf{u}$, so if the segment is shrinking then the rate of change of momentum is $\dot{\mathbf{p}} = \mu \dot{L} \mathbf{u}$. At the junction, $s_i(t)$ labels invariant length so for each string we have $\dot{\mathbf{p}}_i = \mu_i \dot{s}_i(t) \mathbf{x}'_i$. Concentrating on the x -component we have

$$\sum (T_x + \dot{p}_x) = 2\mu \cos \beta \gamma_w^{-1} - 2\mu \dot{s}_1 w_x - \mu_3 = 0, \quad (15)$$

which can be simplified to

$$2\mu \cos \beta \gamma_w - \mu_3 = 0. \quad (16)$$

In view of the CKS constraints described in the beginning of this section, it now becomes obvious that the angle β is indeed the *critical angle* saturating the analogue of Eqs. (3)-(4) for the configuration of Fig. 2, Left. This is as expected: β is the unique angle balancing tensions and momentum transfer, which resolves the original mismatch of tensions at $t = 0$. This configuration of local equilibrium is the reason why the growth of the zipper \dot{s}_3 can be described by considering the external tensions acting on the union of JK_1 and JK_2 thought of as a “rigid” body, Eqs. (5)-(6).

Our motivations for studying the local dynamics of string junctions can be rephrased in this context in terms of studying the stability of the solutions just described. In particular, we are interested in understanding the conditions under which non-trivial dynamics of string junctions could allow for the local equilibrium conditions to evolve so as to decelerate (and potentially invert) the zipping process. More generally, we wish to explore possible mechanisms for the unzipping of string junction configurations, arguably seen in some field theory simulations. In the next section we move on to the general formalism for studying the dynamics of massive junctions.

⁴ We will examine these solutions and their generalisation for $\mu_1 \neq \mu_2$ in Section IV B, where we will also allow for massive junctions and study their dynamics.

III. MASSIVE JUNCTIONS: NAMBU-GOTO ACTION AND DYNAMICS

In this section, we generalise the action of Ref. [38] by including a *massive* junction and derive the equations of motion. We are working with the configuration shown in Figs. 1 and 2, concentrating on the $x > 0$ half x -axis. We parameterise the position of the i -th string ($i = 1, 2, 3$) as

$$x_i^\mu(\tau, \sigma_i) , \quad (17)$$

where τ and σ_i are the world-sheet coordinates; the timelike coordinate τ is chosen to be the same for all three strings. The induced metric on the world-sheet for the string i is

$$\gamma_{ab}^i = \frac{\partial x_i^\mu}{\partial \sigma^a} \frac{\partial x_i^\nu}{\partial \sigma^b} \eta_{\mu\nu} , \quad (18)$$

where $\sigma^a = (\tau, \sigma_i)$ and $\eta_{\mu\nu}$ is the Minkowski metric with signature $(+, -, -, -)$. A dot/dash denotes differentiation with respect to τ/σ_i , respectively. The values of the world-sheet coordinate σ_i at the junction are denoted by s_i and are generally τ -dependent. We choose σ_i to increase towards junction J for all three strings. The position of the junction with mass m is

$$X_m^\mu(\tau) = x_i^\mu(\tau, s_i(\tau)) \quad , \quad \text{with} \quad i = 1, 2, 3 .$$

We are working in the conformal gauge, where

$$\gamma_{\tau\tau}^i + \gamma_{\sigma_i\sigma_i}^i = 0 \quad ; \quad \gamma_{\tau\sigma_i}^i = 0 . \quad (19)$$

The Nambu-Goto action for the three strings of tensions μ_i (with $i = 1, 2, 3$) meeting at junction J with mass m is

$$\begin{aligned} S = & - \sum_i \mu_i \int d\tau d\sigma_i \Theta(s_i(\tau) - \sigma_i) \sqrt{-x_i'^2 \dot{x}_i^2} \\ & + \sum_i \int d\tau f_{i\mu} \cdot [x_i^\mu(\tau, s_i(\tau)) - X_m^\mu(\tau)] - m \int d\tau \sqrt{\dot{X}_m^2} . \end{aligned} \quad (20)$$

Varying the above action, Eq. (20), with respect to x_i^μ yields the usual equation of motion for a string in Minkowski space-time (away from the junction), which is the wave equation

$$\ddot{x}_i^\mu - x_i^{\mu''} = 0 . \quad (21)$$

The boundary terms, i.e. the ones proportional to $\delta(s_i(t) - \sigma_i)$, give

$$\mu_i(x_i^{\mu'} + \dot{s}_i \dot{x}_i^\mu) = -f_i^\mu , \quad (22)$$

where the functions are evaluated at $(\tau, s_i(\tau))$. Varying the Lagrange multipliers f_i^μ provides the boundary condition

$$X_m^\mu(\tau) = x_i^\mu(\tau, s_i(\tau)) , \quad (23)$$

and varying X_m^μ gives

$$m \frac{d}{d\tau} \left(\frac{\dot{X}_m^\mu}{\sqrt{\dot{X}_m^2}} \right) = \sum_i f_i^\mu . \quad (24)$$

Using Eqs. (22) and (24), we obtain

$$m \frac{d}{d\tau} \left(\frac{\dot{X}_m^\mu}{\sqrt{\dot{X}_m^2}} \right) = - \sum_i \mu_i (x_i^{\mu'} + \dot{s}_i \dot{x}_i^\mu) . \quad (25)$$

Let us impose the temporal gauge condition $x^0 \equiv t = \tau$. The conformal gauge conditions, Eq. (19), then reduce to

$$\dot{\mathbf{x}}_i^2 + \mathbf{x}_i'^2 = 1 \quad ; \quad \dot{\mathbf{x}}_i \cdot \mathbf{x}_i' = 0 , \quad (26)$$

where \mathbf{x}_i is the spatial part of x_i^μ , so that $x_i^\mu = (t, \mathbf{x}_i)$. The 4-dimensional wave equation, Eq. (21), reduces to the 3-dimensional wave equation

$$\ddot{\mathbf{x}}_i - \mathbf{x}_i'' = 0 , \quad (27)$$

with solution

$$\mathbf{x}_i(\sigma, t) = \frac{1}{2}[\mathbf{a}_i(\sigma + t) + \mathbf{b}_i(\sigma - t)] , \quad (28)$$

and the gauge conditions in Eq. (26) imply⁵

$$\mathbf{a}_i'^2 = \mathbf{b}_i'^2 = 1 . \quad (29)$$

The equation obtained from the boundary terms, Eq. (22), becomes

$$\mu_i(\mathbf{x}_i' + \dot{s}_i \dot{\mathbf{x}}_i) = -\mathbf{f}_i , \quad (30)$$

where the functions are evaluated at $(t, s_i(t))$. Moreover, the boundary condition, Eq. (23), simplifies to

$$\mathbf{x}_i(t, s_i(t)) = \mathbf{X}_m(t) , \quad (31)$$

which gives

$$\dot{\mathbf{X}}_m = \mathbf{x}_i' \dot{s}_i + \dot{\mathbf{x}}_i . \quad (32)$$

Let us consider Eq. (25): its 0-th component implies the energy conservation equation

$$m\dot{\gamma}_m + \sum_i \mu_i \dot{s}_i = 0 \quad \text{where} \quad \gamma_m = \frac{1}{\sqrt{1 - \dot{\mathbf{X}}_m^2}} , \quad (33)$$

and its i -th components lead to

$$m \frac{d}{dt} (\gamma_m \dot{\mathbf{X}}_m) = - \sum_i \mu_i (\mathbf{x}_i' + \dot{s}_i \dot{\mathbf{x}}_i) . \quad (34)$$

The above equation, Eq. (34), can be written as

$$m\dot{\gamma}_m \dot{\mathbf{X}}_m + m\gamma_m \ddot{\mathbf{X}}_m = - \sum_i \mu_i (\mathbf{x}_i' + \dot{s}_i \dot{\mathbf{x}}_i) , \quad (35)$$

and using Eqs. (32) and (33) we get

$$m\gamma_m \ddot{\mathbf{X}}_m = - \sum_i \mu_i (1 - \dot{s}_i^2) \mathbf{x}_i' . \quad (36)$$

Finally, using the gauge conditions, Eqs. (26), and Eq. (32) above, we obtain

$$\ddot{\mathbf{X}}_m = - \frac{1}{m} \gamma_m^{-3} \sum_i \mu_i \frac{\mathbf{x}_i'}{\mathbf{x}_i'^2} . \quad (37)$$

Equations (33) and (37) agree with the analogous equations found in Refs. [44, 45] for a system of monopoles connected to two strings each.

Note that, using the gauge conditions, Eq. (26), we can write Eq. (32) as

$$\dot{s}_i(t) = \frac{\dot{\mathbf{X}}_m(t) \cdot \mathbf{x}_i'(s_i(t), t)}{|\mathbf{x}_i'^2(s_i(t), t)|} . \quad (38)$$

⁵ Note that, when applied to \mathbf{a} and \mathbf{b} , the prime denotes derivatives with respect to their arguments $(\sigma + t)$ and $(\sigma - t)$, respectively.

Since

$$\mathbf{X}_m = \mathbf{x}_i(s_i(t), t) = \frac{1}{2}[\mathbf{b}_i(s_i(t) - t) + \mathbf{a}_i(s_i(t) + t)] , \quad (39)$$

the vertex velocity can be written as

$$\dot{\mathbf{X}}_m = \frac{1}{2}[-(1 - \dot{s}_i(t))\mathbf{b}'_i + (1 + \dot{s}_i(t))\mathbf{a}'_i] . \quad (40)$$

We can therefore express the outgoing waves, $\mathbf{a}'_i(s_i(t) + t)$, as a function of the incoming waves, $\mathbf{b}'_i(s_i(t) - t)$, and the vertex velocity, $\dot{\mathbf{X}}_m$, in the following way:

$$\mathbf{a}'_i(s_i(t) + t) = \frac{2\dot{\mathbf{X}}_m + (1 - \dot{s}_i)\mathbf{b}'_i(s_i(t) - t)}{1 + \dot{s}_i} . \quad (41)$$

Starting from Eq. (38) and using Eq. (41) and $\mathbf{x}' = (\mathbf{b}' + \mathbf{a}')/2$ with $\mathbf{b}'^2 = \mathbf{a}'^2 = 1$, we obtain

$$\dot{s}_i(t) = \frac{\dot{\mathbf{X}}_m^2(t) + \dot{\mathbf{X}}_m(t) \cdot \mathbf{b}'_i(s_i(t) - t)}{\dot{\mathbf{X}}_m(t) \cdot \mathbf{b}'_i(s_i(t) - t) + 1} . \quad (42)$$

We have thus obtained the general evolution equations for 3 strings of different tensions ending on a massive junction, and have derived the equivalent to Eqs. (2) and (12). Note that a general result arising from these equations is a co-planar condition on \mathbf{x}'_i and $\ddot{\mathbf{X}}$. Indeed, from Eq. (36) we get

$$m\gamma_m \ddot{\mathbf{X}}_m \cdot (\mathbf{x}'_2 \times \mathbf{x}'_3) = -\mu_1(1 - \dot{s}_1^2)\mathbf{x}'_1 \cdot (\mathbf{x}'_2 \times \mathbf{x}'_3) , \quad (43)$$

which is satisfied with $\ddot{\mathbf{X}}_m$ and \mathbf{x}'_i co-planar at the point of the junction⁶.

IV. STABILITY OF Y-JUNCTION CONFIGURATIONS

Clearly, all configurations considered in Section II are special solutions of the equations of the previous section with $\ddot{\mathbf{X}}_m = 0$. But some of the features we saw in those special solutions are also present in the most general Y-type configurations. For example, the joining strings are co-planar in the vicinity of the junction by virtue of Eq. (43). Let us briefly consider the generalisation of our discussion of the balance between tension and momentum transfer in Eqs. (15)-(16). This is described by Eq. (34), where $\mu_i \mathbf{x}'_i$ and $\mu_i \dot{s}_i \dot{\mathbf{x}}_i$ are the vector tension and rate of change of momentum for the i -th string. They are now allowed to be unbalanced, resulting in acceleration $\ddot{\mathbf{X}}_m$ of the massive junction. Using our gauge constraints, this equation is equivalent to Eq. (37) which is remarkably simple: for any straight string with velocity w we have that $|\mathbf{x}'| = \gamma_w^{-1}$ so the overall effect of adding the momentum transfer can be effectively described by rescaling the tension from $T = \mu\gamma_w^{-1}$ to $T_{\text{eff}} \equiv \gamma_w^2 T = \mu\gamma_w$, which is what we found in Eq. (15)-(16) for the x -component. The sum of these effective tensions at the junction is then proportional to $\ddot{\mathbf{X}}_m$, according to Eq. (37).

From this simple picture we may then expect the Y-junction configuration to be stable under small perturbations of the angle. Focusing on our familiar example $\mu_1 = \mu_2 \equiv \mu$, Eq. (37) becomes

$$\ddot{\mathbf{X}}_m = -\frac{1}{m}\gamma_m^{-3}(\mu_3 - 2\mu \cos \beta \gamma_w) , \quad (44)$$

so an angle smaller than⁷ $\beta_{\text{crit}} = \arccos(\mu_3/2\mu\gamma_w)$ will result in positive acceleration, which tends to increase the angle towards β_{crit} . We may then expect the junction equilibrium to be stable. In this section we will study this question quantitatively and for more general string configurations, solving the equations of motion numerically.

Perturbations of Y-junction configurations have been studied in Ref. [38] where the authors considered an initially static three-string solution with $\dot{s}_i = 0$ and studied propagation of waves, transmitted and reflected at the junction, leading to oscillatory behaviour for $s_i(t)$. Here, we will instead consider deformations of basic solutions in which the effective tensions at the junction are not balanced and the massive junction feels a non-trivial force.

⁶ Note that in the massless junction case, one finds $\mathbf{x}'_1 \cdot (\mathbf{x}'_2 \times \mathbf{x}'_3) = 0$, which indicates that the \mathbf{x}'_i are co-planar at the point of the junction [39].

⁷ Notice the difference in the placement of Lorentz factors between this critical angle and that of Eq. (3). This can be attributed to the additional transfer of momentum which can be thought of as an effective rescaling of the tension from $\mu\gamma^{-1}$ to $\mu\gamma$.

A. Equal tensions $\mu_1 = \mu_2$

We start by considering the simplest case we have discussed above, where the colliding strings have equal tension $\mu_1 = \mu_2 \equiv \mu$ and the zipper stays on the x -axis. We first construct the unperturbed solution on which our analysis will be based on.

Unperturbed Solution

We construct the unperturbed solution along the lines sketched in Section II. The idea is to write the solution along each of the segments JK_1 and JK_2 (see Fig. 3 below) in terms of a unit direction vector and a transverse velocity, as in Eq. (13), and then determine \mathbf{d}_i and \mathbf{w}_i from the geometry. Note that for $v \neq 0$ both \mathbf{d} and \mathbf{w} also have finite z -components, which are not shown in perspective in Fig. 3 (see also Fig. 2, *Left*).

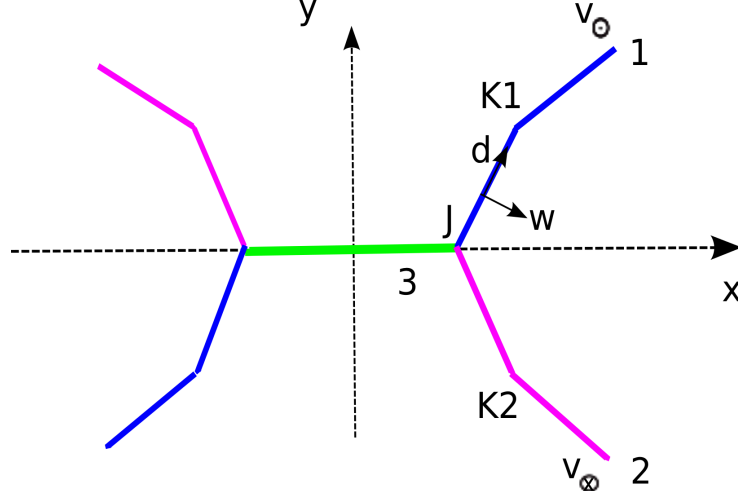


FIG. 3: Straight strings collision.

The complete solution (for the $x > 0$ half x -axis) in the conformal gauge is:

$$\begin{aligned} \mathbf{x}_1 &= \begin{cases} (-\sigma\gamma_v^{-1}\cos\alpha, -\sigma\gamma_v^{-1}\sin\alpha, +vt) \equiv \mathbf{x}_{1\infty} & , \quad \sigma \leq -t \\ -\sigma\gamma_w^{-1}\mathbf{d}_1 + \mathbf{w}_1 t \equiv \mathbf{x}_{1f} & , \quad -t \leq \sigma \leq s_1(t) \end{cases} \\ \mathbf{x}_2 &= \begin{cases} (-\sigma\gamma_v^{-1}\cos\alpha, +\sigma\gamma_v^{-1}\sin\alpha, -vt) \equiv \mathbf{x}_{2\infty} & , \quad \sigma \leq -t \\ -\sigma\gamma_w^{-1}\mathbf{d}_2 + \mathbf{w}_2 t \equiv \mathbf{x}_{2f} & , \quad -t \leq \sigma \leq s_2(t) \end{cases} \\ \mathbf{x}_3 &= (\sigma, 0, 0) & , \quad 0 \leq \sigma \leq s_3(t) \end{aligned}$$

Let us now determine \mathbf{d}_i , \mathbf{w}_i . First note that if $\mathbf{d}_1 = (d_x, d_y, d_z)$ and $\mathbf{w}_1 = (w_x, w_y, w_z)$, then symmetry implies $\mathbf{d}_2 = (d_x, -d_y, -d_z)$ and $\mathbf{w}_2 = (w_x, -w_y, -w_z)$, so we only need to consider \mathbf{x}_1 and \mathbf{x}_3 . Continuity at the position of the junction \mathbf{r}_J requires $\mathbf{x}_1(s_1(t), t) = \mathbf{x}_3(s_3(t), t)$ so that

$$\mathbf{r}_J = \mathbf{w}t - \frac{\sigma_J}{\gamma_w}\mathbf{d} = (\dot{s}_3, 0, 0)t, \quad (45)$$

with $\sigma_J = s_1(t) = \dot{s}_1 t$. Similarly, continuity at the kink implies $\mathbf{x}_{1f}(-t, t) = \mathbf{x}_{1\infty}(-t, t)$, that is

$$\mathbf{r}_{K_1} = \mathbf{w}t - \frac{\sigma_{K_1}}{\gamma_w}\mathbf{d} = \left(\frac{\cos\alpha}{\gamma_v}, \frac{\sin\alpha}{\gamma_v}, v \right) t, \quad (46)$$

with $\sigma_{K_1} = -t$. Since we are working in the conformal gauge ($\mathbf{d} \cdot \mathbf{w} = 0$, $\mathbf{d}^2 = 1$) the invariant length of the segment $J\vec{K}_1 = \mathbf{r}_{K_1} - \mathbf{r}_J$ is just $|\sigma_{K_1} - \sigma_J| = (1 + \dot{s}_1)t$. From energy conservation (cf. Eq. (33) with $\dot{\gamma}_m = 0$) we have

$$\dot{s}_1 = -\frac{\mu_3}{2\mu}\dot{s}_3 \equiv -R\dot{s}_3, \quad (47)$$

so we can express \mathbf{d} and \mathbf{w} in terms of the original angle α , γ_v , \dot{s}_3 and the tension ratio $R = \mu_3/2\mu < 1$ (but note these four quantities are related through Eq. (2)). Multiplying Eq. (46) by \dot{s}_1 and adding it to Eq. (45) we find

$$(1 - R\dot{s}_3)\mathbf{w} = (\dot{s}_3 - R\dot{s}_3\gamma_v^{-1}\cos\alpha, -R\dot{s}_3\gamma_v^{-1}\sin\alpha, -R\dot{s}_3v), \quad (48)$$

from which we read directly the components of \mathbf{w} :

$$w_x = \frac{(1 - R\gamma_v^{-1}\cos\alpha)}{1 - R\dot{s}_3}\dot{s}_3, \quad (49)$$

$$w_y = -\frac{R\gamma_v^{-1}\sin\alpha}{1 - R\dot{s}_3}\dot{s}_3, \quad (50)$$

$$w_z = -\frac{Rv}{1 - R\dot{s}_3}\dot{s}_3. \quad (51)$$

The corresponding Lorentz factor is

$$\gamma_w = \frac{1 - R\dot{s}_3}{\sqrt{A}}, \quad (52)$$

with

$$A = 1 - \dot{s}_3^2 - 2R\dot{s}_3(1 - \gamma_v^{-1}\cos\alpha\dot{s}_3). \quad (53)$$

Since $1 - R\dot{s}_3$ is the invariant length of the segment JK_1 in units of t , from Eq. (52) it follows immediately that the quantity \sqrt{A} is the physical length of JK_1 , in units of t . Equivalence with $|J\vec{K}_1| = |\vec{\mathbf{x}}_1(-t, t) - \mathbf{x}_3(s_3(t), t)| = t\sqrt{1 + \dot{s}_3[\dot{s}_3 - 2\cos\alpha/\gamma_v]}$ can be easily checked using Eq. (2). Finally, for the components of \mathbf{d} we find

$$d_x = -\frac{(\gamma_v^{-1}\cos\alpha - \dot{s}_3)}{\sqrt{A}}, \quad (54)$$

$$d_y = -\frac{\gamma_v^{-1}\sin\alpha}{\sqrt{A}}, \quad (55)$$

$$d_z = -\frac{v}{\sqrt{A}}. \quad (56)$$

The signs are in agreement with our convention that σ is negative and increasing towards the junction on \mathbf{x}_1 , so that d_x is identified with $-\cos\beta$ in Eq. (14). The angle ϕ , describing the rotation of the plane spanned by \mathbf{x}_{1f} and \mathbf{x}_{2f} with the x -axis, is given by

$$\tan\phi = d_z/d_y = v\gamma_v \csc\alpha. \quad (57)$$

In terms of the angles β and ϕ the solution between the junction and the kink, $-t \leq \sigma \leq s_1(t)$, can be written as

$$\mathbf{x}_{1f} = (-\sigma\gamma_w^{-1}\cos\beta + w\sin\beta t, (-\sigma\gamma_w^{-1}\sin\beta - w\cos\beta t)\cos\phi, (-\sigma\gamma_w^{-1}\sin\beta - w\cos\beta t)\sin\phi), \quad (58)$$

and similarly for \mathbf{x}_{2f} . This can now be compared directly to the solution (11) which corresponds to $v = 0 \Rightarrow \phi = 0$.

We will next introduce a mass on the junction which will allow us to deform this solution by breaking the effective tension balance at the expense of having non trivial dynamics for the junction $\ddot{\mathbf{X}}_m \neq 0$.

Massive Junction and Deformed Solution

Keeping the enhanced symmetry of the problem for $\mu_1 = \mu_2$, we look for a more general ansatz for \mathbf{x}_{1f} (equivalently \mathbf{x}_{2f}) in Eq. (45), which will allow for non-trivial acceleration at $\mathbf{r}_J = \mathbf{x}_{1f}(s_1(t), t) = \mathbf{X}_m$. Since the junction can now have $\dot{\gamma}_m \neq 0$ we cannot assume $\dot{s}_1 = -R\dot{s}_3$, but instead $s_1(t)$ and $s_3(t)$ (we still have $\dot{s}_2 = \dot{s}_1$) must satisfy the evolution (37) and the constraint equation (33). Note that the kink is still moving with the speed of light along the original strings.

Let us write the equation for the segment JK_1 :

$$\mathbf{x}_1 = \mathbf{W}(t) - \frac{\sigma}{\gamma_w}\mathbf{d}(t), \quad (59)$$

and for the junction J :

$$\mathbf{r}_J = \mathbf{W}(t) - \frac{s_1(t)}{\gamma_w} \mathbf{d}(t) = (s_3(t), 0, 0) , \quad (60)$$

while for the kink K_1 :

$$\mathbf{r}_{K_1} = \mathbf{W}(t) + \frac{t}{\gamma_w} \mathbf{d}(t) = \left(\frac{\cos \alpha}{\gamma_v}, \frac{\sin \alpha}{\gamma_v}, v \right) t . \quad (61)$$

Hence, subtracting we get

$$[t + s_1(t)] \mathbf{d}(t) = \gamma_w \left(\frac{\cos \alpha}{\gamma_v} t - s_3(t), \frac{\sin \alpha}{\gamma_v} t, vt \right) , \quad (62)$$

from which, after squaring and using $\mathbf{d}^2 = 1$, we obtain

$$t + s_1(t) = \gamma_w \sqrt{t^2 + [s_3(t)]^2 - 2(\cos \alpha / \gamma_v) t s_3(t)} . \quad (63)$$

Energy conservation, Eq. (33), leads to

$$2\mu \dot{s}_1 + \mu_3 \dot{s}_3 + m(1 - \dot{s}_3^2)^{-3/2} \dot{s}_3 \ddot{s}_3 = 0 , \quad (64)$$

and from Eq. (37) we get

$$\ddot{s}_3 = -\frac{1}{m} (1 - \dot{s}_3^2)^{3/2} [\mu_3 - 2\mu \gamma_w(t) d_x(t)] . \quad (65)$$

We can now solve numerically for $s_3(t)$. For initial conditions corresponding to tension and momentum transfer balance, Eq. (44) with $\ddot{\mathbf{X}}_m = 0$ ($\beta = \beta_{\text{crit}}$), the solution is identical to the one considered above. Let us now consider a deformation of the initial configuration with $\ddot{\mathbf{X}}_m \neq 0$, $\beta \neq \beta_{\text{crit}}$, and study its evolution. In this case, we find that $\dot{s}_3(t)$ oscillates around the CKS value $(2\mu \gamma_v^{-1} \cos \alpha - \mu_3) / (2\mu - \mu_3 \gamma_v^{-1} \cos \alpha)$ with decaying amplitude. Hence, as we may have expected from the discussion following Eq. (44), the local angle β exhibits damped oscillations around β_{crit} . This is shown in Fig. 4.

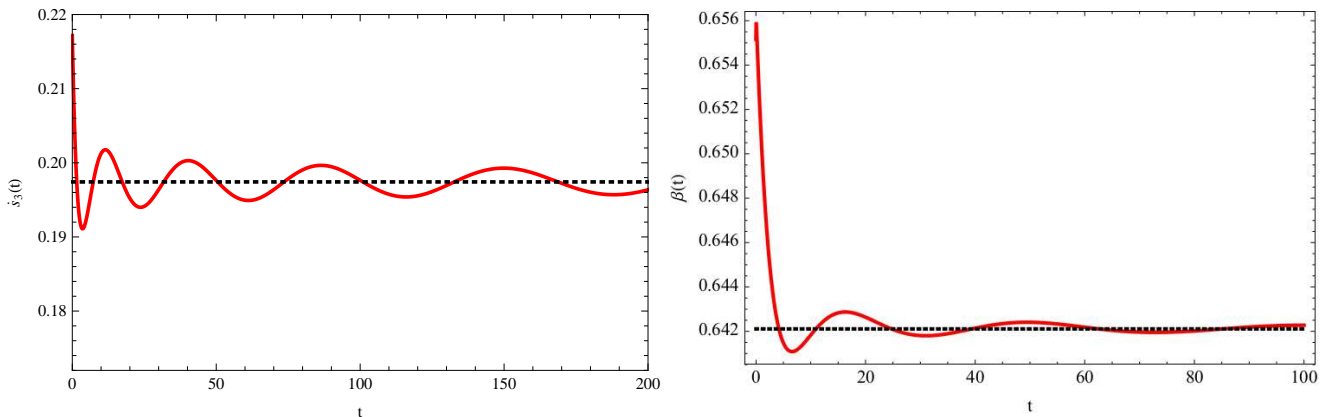


FIG. 4: *Left*: The evolution of \dot{s}_3 for $\mu = 1$, $\mu_3 = \sqrt{2}\mu$, $m = \sqrt{\mu}$, $\alpha = \pi/6$ and $v = 0.4$, shown together with the unperturbed case $\dot{s}_3 = \text{const.}$, given by the CKS solution. *Right*: Evolution of the angle $\beta(t)$ for the same parameters. The dotted line shows the critical angle for tension and momentum transfer balance.

Therefore, even though the constant zipper growth prediction, $\dot{s}_3 = \text{const.}$, was based on special string configurations satisfying the balance condition (16), these configurations are stable and deformations of the balance condition lead to identical asymptotic behaviour. In conclusion, unzipping cannot occur by simply destabilising the angle β_{crit} .

B. Unequal tensions $\mu_1 \neq \mu_2$

Let us consider the general case where the colliding strings have unequal tensions $\mu_1 \neq \mu_2$. For a massless junction, this has been studied in Refs. [38, 39, 41]. The geometry of the problem involves two extra parameters, the angle of the zipper with the x -axis and the zipper velocity u along the z -axis (see Fig. 5), which are found by solving [38]

$$0 = u^4 S^2 \sin^2 \alpha + u^2 [R^2 (1 - v^2) + S^2 (v^2 \cos^2 \alpha - \sin^2 \alpha)] - S^2 v^2 \cos^2 \alpha \quad (66)$$

and

$$\tan \theta = \frac{u}{v} \tan \alpha, \quad (67)$$

where $R = \mu_3/(\mu_1 + \mu_2)$ and $S = (\mu_1 - \mu_2)/(\mu_1 + \mu_2)$. Note that due to the asymmetry of the configuration, we no longer have $s_1(t) = s_2(t)$. The general ansatz for the segment JK_1 is

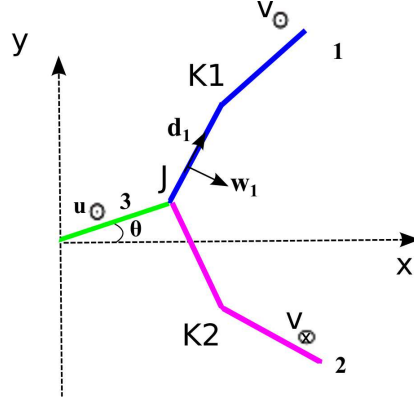


FIG. 5: Schematic representation of the three-string configuration in the general case where the colliding strings have unequal tensions $\mu_1 \neq \mu_2$. Note we only show the $x > 0$ part of the configuration.

$$\mathbf{x}_1 = \mathbf{W}_1(t) - \frac{\sigma}{\gamma_{w_1}} \mathbf{d}_1(t), \quad (68)$$

while for JK_2 it reads

$$\mathbf{x}_2 = \mathbf{W}_2(t) - \frac{\sigma}{\gamma_{w_2}} \mathbf{d}_2(t). \quad (69)$$

For the junction J we have

$$\mathbf{r}_J = \mathbf{W}_1(t) - \frac{s_1(t)}{\gamma_{w_1}} \mathbf{d}_1(t) = (\gamma_u^{-1} s_3(t) \cos \theta, \gamma_u^{-1} s_3(t) \sin \theta, ut), \quad (70)$$

while for the kink K_1 :

$$\mathbf{r}_{K_1} = \mathbf{W}_1(t) + \frac{t}{\gamma_{w_1}} \mathbf{d}_1(t) = \left(\frac{\cos \alpha}{\gamma_v}, \frac{\sin \alpha}{\gamma_v}, v \right) t. \quad (71)$$

Following the same procedure as before, we find

$$[t + s_1(t)] \mathbf{d}_1(t) = \gamma_{w_1} (\gamma_v^{-1} \cos \alpha t - \gamma_u^{-1} s_3(t) \cos \theta, \gamma_v^{-1} \sin \alpha t - \gamma_u^{-1} s_3(t) \sin \theta, vt - ut) \quad (72)$$

and

$$t + s_1(t) = \gamma_{w_1} \sqrt{t^2 (1 + u^2 - 2uv) + \gamma_u^{-2} [s_3(t)]^2 - 2ts_3(t) \gamma_u^{-1} \gamma_v^{-1} \cos(\alpha - \theta)}. \quad (73)$$

Note that one can easily write down the analogous equations for $\mathbf{d}_2(t)$ and γ_{w_2} . From Eq. (37) we get

$$\gamma_u^{-1} \ddot{s}_3 \cos \theta = -\frac{1}{m} (1 - \dot{s}_3^2 \gamma_u^{-2} - u^2)^{3/2} [\mu_3 \gamma_u \cos \theta - \mu_1 \gamma_{w_1}(t) d_{x_1}(t) - \mu_2 \gamma_{w_2}(t) d_{x_2}(t)]. \quad (74)$$

The above equation is numerically solved to find the evolution of $s_3(t)$, together with Eq. (38) for $s_1(t)$ and $s_2(t)$. Taking initial conditions corresponding to tension and momentum transfer balance, as before, the solution is identical to the massless case. Considering a deformation of the initial configuration we again find that $\dot{s}_3(t)$ oscillates around the CKS value with decaying amplitude, as expected from the results of the symmetric case. This is shown in Fig. 6.

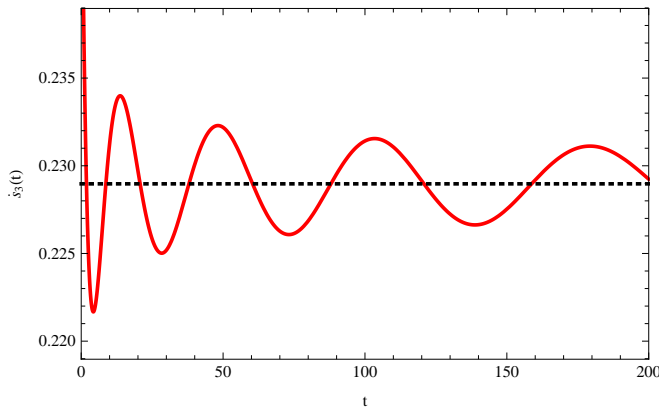


FIG. 6: The evolution of \dot{s}_3 for $\mu_1 = 1$, $\mu_2 = 0.7$, $\mu_3 = 1.2$, $m = \sqrt{\mu_1}$, $\alpha = \pi/6$ and $v = 0.4$, shown together with the unperturbed case $\dot{s}_3 = \text{const.}$ given by the CKS solution.

V. UNZIPPING MECHANISMS

In this section we investigate whether monopole or string forces, and string curvature for loops with junctions, are viable unzipping mechanisms.

A. Monopole Forces

In the previous sections we saw that unzipping cannot occur by simply perturbing the angle β_{crit} . The dynamics of free junctions is such that perturbations get damped and the configuration stabilises at the critical angle. This could of course change if the junction was subject to external forces. We now move on to allow for such forces exerted on the junction and ask whether this can lead to non-trivial evolution of string-junction configurations. We are interested in solutions exhibiting accelerated zipping and, more interestingly, decelerated zipping possibly leading to unzipping.

Physically, junction forces can arise, for example, in hybrid string-monopole networks, where the monopoles are subject to long-range interactions. In this context, we consider the junction of our three-string configuration as a monopole of mass m , and introduce a force due to another monopole (or anti-monopole) located at some distance apart. For local monopoles, the force is just electromagnetic (Coulomb) interaction due to their magnetic charge, and for global monopoles the force is independent of distance. One can also consider the drag force felt by the monopoles due to their interaction with charged particles in a plasma.

We will start with the action (20), having both string world-volume and monopole world-line contributions, but we will now introduce an additional world-line piece, giving rise to a force term for the monopole at the junction. The force can be conveniently described through the integral of a 1-form, say A , on the world-line. A is to be thought of as a background field (a 1-form gauge potential) which is pulled-pack on the world-line, yielding the following reparametrisation-invariant piece:

$$S_{\text{monopole}} = -q \int A = -q \int d\tau A_\nu \dot{X}_m^\nu, \quad (75)$$

with A_ν evaluated on the monopole world-line $X_m^\mu(\tau)$. This is the standard coupling of a relativistic particle with electric charge q to the electromagnetic gauge potential, giving rise to the Lorentz force $q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ in standard notation classical electrodynamics. Equivalently, it also describes the coupling of a monopole of magnetic charge q to the ‘magnetoelectric’ potential, giving rise to the corresponding Lorentz force $q(\mathbf{B} - \mathbf{v} \times \mathbf{E})$. Here, we will use it as a convenient phenomenological action to construct a monopole force and study its effect on junction dynamics.

The total action is

$$\begin{aligned} S = & - \sum_i \mu_i \int d\tau d\sigma_i \Theta(s_i(\tau) - \sigma_i) \sqrt{-x_i'^2 \dot{x}_i^2} \\ & + \sum_i \int d\tau f_{i\mu} \cdot [x_i^\mu(\tau, s_i(\tau)) - X_m^\mu(\tau)] - m \int d\tau \sqrt{\dot{X}_m^2} \\ & - q \int d\tau A_\nu(X_m^\mu(\tau)) \dot{X}_m^\nu. \end{aligned} \quad (76)$$

Varying this action with respect to x_i^μ , the terms proportional to $\Theta(s_i(\tau) - \sigma)$ give the wave equation

$$\ddot{x}_i^\mu - x_i^{\mu''} = 0, \quad (77)$$

and the boundary terms proportional to $\delta(s_i(t) - \sigma)$ give

$$\mu_i(x_i^{\mu''} + \dot{s}_i \dot{x}_i^\mu) = -f_i^\mu \quad (78)$$

at the junction, as before. Varying the Lagrange multipliers f_i^μ we obtain the boundary condition

$$X_m^\mu(\tau) = x_i^\mu(\tau, s_i(\tau)), \quad (79)$$

while varying X_m^μ we get

$$m \frac{d}{d\tau} \left(\frac{\dot{X}_m^\mu}{\sqrt{\dot{X}_m^2}} \right) = \sum_i f_i^\mu + q F^{\mu\nu} (\dot{X}_m)_\nu, \quad (80)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We can therefore write

$$m \frac{d}{d\tau} \left(\frac{\dot{X}_m^\mu}{\sqrt{\dot{X}_m^2}} \right) = - \sum_i \mu_i (x_i^{\mu''} + \dot{s}_i \dot{x}_i^\mu) + q F^{\mu\nu} (\dot{X}_m)_\nu. \quad (81)$$

We now have an extra term in the equations of motion for the vertex, which we will use to model the contribution of monopole forces at the junction. Writing the 4-potential in terms of a scalar potential and 3-vector potential, $A^\mu = (\phi, \mathbf{A})$, and defining

$$\mathcal{E} = -\dot{\mathbf{A}} - \nabla\phi, \quad \mathcal{B} = \nabla \times \mathbf{A}, \quad (82)$$

the energy conservation equation (i.e. the $\mu = 0$ component of Eq. (81)) becomes

$$m\dot{\gamma}_m + \sum_i \mu_i \dot{s}_i = q \mathcal{E} \cdot \dot{\mathbf{X}}_m, \quad (83)$$

while the i -th components give

$$m \frac{d}{dt} (\gamma_m \dot{\mathbf{X}}_m) = - \sum_i \mu_i (\mathbf{x}_i' + \dot{s}_i \dot{\mathbf{x}}_i) + q (\mathcal{E} + \dot{\mathbf{X}}_m \times \mathcal{B}). \quad (84)$$

After a little algebra using Eqs. (26) and (32) as well as Eq. (83), we find

$$\ddot{\mathbf{X}}_m = -\frac{1}{m} \gamma_m^{-3} \sum_i \mu_i \frac{\mathbf{x}_i'}{\mathbf{x}_i'^2} - \frac{q}{m} \gamma_m^{-1} \left[\gamma_m^{-2} \mathcal{E}_\parallel + \mathcal{E}_\perp + \dot{\mathbf{X}}_m \times \mathcal{B} \right], \quad (85)$$

where $\mathcal{E}_\parallel, \mathcal{E}_\perp$ are the components of \mathcal{E} parallel and transverse to $\dot{\mathbf{X}}_m$ respectively.

With A the standard electromagnetic gauge 4-potential, we have $\mathcal{E} = \mathbf{E}$ and $\mathcal{B} = \mathbf{B}$ in standard notation, so the rightmost quantity on the right hand side of Eq. (85) is the Lorentz force for an electrically charged particle, as we expect from the above discussion. Similarly, if A is the magnetoelectric 4-potential then $\mathcal{E} = \mathbf{B}$ and $\mathcal{B} = -\mathbf{E}$ in standard notation, giving rise to the Lorentz force for a magnetically charged particle. As mentioned above, here we will use A as a phenomenological potential to construct a monopole force of the desired type. Let us for example consider a constant force, as is the case for global monopoles, and take it for simplicity to be aligned with string 3 (i.e. the zipper along the x -axis). Then $\mathcal{E}_\parallel = \mathcal{E}$, $\mathcal{E}_\perp = 0$ in (85), and in the equal tension case the junction only moves along the x -axis. We choose $\mathcal{B} = 0$ and \mathcal{E} a constant vector tangent to string 3, such that

$$\mathcal{E} = \epsilon \hat{x}, \quad (86)$$

with ϵ a constant. We can then write the x -component of Eq. (85) as

$$\ddot{s}_3 = -\frac{1}{m} (1 - \dot{s}_3^2)^{3/2} [\mu_3 + q\epsilon - 2\mu\gamma_w(t)d_x(t)], \quad (87)$$

and the energy conservation equation (83) as

$$m\dot{\gamma}_m + \sum_i \mu_i \dot{s}_i = q\epsilon \dot{s}_3 . \quad (88)$$

The last two equations generalise Eqs. (65) and (33) by including a constant force along the zipper. We now investigate numerically the effect of this additional force. The initial conditions we use are the ones corresponding to tension and momentum transfer balance, i.e. in our initial configuration the angle β has the critical value satisfying Eq. (44) for $\dot{\mathbf{X}}_m = 0$. The force is felt for $t > 0$, and, for positive ϵ , its effect is to cause unzipping of the initial configuration over a timescale determined by the mass of the junction m and the magnitude of the force $q\epsilon$. In Fig. 7 we show the evolution of $s_3(t)$ for the three-string symmetric configuration studied before, but we have now included a monopole-like force term with a fixed magnitude $q\epsilon = 0.5$. Having fixed $q\epsilon$, the unzipping timescale is controlled by the monopole mass. For lower mass the unzipping starts earlier and takes shorter to complete, while for larger masses, unzipping starts later and completes over a longer period. In the figure we show examples for $m = \sqrt{\mu}/2$ (top left), $m = \sqrt{\mu}/4$ (top right), $m = 2\sqrt{\mu}$ (bottom). The $q\epsilon = 0$ case, with $\dot{s}_3(t) = \text{const.}$ solution, is shown by the dashed black lines.

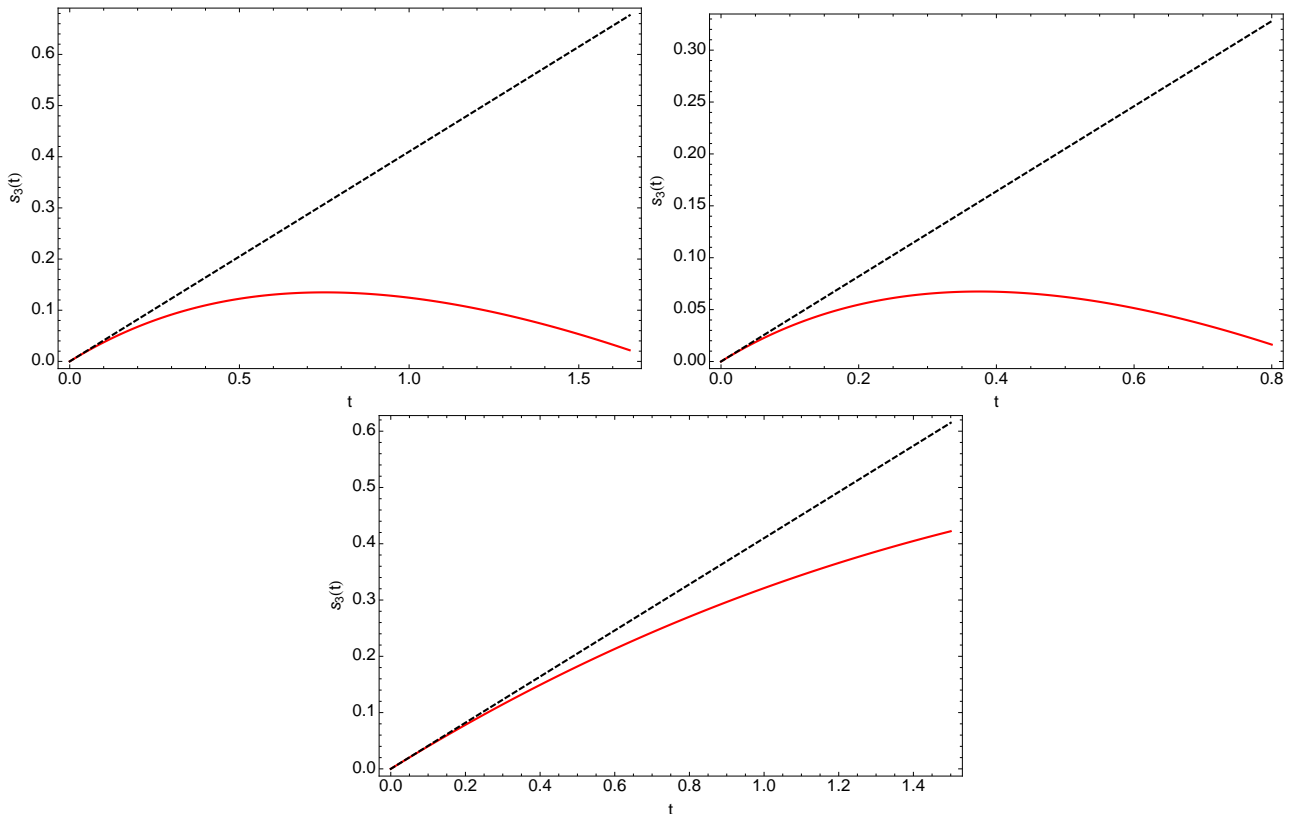


FIG. 7: The evolution of s_3 including an extra force towards the junction with magnitude $q\epsilon = 0.5$ (solid red lines), for $\mu = 1$, $\mu_3 = \sqrt{2}\mu$, $\alpha = \pi/6$ and $v = 0$, for $m = \sqrt{\mu}/2$ (top left), $m = \sqrt{\mu}/4$ (top right), $m = 2\sqrt{\mu}$ (bottom), shown together with the unperturbed case given by the CKS solution (dashed black lines).

The situation is analogous for a force also having components transverse to the zipper. The projection of the force along the zipper produces an acceleration for $s_3(t)$, as above. From Eq. (85) it is clear that allowing for a non-trivial component of the monopole force in the direction transverse to the zipper, $\mathcal{E}_\perp \neq 0$, breaks the symmetry of the problem leading to a situation analogous to the case $\mu_1 \neq \mu_2$.

B. String Forces

Let us move on to consider the effect of string forces on the evolution of junctions. Such forces can arise in a wide range of physically relevant situations. Examples include long range interactions between field theory solitons (e.g.

global string interactions), friction due to particle scattering in a plasma, exchange of dilatons for cosmic superstrings, and D-brane forces arising from fundamental strings stretching between two branes. In analogy to our approach for modelling monopole forces in the previous subsection, we will phenomenologically describe string forces through a background field.

A string force can be readily described by the integral of a background 2-form field pulled-back on the string worldsheet. For each string, i , we introduce the following reparametrisation-invariant worldsheet contribution:

$$S_{\text{string}} = q_i \int B = q_i \int d\tau d\sigma_i B_{\mu\nu} \epsilon^{\alpha\beta} \partial_\alpha x_i^\mu \partial_\beta x_i^\nu, \quad (89)$$

with $B_{\mu\nu}$ evaluated on the i -string world-sheet $x^\lambda(\tau, \sigma_i)$ and q_i constants. This is, for example, how the fundamental string couples to the Neveu-Schwarz 2-form and the D1-brane to the Ramond-Ramond 2-form, giving rise to a stringy interaction analogous to electromagnetism. Here, we will use this action as a phenomenological tool to construct a convenient string force.

The full action reads

$$\begin{aligned} S = & - \sum_i \mu_i \int d\tau d\sigma_i \Theta(s_i(\tau) - \sigma_i) \sqrt{-\dot{x}_i'^2} \\ & + \sum_i \int d\tau f_{i\mu} \cdot [x_i^\mu(\tau, s_i(\tau)) - X_m^\mu(\tau)] - m \int d\tau \sqrt{\dot{X}_m^2} \\ & + \sum_i q_i \int d\tau d\sigma_i \Theta(s_i(\tau) - \sigma_i) B_{\mu\nu}(x^\lambda(\tau, \sigma_i)) \epsilon^{\alpha\beta} \partial_\alpha x_i^\mu \partial_\beta x_i^\nu, \end{aligned} \quad (90)$$

where we have introduced a Heaviside Θ -function in the last term accounting for the moving junction. Upon variation, this produces a δ -function contribution localised at the junction so the boundary term in Eq. (22) acquires a term proportional to $q_i B_\nu^\mu(\dot{x}_i^\nu + x_i'^\nu \dot{s}_i(t))$. Consequently, Eq. (24) receives a contribution proportional to $\sum_i q_i B_\nu^\mu(\dot{x}_i^\nu + x_i'^\nu \dot{s}_i(t))$.

The conformal temporal gauge we have adopted in the preceding sections is too constraining for the present case. For a non-trivial string force there is no residual freedom to impose the temporal condition on top of the conformal gauge – doing so would violate the equations of motion. It is then convenient to keep the temporal ($x^0 = \tau$) and transverse ($\dot{\mathbf{x}}_i \cdot \mathbf{x}_i' = 0$) conditions, but relax the tracelessness condition $\dot{\mathbf{x}}_i^2 + \mathbf{x}_i'^2 = 1$. We then define the scalar quantities

$$\varepsilon_i \equiv \frac{\sqrt{-\gamma_i}}{\dot{x}_i^2} = \sqrt{\frac{\mathbf{x}_i'^2}{1 - \dot{\mathbf{x}}_i^2}}. \quad (91)$$

In the conformal temporal gauge these quantities are effectively set to unity, but in this transverse temporal gauge they are dynamical and their dynamics is governed by the 0-th component of the equations of motion. In this gauge the invariant length of the i -th string is given by $\int \varepsilon_i d\sigma$, while physical velocities remain transverse to the strings.

The string equation of motion in the presence of our string force becomes

$$\frac{\mu_i}{\sqrt{-\gamma_i}} \left[\frac{\partial}{\partial \tau} (\varepsilon_i \dot{x}_i^\mu) - \frac{\partial}{\partial \sigma} \left(\frac{x_i^{\mu'}}{\varepsilon_i} \right) \right] = q_i \epsilon^{\alpha\beta} \partial_\alpha x_i^\lambda \partial_\beta x_i^\nu H_{\lambda\nu}^\mu = q_i (\dot{x}_i^\lambda x_i'^\nu - x_i'^\lambda \dot{x}_i^\nu) H_{\lambda\nu}^\mu, \quad (92)$$

where $H_{\mu\nu\lambda} = 3 \partial_{[\mu} B_{\nu\lambda]}$, the square brackets denoting antisymmetrisation with respect to the enclosed indices. This splits into evolution equations for ε_i and \mathbf{x}_i :

$$\begin{cases} \dot{\varepsilon}_i = \frac{q_i}{\mu_i} (\dot{x}_i^\lambda x_i'^\nu - x_i'^\lambda \dot{x}_i^\nu) H_{\lambda\nu}^0 \mathbf{x}_i'^2 \varepsilon_i \\ (\varepsilon_i \dot{\mathbf{x}}_i) - \left(\frac{\mathbf{x}_i'}{\varepsilon_i} \right)' = \frac{q_i}{\mu_i} \sqrt{-\gamma_i} (\dot{x}_i^\lambda x_i'^\nu - x_i'^\lambda \dot{x}_i^\nu) \mathbf{H}_{\lambda\nu} \end{cases}, \quad (93)$$

where we have denoted the spatial components of $H_{\lambda\nu}^\mu$ as $\mathbf{H}_{\lambda\nu}$. In this gauge the boundary term at the junction (cf. equation (22)) reads

$$\mu_i \left(\frac{x_i^{\mu'}}{\varepsilon_i} + \varepsilon_i \dot{s}_i \dot{x}_i^\mu \right) - 2q_i B_\nu^\mu (\dot{x}_i^\nu + x_i'^\nu \dot{s}_i(t)) = -f_i^\mu, \quad (94)$$

leading to the following equation of motion for the massive junction (cf. equation (24)):

$$m \frac{d}{d\tau} \left(\frac{\dot{X}_m^\mu}{\sqrt{\dot{X}_m^2}} \right) = 2 \sum_i q_i B_\nu^\mu (\dot{x}_i^\nu + x_i'^\nu \dot{s}_i(t)) - \sum_i \mu_i \left(\frac{x_i^{\mu'}}{\varepsilon_i} + \varepsilon_i \dot{s}_i \dot{x}_i^\mu \right). \quad (95)$$

The toy model we would like to construct in the context of our 3-string Y-junction configuration in Fig. 2 (*Left*) is a “Hook-Yukawa” force between strings 1 and 2. In other words, we are taking the magnitude of the force to scale with the product of a linear factor and an exponentially damped factor in the $x^2 \equiv y$ direction. This is a phenomenological choice motivated from the requirement that the force be localised near the junction. Close to the junction the force is taken to be linear (Hook-like), but it gets exponentially damped away from the junction, as the y -separation of the strings becomes large:

$$F = -k(y_1 - y_2)e^{-M(y_1 - y_2)}, \quad (96)$$

with k and M positive constants.

With our choice of gauge, for which the physical velocities are transverse to the string segments, it is convenient to construct the force transverse to the segments too, which is consistent with the 2-form nature of the force potential. Remembering that $x^0 = \tau$ and setting $x^1 \equiv x$, $x^2 \equiv y$ and $x^3 \equiv z$, the equations of motion (92) for string 1 in our simple planar configuration of Fig. 2 (*Left*) read

$$\begin{aligned} \dot{\epsilon}_1 &= \frac{2q_1}{\mu_1} (\dot{x}_1 y_1' - x_1' \dot{y}_1) H_{12}^0 (1 - \dot{x}_1^2 - \dot{y}_1^2) \epsilon_1, \\ \mu_1 \left[\ddot{x}_1 - \frac{1}{\epsilon_1^2} x_1'' - \frac{1}{\epsilon_1} \left(\frac{1}{\epsilon_1} \right)' x_1' \right] &= 2q_1 y_1' (1 - \dot{x}_1^2 - \dot{y}_1^2) H_{02}^1 \equiv q_1 k (y_1 - y_2) e^{-M(y_1 - y_2)} (1 - \dot{x}_1^2 - \dot{y}_1^2) \sin \beta, \\ \mu_1 \left[\ddot{y}_1 - \frac{1}{\epsilon_1^2} y_1'' - \frac{1}{\epsilon_1} \left(\frac{1}{\epsilon_1} \right)' y_1' \right] &= 2q_1 x_1' (1 - \dot{x}_1^2 - \dot{y}_1^2) H_{01}^2 \equiv -q_1 k (y_1 - y_2) e^{-M(y_1 - y_2)} (1 - \dot{x}_1^2 - \dot{y}_1^2) \cos \beta, \\ z_1 &= 0, \end{aligned} \quad (97)$$

where β is the local string orientation, $\epsilon_1 = \sqrt{(x_1'^2 + y_1'^2)/(1 - \dot{x}_1^2 - \dot{y}_1^2)}$, and we have used $H_{12}^0 = H_{02}^1 = -H_{21}^0$ in our conventions. We have chosen H_{12}^0 to correspond to the phenomenological force (96), which can be generated by a static potential, $\partial_0 B_{\mu\nu} = 0$, with

$$B_{01} = -\frac{k\gamma_v}{2} \left(\frac{1}{M^2} + \frac{y_1 - y_2}{M} \right) e^{-M(y_1 - y_2)}, \quad (98)$$

and all unrelated components chosen to be 0. The force can be attractive or repulsive depending on the sign of q_1 .

We solve equations (97) for string 1 numerically (and similarly the corresponding equations for string 2) starting from the simple configuration:

$$\begin{cases} \mathbf{x}_1 = (-\sigma \cos \alpha, -\sigma \sin \alpha, 0) \\ \mathbf{x}_2 = (-\sigma \cos \alpha, +\sigma \sin \alpha, 0) \end{cases}, \quad (99)$$

with $\alpha = \beta_{\text{crit}}$, that is, at $t = 0$ we have $\beta = \beta_{\text{crit}}$ in (97). From the above discussion we expect that the force acting locally at the junction will distort the initial configuration leading to a non-trivial evolution of the local angle β . An attractive force can be expected to reduce the angle locally, bending the string segments from their initial straight segment configuration, and accelerating the junction towards the right. A repulsive force can be expected to have the opposite effect, increasing the angle from its equilibrium value and resulting in deceleration of the junction.

Figure 8 shows two such examples, for $M = 12$ and $k = 65$. On the left plot, the force is attractive ($q = q_1 = q_2 = 1$) and on the right it is repulsive ($q = -1$). The attractive force leads to $\beta < \beta_{\text{crit}}$, as expected, bending the two strings towards each other near the junction, while sufficiently far away from the junction the strings retain their straight profiles at angle $\alpha = \beta_{\text{crit}}$ from the x -axis. Similarly, the repulsive force produces $\beta > \beta_{\text{crit}}$ near the junction. The former case gives rise to accelerated zipping, while for the latter, we expect unzipping to occur. Indeed, as we can see in Fig. 9, where we have numerically solved for the evolution of $s_3(t)$, unzipping starts at $t \sim 0.25$.

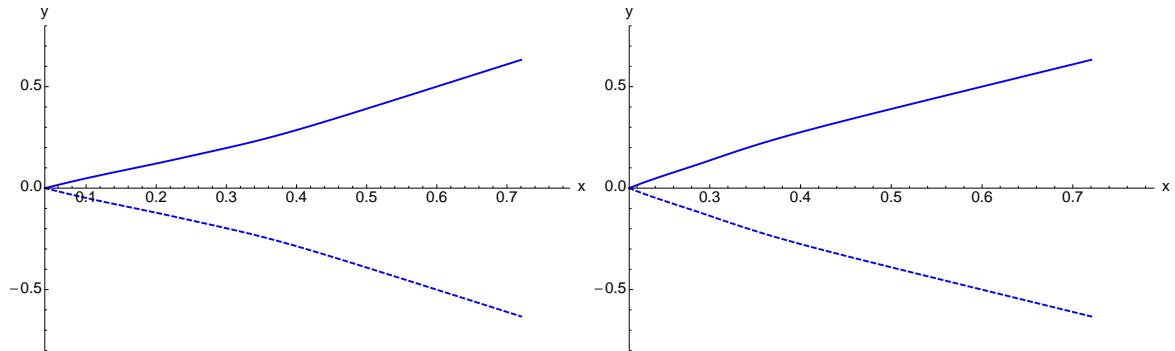


FIG. 8: A snapshot in the evolution of the 3-string configuration (99), under the influence of the local string force (96), for $M = 12$ and $k = 65$. On the left the force is attractive ($q = 1$) leading to $\beta < \beta_{\text{crit}}$ near the junction, while on the right it is repulsive ($q = -1$) producing $\beta > \beta_{\text{crit}}$.

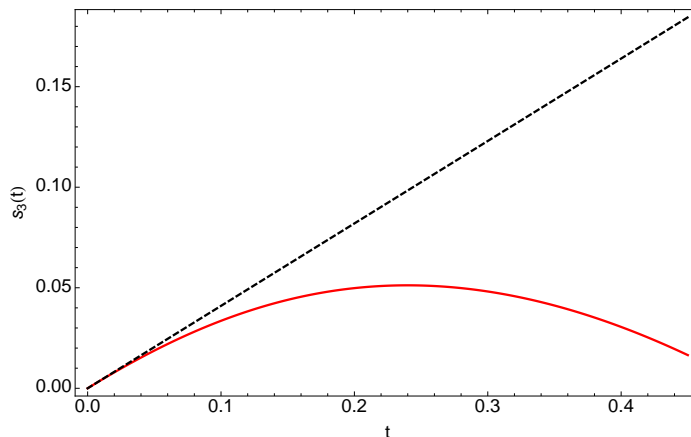


FIG. 9: The evolution of $s_3(t)$ under the influence of the local string force (96), for the repulsive case $M = 12$ and $k = 65$ (solid red line), shown together with the unperturbed case given by the CKS solution (dashed black line). Unzipping happens at $t \sim 0.25$.

C. String Curvature – Loops

In this subsection we discuss the unzipping effects of string curvature. The evolution and stability of cosmic string loops with junctions was studied in Ref. [35], using both the field theoretical and the Nambu-Goto approaches. In the field theory studies of Ref. [35], a new phenomenon occurred: the composite vortices could unzip, producing in the process new junctions whose separation could grow, destabilizing the configuration. This phenomenon was then successfully modeled within the Nambu-Goto dynamics, and the results showed that it is the initial *local curvature* around a given junction that affects its evolution.

The study of junction formation and evolution following the collision of cosmic string loops has shown that unzipping phenomena can occur (see Refs. [42, 43]). In Ref. [42] it was shown that for colliding loops in a flat background zipping and unzipping generically happen, while in Ref. [43] the effect of expansion of the Universe was also taken into account. In what follows we briefly review the aforementioned study and then generalise it to include loops of unequal tensions, which is more relevant to the case of cosmic superstring networks.

The basic picture is that of two co-planar loops extended in the (x, y) -plane which are moving in the z -direction with opposite velocities (Fig. 10). After collision, four junctions are formed, A, B, C and D . Certainly, due to the symmetry of the problem, one can study the evolution of only two junctions, for instance B and D ; A and C are just their mirror images. In what follows we concentrate on junction B .⁸

⁸ The formalism for both junctions is essentially the same. Note that junction B is found to unzip faster than D in Ref. [43].

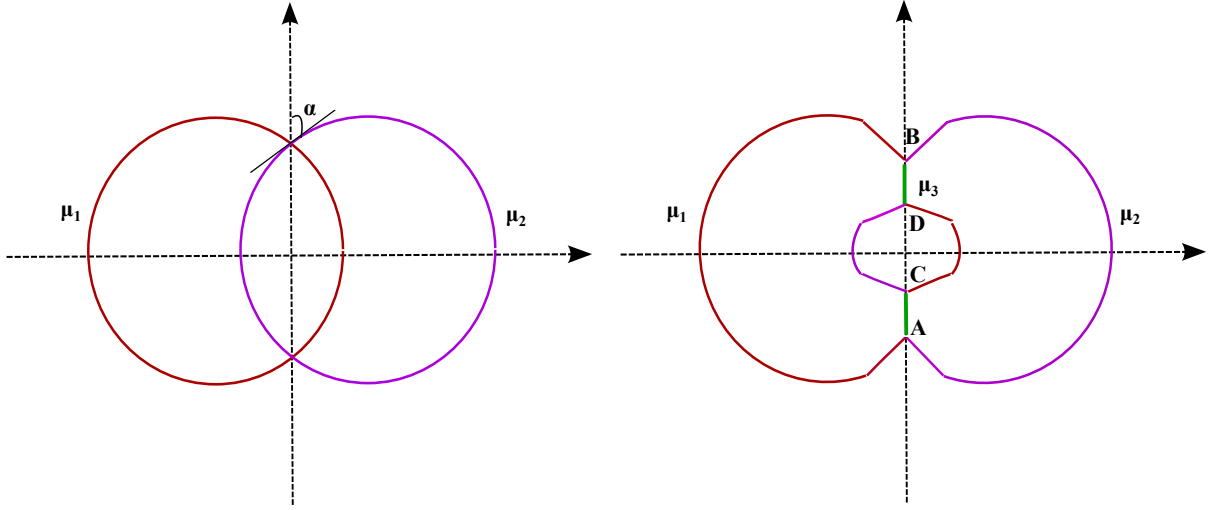


FIG. 10: *Left*: Two co-planar loops which are moving in the z -direction with opposite velocities collide. *Right*: After the collision four junctions and eight kinks are formed.

We consider an expanding Universe described by Friedmann-Lemaître-Robinson-Walker (FLRW) metric

$$ds^2 = a^2(\tau)(d\tau^2 - d\mathbf{x}^2), \quad (100)$$

where τ denotes the conformal time related to the cosmic time t via $dt = a d\tau$ and $a(\tau)$ stands for the scale factor. We have also assumed that the background space-time is spatially flat. The two loops collide at $\tau = \tau_0$. This system is described by an action which generalizes the one of Ref. [38] to an expanding background. Since all details can be found in Ref. [43], here we only present the equations which determine the evolution of the system. Following the conventions of [43], we denote the initial incoming strings by \mathbf{x}_i with $i = 1, 2$, whereas the newly formed strings are denoted by \mathbf{y}_a with $a = 1, 2, 3$. We also denote with s_i^J the value of the σ coordinate of a string at a junction J (where J can represent all four junctions A, B, C, D , but here we will concentrate on $J = B$), and with ω_i^J the value of the σ coordinate of a string at a kink. Note that, in general, s and ω vary with time.

The segments of old strings, which are not influenced by the junctions, will obey the usual equations of motion in a FLRW background, namely

$$\frac{\partial}{\partial \tau}(\dot{\mathbf{x}}_i \epsilon_{\mathbf{x}_i}) + 2 \frac{\dot{a}}{a} \dot{\mathbf{x}}_i \epsilon_{\mathbf{x}_i} = \frac{\partial}{\partial \sigma} \left(\frac{\mathbf{x}'_i}{\epsilon_{\mathbf{x}_i}} \right), \quad (101)$$

where $\epsilon_{\mathbf{x}_i}$ is defined by $\epsilon_{\mathbf{x}_i} \equiv \sqrt{\frac{\mathbf{x}_i'^2}{1 - \dot{\mathbf{x}}_i^2}}$. The same is true for the new strings stretched between a junction and a nearby kink:

$$\frac{\partial}{\partial \tau}(\dot{\mathbf{y}}_a \epsilon_{\mathbf{y}_a}) + 2 \frac{\dot{a}}{a} \dot{\mathbf{y}}_a \epsilon_{\mathbf{y}_a} = \frac{\partial}{\partial \sigma} \left(\frac{\mathbf{y}'_a}{\epsilon_{\mathbf{y}_a}} \right). \quad (102)$$

It can be also shown that $\epsilon_{\mathbf{x}_i} = \epsilon_{\mathbf{y}_i} = \epsilon_i$, and $\dot{\omega}_i^J \epsilon_i = -1$. We can define the right- and left-moving momenta \mathbf{p}_a^\pm as

$$\mathbf{p}_{\mathbf{y}_a}^{\pm J} = \frac{\mathbf{y}'_a}{\epsilon_a} \pm \dot{\mathbf{y}}_a, \quad \mathbf{p}_{\mathbf{x}_i}^{\pm J} = \frac{\mathbf{x}'_i}{\epsilon_i} \pm \dot{\mathbf{x}}_i, \quad (103)$$

with $\mathbf{p}_a^{\pm 2} = 1$. After some algebra [43], we can express the unknown $\mathbf{p}_{\mathbf{y}_a}^+$ in terms of the known $\mathbf{p}_{\mathbf{y}_b}^-$ and derive a system of equations for the evolution of \dot{s}_i^J . Defining $c_1 \equiv \mathbf{p}_{\mathbf{y}_2}^- \cdot \mathbf{p}_{\mathbf{y}_3}^-$ and similarly c_2 and c_3 , we obtain

$$1 - \epsilon_1 \dot{s}_1^J = \frac{\bar{\mu} M_1 (1 - c_1)}{\mu_1 [M_1 (1 - c_1) + M_2 (1 - c_2) + M_3 (1 - c_3)]}, \quad (104)$$

where $\bar{\mu} = \mu_1 + \mu_2 + \mu_3$, $M_1 \equiv \mu_1^2 - (\mu_2 - \mu_3)^2$ with a similar definition for M_2 and M_3 , and $\dot{s}_{2,3}$ given by the same equation with appropriate permutations of the indices. Moreover, for the position of the vertex \mathbf{Y} we find

$$\mathcal{M} \dot{\mathbf{Y}} = -M_1 (1 - c_1) \mathbf{p}_{\mathbf{y}_1}^{-J} - M_2 (1 - c_2) \mathbf{p}_{\mathbf{y}_2}^{-J} - M_3 (1 - c_3) \mathbf{p}_{\mathbf{y}_3}^{-J}, \quad (105)$$

with $\mathcal{M} = M_1(1 - c_1) + M_2(1 - c_2) + M_3(1 - c_3)$. Finally, the energy conservation at the junctions implies

$$\mu_1 \epsilon_1 \dot{s}_1^J + \mu_2 \epsilon_2 \dot{s}_2^J + \mu_3 \epsilon_3 \dot{s}_3^J = 0 . \quad (106)$$

In Ref. [43], the previous analysis was applied to the case of two identical loops colliding. We are going to generalize it assuming the colliding loops have different tensions, $\mu_1 \neq \mu_2$, which is more relevant to the case of cosmic superstring networks. The collision happens at $\tau = \tau_0$ and the expansion law for the scale factor is $a(\tau) = (\tau/\tau_0)^n$, with $n = 1, 2$ for radiation, matter domination respectively. Using our conventions, with σ increasing towards junction B , we parameterize the two loops as follows (note they have the same size, for simplicity)

$$\begin{aligned} \mathbf{x}_1 &= \left(b + f(\tau) \cos \frac{\sigma_1}{R_0}, f(\tau) \sin \frac{\sigma_1}{R_0}, z(\tau) \right) , \\ \mathbf{x}_2 &= \left(-b - f(\tau) \cos \frac{\sigma_2}{R_0}, f(\tau) \sin \frac{\sigma_2}{R_0}, -z(\tau) \right) , \end{aligned} \quad (107)$$

where $2b$ is the separation between the centres of the loops, $R(\tau) \equiv a(\tau)f(\tau)$ is the physical radius of each loop and $R_0 = f(\tau_0)$ represents the size of each loop at the time of collision. The independent equations of motion read

$$F'' + \frac{2n}{x} F' (1 - v^2 - F'^2) + (1 - v^2 - F'^2) F^{-1} = 0 , \quad (108)$$

$$v' + \frac{2n}{x} v (1 - v^2 - F'^2) = 0 , \quad (109)$$

where the loop center of mass velocity is defined by $v = \dot{z}(\tau)$, and for the ease of the numerical investigations, we introduced the dimensionless time variable $x \equiv \tau/\tau_0$ and $F(x) \equiv f(\tau)/\tau_0$, following [43]. Note that prime represents derivatives with respect to the dimensionless time x .

From the continuity of the left-moving momenta we find

$$\mathbf{p}_{\mathbf{y}_1}^{-B} = \left(-\sqrt{1 - F'^2 - v^2} \sin \frac{\sigma_1}{R_0} - F' \cos \frac{\sigma_1}{R_0}, \sqrt{1 - F'^2 - v^2} \cos \frac{\sigma_1}{R_0} - F' \sin \frac{\sigma_1}{R_0}, -v \right) ,$$

$$\mathbf{p}_{\mathbf{y}_2}^{-B} = \left(\sqrt{1 - F'^2 - v^2} \sin \frac{\sigma_2}{R_0} + F' \cos \frac{\sigma_2}{R_0}, \sqrt{1 - F'^2 - v^2} \cos \frac{\sigma_2}{R_0} - F' \sin \frac{\sigma_2}{R_0}, v \right) .$$

Since the colliding loops have unequal tensions, we first need to determine the velocity and orientation of the joining string after collision. In order to do that, we follow a similar treatment to the one presented in Ref. [38] for the case of straight strings colliding in a flat background. We let string labeled 3 to lie at an angle θ to the y -axis, and to move in the z -direction with velocity u (assuming $\mu_1 > \mu_2$). This gives

$$\mathbf{p}_{\mathbf{y}_3}^{-B} = (\sqrt{1 - u^2} \sin \theta, \sqrt{1 - u^2} \cos \theta, -u) ,$$

and

$$u' + \frac{2n}{x} (1 - u^2) u = 0 . \quad (110)$$

The position of the vertex is

$$\mathbf{Y} = \mathbf{x}_3(s_3(\tau), \tau) , \quad (111)$$

which gives

$$\dot{\mathbf{Y}} = (\dot{s}_3 \sin \theta, \dot{s}_3 \cos \theta, u) . \quad (112)$$

At collision, the three components of the vector Eq. (105) are

$$[\mathcal{M} \dot{s}_3 + M_3(1 - c_3) \sqrt{1 - u^2}] \sin \theta|_{\tau=\tau_0} = (\sqrt{1 - F'^2 - v^2} \sin a - F' \cos a) [M_1(1 - c_1) - M_2(1 - c_2)]|_{\tau=\tau_0} ,$$

$$[\mathcal{M} \dot{s}_3 + M_3(1 - c_3) \sqrt{1 - u^2}] \cos \theta|_{\tau=\tau_0} = (\sqrt{1 - F'^2 - v^2} \cos a + F' \sin a) [M_1(1 - c_1) + M_2(1 - c_2)]|_{\tau=\tau_0} ,$$

and

$$[M_1(1 - c_1) + M_2(1 - c_2)]|_{\tau=\tau_0} u = [M_1(1 - c_1) - M_2(1 - c_2)]|_{\tau=\tau_0} v .$$

After some algebra we get

$$\frac{u_0}{v} = \frac{M_1(1 - c_1) - M_2(1 - c_2)}{M_1(1 - c_1) + M_2(1 - c_2)}|_{\tau=\tau_0} , \quad (113)$$

and

$$\tan \theta = \left(\frac{u_0}{v} \right) \left(\frac{\sqrt{1 - F'^2 - v^2} \sin a - F' \cos a}{\sqrt{1 - F'^2 - v^2} \cos a + F' \sin a} \right)|_{\tau=\tau_0} . \quad (114)$$

As a first check, solving the above system of equations for $\mu_1 = \mu_2$, we find $u_0 = \theta = 0$, as expected, and solving the derived equations numerically we reproduce the results obtained in Ref. [43].

Now let us try an asymmetric example, assuming we are in the radiation dominated era. As initial conditions we choose $a = \pi/9$, $\mu_1 = 2$, $\mu_2 = 1$, $\mu_3 = 2.5$, $v(x = 1) = 0.4$, and $F'(x = 1) = 0.1$, and we define $S_i \equiv s_i/R_0$ [43] to simplify the equations and the numerical analysis. We indeed find two real and positive solutions, which are $u(x = 1) \rightarrow 0.168$ and $\theta \rightarrow 0.102$. Now we can proceed to solve for the evolution of the junction (with $F(x = 1) = 100$). We see that S_3 initially increases but after some time the growth of the zipper stops and unzipping occurs — note that this happens before the loops shrink to zero.

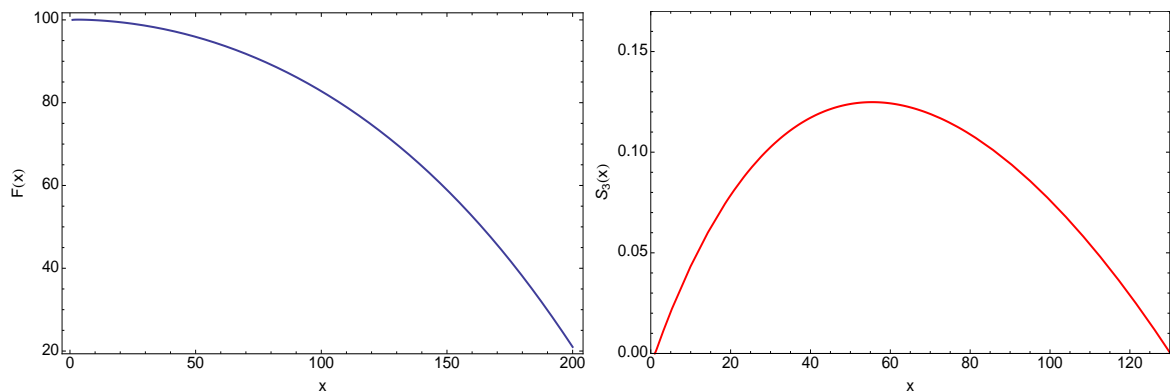


FIG. 11: *Left panel:* The background evolution for $F(x)$ with $F(x = 1) = 100$, $F'(x = 1) = 0.1$, and $v(x = 1) = 0.4$ for the radiation era. *Right panel:* The evolution of junction B for the radiation dominated background, for $a = \pi/9$, $\mu_1 = 2$, $\mu_2 = 1$, $\mu_3 = 2.5$, $v(x = 1) = 0.4$, $u(x = 1) = 0.168$, $\theta = 0.102$.

As in Ref. [43], the initial condition $F(x = 1)$ is a measure of the physical radius of the colliding loops compared to the Hubble radius. In our case, where $F(x = 1) = 100$, we considered loops of superhorizon size. The qualitative results of our study are the same as the ones of Ref. [43]: As we know, large superhorizon loops can be approximated with straight strings. Consequently, if the initial conditions are appropriate for junction formation the junction will grow, following the usual behavior of straight infinite strings. However, at some point these loops will reenter the horizon, their velocities will rapidly reduce and unzipping will occur. In our example (Fig. 11), after junction formation s_3^B reaches a maximum value indicating its unzipping. Similarly, junction D also unzips, although the unzipping of junction B happens sooner. When junctions B and D meet, the loops disentangle. However, there is also the possibility that the loops shrink to zero before B and D meet. For intermediate and small size loops the results are similar, but the smaller the loop is the less they depend on the background expansion, as expected. In general, we can conclude that string curvature and loop dynamics are an effective unzipping mechanism.

VI. CONCLUSIONS

In this paper we have studied the dynamics of string junctions in an attempt to identify dynamical mechanisms that could trigger the unzipping of Y-junction configurations. We are motivated by field theory simulations of string

networks with junctions [32, 33], where one observes a lower than expected abundance of heavy string segments. This may suggest that junction formation may be dynamically obstructed or that there is a tendency for heavy string zippers to unzip after they form. One could entertain the possibility that a Y-junction formed by the collision of two string segments could unzip for high enough collision velocities, as the colliding strings retain their original motion beyond the kinks, effectively pulling the zipper apart. However, a simple exercise shows that this can only work for non-relativistic strings as it requires the collision velocity to be larger than the speed of propagation of kinks along the string. For relativistic strings, having worldsheet Lorentz invariance, kinks propagate at the speed of light and this kinematic mechanism cannot work. A more relevant physical effect for string networks is perhaps velocity damping due to cosmic expansion, but this can only slow down (not invert) the zipping process and is also suppressed at sub-horizon scales.

Having in mind cosmic superstring models that are generally expected to form junctions, we have in this paper investigated a number of potential dynamical mechanisms that could be responsible for unzipping in string networks. Firstly, we have studied, within the Nambu-Goto approximation, the stability of massive string junctions under the influence of the tensions of three strings joining in a Y-type configuration, and concluded that these configurations are stable under deformations of the tension balance condition at the junction. This justifies the usual assumption of string evolution models with junctions that zippers grow according to the special solutions of [37–39] once formed. Secondly, we have investigated whether monopole or string forces, and string curvature for loops with junctions, can lead to unzipping of string junctions. In each case we have found solutions exhibiting decelerating zipping leading to the unzipping of Y-type configurations, and we have discussed the conditions under which unzipping happens in the context of our simple 3-string Nambu-Goto modelling.

It remains unclear at this stage whether these mechanisms can play a significant rôle in realistic string networks. If such an unzipping mechanism gets realised within cosmic (super)string networks it would affect the relative abundance between the two lightest string species, which, as was shown in [28, 31], controls the characteristically stringy B-mode signal of these networks. To date, the incorporation of realistic string interactions in the modelling of these networks has not been achieved and our results provide further motivation for this.

Note that the Nambu-Goto equations for a massive junction, which we have obtained, do not have a well-defined massless ($m \rightarrow 0$) limit. This has been already discussed in Ref. [45] for a system of (anti)monopoles connecting strings, which are expected to eventually collide and annihilate being constrained to live on a string. A similar situation may be expected in the case of a massive junction after unzipping takes places (i.e. when a massive junction meets its mirror).

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